
**Determination of the detection limit and
decision threshold for ionizing radiation
measurements —**

**Part 7:
Fundamentals and general applications**

*Détermination de la limite de détection et du seuil de décision des
mesurages de rayonnements ionisants —*

Partie 7: Principes fondamentaux et leurs applications générales



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Published in Switzerland

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 11929-7 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, Subcommittee SC 2, *Radiation protection*.

ISO 11929 consists of the following parts, under the general title *Determination of the detection limit and decision threshold for ionizing radiation measurements*:

- *Part 1: Fundamentals and application to counting measurements without the influence of sample treatment*
- *Part 2: Fundamentals and application to counting measurements with the influence of sample treatment*
- *Part 3: Fundamentals and application to counting measurements with high resolution gamma spectrometry without the influence of sample treatment*
- *Part 4: Fundamentals and application to measurements by use of linear-scale analogue ratemeters, without the influence of sample treatment*
- *Part 5: Fundamentals and applications to counting measurements on filters during accumulation of radioactive material*
- *Part 6: Fundamentals and applications to measurements by use of transient mode*
- *Part 7: Fundamentals and general applications*
- *Part 8: Fundamentals and applications to unfolding of spectrometric measurements without the influence of sample treatment*

Introduction

This part of ISO 11929 gives basic information on the statistical principles for the determination of the detection limit, of the decision threshold and of the limits of the confidence interval for general applications of nuclear radiation measurements.

ISO 11929-1 and ISO 11929-2, respectively, deal with integral counting measurements with or without consideration of the sample treatment. High-resolution spectrometric measurements are covered in ISO 11929-3. ISO 11929-4 deals with measurements using linear-scale analogue ratemeters, ISO 11929-5 with monitoring of the concentration of aerosols in exhaust gas, air or waste water, ISO 11929-6 with measurements by use of a transient measuring mode, and ISO 11929-8 with unfolding of spectrometric measurements.

Whereas the earlier parts 1 to 4 were elaborated for special measuring tasks in nuclear radiation measurements based on the principles defined by Altschuler and Pasternack^[1], Nicholson^[2], Currie^[3], this restriction does not apply to this part, or to part 5, part 6 and part 8. The determination of the characteristic limits mentioned above is separated from the evaluation of the measurement. Consequently, this part of ISO 11929 is generally applicable and can be applied to any suitable procedure for the evaluation of a measurement. Since the uncertainty of measurement plays a fundamental role in this part of ISO 11929, evaluations of measurements and the determination of the uncertainties of measurement have to be performed according to the Guide for the Expression of Uncertainty in Measurement.

This part, as well as parts 5, 6 and 8, of ISO 11929 is based on methods of Bayesian statistics (see [4] to [6] in the Bibliography) in order to be able to account also for such uncertain quantities and influences which do not behave randomly in repeated or counting measurements.

For this purpose, Bayesian statistical methods are used to specify the following statistical values, called "characteristic limits":

- the *decision threshold*, which allows a decision to be made for a measurement with a given probability of error as to whether the result of the measurement indicates the presence of the physical effect quantified by the measurand.
- the *detection limit*, which specifies the minimum true value of the measurand which can be detected with a given probability of error using the measuring procedure in question. This consequently allows a decision to be made as to whether or not a measuring method checked using this part of ISO 11929 satisfies certain requirements and is consequently suitable for the given purpose of measurement.
- the *limits of the confidence interval*, which define an interval which contains the true value of the measurand with a given probability, in the case that the result of the measurement exceeds the decision threshold.

Determination of the detection limit and decision threshold for ionizing radiation measurements —

Part 7: Fundamentals and general applications

1 Scope

This part of ISO 11929 specifies suitable statistical values which allow an assessment of the detection capabilities in ionizing radiation measurements and of the physical effect quantified by the measurand. For this purpose, Bayesian statistical methods are used to specify characteristic limits.

This part of ISO 11929 deals with fundamentals and general applications.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11929-1:2000, *Determination of the detection limit and decision threshold for ionizing radiation measurements — Part 1: Fundamentals and application to counting measurements without the influence of sample treatment*

ISO 11929-2:2000, *Determination of the detection limit and decision threshold for ionizing radiation measurements — Part 2: Fundamentals and application to counting measurements with the influence of sample treatment*

BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML, *Guide to the Expression of Uncertainty in Measurement*, Geneva, 1993

BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML, *International Vocabulary of Basic and General Terms in Metrology*, 2nd edition, Geneva, 1993

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1

measuring method

any logical sequence of operations, described generically, used in the performance of measurements

NOTE Adapted from the International Vocabulary of Basic and General Terms in Metrology:1993.

3.2

measurand

particular quantity subject to measurement

[International Vocabulary of Basic and General Terms in Metrology:1993]

NOTE In this part of ISO 11929, a measurand is non-negative and quantifies a nuclear radiation effect. The effect is not present if the value of the measurand is zero. Examples for a measurand are the net count rate of a sample of radioactive material, the net activity of a sample of radioactive material given the activity of a blank sample, the increase of the specific activity or activity concentration of a gas flow, or the intensity of a line in a spectrum above the background in a spectrometric measurement.

3.3

uncertainty (of measurement)

parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand

[Guide for the Expression of Uncertainty in Measurement:1993]

NOTE The uncertainty of measurement defined in the Guide for the Expression of Uncertainty in Measurement comprises, in general, many components. Some of these components may be evaluated from the results of series of measurements or counting measurements and can be characterized by experimental standard deviations. The other components, which also can be characterized by standard deviations, are evaluated from assumed or known probability distributions based on experience and other information.

3.4

mathematical model of the evaluation

a set of mathematical relationships between all measured and other quantities involved in the evaluation of measurements

3.5

decision quantity

random variable for the decision whether or not the physical effect to be measured is present

3.6

decision threshold

fixed value of the decision quantity by which, when exceeded by the result of an actual measurement of a measurand quantifying a physical effect, one decides that the physical effect is present

NOTE The decision threshold is the critical value of a statistical test to decide between the hypothesis that the physical effect is not present and the alternative hypothesis that it is present. When the critical value is exceeded by the result of an actual measurement, this is taken to indicate that the hypothesis should be rejected. The statistical test will be designed such that the probability of wrongly rejecting the hypothesis (error of the first kind) is equal at most to a given value α .

3.7

detection limit

smallest true value of the measurand which is detectable by the measuring method

NOTE 1 The detection limit is the smallest true value of the measurand which is associated with the statistical test and hypotheses according to 3.6 by the following characteristics: if in reality the true value is equal to or exceeds the detection limit, the probability of wrongly not rejecting the hypothesis (error of the second kind) will be at most equal to a given value β .

NOTE 2 The difference between using the decision threshold and using the detection limit is that measured values are to be compared with the decision threshold, whereas the detection limit is to be compared with the guideline value.

3.8

confidence limits

values which define a confidence interval to be specified for the measurand in question which, if the result exceeds the decision threshold, includes the true value of the measurand with the given probability $(1 - \gamma)$

3.9

guideline value

value which corresponds to scientific, legal or other requirements for which the measuring procedure is intended to assess

EXAMPLE Activity, specific activity or activity concentration, surface activity, or dose rate.

4 Quantities and symbols

$\hat{\xi}$	Random variable as an estimator for a non-negative measurand quantifying the physical effect in question
ξ	Value of the estimator $\hat{\xi}$ of the measurand; true value of the measurand
X	Random variable as decision quantity; estimator of the measurand
x	Primary result of a measurement of the measurand; obtained value of the decision quantity X ; primary estimate of the measurand
$u(x)$	Standard uncertainty of the measurand associated with the primary result x of a measurement
$\tilde{u}(\xi)$	Standard uncertainty of the decision quantity X as a function of the true value ξ of the measurand
z	Best estimate of the measurand
$u(z)$	Standard uncertainty associated with the best estimate z of the measurand
x^*	Decision threshold for the measurand
ξ^*	Detection limit for the measurand; ξ_l^* , ξ_u^* , respectively, the lower and upper limit of the confidence interval for the measurand
α	Probability of the error of the first kind; the probability of rejecting a hypothesis if it is true
β	Probability of the error of the second kind; the probability of accepting a hypothesis if it is false
$1 - \gamma$	Probability attributed to the confidence interval of the measurand; probability that the true value of the measurand is included by the confidence interval
k_p	Quantiles of the standardized normal distribution for the probability p (see Table 1); ($p = 1 - \alpha$), ($1 - \beta$), ($1 - \gamma/2$)
E	Operator for the formation of the expectation value of a random variable
Var	Operator for the formation of the variance of a random variable
Φ	Distribution function of the standardized normal (Gaussian) distribution

5 Statistical values and confidence interval

5.1 General aspects

For a particular task involving nuclear radiation measurements, first the particular physical effect which is the objective of the measurement has to be described. Then a non-negative measurand has to be defined which quantifies the physical effect and which assumes the value zero if the effect is not present in an actual case.

A random variable, called a decision quantity X has to be attributed to the measurand. It is also an estimator of the measurand. It is required that the expectation value EX of the decision quantity X equals the true value ξ of the measurand. A value x of the estimator X derived from measurements is a primary estimate of the measurand. The primary estimate x of the measurand, and its associated standard uncertainty $u(x)$, have to be calculated as the primary complete result of the measurement according to the Guide for the Expression of Uncertainty in Measurement, by evaluation of measured quantities and of other information using a mathematical model of the evaluation which takes into account all relevant quantities. Generally, the fact that the measurand is non-negative will not be explicitly made use of. Therefore, x may become negative, in particular, if the true value of the measurand is close to zero.

NOTE The model of the evaluation of the measurement need not necessarily be given in the form of explicit mathematical formulas. It can also be represented by an algorithm or a computer code (see A.2).

For the determination of the decision threshold and the detection limit, the standard uncertainty of the decision quantity has to be calculated, if possible, as a function $\tilde{u}(\xi)$ of the true value ξ of the measurand. In the case that this is not possible, approximate solutions are described below.

ξ is the value of another, non-negative estimator $\hat{\xi}$ of the measurand. The estimator $\hat{\xi}$, in contrast to X , makes use of the knowledge that the measurand is non-negative. The limits of the confidence interval to be determined refer to this estimator $\hat{\xi}$ (compare 5.4). Besides the limits of the confidence interval, the expectation value $E\hat{\xi}$ of this estimator as a best estimate z of the measurand, and the standard deviation $[\text{Var}(\hat{\xi})]^{1/2}$ as the standard uncertainty $u(z)$ associated with the best estimate z of the measurand have to be calculated (see 6.3).

For the numerical calculation of the decision threshold and the detection limit, the function $\tilde{u}(\xi)$ is needed, which is the standard uncertainty of the decision quantity X as a function of the true value ξ of the measurand. The function $\tilde{u}(\xi)$ generally has to be determined by the user of this part of ISO 11929, in the course of the evaluation of the measurement according to the Guide for the Expression of Uncertainty in Measurement. For examples see Annex A. This function is often only slowly increasing. Therefore, it is justified in many cases to use the approximation $\tilde{u}(\xi) = u(x)$. This applies, in particular, if the primary estimate x of the measurand is not much larger than its standard uncertainty $u(x)$ associated with x . If the value x is calculated as the difference (net effect) of two approximately equal values y_1 and y_0 obtained from independent measurements, that is $x = y_1 - y_0$, one gets $\tilde{u}^2(\xi) = u^2(y_1) + u^2(y_0)$ with the standard uncertainties $u(y_1)$ and $u(y_0)$ associated with y_1 and y_0 , respectively.

If only $\tilde{u}(0)$ and $u(x)$ are known, an approximation by linear interpolation is often sufficient for $x > 0$ according to:

$$\tilde{u}^2(\xi) = \tilde{u}^2(0) \cdot (1 - \xi/x) + u^2(x) \cdot \xi/x \quad (1)$$

NOTE In many practical cases, $\tilde{u}^2(\xi)$ is a slowly increasing linear function of ξ . This justifies the approximations above, in particular, the linear interpolation of $\tilde{u}^2(\xi)$ instead of $\tilde{u}(\xi)$ itself.

5.2 Decision threshold

The decision threshold x^* of a non-negative measurand quantifying the physical effect, according to 5.1, is a value of the decision quantity X which, when it is exceeded by a result x of a measurement, indicates that the physical effect is present. If $x \leq x^*$ one decides that the physical effect is not present. If this decision rule is

observed, a wrong decision in favour of the presence of the physical effect occurs with a probability not greater than α (error of the first kind).

The decision threshold is given by:

$$x^* = k_{1-\alpha} \cdot \tilde{u}(0) \quad (2)$$

Values of the quantiles $k_{1-\alpha}$ of the standardized normal distribution are given in Table 1. It is $\Phi(k_{1-\alpha}) = 1 - \alpha$.

If the approximation $\tilde{u}(\xi) = u(x)$ is sufficient, one gets $x^* = k_{1-\alpha} \cdot u(x)$.

5.3 Detection limit

The detection limit ξ^* , which is the smallest true value of the measurand detectable with the measuring method, is so much larger than the decision threshold that the probability of an error of the second kind is not greater than β . The detection limit is given by:

$$\xi^* = x^* + k_{1-\beta} \cdot \tilde{u}(\xi^*) \quad (3)$$

Equation (3) is an implicit one. The detection limit can be calculated from it by iteration using, for example, the starting approximation $\xi^* = 2x^*$. The iteration converges in most cases. Equation (3) may have multiple solutions. In this case the detection limit is the smallest one. If Equation (3) has no solution, the measuring procedure is not suited for the measuring purpose.

If the approximation $\tilde{u}(\xi) = u(x)$ is sufficient, then $\xi^* = (k_{1-\alpha} + k_{1-\beta}) \cdot u(x)$ is valid. If $\tilde{u}(\xi)$ is not explicitly known for $\xi > 0$, one gets with $\tilde{u}(0)$ and with a result x of a measurement and its associated uncertainty $u(x)$, an approximation of ξ^* using the interpolation formula according to Equation (1):

$$\xi^* = a + \sqrt{a^2 + (k_{1-\beta}^2 - k_{1-\alpha}^2) \cdot \tilde{u}^2(0)} \quad \text{with} \quad a = k_{1-\alpha} \cdot \tilde{u}(0) + \frac{1}{2}(k_{1-\alpha}^2 / x) \cdot [u^2(x) - \tilde{u}^2(0)] \quad (4)$$

For $\alpha = \beta$ one obtains $\xi^* = 2a$.

When using the approximation of Equation (4) to calculate the detection limit ξ^* and when type B uncertainties are not negligible, a measurement result $x \approx 2x^*$ shall be chosen. If $x \gg 2x^*$ holds, one obtains an unreasonably high detection limit. In this case, the approximation yields only an upper limit of ξ^* . If type B uncertainties are negligible, Equations (3) and (4) converge to the same result for the detection limit.

Values of the quantiles $k_{1-\alpha}$, $k_{1-\beta}$ of the standardized normal distribution are given in Table 1. It is $\Phi(k_{1-\alpha}) = 1 - \alpha$ and $\Phi(k_{1-\beta}) = 1 - \beta$.

5.4 Confidence limits

For a result x of a measurement which exceeds the decision threshold x^* , the confidence interval includes the true value of the measurand with the given probability $(1 - \gamma)$. It is enclosed by the confidence limits ξ_l and ξ_u according to:

$$\xi_l = x - k_p \cdot u(x) \quad \text{with} \quad p = \kappa \cdot (1 - \gamma / 2) \quad (5)$$

$$\xi_u = x + k_p \cdot u(x) \quad \text{with} \quad q = 1 - (\kappa \cdot \gamma / 2) \quad (6)$$

κ is given by

$$\kappa = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x/u(x)} \exp(-z^2/2) dz = \Phi[x/u(x)] \quad (7)$$

Values of the function $\Phi(t)$ are tabulated (see [7] in the Bibliography) and given in Table 1. It is $\Phi(k_p) = p$ and $\Phi(k_q) = q$.

The confidence limits are not symmetrical around the expectation $E\hat{\xi}$. The probabilities of $\hat{\xi} < \xi_l$ and $\hat{\xi} > \xi_u$ however, are both equal to $\gamma/2$ and the relationship $0 < \xi_l < \xi_u$ is valid. For $x \gg u(x)$, the approximation

$$\xi_{l,u} = x \pm k_{1-\gamma/2} \cdot u(x) \quad (8)$$

is applicable if $x > \approx 2 \cdot k_{1-\gamma/2} u(x)$

6 Application of this part of ISO 11929

6.1 Specific values

The probabilities α , β and $(1 - \gamma)$ shall be specified in advance by the user of this part of ISO 11929. Commonly used values are $\alpha = \beta = 0,05$ and $\gamma = 0,05$.

6.2 Assessment of a measuring method

To check whether a measuring method (see 3.1) is suitable for the measurement of a physical effect, the detection limit shall be compared with a specified guideline value (e.g. specified requirements on the sensitivity of the measuring procedure for scientific, legal or other reasons; see 3.9).

The detection limit shall be calculated by means of Equation (3). If the detection limit thus determined is greater than the guideline value, the measuring procedure is not suitable for the measurement.

6.3 Assessment of a measured result

A measured result has to be compared with the decision threshold calculated by means of Equation (2). If the result of the measurement x is larger than the decision threshold x^* , it is decided that the physical effect quantified by the measurand is present.

If this is the case, the best estimate z of the measurand is calculated using κ from Equation (7) by:

$$z = E\hat{\xi} = x + \frac{u(x) \cdot \exp\{-x^2/[2u^2(x)]\}}{\kappa \cdot \sqrt{2\pi}} \quad (9)$$

with the standard uncertainty $u(z)$ associated with z :

$$u(z) = \sqrt{\text{Var}(\hat{\xi})} = \sqrt{u^2(x) - (z - x) \cdot z} \quad (10)$$

The following relationships: $z \geq x$ and $z \geq 0$, as well as $u(z) \leq u(x)$, are valid and for $x \gg u(x)$, i.e. $x > 4 \cdot u(x)$, the approximations $z = x$ and $u(z) = u(x)$ hold true.

6.4 Documentation

The documentation of measurements in accordance with this part of ISO 11929 shall contain details of the probabilities α , β and $(1 - \gamma)$, the decision threshold x^* , the detection limit ξ^* , and the guideline value.

For a result x of the measurement exceeding the decision threshold x^* , the standard uncertainty $u(x)$ associated with x and the limits of the confidence interval ξ_l and ξ_u have to be given. If the result x of the measurement is below the decision threshold ξ^* , it shall be documented as “below the decision threshold”.

If the detection limit exceeds the guideline value, it shall be documented that the method is not suitable for the measurement purpose.

In addition, the best estimate z of the measurand and the standard uncertainty $u(z)$ associated with z may be specified if $x/u(x) < 4$.

7 Values of the distribution function of the standardized normal distribution

Values $\Phi(t) = \int_{-\infty}^t \phi(v) dv$ with $\phi(z) = (1/\sqrt{2\pi}) \cdot \exp(-z^2/2)$ are given in Table 1. For the distribution function of

the standardized normal distribution, $\Phi(-t) = 1 - \Phi(t)$ is valid. Quantiles of the standardized normal distribution can also be obtained from this Table 1 since $t = k_p$ for $p = \Phi(t)$, i.e. $\Phi(k_p) = p$.

For $t \geq 0$, the approximation (see [8] in the Bibliography):

$$\Phi(t) = 1 - \frac{\exp(-t^2/2)}{\sqrt{2\pi}} \cdot (a_1 \cdot \zeta + a_2 \cdot \zeta^2 + a_3 \cdot \zeta^3) + \varepsilon; \quad \zeta = \frac{1}{1 + a_0 \cdot t}$$

is valid with $|\varepsilon| < 10^{-5}$ and

$$a_0 = 0,332\,67; \quad a_1 = 0,436\,183\,6; \quad a_2 = -0,120\,167\,6; \quad a_3 = 0,937\,298\,0$$

For $t < 0$, one obtains $\Phi(t)$ from the relationship $\Phi(t) = 1 - \Phi(-t)$.

For $0,5 \leq p < 1$ the approximation (see [8] in the Bibliography):

$$k_p = t - \frac{b_0 + b_1 \cdot t + b_2 \cdot t^2}{1 + c_1 \cdot t + c_2 \cdot t^2 + c_3 \cdot t^3} + \varepsilon; \quad t = \sqrt{-2 \cdot \ln(1 - p)}$$

is valid with $|\varepsilon| < 4,5 \times 10^{-4}$ and

$$b_0 = 2,515\,517; \quad b_1 = 0,802\,853; \quad b_2 = 0,010\,328$$

$$c_1 = 1,432\,788; \quad c_2 = 0,189\,269; \quad c_3 = 0,001\,308$$

For $0 < p < 0,5$, one obtains k_p from the relationship $k_p = -k_{1-p}$.

Table 1 — Values of the distribution function of the standardized normal distribution $\Phi(t)$
(see [7] in the Bibliography)

t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$
0,00	0,500 0	0,70	0,758 0	1,40	0,919 2	2,10	0,982 1	2,80	0,997 4
0,02	0,508 0	0,72	0,764 2	1,42	0,922 2	2,12	0,983 0	2,90	0,998 1
0,04	0,516 0	0,74	0,770 4	1,44	0,925 1	2,14	0,983 8	3,00	0,998 6
0,06	0,523 9	0,76	0,776 4	1,46	0,927 8	2,16	0,984 6	3,10	0,999 0
0,08	0,531 9	0,78	0,782 3	1,48	0,930 6	2,18	0,985 4	3,20	0,999 3
0,10	0,539 8	0,80	0,788 1	1,50	0,933 2	2,20	0,986 1	3,30	0,999 5
0,12	0,547 8	0,82	0,793 9	1,52	0,935 7	2,22	0,986 8	3,40	0,999 7
0,14	0,555 7	0,84	0,799 6	1,54	0,938 2	2,24	0,987 4	3,50	0,999 8
0,16	0,563 6	0,86	0,805 1	1,56	0,940 6	2,26	0,988 1	3,60	0,999 8
0,18	0,571 4	0,88	0,810 6	1,58	0,943 0	2,28	0,988 7	3,80	0,999 9
0,20	0,579 3	0,90	0,815 9	1,60	0,945 2	2,30	0,989 3	4,00	1,000 0
0,22	0,587 1	0,92	0,821 2	1,62	0,947 4	2,32	0,989 8		
0,24	0,594 8	0,94	0,826 4	1,64	0,949 5	2,34	0,990 4		
0,26	0,602 6	0,96	0,831 5	1,66	0,951 5	2,36	0,990 9		
0,28	0,610 3	0,98	0,836 5	1,68	0,953 5	2,38	0,991 3		
0,30	0,617 9	1,00	0,841 3	1,70	0,955 4	2,40	0,991 8		
0,32	0,625 5	1,02	0,846 1	1,72	0,957 3	2,42	0,992 2		
0,34	0,633 1	1,04	0,850 8	1,74	0,959 1	2,44	0,992 7		
0,36	0,640 6	1,06	0,855 4	1,76	0,961 0	2,46	0,993 0		
0,38	0,648 0	1,08	0,859 9	1,78	0,962 5	2,48	0,993 4		
0,40	0,655 4	1,10	0,864 3	1,80	0,964 1	2,50	0,993 8		
0,42	0,662 8	1,12	0,868 6	1,82	0,965 6	2,52	0,994 1		
0,44	0,670 0	1,14	0,872 9	1,84	0,967 1	2,54	0,994 5		
0,46	0,677 2	1,16	0,877 0	1,86	0,968 6	2,56	0,994 8		
0,48	0,684 4	1,18	0,881 0	1,88	0,970 0	2,58	0,995 1		
0,50	0,691 5	1,20	0,884 9	1,90	0,971 3	2,60	0,995 3		
0,52	0,698 5	1,22	0,888 8	1,92	0,972 6	2,62	0,995 6		
0,54	0,705 4	1,24	0,892 5	1,94	0,973 8	2,64	0,995 9		
0,56	0,712 3	1,26	0,896 1	1,96	0,975 0	2,66	0,996 1		
0,58	0,719 0	1,28	0,899 7	1,98	0,976 2	2,68	0,996 3		
0,60	0,725 8	1,30	0,903 2	2,00	0,977 2	2,70	0,996 5		
0,62	0,732 4	1,32	0,906 6	2,02	0,978 3	2,72	0,996 7		
0,64	0,738 9	1,34	0,909 9	2,04	0,979 3	2,74	0,996 9		
0,66	0,745 4	1,36	0,913 1	2,06	0,980 3	2,76	0,997 1		
0,68	0,751 8	1,38	0,916 2	2,08	0,981 2	2,78	0,997 3		
NOTE $t = k_p$ for $p = \Phi(t)$, i.e. $\Phi(k_p) = p$.									

Annex A (informative)

Example 1 of application of this part of ISO 11929

A.1 Explanation of the principles of a Bayesian theory of uncertainty in measurement

This part of ISO 11929 is based on methods of Bayesian statistics (see [5] to [7] in the Bibliography), in order to be able to consider uncertain quantities and influences which do not behave randomly in repeated measurements. Random variables are assigned as estimators to all incompletely known physical quantities. The probability distributions of these estimators must, however, in general not be interpreted as frequency distributions of events in frequently repeated measurements, as is done in conventional statistics. The probability distributions reflect the status of incomplete knowledge of the particular physical quantity in question. They can be derived by using the Principle of Maximum Entropy from the measured data and from other information (see [5] in the Bibliography) and therefore are known together with their parameters. Their expectation are taken as best estimates of the physical quantities. Their variances and covariances serve as measures of the measuring uncertainty. In those cases, where both Bayesian statistics and conventional statistics can be applied, the numerical results obtained by analogous procedures of both approaches are at least asymptotically identical.

In 5.2 and 5.3, a Gaussian distribution is used for the decision quantity X . In 5.4 and 6.3, a Gaussian distribution is truncated at $\xi = 0$ as an estimator of the measurand. These distributions are not approximations, but are direct consequences of the Principle of Maximum Entropy using all available information (see [5] in the Bibliography). According to the preceding paragraph they must, however, not be interpreted as frequency distributions.

In Bayesian statistics, the probability $(1 - \gamma)$ represents the degree of incomplete knowledge of the confidence interval, whose limits are calculated from measured data and from other information to contain the true value of the measurand. The confidence level corresponding to this probability in conventional statistics is not a probability, but must be interpreted as the portion of different confidence intervals which can be calculated, each in an analogous way, from the data measured in frequently repeated measurements and which each contain the true value of the measurand.

A.2 General numerical calculation of uncertainty in measurement

There are two classes of physical quantities to be distinguished in the evaluation of measurements.

Resulting quantities (later called output quantities) Y_k ($k = 1, \dots, n$) are quantities (for instance the parameters of an unfolding procedure) which have to be determined by the evaluation of the measurement. The decision quantity X is one of them. The task is to calculate the estimates y_k of the output quantities as the results of the measurements, the standard uncertainties $u(y_k)$ associated with y_k , and the covariances of the measurement uncertainties $u(y_k, y_l)$. It holds that $u^2(y_k) = u(y_k, y_k)$.

Input quantities X_i ($i = 1, \dots, m$) are quantities which are, for instance, derived by counting measurements. In addition, they are repeatedly measured quantities, influence quantities and output quantities of previous evaluations. The estimators x_i of these input quantities and the standard uncertainties $u(x_i)$ associated with the x_i and the covariances $u(x_i, x_j)$ are either given, or have to be determined following the procedures of the Guide to the Expression of Uncertainty in Measurement. In counting measurements, one obtains for the quantities ρ_i , derived according to A.3 and Equation (A.9), with the counting result n_i and the counting time (or channel width) t_i :

$$x_i = n_i / t_i \cdot u^2(x_i) = n_i / t_i^2 = x_i / t_i, \quad u(x_i, x_j) = 0, \quad \text{for } i \neq j \quad (\text{with respect to } n_i = 0, \text{ see A.3}).$$

The model of the evaluation connects the output quantities mathematically with the input quantities:

$$Y_k = G_k(X_1, \dots, X_m); (k = 1, \dots, n) \quad (\text{A.1})$$

The functions G_k do not need to be explicitly available as mathematical expressions. They may also be an algorithm, for instance, in the form of a computer code of the evaluation.

The measuring results y_k are obtained by substituting the input quantities X_i in the model equations containing G_k by their estimates x_i :

$$y_k = G_k(x_1, \dots, x_m); (k = 1, \dots, n) \quad (\text{A.2})$$

The covariance uncertainty components $u(y_k, y_l)$ are given by:

$$u(y_k, y_l) = \sum_{i,j=1}^m \frac{\partial G_k}{\partial X_i} \cdot \frac{\partial G_l}{\partial X_j} \cdot u(x_i, x_j); (k, l = 1, \dots, n) \quad (\text{A.3})$$

$u(y_k)$ is the positive square root of $u(y_k, y_k)$.

The partial derivatives need not to be explicitly calculated. This is particularly advantageous if such a calculation is difficult, or if the model equations are only available as a computer code. It is sufficient to calculate first the difference quotients:

$$\Delta_i G_k = \frac{1}{u(x_i)} \left\{ G_k[x_1, \dots, x_i + \frac{1}{2}u(x_i), \dots, x_m] - G_k[x_1, \dots, x_i - \frac{1}{2}u(x_i), \dots, x_m] \right\} \quad (\text{A.4})$$

and then:

$$u(y_k, y_l) = \sum_{i,j=1}^m (\Delta_i G_k) \cdot (\Delta_j G_l) \cdot u(x_i, x_j); (k, l = 1, \dots, n) \quad (\text{A.5})$$

This procedure is particularly advantageous in computerized evaluation. Examples of computer codes are given, for instance, in references [9] and [10] in the Bibliography. The partial derivatives may also be obtained in an analogous way experimentally, by changing the input quantities by Δx_i , because one can approximate Equation (A.4) by:

$$\Delta_i G_k = [G_k(x_1, \dots, x_j + \Delta x_j, \dots, x_m) - y_k] / \Delta x_j \quad (\text{A.6})$$

Note that Equation (A.6) has a lower accuracy than Equation (A.4).

Let Y_1 be the decision quantity X . Then $x = y_1$ and $u(x) = u(y_1)$. In order to calculate $\tilde{u}(\xi)$ an (at least approximatively) inverse of the model shall be given for $m' \leq m$ quantities X_i ($i = 1, \dots, m'$), the uncertainties of which depend on the true value ξ of the measurand (such quantities are, for instance, ρ_g or ρ_r in A.3.3):

$$X_i = M_i(Y_1, \dots, Y_n, X_{m'+1}, \dots, X_m); (i = 1, \dots, m') \quad (\text{A.7})$$

In this case, the fixed value ξ has to be substituted for $Y_1 = X$. One obtains the modified estimates:

$$x'_i = M_i(\xi, y_2, \dots, y_n, x_{m'+1}, \dots, x_m); (i = 1, \dots, m') \quad (\text{A.8})$$

which then lead to changed covariance uncertainty components $u(x'_i, x'_i)$. With these modified covariances, the entire calculation according to Equations (11) to (15) has to be repeated. However, one only needs to calculate $\tilde{u}(\xi) = u(y_1)$. If the computer code used operates iteratively, one iterative step is frequently sufficient for repetition.

A.3 Examples of application of this part of ISO 11929

A.3.1 General

In general, a measurement of nuclear radiation consists, at least partially, in counting electronic pulses caused by nuclear radiation events. Such a counting measurement usually comprises several individual counting measurements. Examples are counting measurements of individual radioactive samples or blank materials, counting measurements of the natural radiation background or of another background effect, and counting of events in the individual channels of a multi-channel spectrum or a time sequence of events under identical measuring conditions. In each counting measurement, either the counting time or maximum number of counts is preselected. On the basis of Bayesian statistics, all counting measurements are treated as described below (see [6] in the Bibliography).

An individual random variable N is assigned to the number of pulses counted in each individual counting measurement. n is the result of the measurement and t the counting time. N has the expectation $\rho \cdot t$, with ρ being the count rate, respectively the spectral density in spectrometric measurements. In the latter case, t is the channel width of the corresponding physical quantity, for example particle energy. The measurand is either ρ or $\rho \cdot t$. It is supposed that deadtime or lifetime effects, pile-up of pulses and instrumental instabilities can be neglected during the entire measurement, and that the pulses counted originate from different nuclear radiation events which are physically independent from each other. Then, the number of counts N is Poisson distributed and the number of counts of the individual countings are independent.

The quantity $\rho \cdot t$ obeys a Gamma distribution, whereby ρ is considered to be a random variable. This is independent of whether n counts are registered during a preselected counting time (or a fixed channel width) t , or whether the time t is measured which is required to accumulate a preselected maximum number of counts. Then one obtains the best estimate r of the count rate (or the spectral density) ρ and the standard uncertainty $u(r)$ associated with r

$$r = E\rho = n/t; u^2(r) = \text{Var}(\rho) = n/t^2 \quad (\text{A.9})$$

In the case of $n = 0$, one obtains $u(r) = 0$. This is not realistic, since one cannot be sure that $\rho = 0$ if no count has been obtained during a measurement with finite counting time. This case leads also to a division by zero if the method of least squares is applied (according to [10] in the Bibliography) when one has to divide by $u^2(r)$. This difficulty can be avoided by replacing all counting results n by $n + 1$, or by a suitable combination of channels. Thereby, it is presumed that the counting time (or the channel width) has been chosen in such a way that at least some counts can be expected if $\rho > 0$.

A.3.2 Counting measurements of nuclear radiation and determination of activity

In counting measurements, the measurand is the net count rate with the true value ξ of a sample containing radioactive materials. This net count rate has to be determined by counting measurements of the gross effect and of the background effect. The respective symbols are denoted by the indices "g" and "0", respectively.

The decision quantity can be expressed as:

$$X = \rho_g - \rho_0 \quad (\text{A.10})$$

The primary measuring result x , and the standard uncertainty $u(x)$ associated with x , are derived according to Equation (A.9):

$$x = EX = n_g/t_g - n_0/t_0; u^2(x) = \text{Var}(x) = n_g/t_g^2 + n_0/t_0^2 \quad (\text{A.11})$$

X corresponds to the expectation ρ_n of the net counting rate R_n in ISO 11929-1:2000. However, in this part of ISO 11929 it is a random variable in contrast to its use in ISO 11929-1.

For a given value ξ of the net count rate, x has to be replaced by ξ . Thereby, n_g or t_g can be eliminated in Equation (A.11), which are not available in measurements with preselection of counting time or of maximum number of counts, respectively, if ξ is given. Thus, one obtains in the case of preselection of counting time:

$$\tilde{u}^2(\xi) = (\xi + n_0/t_0)/t_g + n_0/t_0^2 = \xi/t_g + (n_0/t_0) \cdot (1/t_g + 1/t_0) \quad (\text{A.12})$$

and in the case of preselection of a maximum number of counts:

$$\tilde{u}^2(\xi) = (\xi + n_0/t_0)^2 / n_g + n_0/t_0^2 \quad (\text{A.13})$$

The decision thresholds derived by Equations (A.12) and (A.13) are identical for $\xi = 0$, if t_g and n_g , respectively, are chosen so that $n_g/t_g = n_0/t_0$. When preselection of a maximum number of counts is chosen, n_g has to be selected so that $n_g > k_{1-\alpha}^2$ in order to assure that Equation (3) has a solution. The resulting characteristic limits are numerically equal to those derived according to ISO 11929-1 for sufficiently large n_g and n_0 .

If the value ξ of the net count rate is to be translated into the value $A = \xi/\varepsilon$ of the net activity, it is also possible to consider the standard uncertainty $u(\varepsilon)$ or the relative standard uncertainty $u_{\text{rel}}(\varepsilon) = u(\varepsilon)/\varepsilon$ associated with the counting efficiency ε which are known from other sources. (In this example, branching ratios are supposed to be unity. However, in addition branching ratios differing from this value and the standard uncertainties associated with them can be considered analogously.)

Because of the propagation of uncertainties according to Equation (A.13) and, in particular, because of Equation (A.3), one obtains:

$$\tilde{u}^2(A) = \tilde{u}^2(\varepsilon \cdot A)/\varepsilon^2 + u_{\text{rel}}^2(\varepsilon) \cdot A^2 \quad (\text{A.14})$$

from which the characteristic limits can be calculated. The function $\tilde{u}^2(A)$ is that for the net activity. It corresponds to the function $\tilde{u}^2(\xi)$ according to Equation (A.12) or to Equation (A.13) for the net count rate which is $\xi = \varepsilon \cdot A$ here. The standard uncertainty $u_{\text{rel}}(\varepsilon)$ associated with ε does not depend on A . It is not necessarily determined by uncertainties derived from multiple repeated measurements or counting measurements, it can also be a consequence of (for example systematic) influences which are only partially known. Thus, Equation (A.14) represents an example where such influences can also be considered.

A.3.3 Counting measurements of nuclear radiation in the presence of non-Poisson sources of uncertainty

A.3.3.1 Any sample treatment causes uncertainties which can be different from one sample to another. Therefore, the counting results n_i of the counting measurements performed on different samples of a radioactive material under investigation, on different blank samples or on different reference samples of radioactively spiked blank material or of standard reference materials have each to be averaged. The measurand with the true value ξ is the mean net count rate of the samples. The symbols used in the following for the counting of samples, blanks and reference samples are distinguished by the subscripts "g", "0" and "r", respectively. Arithmetic means of m counting measurements, which are performed with preselection of counting time t , are each indicated by a horizontal bar above the symbol. For $m > 1$ counting measurements to be averaged, which each have been performed with the same counting time, one obtains the empirical variance s^2 of the count rates n_i/t :

$$s^2 = \frac{1}{(m-1) \cdot t^2} \sum_{i=1}^m (n_i - \bar{n})^2 \quad (\text{A.15})$$

The decision quantity can be expressed as:

$$X = \bar{\rho}_g - \bar{\rho}_0 \quad (\text{A.16})$$

X corresponds to the expectation $\bar{\rho}_n$ of the mean count rate in ISO 11929-2:2000. However, it is a random variable in contrast to its use in ISO 11929-2. The following equations are approximations for sufficiently large counting results n_i and for $\bar{n}/t \gg s/\sqrt{m}$.

A.3.3.2 The primary result x , for the mean count rate of the measurements and its standard uncertainty $u(x)$ are given by:

$$x = E X = \bar{n}_g / t_g - \bar{n}_0 / t_0 \quad (\text{A.17})$$

$$u^2(x) = \text{Var}(X) = s_g^2 / m_g + s_0^2 / m_0 \quad (\text{A.18})$$

The function $\tilde{u}^2(\xi)$ is derived from Equation (1). The value $\tilde{u}^2(0)$ results from Equation (A.18), because the variance s_g^2 can be substituted by s_0^2 in the case of $\xi = 0$.

A.3.3.3 If there is just one sample measurement ($m_g = 1$), s_g^2 cannot be calculated according to Equation (A.15). Also, in the case when there are just a few counting measurements of the sample material to be investigated and for the calculation of $\tilde{u}^2(\xi)$, it is advisable to determine s_g^2 by interpolation from the results of counting measurements of a larger number n_r of reference samples:

$$s_g^2 = s_0^2 + (s_r^2 - s_0^2) \cdot \frac{\bar{n}_g / t_g - \bar{n}_0 / t_0}{\bar{n}_r / t_r - \bar{n}_0 / t_0} = s_0^2 + \frac{(s_r^2 - s_0^2) \cdot x}{\bar{n}_r / t_r - \bar{n}_0 / t_0} \quad (\text{A.19})$$

For a given value ξ of the mean net count rate, x has to be substituted by ξ . Inserting s_g^2 , calculated according to Equation (A.19), into Equation (A.18) finally yields:

$$\tilde{u}^2(\xi) = \frac{(s_r^2 - s_0^2) \cdot \xi / m_g}{\bar{n}_r / t_r - \bar{n}_0 / t_0} + \left(\frac{1}{m_g} + \frac{1}{m_0} \right) \cdot s_0^2 \quad (\text{A.20})$$

A.3.3.4 Another procedure which is advisable in the case of only minor influence of sample treatment is based on the relation:

$$s^2 = \left(\bar{n} + g^2 \cdot \bar{n}^2 \right) / t^2 \quad (\text{A.21})$$

In Equation (A.21), the term which is linear in $\bar{n}$ is a consequence of the Poisson distributions of the numbers of counts N_i , in the case of negligible influence of sample treatment. The latter influence is described by the term which is quadratic in $\bar{n}$, under the assumption that there is a relative standard deviation g of all measurement uncertainties caused by the influence of sample treatment which is constant for all samples measured and for all count rates. g can be determined by counting measurements of reference samples and is calculated using Equation (A.21) and equating with Equation (A.15) by:

$$g^2 = \left(s_r^2 \cdot t_r^2 - \bar{n}_r \right) / \bar{n}_r^2 \quad (\text{A.22})$$

If this results in $g^2 < 0$, the approach used and the data are not compatible. Then, the number m_r of reference samples should be increased or $g = 0$ should be assumed. Generally, $g < \approx 0,2$ should hold, or else one should proceed according to A.3.3.3.

Instead of Equation (A.18), one now obtains together with Equation (A.21):

$$u^2(x) = (\bar{n}_g + g^2 \cdot \bar{n}_g^2) / (m_g \cdot t_g^2) + (\bar{n}_0 + g^2 \cdot \bar{n}_0^2) / (m_0 \cdot t_0^2) \quad (\text{A.23})$$

$\tilde{u}^2(\xi)$ is calculated by replacing x by ξ and inserting in Equation (A.23) $\bar{n}_g = (\xi + \bar{n}_0 / t_0) \cdot t_g$, derived from Equation (A.11). The cases $m_g = 1$ and $m_0 = 1$ are now allowed.

Annex B (informative)

Example 2 of application of this part of ISO 11929

This annex gives an example of a contamination measurement using a wipe test and counting measurements.

This example describes a contamination measurement using a wipe test, by which a fraction η of an activity on a surface is removed from an area $F = 100 \text{ cm}^2$ and measured later on by a counting measurement for $t_g = 360 \text{ s}$, using a detector with counting efficiency $\varepsilon = 0,31$. The measurement yielded $N_g = 2\,591$ counted events equivalent to a gross counting rate $R_g = 7,197 \text{ s}^{-1}$ with a standard uncertainty $u(R_g) = \sqrt{N_g}/t_g = 0,141 \text{ s}^{-1}$.

The background count rate was measured earlier with a counting time $t_0 = 7\,200 \text{ s}$. One obtained $N_0 = 41\,782$ counts, resulting in a background count rate $R_0 = 5,803 \text{ s}^{-1}$ with a standard uncertainty $u(R_0) = \sqrt{N_0}/t_0 = 0,028\,4 \text{ s}^{-1}$.

The detector efficiency ε was determined using a calibration source with a certified relative standard uncertainty of 5 %. The Poisson uncertainty of the measurement of the detector efficiency can be neglected compared to the certified uncertainty and one obtains a standard uncertainty $u(\varepsilon) = 0,015\,5$.

From previous experiments, it is known that the wiping efficiency η , i.e. the fractional activity removed by wiping a surface, varies randomly between 0,06 and 0,62 for the material under investigation. Therefore, a rectangular distribution can be adopted for its probability. According to the Guide for the Expression of Uncertainty in Measurement, this corresponds to a standard uncertainty of $u(\eta) = 0,16$ associated with the mean wiping efficiency of $\eta = 0,34$.

The relative standard uncertainty of the wiped area $F = 100 \text{ cm}^2$ is assumed, by experience, to be 10 %, i.e. $u(F) = 10 \text{ cm}^2$.

The measurand is the activity A_F per unit area, in becquerels per square centimetre, calculated by the model of Equation (B.1):

$$A_F = \frac{R_g - R_0}{\varepsilon \cdot \eta \cdot F} = \frac{R_n}{\varepsilon \cdot \eta \cdot F} \quad (\text{B.1})$$

The standard uncertainty $u(A_F)$ associated with A_F is given by Equation (B.2):

$$u^2(A_F) = u^2(R_g) \cdot \left(\frac{1}{\varepsilon \cdot \eta \cdot F} \right)^2 + u^2(R_0) \cdot \left(\frac{1}{\varepsilon \cdot \eta \cdot F} \right)^2 + \left(\frac{u^2(\varepsilon)}{\varepsilon^2} \right) \cdot \left(\frac{R_g - R_0}{\eta \cdot \varepsilon \cdot F} \right)^2 + \frac{u^2(\eta)}{\eta^2} \cdot \left(\frac{R_g - R_0}{\eta \cdot \varepsilon \cdot F} \right)^2 + \frac{u^2(F)}{F^2} \cdot \left(\frac{R_g - R_0}{\eta \cdot \varepsilon \cdot F} \right)^2 \quad (\text{B.2})$$

$$u^2(A_F) = [u^2(R_g) + u^2(R_0)] \cdot \left(\frac{1}{\varepsilon \cdot \eta \cdot F} \right)^2 + \left(\frac{u^2(\varepsilon)}{\varepsilon^2} + \frac{u^2(\eta)}{\eta^2} + \frac{u^2(F)}{F^2} \right) \cdot A_F^2$$