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## Hydrometry — Open channel flow measurement using thin-plate weirs

*Hydrométrie — Mesure de débit dans les canaux découverts au moyen  
de déversoirs à paroi mince*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: [www.iso.org/iso/foreword.html](http://www.iso.org/iso/foreword.html).

This document was prepared by Technical Committee ISO/TC 113, *Hydrometry*, Subcommittee SC 2, *Flow measurement structures*.

This third edition cancels and replaces the second edition (ISO 1438:2008), which has been technically revised. It also incorporates the Technical Corrigendum ISO 1438:2008/Cor 1:2008.

The major changes from ISO 1438:2008 are as follows:

- a) the modular flow discharge formula for weirs with weir plate height of  $1 \text{ m} \leq p \leq 2,5 \text{ m}$  has been supplemented in [9.7.1](#);
- b) the  $C_d$  formula for rectangular weir with  $b/B = 1,0$ , [Formula \(5\)](#), has been corrected to the same formula as the full-width weir, [Formula \(15\)](#);
- c) subclause numbers of [9.6](#) have been re-numbered.

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# Hydrometry — Open channel flow measurement using thin-plate weirs

## 1 Scope

This document defines the requirements for the use of rectangular and triangular (V-notch) thin-plate weirs for the measurement of flow of clear water in open channels under free flow conditions. It includes the requirements for the use of full-width rectangular thin-plate weirs in submerged (drowned) flow conditions.

## 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 772, *Hydrometry — Vocabulary and symbols*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 772 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <http://www.iso.org/obp>

## 4 Symbols and abbreviated terms

| Symbol           | Unit                  | Description                                                                                |
|------------------|-----------------------|--------------------------------------------------------------------------------------------|
| $A$              | $\text{m}^2$          | Area of approach channel                                                                   |
| $B$              | $\text{m}$            | Width of approach channel                                                                  |
| $b$              | $\text{m}$            | Measured width of the notch                                                                |
| $b_{\text{max}}$ | $\text{m}$            | Width of notch at maximum head (V-notch)                                                   |
| $C$              |                       | Discharge coefficient (gauged head)                                                        |
| $C_d$            |                       | Coefficient of discharge                                                                   |
| $f$              |                       | Drowned flow reduction factor                                                              |
| $C_v$            |                       | Coefficient of velocity                                                                    |
| $e_b$            | $\text{m}$            | Random uncertainty in the width measurement                                                |
| $g$              | $\text{m/s}^2$        | Acceleration due to gravity                                                                |
| $H$              | $\text{m}$            | Total head above crest level                                                               |
| $h$              | $\text{m}$            | Upstream gauged head above crest level (upstream head is inferred if no subscript is used) |
| $J$              |                       | Numerical constant                                                                         |
| $l$              | $\text{m}$            | Distance of the head measurement section upstream of the weir                              |
| $n$              |                       | Number of measurements in a set                                                            |
| $p$              | $\text{m}$            | Height of the crest relative to the floor                                                  |
| $Q$              | $\text{m}^3/\text{s}$ | Volumetric rate of flow                                                                    |

| Symbol     | Unit | Description                                 |
|------------|------|---------------------------------------------|
| $S$        |      | Submergence ratio, $h_2/h_1$                |
| $S_1$      |      | Modular limit                               |
| $\bar{V}$  | m/s  | Mean velocity                               |
| $U$        | %    | Expanded percentage uncertainty             |
| $u^*(b)$   | %    | Percentage uncertainty in $b$               |
| $u^*(C)$   | %    | Percentage uncertainty in $C$               |
| $u^*(E)$   | %    | Percentage uncertainty in datum measurement |
| $u^*(h_1)$ | %    | Percentage uncertainty in $h_1$             |
| $u^*(Q)$   | %    | Percentage uncertainty in $Q$               |
| $\alpha$   | °    | Notch angle                                 |

### Subscripts

- 1 upstream
- 2 downstream
- e effective
- r rectangular
- t triangular

## 5 Principle

The discharge over thin-plate weirs is a function of the upstream head on the weir (for free-flow), upstream and downstream head (for drowned flow), the size and shape of the discharge area, and an experimentally determined coefficient which takes into account the head, the geometrical properties of the weir and approach channel, and the dynamic properties of the water.

## 6 Installation

### 6.1 General

General requirements of weir installations are described in the following clauses. Special requirements of different types of weirs are described in clauses which deal with specific weirs (see [Clause 9](#) and [Clause 10](#)).

### 6.2 Selection of site

The type of weir to be used for discharge measurement is determined in part by the nature of the proposed measuring site. Under some conditions of design and use, weirs shall be located in rectangular flumes or in weir boxes which simulate flow conditions in rectangular flumes. Under other conditions, weirs may be located in natural channels, as well as flumes or weir boxes, with no significant difference in measurement accuracy. Specific site-related requirements of the installation are described in [6.3](#).

### 6.3 Installation conditions

#### 6.3.1 General

Weir discharge is critically influenced by the physical characteristics of the weir and the weir channel. Thin-plate weirs are especially dependent on installation features which control the velocity

distribution in the approach channel and on the construction and maintenance of the weir crest in meticulous conformance with standard specifications.

### 6.3.2 Weir

Thin-plate weirs shall be vertical and perpendicular to the walls of the channel. The intersection of the weir plate with the walls and floor of the channel shall be watertight and firm, while the weir shall be capable of withstanding the maximum flow without distortion or damage.

Stated practical limits associated with different discharge formulae such as minimum width, minimum weir height, minimum head, and maximum values of  $h/p$  and  $b/B$  (where  $h$  is the measured head,  $p$  is the height of crest relative to floor,  $b$  is the measured width of the notch and  $B$  is the width of the approach channel), are factors which influence both the selection of weir type and the installation.

### 6.3.3 Approach channel

For the purposes of this document, the approach channel is the portion of the weir channel which extends upstream from the weir a distance not less than five times the width of the nappe at maximum head. If the weir is located in a weir tank, ideally, the length of the tank should equal up to 10 times the width of the nappe at maximum head. Information on the use of small weir tanks is given in [Annex A](#).

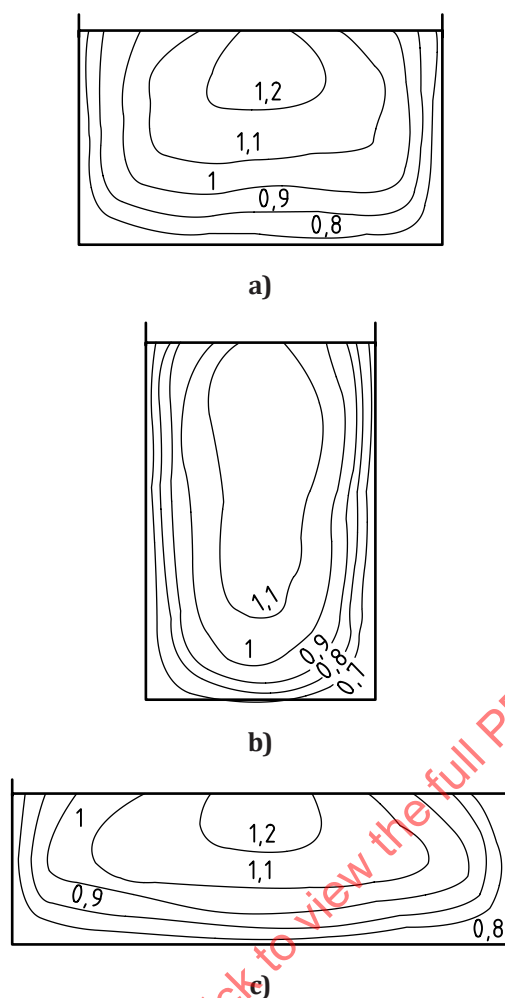
The flow in the approach channel shall be uniform and steady, with the velocity distribution approximating that in a channel of sufficient length to develop satisfactory flow in smooth, straight channels. [Figure 1](#) shows measured velocity distributions perpendicular to the direction of flow in rectangular channels, upstream from the influence of a weir. Baffles and flow straighteners can be used to simulate satisfactory velocity distribution, but their location with respect to the weir shall be not less than the minimum length prescribed for the approach channel.

The influence of approach-channel velocity distribution on weir flow increases as  $h/p$  and  $b/B$  increase in magnitude. If a weir installation unavoidably results in a velocity distribution that is appreciably non-uniform, the possibility of error in calculated discharge should be checked by means of an alternative discharge-measuring method for a representative range of discharges.

If the approach conditions are judged to be unsatisfactory, then flow straighteners shall be introduced in accordance with [Annex B](#).

If the maximum head to be measured is restricted to  $(2/3)p$  for all types of weirs, flow straighteners can be used to reduce the effective length of the approach channel to  $B + 3h_{\max}$  for triangular and rectangular weirs and to  $B + 5h_{\max}$  for full-width weirs.

**NOTE** This restriction on the maximum head to be measured is necessary due to distortion of the velocity near the water surface in the approach channel that results from flow coming through the openings in the baffle of the flow straightener.



NOTE The contours refer to values of local flow velocity relative to the mean cross-sectional velocity.

**Figure 1 — Examples of normal velocity distribution in rectangular channels**

#### 6.3.4 Downstream channel

For most applications, the level of the water in the downstream channel shall be a sufficient vertical distance below the crest to ensure free, fully ventilated discharges. Free (non-submerged) discharge occurs when the discharge is independent of the downstream water level. Fully ventilated discharge is ensured when the air pressure on the lower surface of the nappe is fully ventilated. Drowned flow operation is permitted for full-width weirs under certain conditions (see 9.7.2). Under these circumstances, downstream water levels may rise above crest level.

## 7 Measurement of head

### 7.1 Head-measuring devices

In order to obtain the discharge measurement accuracies specified for the standard weirs, the head on the weir shall be measured with a laboratory-grade hook gauge, point gauge, manometer, or other gauge of equivalent accuracy. For a continuous record of head variants, precise float gauges and servo-operated point gauges can be used. Staff and tape gauges can be used when less accurate measurements are acceptable.

Additional specifications for head-measuring devices are given in ISO 4373.

## 7.2 Stilling or float well

For the exceptional case where surface velocities and disturbances in the approach channel are negligible, the headwater level can be measured directly (for example, by means of a point gauge mounted over the water surface). Generally, however, to avoid water-level variations caused by waves, turbulence or vibration, the headwater level should be measured in a separate stilling well.

Separate stilling wells are connected to the approach channel by means of a suitable conduit, equipped if necessary with a throttle valve to damp oscillations. At the channel end of the conduit, the connection is made to floor or wall piezometers or a static tube at the head-measurement section.

Additional specifications for stilling wells are given in ISO 18365.

## 7.3 Head-measurement section

### 7.3.1 Upstream head-measurement

The head-measurement section shall be located a sufficient distance upstream from the weir to avoid the region of surface drawdown caused by the formation of the nappe. On the other hand, it shall be sufficiently close to the weir that the energy loss between the head-measurement section and the weir is negligible. For the weirs included in this document, the location of the head-measurement section will be satisfactory if it is at a distance equal to two to four times the maximum head ( $2h_{\max}$  to  $4h_{\max}$ ) upstream from the weir.

If high velocities occur in the approach channel or if water-surface disturbances or irregularities occur at the head-measurement section because of high values of  $h/p$  or  $b/B$ , it may be necessary to install several pressure intakes to ensure that the head measured in the gauge well is representative of the average head across the measurement section.

In the case of a full-width thin-plate weir, the effect of frictional effects upon the upstream channel requires an adjustment to the standard coefficient of discharge. The correction is in terms of both  $l/h$  and  $h/p$  and given in [Table 1](#).

**Table 1 — Factors to be applied to the standard discharge coefficient values**

| $h/p$         | $l/h$ |      |      |      |
|---------------|-------|------|------|------|
|               | 2     | 4    | 6    | 8    |
| 3,5 to 4,0    | 1,00  | 1,00 | 0,96 | 0,92 |
| 3,0 to 3,5    | 1,00  | 1,00 | 0,97 | 0,94 |
| 2,5 to 3,0    | 1,00  | 1,00 | 0,98 | 0,96 |
| 2,0 to 2,5    | 1,00  | 1,00 | 0,99 | 0,98 |
| Less than 2,0 | 1,00  | 1,00 | 1,00 | 1,00 |

### 7.3.2 Downstream head measurement

If the weir is to be operated in the submerged (drowned) flow range, a measurement of downstream head is required in addition to the upstream. The downstream head measurement position shall be  $10 h_{\max}$  downstream from the upstream face of the weir. If a stilling well is included in the design, it is recommended that the downstream head measurement be located no closer to the weir than  $4 h_{\max}$ .

## 7.4 Head-gauge datum (gauge zero)

Accuracy of head measurements is critically dependent upon the determination of the head-gauge datum or gauge zero, which is defined as the gauge reading corresponding to the level of the weir crest (rectangular weirs) or the level of the vertex of the notch (triangular-notch weirs). When necessary, the gauge zero shall be checked. Numerous acceptable methods of determining the gauge zero are in

use. Typical methods are described in subsequent clauses dealing specifically with rectangular and triangular weirs. See [Clause 9](#) and [Clause 10](#).

Because of surface tension, the gauge zero cannot be determined with sufficient accuracy by reading the head gauge with the water in the approach channel drawn down to the apparent crest (or notch) level.

## 8 Maintenance

Maintenance of the weir and the weir channel is necessary to ensure accurate measurements.

The approach channel shall be kept free of silt, vegetation and obstructions which might have deleterious effects on the flow conditions specified for the standard installation. The downstream channel shall be kept free of obstructions which might cause submergence or inhibit full ventilation of the nappe under all conditions of flow.

The weir plate shall be kept clean and firmly secured. In the process of cleaning, care shall be taken to avoid damage to the crest or notch, particularly the upstream edges and surfaces. Construction specifications for these most sensitive features should be reviewed before maintenance is undertaken.

Head-measurement piezometers, connecting conduits and the stilling well shall be cleaned and checked for leakage. The hook or point gauge, manometer, float or other instrument used to measure the head shall be checked periodically to ensure accuracy.

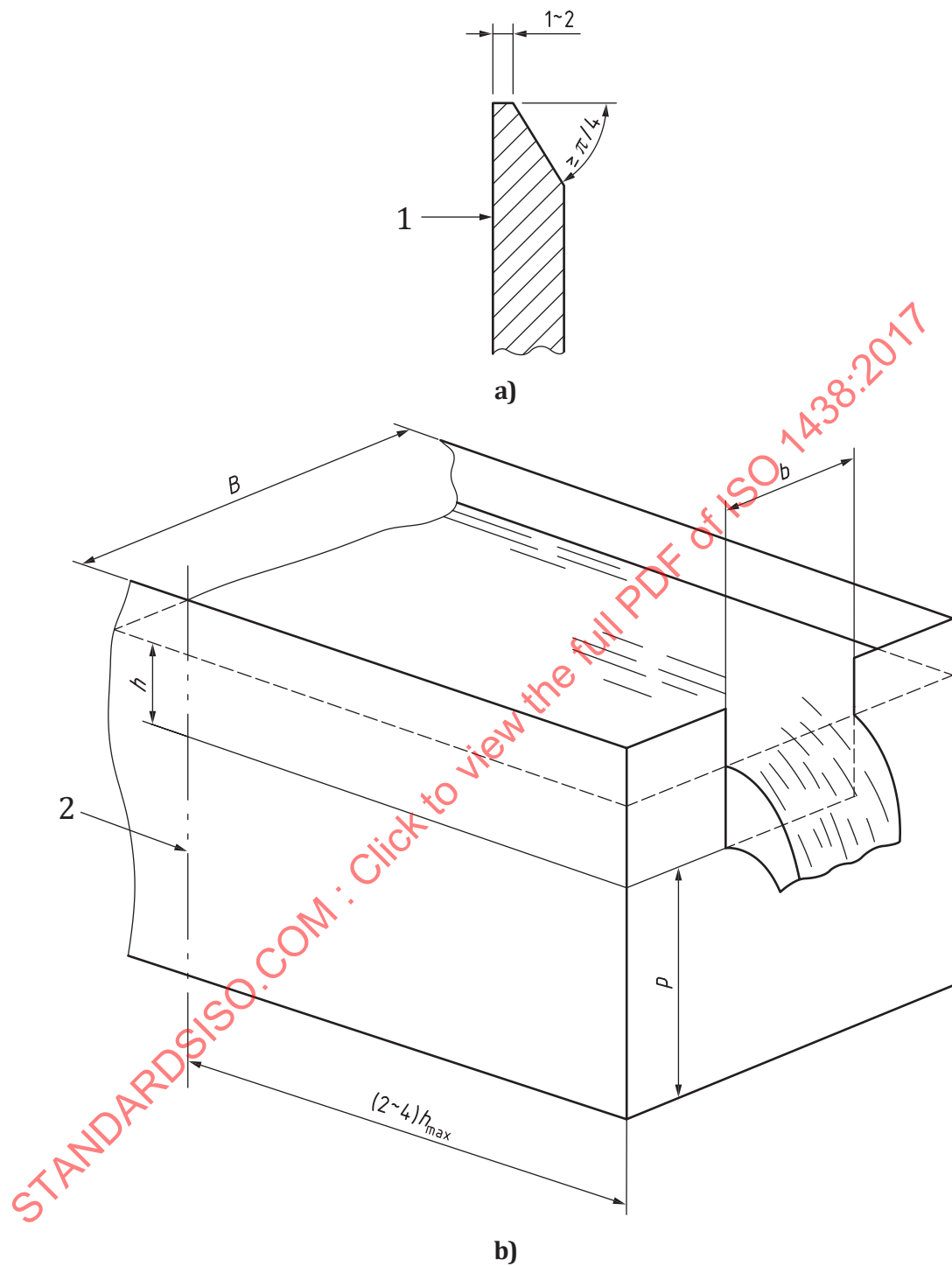
If a flow straightener is used in the approach channel, perforated plates shall be kept clean so that the percentage open area remains greater than 40 %.

## 9 Rectangular thin-plate weir

### 9.1 Types

The rectangular thin-plate weir is a general classification in which the rectangular-notch weir is the basic form and the full-width weir is a limiting case. A diagrammatic illustration of the basic weir form is shown in [Figure 2](#) with intermediate values of  $b/B$  and  $h/p$ . When  $b/B = 1,0$ , that is, when the width of the weir ( $b$ ) is equal to the width of the channel at the weir section ( $B$ ), the weir is of full-width type (also referred to as a “suppressed” weir, because its nappe lacks side contractions).

Dimensions in millimetres

**Key**

- 1 upstream face of weir plate
- 2 head measurement section, measured value  $h$

**Figure 2 — Rectangular-notch, thin-plate weir**

## 9.2 Specifications for the standard weir

The basic weir form consists of a rectangular notch in a vertical thin plate. The plate shall be plane and rigid and perpendicular to the walls and the floor of the approach channel. The upstream face of the plate shall be smooth (in the vicinity of the notch, it shall be equivalent in surface finish to that of rolled sheet-metal).

The vertical bisector of the notch shall be equidistant from the two walls of the channel. The crest surface of the notch shall be a horizontal, plane surface, which shall form a sharp edge at its intersection with the upstream face of the weir plate. The width of the crest surface, measured perpendicular to the face of the plate, shall be between 1 mm and 2 mm. The side surfaces of the notch shall be vertical plane surfaces which shall make sharp edges at their intersection with the upstream face of the weir plate. For the limiting case of the full-width weir, the crest of the weir shall extend to the walls of the channel, which, in the vicinity of the crest, shall be plane and smooth (see also [9.3](#)).

To ensure that the upstream edges of the crest and the sides of the notch are sharp, they shall be machined or filed, perpendicular to the upstream face of the weir plate, free of burrs or scratches, and untouched by abrasive cloth or paper. The downstream edges of the notch shall be chamfered if the weir plate is thicker than the maximum allowable width of the notch surface. The surface of the chamfer shall make an angle of not less than  $\pi/4$  radians ( $45^\circ$ ) with the crest and side surfaces of the notch (see detail shown in [Figure 2](#)). The weir plate in the vicinity of the notch preferably shall be made of corrosion-resistant metal; but if it is not, all specified smooth surfaces and sharp edges shall be kept coated with a thin protective film (for example, oil, wax and silicone) applied with a soft cloth.

## 9.3 Specifications for installation

The specifications stated in [6.3](#) shall apply. In general, the weir shall be located in a straight, horizontal, rectangular approach channel if possible. However, if the effective opening of the notch is so small in comparison with the area of the upstream channel that the approach velocity is negligible, the shape of the channel is not significant. In any case, the flow in the approach channel shall be uniform and steady, as specified in [6.3.3](#).

If the width of the weir is equal to the width of the channel at the weir section (i.e. a full-width weir), the sides of the channel upstream from the plane of the weir shall be vertical, plane, parallel and smooth (equivalent in surface finish to that of rolled sheet-metal). The sides of the channel above the level of the crest of a full-width weir shall extend at least  $0,3 h_{\max}$  downstream from the plane of the weir. Fully ventilated discharge shall be ensured as specified in [6.3.4](#).

The approach channel floor shall be smooth, flat and horizontal when the height of the crest relative to the floor ( $p$ ) is small and/or  $h/p$  is large. For rectangular weirs, the floor should be smooth, flat and horizontal, particularly when  $p$  is less than 0,1 m and/or  $h_{\max}/p$  is greater than 1. Additional conditions are specified in connection with the recommended discharge formulae.

## 9.4 Determination of gauge zero

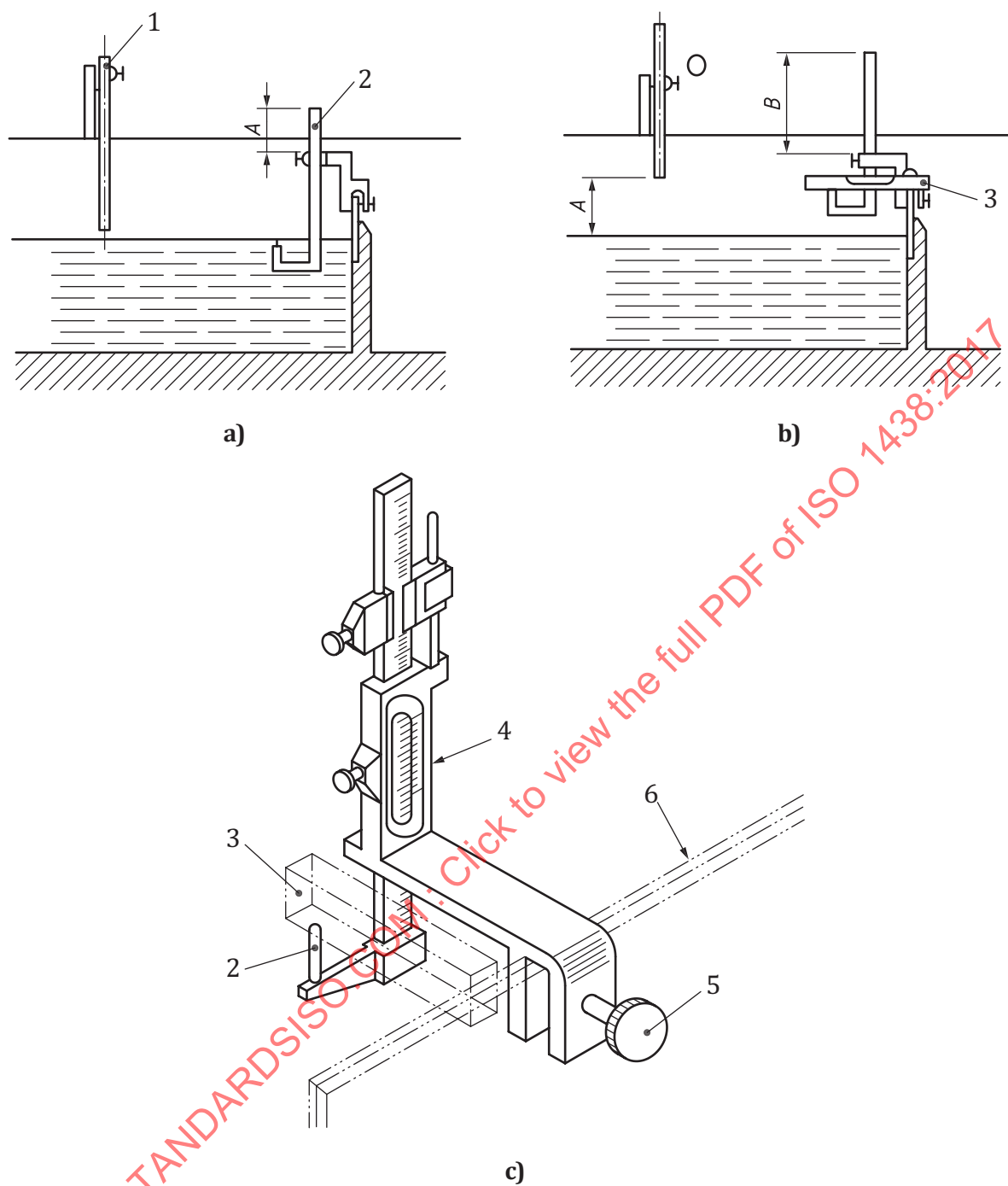
The head-gauge datum or gauge zero shall be determined with great care and it shall be checked when necessary. A typical, acceptable method of determining the gauge zero for rectangular weirs is described as follows.

- a) Still water in the approach channel is drawn to a level below the weir crest.
- b) A temporary hook gauge is mounted over the approach channel, a short distance upstream from the weir crest.
- c) A precise machinists' level is placed with its axis horizontal, with one end lying on the weir crest and the other end on the point of the temporary hook gauge (the gauge having been adjusted to hold the level in this position). The reading of the temporary gauge is recorded.

- d) The temporary hook gauge is lowered to the water surface in the approach channel and its reading is recorded. The permanent gauge is adjusted to read the level in the gauge well and this reading is recorded.
- e) The computed difference between the two readings of the temporary gauge is added to the reading of the permanent gauge. The sum is the gauge zero for the permanent gauge.

[Figure 3](#) illustrates the use of this procedure with a form of temporary hook gauge which is conveniently mounted on the weir plate.

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**Key**

- 1 permanent gauge
- 2 temporary hook gauge
- 3 precision level
- 4 vernier micrometer
- 5 set screw
- 6 weir crest

**Figure 3 — Determination of gauge zero for rectangular weir**

## 9.5 Discharge formulae — General

Recommended discharge formulae for rectangular thin-plate weirs are presented in three categories:

- a) modular discharge formula for the basic weir form (all values of  $b/B$ );
- b) modular discharge formula for full-width weirs ( $b/B = 1,0$ );
- c) non-modular discharge formula for full-width weirs.

## 9.6 Formulae for the basic weir form (all values of $b/B$ )

### 9.6.1 Kindsvater-Carter formula

The Kindsvater-Carter formula for the basic weir form is given in [Formula \(1\)](#):

$$Q = C_d \frac{2}{3} \sqrt{2g} b_e h_e^{3/2} \quad (1)$$

where

$C_d$  is the coefficient of discharge;

$b_e$  is the effective width;

$h_e$  is the effective head.

### 9.6.2 Evaluation of $C_d$ , $k_b$ and $k_h$

[Figure 4](#) shows experimentally determined values of  $C_d$  as a function of  $h/p$  for representative values of  $b/B$ . Values of  $C_d$  for intermediate values of  $b/B$  can be determined by interpolation.

The coefficient of discharge  $C_d$  has been determined by experiment as a function of two variables from [Formula \(2\)](#):

$$C_d = f\left(\frac{b}{B}, \frac{h}{p}\right) \quad (2)$$

The effective width and head are defined by [Formula \(3\)](#) and [Formula \(4\)](#):

$$b_e = b + k_b \quad (3)$$

$$h_e = h + k_h \quad (4)$$

in which  $k_b$  and  $k_h$  are experimentally determined quantities, in metres, which compensate for the combined effects of viscosity and surface tension.

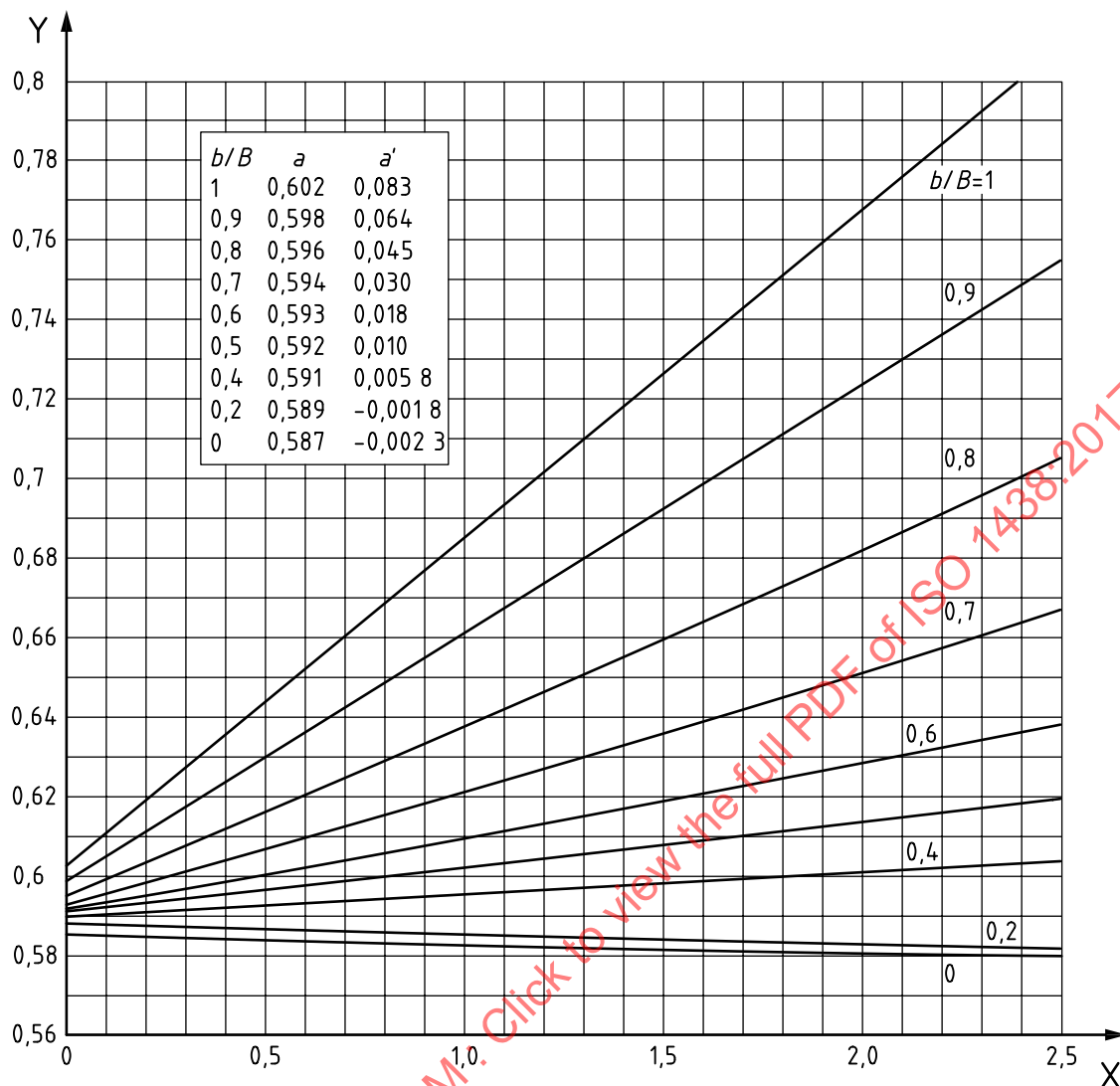
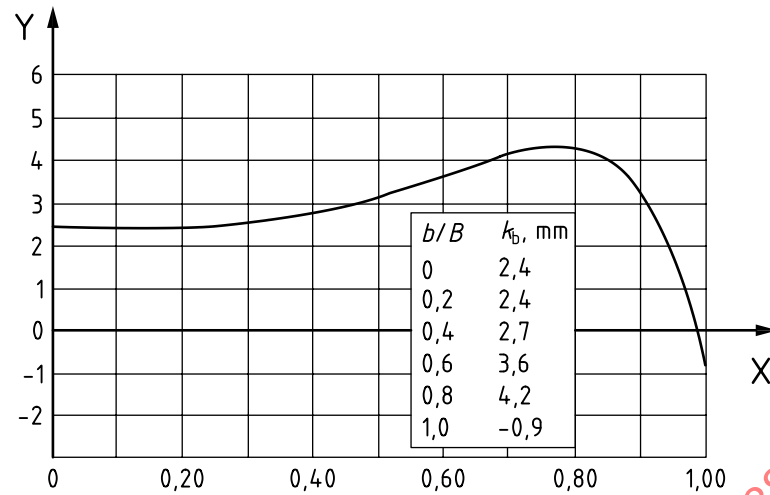
**Key**X value of  $\frac{h}{p}$ Y value of  $C_d$ **Figure 4 — Coefficient of discharge**

Figure 5 shows values of  $k_b$ , which have been experimentally determined as a function of  $b/B$ .

Experiments have shown that  $k_h$  can be taken to have a constant value of 0,001 m for weirs constructed in strict conformance with recommended specifications.

**Key**X  $b/B$ Y  $k_b$ , in millimetres**Figure 5 — Value of  $k_b$  related to  $b/B$** **9.6.3 Formulae for  $C_d$** 

For specific values of  $b/B$ , the relationship between  $C_d$  and  $h/p$  has been shown by experiment (see [Figure 4](#)) to be of the linear form  $C_d = a + a' \left( \frac{h}{p} \right)$ .

Thus, for the values of  $b/B$  shown on [Figure 4](#), formulae for  $C_d$  can be written as given in [Formula \(5\)](#) to [Formula \(13\)](#):

$$\left( \frac{b}{B} = 1,0 \right) : C_d = 0,602 + 0,083 \frac{h}{p} \quad (5)$$

$$\left( \frac{b}{B} = 0,9 \right) : C_d = 0,598 + 0,064 \frac{h}{p} \quad (6)$$

$$\left( \frac{b}{B} = 0,8 \right) : C_d = 0,596 + 0,045 \frac{h}{p} \quad (7)$$

$$\left( \frac{b}{B} = 0,7 \right) : C_d = 0,594 + 0,030 \frac{h}{p} \quad (8)$$

$$\left( \frac{b}{B} = 0,6 \right) : C_d = 0,593 + 0,018 \frac{h}{p} \quad (9)$$

$$\left( \frac{b}{B} = 0,5 \right) : C_d = 0,592 + 0,010 \frac{h}{p} \quad (10)$$

$$\left( \frac{b}{B} = 0,4 \right) : C_d = 0,591 + 0,0058 \frac{h}{p} \quad (11)$$

$$\left( \frac{b}{B} = 0,2 \right) : C_d = 0,589 - 0,0018 \frac{h}{p} \quad (12)$$

$$\left(\frac{b}{B}=0\right): C_d = 0,587 - 0,0023 \frac{h}{p} \quad (13)$$

For intermediate values of  $b/B$ , formulae for  $C_d$  can be determined satisfactorily by interpolation.

#### 9.6.4 Practical limitations on $h/p$ , $h$ , $b$ and $p$

Practical limits are placed on  $h/p$  because head-measurement difficulties and errors result from surges and waves which occur in the approach channel at larger values of  $h/p$ . Limits are placed on  $h$  to avoid the “clinging nappe” phenomenon which occurs at very low heads. Limits are placed on  $b$  because of uncertainties regarding the combined effects of viscosity and surface tension represented by the quantity of  $k_b$  at very small values of  $b$ . Limits are placed on  $p$  and  $B - b$  to avoid the instabilities which result from eddies that form in the corners between the channel boundaries and the weir when values of  $p$  and  $B - b$  are small.

For conservative practice, limitations applicable to the use of the Kindsvater-Carter formulae are:

- a)  $h/p$  shall be not greater than 2,5;
- b)  $h$  shall be not less than 0,03 m;
- c)  $b$  shall be not less than 0,15 m;
- d)  $p$  shall be not less than 0,10 m;
- e) either  $(B - b)/2 = 0$  (full-width weir) or  $(B - b)/2$  shall not be less than 0,10 m (contracted weir).

### 9.7 Formulae for full-width weirs ( $b/B = 1,0$ )

#### 9.7.1 Modular flow discharge formula

The Rehbock formula in the form proposed in 1929<sup>[8]</sup> is of the effective-head variety and is given in [Formula \(14\)](#):

$$Q = C_d \frac{2}{3} \sqrt{2g} b h_e^{3/2} \quad (14)$$

in which for the case of  $p \leq 1$  m, [Formula \(15\)](#) and [Formula \(16\)](#) apply:

$$C_d = 0,602 + 0,083 \frac{h}{p} \quad (15)$$

$$h_e = h + 0,0012 \quad (16)$$

where practical limitations applicable to the use of the Rehbock formula are:

- a)  $h/p$  shall be not greater than 4,0<sup>[3]</sup>;
- b)  $h$  shall be between 0,03 and 1,0 m;
- c)  $b$  shall be not less than 0,30 m;
- d)  $p$  shall be between 0,06 and 1 m;

and for the case of  $1 \text{ m} \leq p \leq 2,5 \text{ m}$ <sup>[7]</sup>,

$$C_d = 0,602 + 0,004(p - 1) + \{0,083 + 0,036(p - 1)\} \frac{h}{p} \quad (17)$$

$$h_e = h + 0,0012 \text{ same as Formula (16)}$$

where practical limitations for this case are:

- a)  $h$  shall be between 0,03 and 0,80 m but not greater than  $b/4$ ;
- b)  $b$  shall be not less than 0,50 m;
- c)  $p$  shall be between 1,0 and 2,5 m.

### 9.7.2 Non-modular flow discharge formula

Submerged (drowned) flow occurs when the tailwater level downstream from a weir affects the flow. The weir operates in the non-modular condition. For this condition, an additional downstream measurement of head ( $h_2$ ) is required and a drowned flow reduction factor ( $f$ ) is applied to the modular discharge formula.

Since the modular limit of a full-width thin-plate weir is significantly influenced by the ratio  $h/p$ , the modular limit increasing with  $h/p$ , drowned flow performance of the typical full-width thin-plate weir is shown in [Figure 6](#) and defined by the formulae below:

For  $h/p = 0,5$ , then  $f = 1,007 [0,975 - (h_2/h)^{1,45}]^{0,265}$  in the range  $0,00 < h_2/h < 0,97$

For  $h/p = 1,0$ , then  $f = 1,026 [0,960 - (h_2/h)^{1,55}]^{0,242}$  in the range  $0,20 < h_2/h < 0,97$

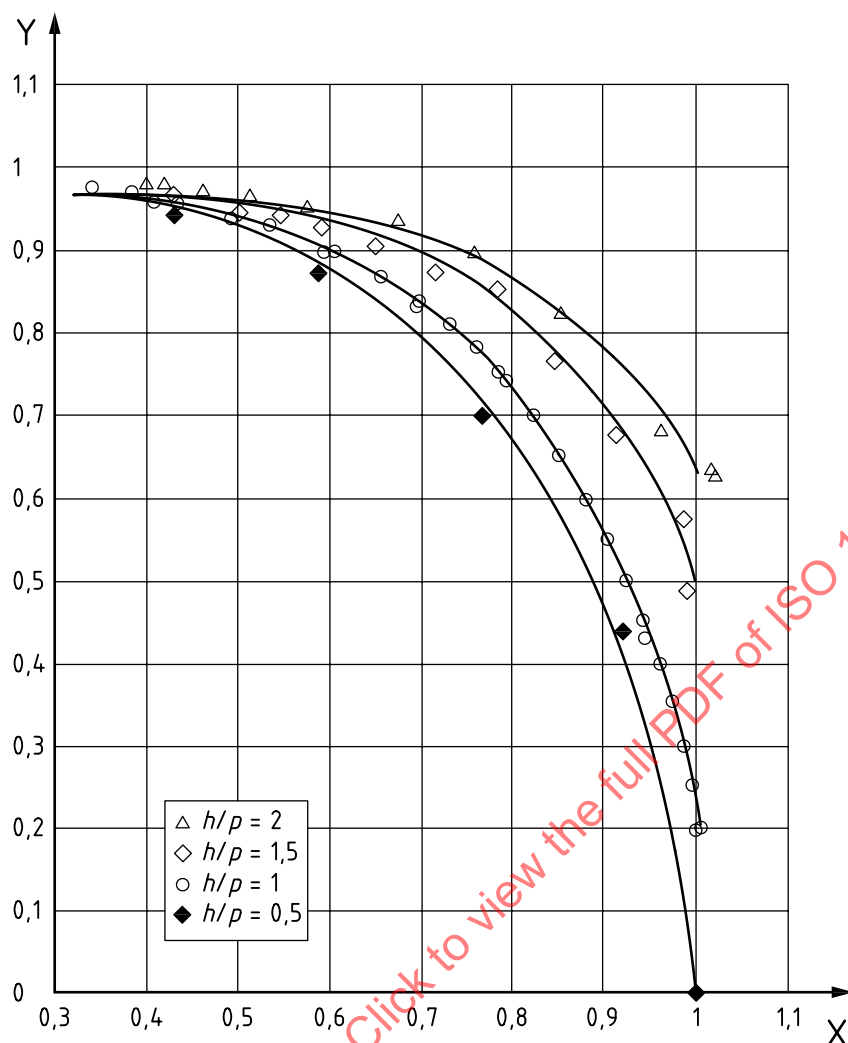
For  $h/p = 1,5$ , then  $f = 1,098 [0,952 - (h_2/h)^{1,75}]^{0,220}$  in the range  $0,50 < h_2/h < 0,97$

For  $h/p = 2,0$ , then  $f = 1,155 [0,950 - (h_2/h)^{1,85}]^{0,219}$  in the range  $0,63 < h_2/h < 0,97$

Thus, the Rehbock Formula (1929) for drowned flow becomes [Formula \(18\)](#):

$$Q = f C_d \frac{2}{3} \sqrt{2g} b h_e^{3/2} \quad (18)$$

NOTE This adjustment only applies where the upstream and downstream measurements are in the same horizontal plane, i.e. there is no drop in the channel bottom at, or downstream, of the weir.



### Key

X value of  $\frac{h_2}{h}$

Y value of  $f$

Figure 6 — Drowned flow performance of the full-width thin-plate weir

## 10 Triangular-notch thin-plate weir

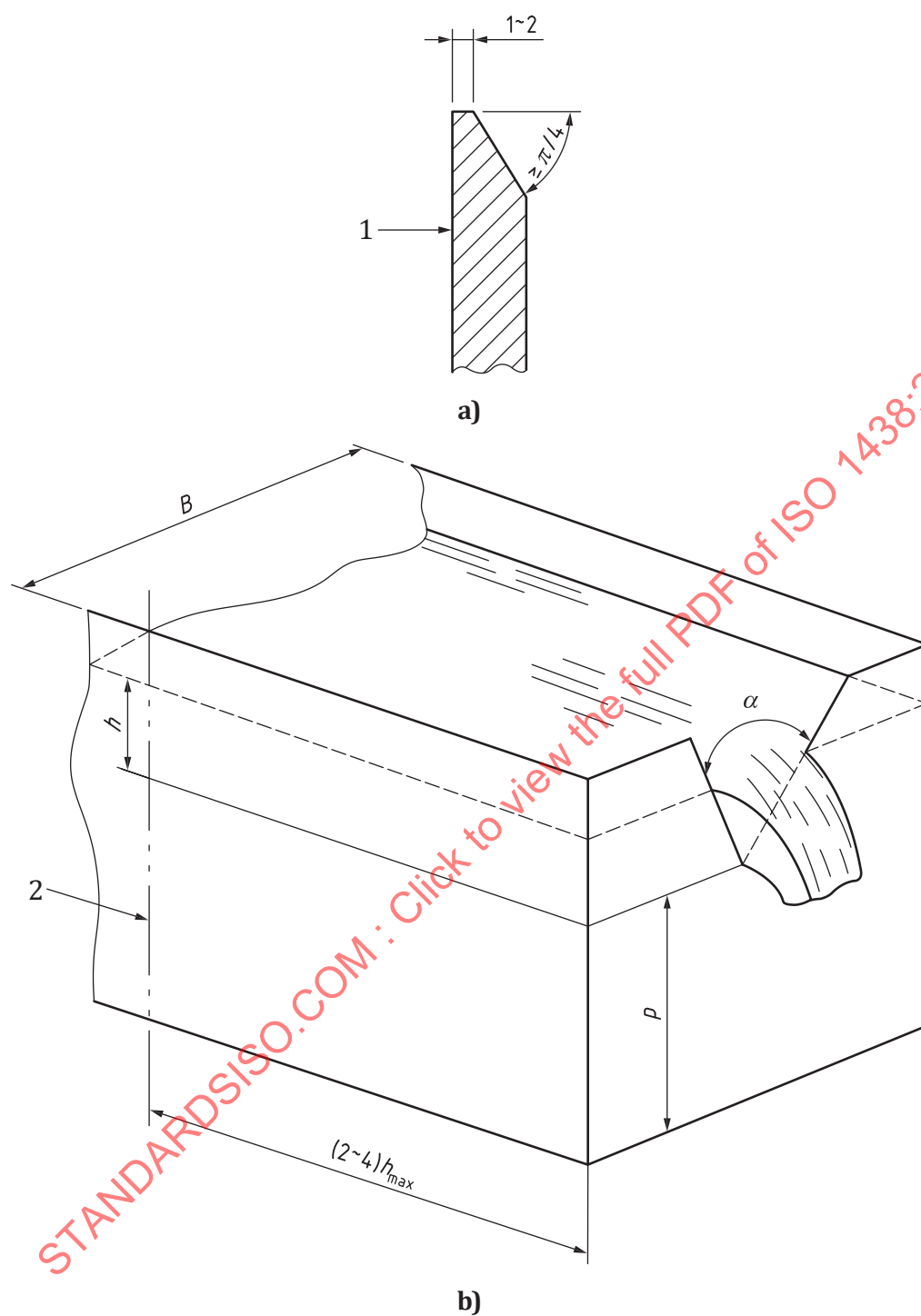
### 10.1 Specifications for the standard weir

The triangular-notch thin-plate weir consists of a V-shaped notch in a vertical, thin plate. A diagrammatic illustration of the triangular-notch weir is shown in [Figure 7](#). The weir plate shall be plane and rigid and perpendicular to the walls and the floor of the channel. The upstream face of the plate shall be smooth (in the vicinity of the notch, it shall be equivalent in surface finish to that of rolled sheet-metal).

The bisector of the notch shall be vertical and equidistant from the two walls of the channel. The surfaces of the notch shall be plane surfaces, which shall form sharp edges at their intersection with the upstream face of the weir plate. The width of the notch surfaces, measured perpendicular to the face of the plate, shall be between 1 mm and 2 mm.

To ensure that the upstream edges of the notch are sharp, they shall be machined or filed, perpendicular to the upstream face of the plate, free of burrs or scratches and untouched by abrasive cloth or paper. The downstream edges of the notch shall be chamfered if the weir plate is thicker than the maximum allowable width of the notch surface. The surface of the chamfer shall make an angle of not less than  $\pi/4$  radians ( $45^\circ$ ) with the surface of the notch (see detail, [Figure 7](#)). The weir plate in the vicinity of the notch preferably shall be made of corrosion-resistant metal; but if it is not, all specified smooth surfaces shall be kept coated with a thin protective film (for example, oil, wax, silicone) applied with a soft cloth.

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**Key**

- 1 upstream face of weir plate
- 2 head measurement section

**Figure 7 — Triangular-notch thin-plate weir**

## 10.2 Specifications for the installation

The specifications stated in 6.3 shall apply. In general, the weir shall be located in a straight, horizontal, rectangular channel if possible. However, if the effective opening of the notch is so small in comparison with the area of the upstream channel that the approach velocity is negligible, the shape of the channel is not significant. In any case, the flow in the approach channel shall be uniform and steady, as specified in 6.3.3.

If the top width of the nappe at maximum head is large in comparison with the width of the channel, the channel walls shall be straight, vertical and parallel. If the height of the vertex relative to the level of the floor is small in comparison with the maximum head, the channel floor shall be smooth, flat and horizontal. In general, the approach channel should be smooth, straight and rectangular when  $B/b_{\max}$  is less than 3 and/or  $h_{\max}/p$  is greater than 1. Additional conditions are specified in connection with the recommended discharge formulae.

## 10.3 Specifications for head measurement

### 10.3.1 General

The conditions specified in 7.1, 7.2 and 7.3 shall apply without exception.

### 10.3.2 Determination of notch angle

Precise head measurements for triangular-notch weirs require that the notch angle (angle included between sides of the notch) be measured accurately. One of several satisfactory methods is described as follows.

- Two true disks of different, micrometered diameters are placed in the notch with their edges tangent to the sides of the notch.
- The vertical distance between the centres (or two corresponding edges) of the two disks is measured with a micrometer caliper.
- The notch angle  $\alpha$  is twice the angle whose sine is equal to the differences between the radii of the disks divided by the distance between the centres of the disks.

### 10.3.3 Determination of gauge zero

The head-gauge datum or gauge zero shall be determined with great care and it shall be checked when necessary. A typical acceptable method of determining the gauge zero for triangular notch weirs is described as follows.

- Still water in the approach channel is drawn to a level below the vertex of the notch.
- A temporary hook gauge is mounted over the approach channel, with its point a short distance upstream from the vertex of the notch.
- A true cylinder of known (micrometered) diameter is placed with its axis horizontal, with one end resting in the notch and the other end balanced on the point of the temporary hook gauge. A machinists' level is placed on top of the cylinder, and the hook gauge is adjusted to make the cylinder precisely horizontal. The reading of the temporary gauge is recorded.
- The temporary hook gauge is lowered to the water surface in the approach channel and the reading is recorded. The permanent gauge is adjusted to read the level in the gauge well, and this reading is recorded.
- The distance ( $y$ ) from the top of the cylinder to the vertex of the notch is computed with the known value of the notch angle ( $\alpha$ ) and the radius ( $r$ ) of the cylinder  $\left[ y = \left( r / \sin \frac{\alpha}{2} \right) + r \right]$ . This distance is

then subtracted from the reading recorded in c), the result being the reading of the temporary gauge at the vertex of the notch.

- f) The difference between the computed reading in e) and the reading of the temporary gauge in d) is added to the reading of the permanent gauge in d). The sum is the gauge zero for the permanent gauge.

An advantage of this method is that it refers the gauge zero to the geometrical vertex which is defined by the sides of the notch.

## 10.4 Discharge formulae — General

Recommended discharge formulae for triangular-notch thin-plate weirs are presented in two categories:

- a) formula for all notch angles between  $\pi/9$  and  $5\pi/9$  radians ( $20^\circ$  and  $100^\circ$ );
- b) formulae for specific notch angles (fully contracted weirs).

## 10.5 Formula for all notch angles between $\pi/9$ and $5\pi/9$ radians ( $20^\circ$ and $100^\circ$ )

### 10.5.1 Kindsvater-Shen formula

The Kindsvater-Shen formula for triangular notch weirs is given in [Formula \(19\)](#):

$$Q = C_d \frac{8}{15} \tan \frac{\alpha}{2} \sqrt{2g} h_e^{5/2} \quad (19)$$

where

$C_d$  is the coefficient of discharge;

$h_e$  is the effective head.

The coefficient of discharge,  $C_d$ , has been determined by experiment as a function of three variables (see [Figure 8](#)), and is given in [Formula \(20\)](#):

$$C_d = f\left(\frac{h}{p}, \frac{p}{B}, \alpha\right) \quad (20)$$

where

$p$  is the height of the vertex of the notch with respect to the floor of the approach channel;

$B$  is the width of the approach channel;

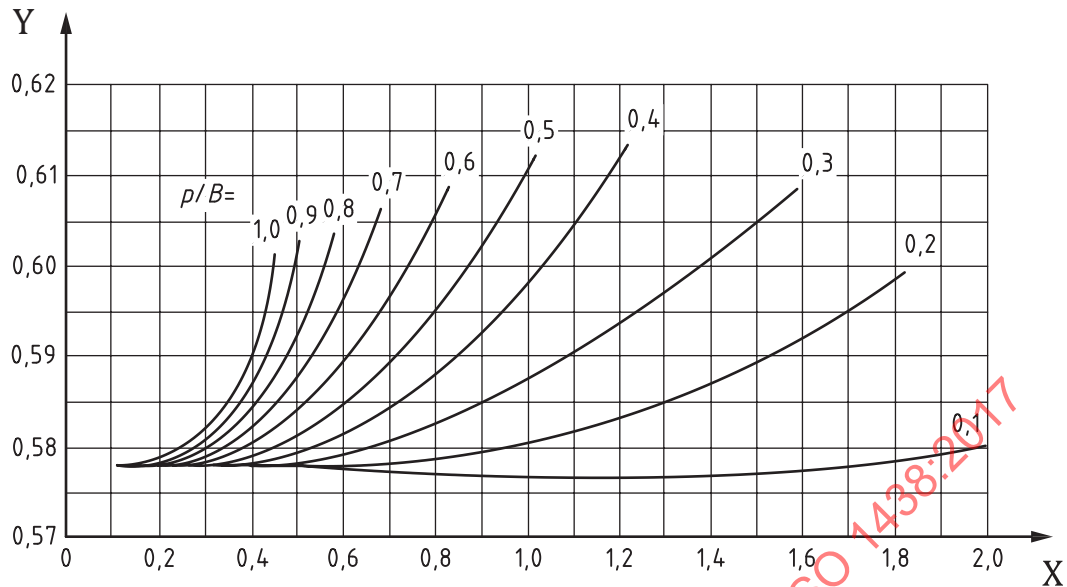
$h_e$  is defined by [Formula \(21\)](#):

$$h_e = h + k_h \quad (21)$$

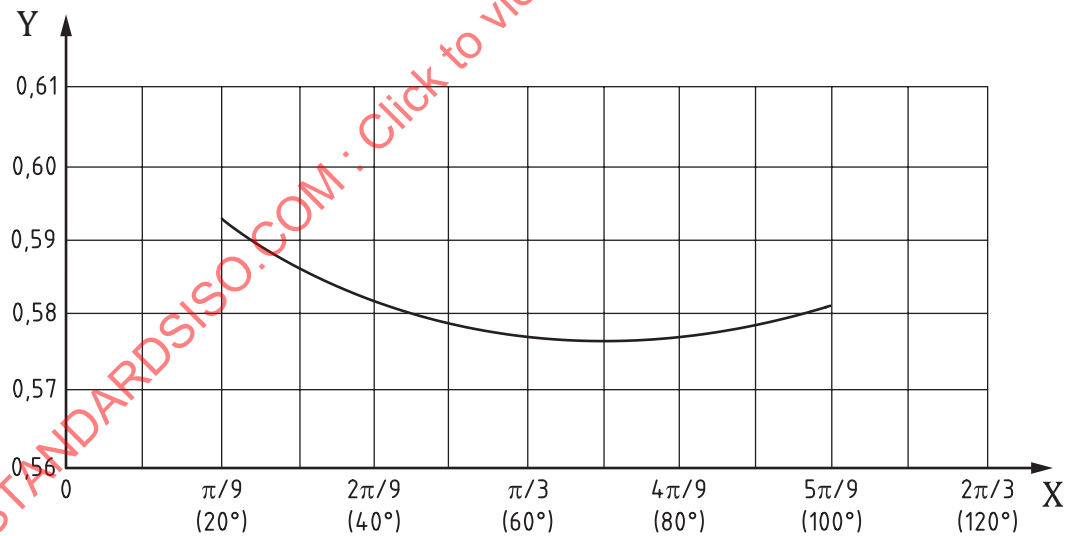
in which  $k_h$  is an experimentally determined quantity, in metres, which compensates for the combined effects of viscosity and surface tension.

### 10.5.2 Evaluation of $C_d$ and $k_h$

For triangular weirs with notch angle  $\alpha$  equal to  $\pi/2$  radians ( $90^\circ$ ), [Figure 8](#) shows experimentally determined values of  $C_d$  for a wide range of values of  $h/p$  and  $p/B$ . For  $\alpha = \pi/2$  radians ( $90^\circ$ ),  $k_h$  has been shown to have a constant value of 0,000 85 m for a corresponding range of values of  $h/p$  and  $p/B$ .

**Key**

X value of  $\frac{h}{p}$   
 Y value of  $C_d$

**Figure 8 — Coefficient of discharge,  $C_d$  ( $\alpha = 90^\circ$ )****Key**

X value of notch angle,  $\alpha$  (radians)  
 Y value of  $C_d$

**Figure 9 — Coefficient of discharge,  $C_d$ , related to notch angle,  $\alpha$** 

For notch angles other than  $\pi/2$  radians ( $90^\circ$ ), experimental data are insufficient to define  $C_d$  as a function of  $h/p$  and  $p/B$ . However, for weir notches which are small relative to the area of the approach channel, the velocity of approach is negligible and the effects of  $h/p$  and  $p/B$  are also negligible. For this condition (the so-called “fully-contracted” condition), Figure 9 shows experimentally determined values of  $C_d$  as a function of  $\alpha$  alone. Corresponding values of  $k_h$  are shown in Figure 10.

### 10.5.3 Practical limitations on $\alpha$ , $h/p$ , $p/B$ , $h$ and $p$

For reasons related to hazards of measurement error and lack of experimental data, the following practical limits are applicable to the use of the Kindsvater-Shen formula:

- a)  $\alpha$  shall be between  $\pi/9$  radians and  $5\pi/9$  radians ( $20^\circ$  and  $100^\circ$ );
- b)  $h/p$  shall be limited to the range shown in [Figure 8](#) for  $\alpha = \pi/2$  radians ( $90^\circ$ );  $h/p$  shall be not greater than 0,35 for other values of  $\alpha$ ;
- c)  $h$  shall be not less than 0,06 m;
- d)  $p$  shall be not less than 0,09 m.

### 10.6 Formula for specific notch angles (fully-contracted weir)

The British Standards Institution (BSI) formula is for three notch angles that have a special geometric relationship to each other:

- a)  $\tan \alpha/2 = 1$  ( $\alpha = \pi/2$  radians or  $90^\circ$ );
- b)  $\tan \alpha/2 = 0,50$  ( $\alpha = 0,927\ 3$  rad or  $53^\circ\ 8'$ );
- c)  $\tan \alpha/2 = 0,25$  ( $\alpha = 0,489\ 9$  rad or  $28^\circ\ 4'$ ).

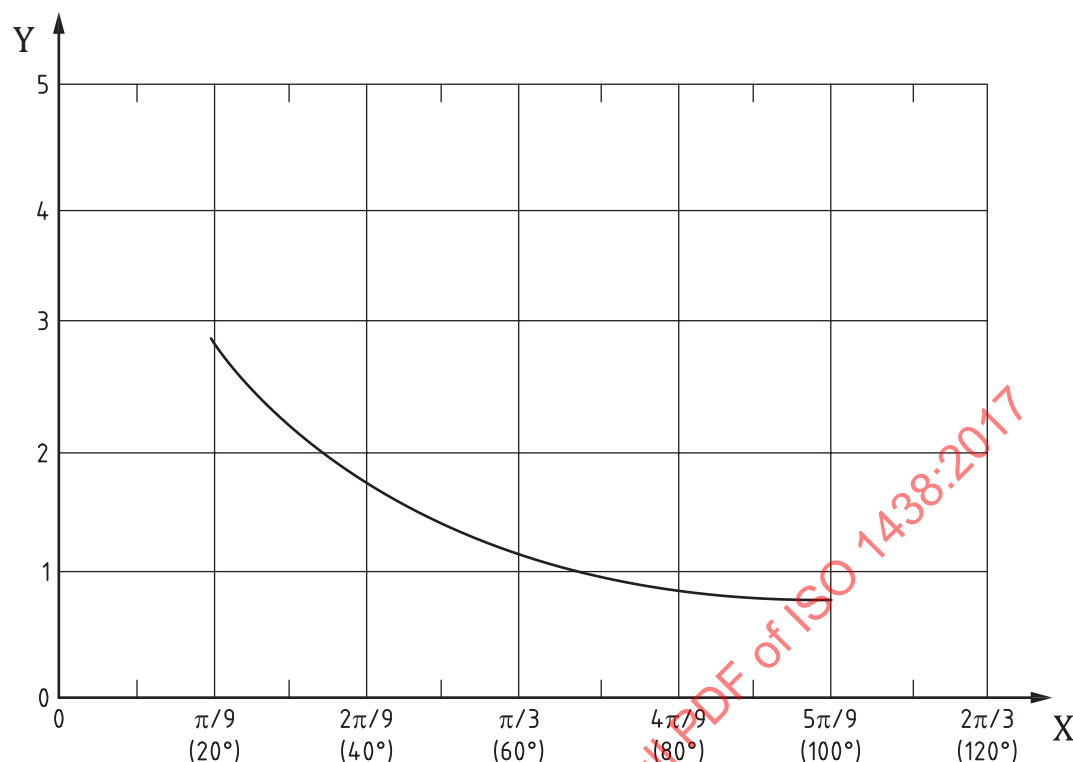
The BSI discharge formula is given in [Formula \(22\)](#):

$$Q = C \frac{8}{15} \tan \frac{\alpha}{2} \sqrt{2g} h_e^{5/2} \quad (22)$$

and the experimentally determined values of  $C$  and  $Q$  for the condition of “full contraction” are shown in [Tables E.1](#), [E.2](#) and [E.3](#).

Practical limitations applicable to the use of this formula are:

- a)  $h/p$  shall be not greater than 0,4;
- b)  $h/B$  shall be not greater than 0,2;
- c)  $h$  shall be between 0,05 and 0,38 m;
- d)  $p$  shall be not less than 0,45 m;
- e)  $B$  shall be not less than 1,0 m.

**Key**X notch angle,  $\alpha$ Y  $k_h$ , in millimetres**Figure 10 — Value of  $k_h$  related to notch angle,  $\alpha$** **10.7 Accuracy of discharge coefficients — Triangular-notch weirs**

The accuracy of discharge measurements made with a triangular-notch thin-plate weir depends primarily on the accuracy of the head and notch-angle measurements and on the applicability of the discharge formula and coefficients used. If great care is exercised in meeting the construction, installation and operational conditions specified in this document, uncertainties (at 95 % confidence level) attributable to the coefficients of discharge will be not greater than 1,0 %. The combination of all uncertainties which contribute significantly to the uncertainty of discharge measurements is treated in [Clause 11](#). Examples of estimated uncertainties in measured discharge are given in [Clause 12](#).

**11 Uncertainties of flow measurement****11.1 General**

**11.1.1** This clause provides information for the user of this document to state the uncertainty of a measurement of discharge.

**11.1.2** [Annex C](#) is an introduction to measurement uncertainty. It provides supporting information based on ISO/IEC Guide 98-3[1] and ISO/TS 25377[5]. Refer to [Annex C](#) for definitions.

Previous versions of this document have expressed the uncertainty of discharge coefficient  $u(C)$  at the 95 % level of confidence. This is equivalent to two standard deviations or twice the value of standard uncertainty.

This document expresses discharge coefficient as standard uncertainty (one standard deviation) to be in accordance with ISO/IEC Guide 98-3.

Hydrometry requires measurements using various techniques, the results of which are used to calculate a value for flow. [Annex D](#) provides sample values for the various techniques. These are presented in tabular form with uncertainty estimates ascribed to each technique for the purpose of illustration only.

These sample values are not to be interpreted as norms of performance.

The example given in [Clause 12](#) uses values from [Annex D](#).

### 11.1.3 A measurement result comprises

- a) an estimate of the measured value, with
- b) a statement of the uncertainty of the measurement.

**11.1.4** A statement of the uncertainty of a flow measurement in a channel has four separate components of uncertainty:

- a) uncertainty of the measurement of head in the channel;
- b) uncertainty of the dimensions of the structure;
- c) uncertainty of the discharge coefficient stated in this document from laboratory calibration of the flow structure being considered;
- d) uncertainty of channel velocity distribution related to the velocity coefficient,  $C_v$ .

This clause does not accommodate component d). It is assumed that the channel hydraulics are substantially equivalent to those existing in the calibration facility at the time of derivation of component c) as defined in [6.3.3](#).

**11.1.5** The estimation of measurement uncertainty associated with items a) and b) of [11.1.4](#) is provided in [Annex D](#).

Values taken from [Annex D](#) are used in the examples in [Clause 12](#). These values are for illustrative purpose only and should not be interpreted as norms of performance for the types of equipment listed. In practice, uncertainty estimates should be taken from test certificates for the equipment, preferably obtained from laboratories which are accredited to ISO/IEC 17025[3].

## 11.2 Combining measurement uncertainties

Refer to [C.7](#).

The proportion in which each flow formula parameter contributes to flow measurement uncertainty,  $U(Q)$ , is derived by analytical solution using partial differentials of the discharge formula.

For this purpose, the formulae for rectangular and triangular forms have been simplified as shown in [Formula \(23\)](#) and [Formula \(24\)](#):

$$Q_r = J_r \sqrt{g} C_d b_e h_e^{1,5} \quad (23)$$

$$Q_t = J_t \sqrt{g} C_d \tan \frac{\alpha}{2} h_e^{2,5} \quad (24)$$

where  $J$  is a numerical constant, dependent on the form of weir but not subject to error. The subscripts r and t denote the rectangular form and the triangular form of weir, respectively. From [Formula \(23\)](#)

and [Formula \(24\)](#), the dispersion of the value  $Q$  of the formula can be written as [Formula \(25\)](#) and [Formula \(26\)](#):

$$\Delta Q_r = J_r \sqrt{g} \left( \frac{\partial Q_r}{\partial C_d} \Delta C_d + \frac{\partial Q_r}{\partial b_e} \Delta b_e + \frac{\partial Q_r}{\partial h_e} \Delta h_e \right) \quad (25)$$

$$\Delta Q_t = J_t \sqrt{g} \left[ \frac{\delta Q_t}{\delta C_d} \Delta C_d + \frac{\partial Q_t}{\partial \tan\left(\frac{\alpha}{2}\right)} \Delta \tan\left(\frac{\alpha}{2}\right) + \frac{\partial Q_t}{\partial h_e} \Delta h_e \right] \quad (26)$$

where the partial derivatives are the sensitivity coefficients described in ISO/TS 25377 and where  $\Delta Q$  is the dispersion of  $Q$  due to small dispersions of  $\Delta C$ ,  $\Delta b$  or  $\Delta \tan\left(\frac{\alpha}{2}\right)$  and  $\Delta h_e$ . Evaluating the partial differentials and using [Formula \(23\)](#) and [Formula \(24\)](#), the relationship can be written as [Formula \(27\)](#) and [Formula \(28\)](#):

$$\frac{\Delta Q_t}{Q_t} = \frac{\Delta C_d}{C_d} + \frac{\Delta b_e}{b_e} + 1,5 \frac{\Delta h_e}{h_e} \quad (27)$$

$$\frac{\Delta Q_t}{Q_t} = \frac{\Delta C_d}{C_d} + \frac{\Delta \tan\frac{\alpha}{2}}{\tan\frac{\alpha}{2}} + 2,5 \frac{\Delta h_e}{h_e} \quad (28)$$

In uncertainty analysis, the values  $\frac{\Delta Q}{Q}$ ,  $\frac{\Delta b}{b}$ ,  $\frac{\Delta C}{C}$ ,  $\frac{\Delta \tan\frac{\alpha}{2}}{\tan\frac{\alpha}{2}}$  and  $\frac{\Delta h}{h}$  are referred to as dimensionless

standard uncertainties and have the notation  $u^*(Q)$ ,  $u^*(C)$ ,  $u^*(b)$ ,  $u^*\left[\tan\left(\frac{\alpha}{2}\right)\right]$  and  $u^*(h)$ .

Note that the value  $u^*\left[\tan\left(\frac{\alpha}{2}\right)\right]$  is derived from the relationship given in [Formula \(29\)](#):

$$\tan\left(\frac{\alpha}{2}\right) = \frac{b_t}{2h_t} \quad (29)$$

where  $b_t$  is the crest width and  $h_t$  is the height of the notch.

Since the uncertainties of  $b$ ,  $\alpha$ ,  $C$  and  $h$  are independent of each other, probability requires summation in quadrature rather than a simple summation, as given in [Formula \(30\)](#) and [Formula \(31\)](#):

$$u_c^*(Q)_r = \sqrt{u^*(C_d)^2 + u^*(b_e)^2 + [1,5 u^*(h_e)]^2} \quad (30)$$

$$u_c^*(Q)_t = \sqrt{u^*(C_d)^2 + u^*\left[\tan\left(\frac{\alpha}{2}\right)\right]^2 + [2,5 u^*(h_e)]^2} \quad (31)$$

### 11.3 Uncertainty of discharge coefficient, $u^*(C_d)$ , for thin-plate weirs

The discharge coefficient,  $C_d$ , of [Clause 9](#) and [Clause 10](#) have been determined from a series of hydraulics tests using a high resolution calibration facility. From these tests, the values of discharge coefficient uncertainty,  $u^*(C_d)$ , are summarized in [Table 2](#).

**Table 2 — Values of discharge coefficient uncertainty,  $u^*(C_d)$ , against head,  $h$** 

| Type        | Head, $h$          | $u^*(C_d)$ |
|-------------|--------------------|------------|
| Rectangular | $h < 1,0p$         | 0,75 %     |
| Rectangular | $1,0p < h < 1,5p$  | 1,00 %     |
| Rectangular | $1,5p < h < 2,5 p$ | 1,50 %     |
| Triangular  | —                  | 0,5 %      |

## 11.4 Uncertainty budget

In reports, an uncertainty budget table may be presented (or referenced) to provide the following information for each source of uncertainty:

- the method of evaluation (from [Annex C](#));
- the determined value of standard uncertainty  $u^*(C_d)$ ,  $u^*\left[\tan\left(\frac{\alpha}{2}\right)\right]$  and  $u^*(h_e)$ , including datum uncertainty of  $u^*(h_e)$ ;
- the relative sensitivity coefficients, [Formula \(27\)](#) and [Formula \(28\)](#).

The values for each source are then applied according to [Formula \(30\)](#) or [Formula \(31\)](#) to give the combined standard uncertainty,  $u^*(Q)$ .

The expanded uncertainty  $U^*(Q)$  for a confidence level of 95 % is calculated using [Table C.1](#).

It is customary to present these steps in tabular form with one row for each source and a column for each of the items a) to c).

The table may include, where appropriate, the critical thinking behind the subjective allocation of uncertainty to the quantities  $b$  and  $h$ . This section of the table may be replicated for a range of values of  $h_1$  to determine a relationship between  $u^*(Q)$  and  $h_1$ .

## 12 Example

### 12.1 General

In presenting examples, the formulae given in [Clause 10](#) and [Clause 11](#) define the relationship between the parameters which determine flow rate.

Uncertainty of the discharge coefficient is a fundamental uncertainty and is defined in [12.3](#). To complete overall uncertainty estimation, practical estimations shall be made of the head measurement uncertainty and the uncertainty of the measurement of physical dimensions.

[Annex D](#) provides a consistent framework for evaluating these uncertainties for the commonly used measurement techniques.

One such technique is selected in [12.3](#) for the example that follows.

### 12.2 Characteristics — Gauging structure

The example relates to modular flow conditions for a 90° V-notch weir. The crest height  $p$  above the bed of the approach channel is 0,151 m. The channel is 0,503 m wide. The angle of the V-notch is estimated to lie between 89,5° and 90,5°.

### 12.3 Characteristics — Gauged head instrumentation

In this example, a pressure transducer is used to determine head. The transducer is located in the approach channel about 1 m upstream of the weir.

- The signal indicates a head of 0,212 m. Referring to [Annex D](#), the measurement uncertainty from [Table D.1](#), at this head, is  $u(h_1) = 0,002$  m.
- The transducer is susceptible to drift over a period of time. Over a period of time, it has been noted that the nominal datum signal varies in the range 0,000 m to 0,007 m. Datum uncertainty is estimated according to the rectangular distribution given in [Formula \(C.5\)](#).

$$u(E) = \frac{1}{\sqrt{3}} \left( \frac{0,007 - 0,000}{2} \right)$$

$$u(E) = 0,002 \text{ m}$$

### 12.4 Discharge coefficient

The value of the gauged head discharge coefficient is determined from [Figure 8](#) for the 90° V-notch weir. The key ratios of  $h/p$  and  $p/B$  are:

$$\frac{h}{p} = \frac{0,212}{0,151} = 1,40$$

$$\frac{p}{B} = \frac{0,151}{0,503} = 0,30$$

from which  $C_d = 0,600$ .

### 12.5 Discharge estimate

The flow rate is calculated from [Formula \(19\)](#):

$$Q_t = C_d \frac{8}{15} \tan \frac{\alpha}{2} \sqrt{2g} h_e^{5/2}$$

where  $h_e = h + k_h = 0,212 + 0,00085$

$$Q_t = 0,600 \times 0,5333 \times 4,429 \times 1 \times 0,21285^{2,5}$$

$$\therefore Q_t = 0,0296 \text{ m}^3/\text{s}$$

Specimen tables for the discharge of water over a V-notch with  $\tan 1/\alpha$  equal to 1,  $\frac{1}{2}$  and  $\frac{1}{4}$  are given in [Annex E](#).

### 12.6 Uncertainty statement

**12.6.1** From [Table 2](#), the value for uncertainty of the discharge coefficient is:

$$u^*(C) = 0,50 \%$$

**12.6.2** Using [Formula \(C.4\)](#), the value of uncertainty of the V-angle may be written as follows:

$$u \left[ \tan \left( \frac{\alpha}{2} \right) \right] = \frac{1}{\sqrt{6}} \left[ \frac{\tan \left( \frac{90,5}{2} \right) - \tan \left( \frac{89,5}{2} \right)}{2} \right]$$

$$= 0,0036$$

$$\text{or } u^* \left[ \tan \left( \frac{\alpha}{2} \right) \right] = 100 \times \frac{0,0036}{\tan \left( \frac{90}{2} \right)} = 0,36 \%$$

**12.6.3** The combined uncertainty of gauged head  $u(h)$ , calculated in [12.3](#), is combined with instrumentation measurement uncertainty and datum measurement uncertainty.

$$u(h) = \sqrt{(0,002)^2 + (0,002)^2} \text{ m}$$

$$u(h) = 0,0028 \text{ m}$$

$$u^*(h) = \frac{0,0028}{0,212} \times 100$$

$$u^*(h) = 1,32 \%$$

**12.6.4** The combined uncertainty estimate is determined from [Formula \(31\)](#).

$$u_c^*(Q) = \sqrt{u^*(C_d)^2 + u^* \left[ \tan \left( \frac{\alpha}{2} \right) \right]^2 + [2,5u^*(h_e)]^2}$$

$$u_c^*(Q) = \sqrt{0,50^2 + 0,36^2 + (2,5 \times 1,32)^2}$$

$$u_c^*(Q) = 3,35 \%$$

Therefore, at the 95 % confidence level:

$$u_c^*(Q) = 3,35 \times 2 = 6,7 \%$$

**NOTE** This estimate is dominated by the contribution from head measurement uncertainty and assumes sufficient measurement samples.

**12.6.5** The conventional statement of discharge is therefore:

0,029 3 m<sup>3</sup>/s with an uncertainty of 6,7 % at the 95 % level of confidence based on a coverage factor of  $k = 2$ .

**12.6.6** The uncertainty budget for the example could be expressed as in [Table 3](#).

**Table 3 — Uncertainty budget for the example**

|                       | Type/<br>Evaluation | $u, u^*$<br>Value | Sensitivity<br>coefficients | Comment                           |
|-----------------------|---------------------|-------------------|-----------------------------|-----------------------------------|
| $u^*(C_d)$            | B/Normal            | 0,5 %             | 1,0                         | From laboratory tests             |
| $u^*[\tan(\alpha/2)]$ | B/Triangular        | 0,36 %            | 1,0                         | Using <a href="#">C.6.2</a>       |
| $u(E)$                | B/Rectangular       | 0,002 m           | —                           | Using <a href="#">C.6.3</a>       |
| $u(h_1)$              | B/Rectangular       | 0,002 m           | —                           | From <a href="#">Table D.1</a>    |
| $u^*(h_1)$            | Combined            | 1,32 %            | 2,5                         | From <a href="#">12.6.3</a>       |
| $u_c^*(Q)$            | Combined            | 3,35 %            | —                           | Using <a href="#">Formula (6)</a> |

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## Annex A (informative)

### Flow measurement with small weir tanks

Whenever possible, weir tanks conforming to 6.3.3 should be used to measure flow in the field. When the highest accuracy is not required or where site conditions make it difficult to install or operate large tanks satisfactorily, smaller tanks may be used.

There is a limited amount of data on how the discharge coefficients of weirs are affected by the size of the tank, as well as by non-standard head-measuring positions, asymmetric and unsteady flow conditions at entry and sediment deposits. Further information can be obtained from *The performance of weir tanks fitted with V-notch and rectangular thin-plate weirs*<sup>1)</sup>.

In order to give some guide to the effect that a reduction in the size of the weir tank will have, values of the discharge coefficient have been tabulated for seven different sizes of weir tank. Table A.1 gives values of  $C$  in the BSI formula for a 90° V-notch and of  $C_d$  in the Kindsvater-Carter formula for a contracted rectangular-notch.

Some indication of the influence of sediment deposit is given in Table A.2, which shows values of  $C_d$  for a tank with dimensions conforming to 6.3.3 but with differing amounts of sediment deposited against the weir plate. The uncertainty of these coefficients is approximately 1 %.

Within the range of tank sizes and heads covered in Table A.1, the location of the head-measuring device is relatively unimportant. Positions between 100 mm and 720 mm upstream of the weir produce discharge coefficients which vary by less than 0,5 %. Heads should not be measured, however, near the inlet baffle or in the downstream corner of a narrow tank.

**Table A.1 — Discharge coefficients for 90° V-notches and rectangular notches in small weir tanks**

| Tank size<br>(length, width, height)<br>m                                                                       | C values for 90° V-notches <sup>a</sup> |       |       | C <sub>d</sub> values for rectangular notches <sup>b</sup> |       |       |
|-----------------------------------------------------------------------------------------------------------------|-----------------------------------------|-------|-------|------------------------------------------------------------|-------|-------|
|                                                                                                                 | Head<br>(h)<br>mm                       |       |       | Head<br>(h)<br>mm                                          |       |       |
|                                                                                                                 | 115                                     | 150   | 180   | 65                                                         | 100   | 135   |
| 2,62 × 0,92 × 0,45                                                                                              | 0,593                                   | 0,592 | 0,587 | 0,609                                                      | 0,592 | 0,588 |
| 1,50 × 0,92 × 0,45                                                                                              | 0,603                                   | 0,592 | 0,587 | 0,604                                                      | 0,592 | 0,588 |
| 1,00 × 0,92 × 0,45                                                                                              | 0,603                                   | 0,592 | 0,587 | 0,600                                                      | 0,590 | 0,588 |
| 2,62 × 0,75 × 0,45                                                                                              | 0,597                                   | 0,592 | 0,590 | 0,606                                                      | 0,593 | 0,588 |
| 2,62 × 0,50 × 0,45                                                                                              | 0,605                                   | 0,596 | 0,595 | 0,611                                                      | 0,598 | 0,598 |
| 2,62 × 0,92 × 0,30                                                                                              | 0,600                                   | 0,590 | 0,586 | 0,606                                                      | 0,592 | 0,588 |
| 2,62 × 0,92 × 0,15                                                                                              | 0,602                                   | 0,597 | 0,593 | 0,613                                                      | 0,598 | 0,595 |
| <sup>a</sup> $Q = C \frac{8}{15} \sqrt{2g} h^{5/2}$<br><sup>b</sup> $Q = C \frac{2}{3} \sqrt{2g} b_e h_e^{3/2}$ |                                         |       |       |                                                            |       |       |

1) Available from Hydraulics Research Ltd., Wallingford, Oxon., England, as Report No. EX1243, 1985.

**Table A.2 — Discharge coefficients for rectangular notches in small weir tanks with sediment deposits**

| Tank size<br>(length, width,<br>height)<br>m             | Max. level of<br>deposits relative to<br>weir crest<br>mm | $C_d$ values for rectangular notches <sup>a</sup> |       |       |
|----------------------------------------------------------|-----------------------------------------------------------|---------------------------------------------------|-------|-------|
|                                                          |                                                           | Head<br>( <i>h</i> )<br>mm                        |       |       |
|                                                          |                                                           | 65                                                | 100   | 135   |
| 2,62 × 0,92 × 0,45                                       | –150                                                      | 0,605                                             | 0,591 | 0,588 |
|                                                          | –40                                                       | 0,606                                             | 0,597 | 0,590 |
|                                                          | –40                                                       | 0,613                                             | 0,601 | 0,595 |
|                                                          | (+ 100 at sides)                                          |                                                   |       |       |
| <sup>a</sup> $Q = C \frac{2}{3} \sqrt{2g} b_e h_e^{3/2}$ |                                                           |                                                   |       |       |

## Annex B (normative)

### Guide to the design and installation of a flow straightener

A flow straightener may be used for reducing the approach channel length.

The purpose of a flow straightener is to modify the flow in a shortened approach channel so that the velocity distribution of the flow is normal and steady.

6.3.3 and Figure 1 specify a normal velocity distribution.

A flow straightener should consist of several perforated plates (at least four), installed vertically and perpendicular to the flow direction with a minimum spacing of 0,2 m between adjacent plates. The percentage of the open area of each plate should be between 40 % and 60 % inclusive.

Figure B.1 shows an example of perforation. Holes are distributed in a staggered formation; in the example, the distance between the centres of two neighbouring holes is 30 mm while the hole diameter is 20 mm. This gives a percentage of open area equal to 40,31 %.

The plates should be thick and sufficiently strong to sustain the force exerted by the channel flow. The dimension of the holes may be varied in accordance with the channel width, provided that the spacing between the plates is adjusted in proportion to the hole diameter.

The straightener plates may be fixed on the approach channel, either with the perforations of the different plates aligned on the general direction of the stream (see Figure B.2), or with them positioned in a staggered formation, provided that the distance between adjacent plates is large compared with the hole diameter (see Figure B.3).

Dimensions in millimetres

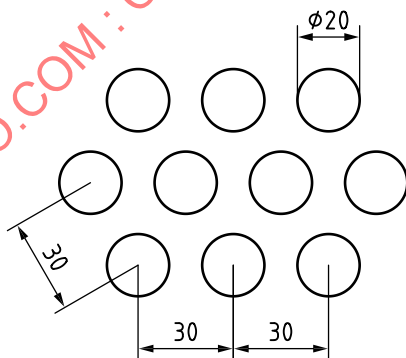


Figure B.1 — Example of perforation

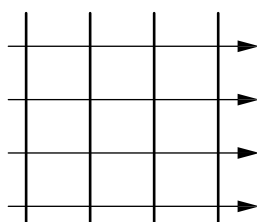
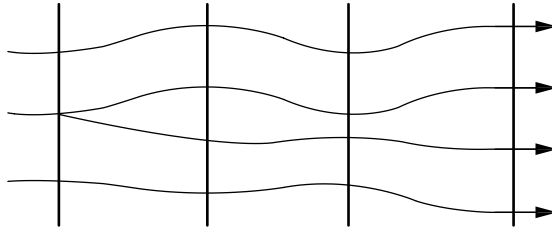


Figure B.2 — Aligned perforations



**Figure B.3 — Staggered perforations**

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## Annex C (informative)

### Introduction to measurement uncertainty

#### C.1 General

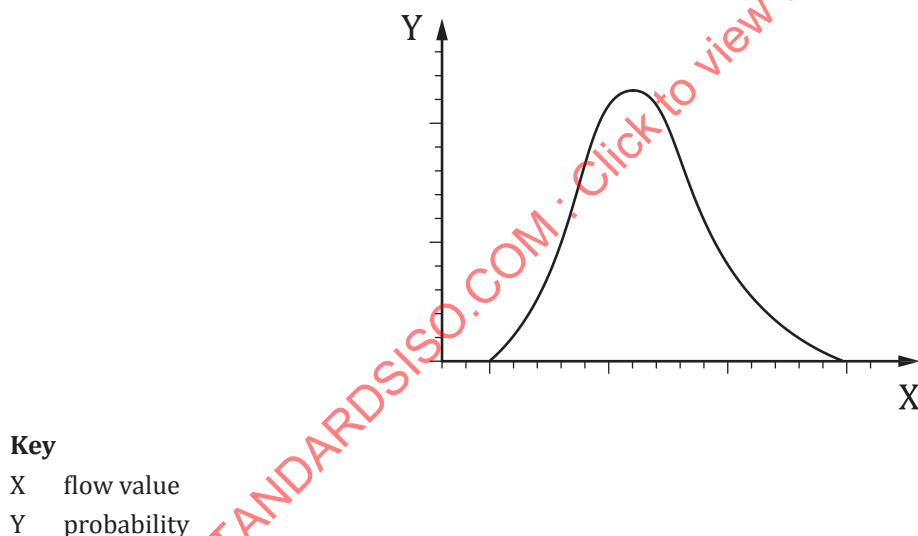
Results of measurements or analysis cannot be exact. The discrepancy between the true value, which is unknowable, and the measured value is the measurement error. The concept of uncertainty is a way of expressing this lack of knowledge. For example, if water is controlled to flow at a constant rate, then a flow meter will exhibit a spread of measurements about a mean value. If attention is not given to the uncertain nature of data, incorrect decisions can be made which have financial or judicial consequences. A realistic statement of uncertainty enhances the quality of information, making it more useful.

The uncertainty of a measurement represents a dispersion of values that could be attributed to it. Statistical methods provide objective values based on the application of theory.

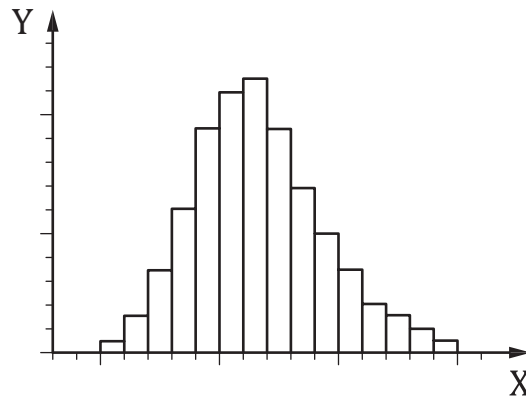
Standard uncertainty is defined as:

*Standard uncertainty equates to a dispersion of measurements expressed as a standard deviation.*

From this definition, uncertainty can be readily calculated for a set of measurements.

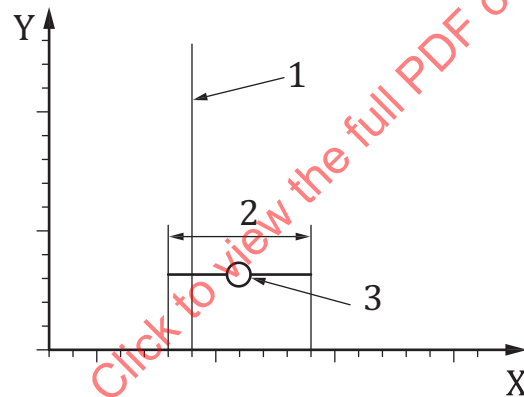


**Figure C.1 — Pictorial representation of some uncertainty paramaters — Example 1**

**Key**

- X flow value  
Y number of samples

**Figure C.2 — Pictorial representation of some uncertainty paramaters — Example 2**

**Key**

- 1 limit  
2 standard deviation  
3 mean value  
X flow value  
Y number of samples

**Figure C.3 — Pictorial representation of some uncertainty paramaters — Example 3**

[Figure C.1](#) shows the probability that a measurement of flow under steady conditions takes a particular value due to the uncertainties of various components of the measurement process, in the form of a probability density function.

[Figure C.2](#) shows sampled flow measurements, in the form of a histogram.

[Figure C.3](#) shows standard deviation of the sampled measurements compared with a limiting value. The mean value is shown to exceed the limiting value but is within the band of uncertainty (expressed as the standard deviation about the mean value).

## C.2 Confidence limits and coverage factors

For a normal probability distribution, analysis shows that 68 % of a large set of measurements lies within one standard deviation of the mean value. Thus, standard uncertainty is said to have a 68 % level of confidence.

However, for some measurement results, it is customary to express the uncertainty at a level of confidence which will cover a larger portion of the measurements; for example, at a 95 % level of confidence (see [Figure C.6](#)). This is done by applying a factor, the coverage factor  $k$ , to the computed value of standard uncertainty.

For a normal probability distribution, 95,45 % (effectively 95 %) of the measurements are covered for a value of  $k = 2$ . Thus, uncertainty at the 95 % level of confidence is twice the standard uncertainty value.

In practice, measurement variances rarely follow closely the normal probability distribution. They may be better represented by triangular, rectangular or bimodal probability distributions and only sometimes approximate to the normal distribution.

So a probability distribution must be selected to model the observed variances. To express the uncertainty of such models at the 95 % confidence limit requires a coverage factor that represents 95 % of the observations. However, the same coverage factor,  $k = 2$ , is used for all models. This simplifies the procedure while ensuring consistency of application within tolerable limits.

## C.3 Random and systematic error

The terms “random” and “systematic” have been applied in hydrometric standards to distinguish between a) random errors that represent an inherent dispersion of values under steady conditions, and b) systematic errors that are associated with inherent limitations of the means of determining the measured quantity.

A difficulty with the concept of a systematic error is that a systematic error cannot be determined without pre-knowledge of true values. If its existence is known or suspected, then steps should be taken to minimize such an error either by recalibration of the equipment or by reversing its effect in the calculation procedure. At which point, a systematic error contributes to uncertainty in the same way as random components of uncertainty.

For this reason, ISO/IEC Guide 98-3 does not distinguish between the treatment of random and systematic uncertainties. Generally, when determining a single discharge, random errors dominate and there is no need to separate random and systematic errors. However, where (say) totalized volume is established over a long time base, the systematic errors, even when reduced, can remain dominant in the estimation of uncertainty.

## C.4 Measurement standards

ISO/IEC Guide 98-3 and ISO/TS 25377 provide rules for the application of the principles of measurement uncertainty, in particular on the identification of components of error, the quantification of their corresponding uncertainties and how these are combined using methods derived from statistical theory into an overall result for the measurement process.

The components of uncertainty are characterized by estimates of standard deviations. There are two methods of estimation.

- a) **Type-A estimation** (by statistical analysis of repeated measurements from which an equivalent standard deviation is derived).

This process may be automated in real-time for depth or for velocity measurement.

- b) **Type-B estimation** (by ascribing a probability distribution to the measurement process).

This is applicable to:

- 1) human judgement of a manual measurement (distance or weight),
- 2) manual readings taken from instrumentation (manufacturer's statement), or
- 3) calibration data (from manufacturer).

## C.5 Evaluation of Type-A uncertainty

Defined in [C.1](#), the term “standard uncertainty” equates to a dispersion of measurements expressed as a standard deviation. Thus, any single measurement of a set of  $n$  measurements has, by definition, an uncertainty given in [Formula \(C.1\)](#):

$$u(x) = t_e \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (\text{C.1})$$

where  $\bar{x}$  is the best estimate of the true mean, as given in [Formula \(C.2\)](#):

$$\bar{x} = \frac{1}{n} (x_1 + x_2 + \dots + x_n) \quad (\text{C.2})$$

and  $t_e$  is a factor derived from statistical theory to account for the increased uncertainty when small numbers of measurements are available (refer to [Table C.1](#)).

If, instead of a single measurement from the set, the uncertainty is to apply to the mean of all  $n$  values, then [Formula \(C.3\)](#) applies:

$$u(\bar{x}) = \frac{t_e}{\sqrt{n}} \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (\text{C.3})$$

For continuous measurement, Type-A evaluations may be derived as a continuous variable from the primary measurement, i.e. from water level or water velocity.

By taking average values over large numbers,  $n$ , of measurements, the uncertainty of the mean value  $u(\bar{x})$  is reduced by a factor of  $\frac{1}{\sqrt{n}}$  compared to the uncertainty  $u(x)$  of an individual measurement. For this reason, monitoring equipment should specify measurement performance in terms including both  $u(\bar{x})$  and  $u(x)$  to show the extent to which averaging is applied.

**Table C.1 —  $t_e$  factors at 90 %, 95 % and 99 % confidence levels**

| Degrees of freedom <sup>a</sup> | Confidence level<br>% |       |       |
|---------------------------------|-----------------------|-------|-------|
|                                 | 90                    | 95    | 99    |
| 1                               | 6,31                  | 12,71 | 63,66 |
| 2                               | 2,92                  | 4,30  | 9,92  |
| 3                               | 2,35                  | 3,18  | 5,84  |
| 4                               | 2,13                  | 2,78  | 4,60  |
| 5                               | 2,02                  | 2,57  | 4,03  |
| 10                              | 1,81                  | 2,23  | 3,17  |
| 15                              | 1,75                  | 2,13  | 2,95  |
| 20                              | 1,72                  | 2,09  | 2,85  |
| 25                              | 1,71                  | 2,06  | 2,79  |

<sup>a</sup> In general, the number of terms in a sum minus the number of constraints on the terms of the sum (GUM).

Table C.1 (continued)

| Degrees of freedom <sup>a</sup> | Confidence level<br>% |      |      |
|---------------------------------|-----------------------|------|------|
|                                 | 90                    | 95   | 99   |
| 30                              | 1,70                  | 2,04 | 2,75 |
| 40                              | 1,68                  | 2,02 | 2,70 |
| 60                              | 1,67                  | 2,00 | 2,66 |
| 100                             | 1,66                  | 1,98 | 2,63 |
| Infinite                        | 1,64                  | 1,96 | 2,58 |

<sup>a</sup> In general, the number of terms in a sum minus the number of constraints on the terms of the sum (GUM).

## C.6 Evaluation of Type-B uncertainty

### C.6.1 General

When there is no access to a continuous stream of measured data or if a large set of measurements is not available, the type-B method of estimation is used to:

- assign a probability distribution to the measurement process to represent the probability of the true value being represented by any single measured value;
- define upper and lower bounds of the measurement;
- determine a standard uncertainty from a standard deviation implied by the assigned probability distribution.

The Type-B methods allow estimates of upper and lower bounding values to be used to derive the equivalent standard deviation.

Four probability distributions are described in ISO/IEC Guide 98-3 and in [C.6.2](#) to [C.6.5](#).

### C.6.2 The triangular distribution

The triangular distribution is represented in [Figure C.4](#).

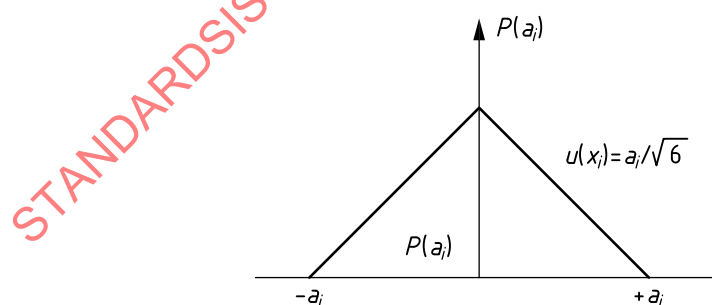


Figure C.4 — Triangular distribution

$$u(\bar{x}) = \frac{1}{\sqrt{6}} \left( \frac{x_{\max} - x_{\min}}{2} \right) \quad (\text{C.4})$$

This usually applies to manual measurements where the mean value is most likely to be closer to the true value than others between the discernible upper and lower limits of the measurement.

### C.6.3 The rectangular distribution

The rectangular distribution is represented in [Figure C.5](#).

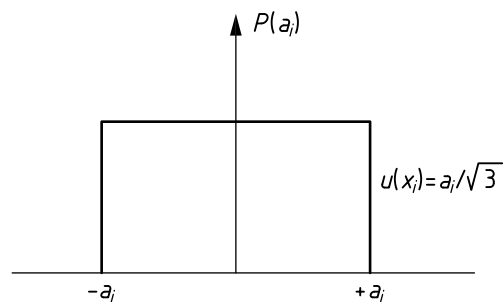


Figure C.5 — Rectangular distribution

$$u(\bar{x}) = \frac{1}{\sqrt{3}} \left( \frac{x_{\max} - x_{\min}}{2} \right) \quad (\text{C.5})$$

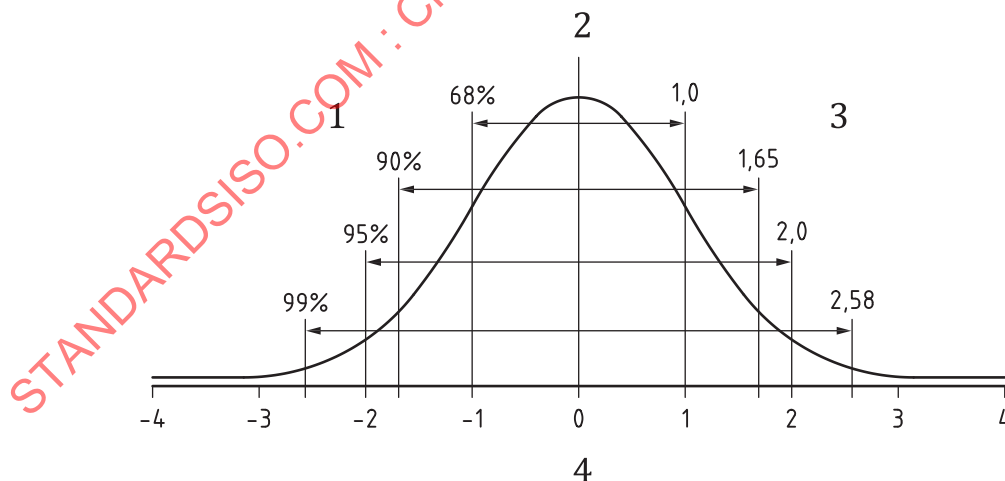
This probability distribution is usually applied to the resolution limit of the measurement instrumentation (i.e. the displayed resolution or the resolution of internal analogue/digital converters).

However, this is not the only source of uncertainty of measurement equipment. There may be uncertainty arising from the measurement algorithm used and/or from the calibration process.

If the equipment measures relative values, then there will also be uncertainty in the determination of its datum.

### C.6.4 The normal probability distribution

The normal probability distribution is represented in [Figure C.6](#).



#### Key

- 1 percent of readings in bandwidth
- 2 probability
- 3 coverage factor
- 4 standard deviations

Figure C.6 — Normal probability distribution

$$u(x_{\text{mean}}) = \frac{u(\text{specified})}{k} \quad (\text{C.6})$$

where  $k$  is the coverage factor applying to the specified uncertainty value.

These are uncertainty statements based on “off-line” statistical analysis, usually as part of a calibration process where they have been derived using a Type-A process. When expressed as standard uncertainty, the uncertainty value is to be used directly with an equivalent coverage factor of  $k = 1$ .

### C.6.5 Bimodal probability distribution

The bimodal probability distribution is represented in [Figure C.7](#).

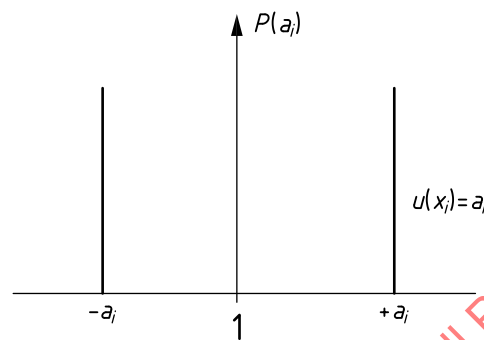


Figure C.7 — Bimodal probability distribution

$$u(x_{\text{mean}}) = \frac{(x_{\text{max}} - x_{\text{min}})}{2} \quad (\text{C.7})$$

Measurement equipment with hysteresis can only exhibit values at the upper and lower bounds of the measurement.

An example of this is the float mechanism where friction and surface tension combine to cause the float to move in finite steps.

### C.7 Combined uncertainty value, $u_c$

For most measurement systems, a measurement result is derived from several variables. For example, flow measurement,  $Q$ , in a rectangular channel can be expressed as a function of independent variables as shown in [Formula \(C.8\)](#):

$$Q = b \times h \times \bar{V} \quad (\text{C.8})$$

where

$b$  is the channel width;

$h$  is the depth of water in the channel;

$\bar{V}$  is the mean velocity.

These three components are measured independently and combined to determine a value for  $Q$ .

Just as  $b$ ,  $h$  and  $\bar{V}$  are combined to determine the value  $Q$ , so each component of uncertainty must be combined to determine a value for  $u_c(Q)$ . This is done by evaluating the sensitivity of  $Q$  to small change,  $\Delta$ , in  $b$ ,  $h$  or  $V$ . Thus, [Formula \(C.9\)](#) applies:

$$\Delta Q = \frac{\partial Q}{\partial b} \Delta b + \frac{\partial Q}{\partial h} \Delta h + \frac{\partial Q}{\partial \bar{V}} \Delta \bar{V} \quad (\text{C.9})$$

where the partial differentials,  $\frac{\partial Q}{\partial b}$ ,  $\frac{\partial Q}{\partial h}$  and  $\frac{\partial Q}{\partial \bar{V}}$  are sensitivity coefficients. For the formula  $Q = b \times h \times \bar{V}$ , this is equal to [Formula \(C.10\)](#):

$$\frac{\Delta Q}{Q} = \frac{\Delta b}{b} + \frac{\Delta h}{h} + \frac{\Delta \bar{V}}{\bar{V}} \quad (\text{C.10})$$

In uncertainty analysis, the values  $\frac{\Delta Q}{Q}$ ,  $\frac{\Delta b}{b}$ ,  $\frac{\Delta h}{h}$  and  $\frac{\Delta \bar{V}}{\bar{V}}$  correspond to dimensionless standard uncertainties. They are given the notation  $u_c^*(Q)$ ,  $u^*(b)$ ,  $u^*(h)$  and  $u^*(V)$ .

Since the uncertainties of  $b$ ,  $V$  and  $h$  are independent of each other, probability considerations require summation in quadrature, as given in [Formula \(C.11\)](#).

$$u_c^*(Q) \equiv \sqrt{u^*(\bar{V})^2 + u^*(b)^2 + u^*(h)^2} \quad (\text{C.11})$$

**Annex D**  
(informative)

**Sample measurement performance for use in  
hydrometric worked examples**

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Table D.1 — Sample measurement performance for use in hydrometric worked examples

| Measurement technologies |                      | Comment                                      | Sym-<br>bol | Uncertainty options |                  | Installed equipment to have corresponding values certified by the manufacturer |           |           |           |           |             |           |           |           |           |
|--------------------------|----------------------|----------------------------------------------|-------------|---------------------|------------------|--------------------------------------------------------------------------------|-----------|-----------|-----------|-----------|-------------|-----------|-----------|-----------|-----------|
|                          |                      |                                              |             |                     |                  | Corresponding standard uncertainty<br>(68 % confidence limit)                  |           |           |           |           |             |           |           |           |           |
| Velocity (continuous)    |                      |                                              |             | A                   | B                | Minimum                                                                        | 25 %      | 50 %      | 75 %      | Maximum   | Minimum     | 25 %      | 50 %      | 75 %      | Maximum   |
| Point velocity           | Propeller            | Calibration certificate                      | $u(V)$      | YES                 | Normal           | 0,005 m/s                                                                      | 1,250 m/s | 2,500 m/s | 3,750 m/s | 5,000 m/s | 0,000 5 m/s | 0,010 m/s | 0,022 m/s | 0,030 m/s | 0,040 m/s |
|                          | Electromag-<br>netic | Calibration certificate                      | $u(V)$      | YES                 | Normal           | 0,005 m/s                                                                      | 0,750 m/s | 1,500 /s  | 2,250 m/s | 3,000 m/s | 0,000 5 m/s | 0,010 m/s | 0,018 m/s | 0,025 m/s | 0,025 m/s |
| Path velocity            | Time of flight sonar | Sonic velocity path angle                    | $u(V)$      | YES                 | Rectangu-<br>lar | 0,030 m/s                                                                      | 0,250 m/s | 0,500 m/s | 0,750 m/s | 1,000 m/s | 0,003 m/s   | 0,005 m/s | 0,007 m/s | 0,007 m/s | 0,010 m/s |
|                          | Gated Doppler sonar  | Particle dependent - low velocity resolution | $u(V)$      | YES                 | Rectangu-<br>lar | 0,030 m/s                                                                      | 0,250 m/s | 0,500 m/s | 0,750 m/s | 1,000 m/s | 0,003 m/s   | 0,005 m/s | 0,007 m/s | 0,007 m/s | 0,010 m/s |
|                          | Sonar correlation    | Particle dependent                           | $u(V)$      | YES                 | Rectangu-<br>lar | 0,030 m/s                                                                      | 0,250 m/s | 0,500 m/s | 0,750 m/s | 1,000 m/s | 0,003 m/s   | 0,005 m/s | 0,007 m/s | 0,007 m/s | 0,010 m/s |
| Section velocity         | EM                   | To be calibrated <i>in situ</i>              | $u(V)$      |                     | Rectangu-<br>lar | 0,030 m/s                                                                      | 0,250 m/s | 0,500 m/s | 0,750 m/s | 1,000 m/s | 0,003 m/s   | 0,005 m/s | 0,007 m/s | 0,007 m/s | 0,010 m/s |

Table D.1 (continued)

| Measurement technologies                        |                       | Comment                               | Symbol   | Uncertainty options |             | Installed equipment to have corresponding values certified by the manufacturer |                   |                   |                   |                   |                                                               |           |           |           |
|-------------------------------------------------|-----------------------|---------------------------------------|----------|---------------------|-------------|--------------------------------------------------------------------------------|-------------------|-------------------|-------------------|-------------------|---------------------------------------------------------------|-----------|-----------|-----------|
| Velocity (continuous)                           |                       |                                       |          | A                   | B           | Nominal range of measurement                                                   |                   |                   |                   |                   | Corresponding standard uncertainty<br>(68 % confidence limit) |           |           |           |
|                                                 |                       |                                       |          |                     |             | Minimum                                                                        | 25 %              | 50 %              | 75 %              | Maximum           | Minimum                                                       | 25 %      | 50 %      | 75 %      |
| Water level (continuous)                        |                       |                                       |          |                     |             |                                                                                |                   |                   |                   |                   |                                                               |           |           |           |
| Relative datum (must be applied to all methods) |                       | Manual process                        | $u(E)$   |                     | Triangular  | Not applicable                                                                 | 0,500 m           | 1,000 m           | 1,500 m           | 2,000 m           | 0,001 m                                                       | 0,001 m   | 0,001 5 m | 0,001 5 m |
|                                                 | In-contact methods    | Requires regular maintenance          | $u(h_1)$ |                     | Bimodal     | Extension 0,200 m                                                              | Extension 1,250 m | Extension 2,500 m | Extension 3,750 m | Extension 5,000 m | 0,001 5 m                                                     | 0,002 0 m | 0,002 5 m | 0,002 5 m |
|                                                 | Pressure transducer   | Datum value drift                     | $u(h_1)$ |                     | Rectangular | 0,010 m                                                                        | 0,500 m           | 1,000 m           | 1,500 m           | 2,000 m           | 0,002 m                                                       | 0,002 m   | 0,002 5 m | 0,003 0 m |
|                                                 | Sonar                 | Surface wave effects                  | $u(h_1)$ | YES                 | Rectangular | 0,050 m                                                                        | 0,500 m           | 1,000 m           | 1,500 m           | 2,000 m           | 0,001 m                                                       | 0,001 m   | 0,001 5 m | 0,001 5 m |
| Non-contact methods                             |                       | Surface wave effects                  |          |                     |             |                                                                                |                   |                   |                   |                   |                                                               |           |           |           |
|                                                 | Pulse echo ultrasound | Air temperature Compensation          | $u(R)$   | YES                 | Rectangular | Range 0,300 m                                                                  | Range 1,250 m     | Range 2,500 m     | Range 3,750 m     | Range 5,000 m     | 0,002 m                                                       | 0,004 m   | 0,010 m   | 0,025 m   |
|                                                 | Pulse echo opto/radar | Surface wave effects                  | $u(R)$   |                     | Rectangular | Range 0,300 m                                                                  | Range 1,250 m     | Range 2,500 m     | Range 3,750 m     | Range 5,000 m     | 0,002 m                                                       | 0,004 m   | 0,010 m   | 0,060 m   |
| Cross-section profile (distance measurement)    |                       |                                       |          |                     |             |                                                                                |                   |                   |                   |                   |                                                               |           |           |           |
|                                                 | Natural channels      | Sonar or dip gauging/GPRS or tracking | $u(B)$   |                     | Rectangular | 0,500 m                                                                        | 5,000 m           | 10,000 m          | 15,000 m          | 20,000 m          | 0,002 m                                                       | 0,020 m   | 0,060 m   | 0,200 m   |
|                                                 | Man-made channels     | Manual measurement                    | $u(B)$   |                     | Triangular  | Not applicable                                                                 | 0,500 m           | 1,000 m           | 1,500 m           | 2,000 m           | 0,001 m                                                       | 0,001 m   | 0,001 5 m | 0,001 5 m |

## Annex E (informative)

### Specimen tables

**Table E.1 — Discharge of water over a V-notch with  $\tan \alpha/2 = 1$  ( $\alpha = \pi/2$  radians or  $90^\circ$ ),  
 $Q = 2,362\,5\,C_d h^{5/2}$ , ( $g = 9,806\,6\,\text{m/s}^2$ )**

| Head<br>$h$<br>m | Coefficient<br>$C_d$ | Discharge<br>$Q$<br>$\text{m}^3/\text{s} \times 10^{-1}$ | Head<br>$h$<br>m | Coefficient<br>$C_d$ | Discharge<br>$Q$<br>$\text{m}^3/\text{s} \times 10^{-1}$ |
|------------------|----------------------|----------------------------------------------------------|------------------|----------------------|----------------------------------------------------------|
| 0,060            | 0,603 2              | 0,012 57                                                 | 0,091            | 0,593 5              | 0,035 03                                                 |
| 0,061            | 0,602 8              | 0,013 09                                                 | 0,092            | 0,593 3              | 0,035 98                                                 |
| 0,062            | 0,602 3              | 0,013 62                                                 | 0,093            | 0,593 1              | 0,036 96                                                 |
| 0,063            | 0,601 9              | 0,014 17                                                 | 0,094            | 0,592 9              | 0,037 95                                                 |
| 0,064            | 0,601 5              | 0,014 73                                                 | 0,095            | 0,592 7              | 0,038 95                                                 |
| 0,065            | 0,601 2              | 0,015 30                                                 | 0,096            | 0,592 5              | 0,039 97                                                 |
| 0,066            | 0,600 8              | 0,015 88                                                 | 0,097            | 0,592 3              | 0,041 01                                                 |
| 0,067            | 0,600 5              | 0,016 48                                                 | 0,098            | 0,592 1              | 0,042 06                                                 |
| 0,068            | 0,600 1              | 0,017 10                                                 | 0,099            | 0,591 9              | 0,043 12                                                 |
| 0,069            | 0,599 8              | 0,017 72                                                 | 0,100            | 0,591 7              | 0,044 20                                                 |
| 0,070            | 0,599 4              | 0,018 36                                                 | 0,101            | 0,591 4              | 0,045 30                                                 |
| 0,071            | 0,599 0              | 0,019 01                                                 | 0,102            | 0,591 2              | 0,046 41                                                 |
| 0,072            | 0,598 7              | 0,019 67                                                 | 0,103            | 0,591 0              | 0,047 54                                                 |
| 0,073            | 0,598 3              | 0,020 35                                                 | 0,104            | 0,590 8              | 0,048 69                                                 |
| 0,074            | 0,598 0              | 0,021 05                                                 | 0,105            | 0,590 6              | 0,049 85                                                 |
| 0,075            | 0,597 8              | 0,021 76                                                 | 0,106            | 0,590 4              | 0,051 03                                                 |
| 0,076            | 0,597 5              | 0,022 48                                                 | 0,107            | 0,590 2              | 0,052 22                                                 |
| 0,077            | 0,597 3              | 0,023 22                                                 | 0,108            | 0,590 1              | 0,053 44                                                 |
| 0,078            | 0,597 0              | 0,023 97                                                 | 0,109            | 0,589 9              | 0,054 67                                                 |
| 0,079            | 0,596 7              | 0,024 73                                                 | 0,110            | 0,589 8              | 0,055 92                                                 |
| 0,080            | 0,596 4              | 0,025 51                                                 | 0,111            | 0,589 7              | 0,057 19                                                 |
| 0,081            | 0,596 1              | 0,026 30                                                 | 0,112            | 0,589 6              | 0,058 47                                                 |
| 0,082            | 0,595 8              | 0,027 10                                                 | 0,113            | 0,589 4              | 0,059 77                                                 |
| 0,083            | 0,595 5              | 0,027 92                                                 | 0,114            | 0,589 2              | 0,061 08                                                 |
| 0,084            | 0,595 3              | 0,028 76                                                 | 0,115            | 0,589 1              | 0,062 42                                                 |
| 0,085            | 0,595 0              | 0,029 61                                                 | 0,116            | 0,589 0              | 0,063 77                                                 |
| 0,086            | 0,594 8              | 0,030 48                                                 | 0,117            | 0,588 9              | 0,065 14                                                 |
| 0,087            | 0,594 5              | 0,031 36                                                 | 0,118            | 0,588 8              | 0,066 53                                                 |
| 0,088            | 0,594 2              | 0,032 25                                                 | 0,119            | 0,588 6              | 0,067 93                                                 |
| 0,089            | 0,594 0              | 0,033 16                                                 | 0,120            | 0,588 5              | 0,069 35                                                 |
| 0,090            | 0,593 7              | 0,034 09                                                 | 0,121            | 0,588 3              | 0,070 79                                                 |
|                  |                      |                                                          | 0,122            | 0,588 2              | 0,072 24                                                 |

Table E.1 (continued)

| Head<br>$h$<br>m | Coefficient<br>$C_d$ | Discharge<br>$Q$<br>$\text{m}^3/\text{s} \times 10^{-1}$ | Head<br>$h$<br>m | Coefficient<br>$C_d$ | Discharge<br>$Q$<br>$\text{m}^3/\text{s} \times 10^{-1}$ |
|------------------|----------------------|----------------------------------------------------------|------------------|----------------------|----------------------------------------------------------|
| 0,123            | 0,588 1              | 0,073 72                                                 | 0,162            | 0,585 6              | 0,146 14                                                 |
| 0,124            | 0,588 0              | 0,075 22                                                 | 0,163            | 0,585 6              | 0,148 40                                                 |
| 0,125            | 0,588 0              | 0,076 73                                                 | 0,164            | 0,585 5              | 0,150 67                                                 |
| 0,126            | 0,587 9              | 0,078 27                                                 | 0,165            | 0,585 5              | 0,152 97                                                 |
| 0,127            | 0,587 8              | 0,079 82                                                 | 0,166            | 0,585 5              | 0,155 29                                                 |
| 0,128            | 0,587 7              | 0,081 39                                                 | 0,167            | 0,585 4              | 0,157 63                                                 |
| 0,129            | 0,587 6              | 0,082 98                                                 | 0,168            | 0,585 4              | 0,159 99                                                 |
| 0,130            | 0,587 6              | 0,084 58                                                 | 0,169            | 0,585 3              | 0,162 37                                                 |
| 0,131            | 0,587 5              | 0,086 21                                                 |                  |                      |                                                          |
| 0,132            | 0,587 4              | 0,087 85                                                 | 0,170            | 0,585 3              | 0,164 77                                                 |
| 0,133            | 0,587 3              | 0,089 51                                                 | 0,171            | 0,585 3              | 0,167 19                                                 |
| 0,134            | 0,587 2              | 0,091 19                                                 | 0,172            | 0,585 2              | 0,169 64                                                 |
| 0,135            | 0,587 2              | 0,092 89                                                 | 0,173            | 0,585 2              | 0,172 10                                                 |
| 0,136            | 0,587 1              | 0,094 61                                                 | 0,174            | 0,585 1              | 0,174 59                                                 |
| 0,137            | 0,587 0              | 0,096 34                                                 | 0,175            | 0,585 1              | 0,177 09                                                 |
| 0,138            | 0,586 9              | 0,098 10                                                 | 0,176            | 0,585 1              | 0,179 63                                                 |
| 0,139            | 0,586 9              | 0,099 87                                                 | 0,177            | 0,585 1              | 0,182 19                                                 |
|                  |                      |                                                          | 0,178            | 0,585 1              | 0,184 78                                                 |
| 0,140            | 0,586 8              | 0,101 67                                                 | 0,179            | 0,585 1              | 0,187 38                                                 |
| 0,141            | 0,586 7              | 0,103 48                                                 |                  |                      |                                                          |
| 0,142            | 0,586 7              | 0,105 32                                                 | 0,180            | 0,585 1              | 0,190 01                                                 |
| 0,143            | 0,586 6              | 0,107 17                                                 | 0,181            | 0,585 1              | 0,192 65                                                 |
| 0,144            | 0,586 6              | 0,109 04                                                 | 0,182            | 0,585 0              | 0,195 31                                                 |
| 0,145            | 0,586 5              | 0,110 93                                                 | 0,183            | 0,585 0              | 0,198 00                                                 |
| 0,146            | 0,586 4              | 0,112 84                                                 | 0,184            | 0,585 0              | 0,200 71                                                 |
| 0,147            | 0,586 3              | 0,114 76                                                 | 0,185            | 0,585 0              | 0,203 45                                                 |
| 0,148            | 0,586 2              | 0,116 71                                                 | 0,186            | 0,585 0              | 0,206 21                                                 |
| 0,149            | 0,586 2              | 0,118 67                                                 | 0,187            | 0,585 0              | 0,208 99                                                 |
|                  |                      |                                                          | 0,188            | 0,585 0              | 0,211 80                                                 |
| 0,150            | 0,586 1              | 0,120 66                                                 | 0,189            | 0,585 0              | 0,214 63                                                 |
| 0,151            | 0,586 1              | 0,122 67                                                 |                  |                      |                                                          |
| 0,152            | 0,586 0              | 0,124 71                                                 | 0,190            | 0,585 0              | 0,217 48                                                 |
| 0,153            | 0,586 0              | 0,126 76                                                 | 0,191            | 0,585 0              | 0,220 34                                                 |
| 0,154            | 0,585 9              | 0,128 83                                                 | 0,192            | 0,584 9              | 0,223 22                                                 |
| 0,155            | 0,585 9              | 0,130 93                                                 | 0,193            | 0,584 9              | 0,226 12                                                 |
| 0,156            | 0,585 9              | 0,133 04                                                 | 0,194            | 0,584 9              | 0,229 06                                                 |
| 0,157            | 0,585 8              | 0,135 17                                                 | 0,195            | 0,584 9              | 0,232 03                                                 |
| 0,158            | 0,585 8              | 0,137 32                                                 | 0,196            | 0,584 9              | 0,235 01                                                 |
| 0,159            | 0,585 7              | 0,139 50                                                 | 0,197            | 0,584 9              | 0,238 02                                                 |
|                  |                      |                                                          | 0,198            | 0,584 9              | 0,241 06                                                 |
| 0,160            | 0,585 7              | 0,141 69                                                 | 0,199            | 0,584 9              | 0,244 11                                                 |
| 0,161            | 0,585 7              | 0,143 91                                                 |                  |                      |                                                          |

Table E.1 (continued)

| Head<br>$h$<br>m | Coefficient<br>$C_d$ | Discharge<br>$Q$<br>$\text{m}^3/\text{s} \times 10^{-1}$ | Head<br>$h$<br>m | Coefficient<br>$C_d$ | Discharge<br>$Q$<br>$\text{m}^3/\text{s} \times 10^{-1}$ |
|------------------|----------------------|----------------------------------------------------------|------------------|----------------------|----------------------------------------------------------|
| 0,200            | 0,584 9              | 0,247 19                                                 | 0,239            | 0,584 6              | 0,385 68                                                 |
| 0,201            | 0,584 9              | 0,250 28                                                 |                  |                      |                                                          |
| 0,202            | 0,584 8              | 0,253 39                                                 | 0,240            | 0,584 6              | 0,389 73                                                 |
| 0,203            | 0,584 8              | 0,256 52                                                 | 0,241            | 0,584 6              | 0,393 80                                                 |
| 0,204            | 0,584 8              | 0,259 69                                                 | 0,242            | 0,584 6              | 0,397 90                                                 |
| 0,205            | 0,584 8              | 0,262 88                                                 | 0,243            | 0,584 6              | 0,402 02                                                 |
| 0,206            | 0,584 8              | 0,266 10                                                 | 0,244            | 0,584 6              | 0,406 17                                                 |
| 0,207            | 0,584 8              | 0,269 34                                                 | 0,245            | 0,584 6              | 0,410 34                                                 |
| 0,208            | 0,584 8              | 0,272 61                                                 | 0,246            | 0,584 6              | 0,414 54                                                 |
| 0,209            | 0,584 8              | 0,275 90                                                 | 0,247            | 0,584 6              | 0,418 77                                                 |
|                  |                      |                                                          | 0,248            | 0,584 6              | 0,423 02                                                 |
| 0,210            | 0,584 8              | 0,279 21                                                 | 0,249            | 0,584 6              | 0,427 30                                                 |
| 0,211            | 0,584 8              | 0,282 54                                                 |                  |                      |                                                          |
| 0,212            | 0,584 8              | 0,285 88                                                 | 0,250            | 0,584 6              | 0,431 60                                                 |
| 0,213            | 0,584 7              | 0,289 24                                                 | 0,251            | 0,584 6              | 0,435 93                                                 |
| 0,214            | 0,584 7              | 0,292 64                                                 | 0,252            | 0,584 6              | 0,440 28                                                 |
| 0,215            | 0,584 7              | 0,296 07                                                 | 0,253            | 0,584 6              | 0,440 66                                                 |
| 0,216            | 0,584 7              | 0,299 53                                                 | 0,254            | 0,584 6              | 0,449 07                                                 |
| 0,217            | 0,584 7              | 0,303 01                                                 | 0,255            | 0,584 6              | 0,453 50                                                 |
| 0,218            | 0,584 7              | 0,306 51                                                 | 0,256            | 0,584 6              | 0,457 96                                                 |
| 0,219            | 0,584 7              | 0,310 04                                                 | 0,257            | 0,584 6              | 0,462 45                                                 |
|                  |                      |                                                          | 0,258            | 0,584 6              | 0,466 96                                                 |
| 0,220            | 0,584 7              | 0,313 59                                                 | 0,259            | 0,584 6              | 0,471 50                                                 |
| 0,221            | 0,584 7              | 0,317 17                                                 |                  |                      |                                                          |
| 0,222            | 0,584 7              | 0,320 77                                                 | 0,260            | 0,584 6              | 0,476 06                                                 |
| 0,223            | 0,584 7              | 0,324 39                                                 | 0,261            | 0,584 6              | 0,480 65                                                 |
| 0,224            | 0,584 7              | 0,328 03                                                 | 0,262            | 0,584 6              | 0,485 27                                                 |
| 0,225            | 0,584 6              | 0,331 68                                                 | 0,263            | 0,584 6              | 0,489 91                                                 |
| 0,226            | 0,584 6              | 0,335 35                                                 | 0,264            | 0,584 6              | 0,494 58                                                 |
| 0,227            | 0,584 6              | 0,339 07                                                 | 0,265            | 0,584 6              | 0,499 28                                                 |
| 0,228            | 0,584 6              | 0,342 82                                                 | 0,266            | 0,584 6              | 0,404 00                                                 |
| 0,229            | 0,584 6              | 0,346 59                                                 | 0,267            | 0,584 6              | 0,508 76                                                 |
|                  |                      |                                                          | 0,268            | 0,584 6              | 0,513 53                                                 |
| 0,230            | 0,584 6              | 0,350 39                                                 | 0,269            | 0,584 6              | 0,518 34                                                 |
| 0,231            | 0,584 6              | 0,354 21                                                 |                  |                      |                                                          |
| 0,232            | 0,584 6              | 0,358 06                                                 | 0,270            | 0,584 6              | 0,523 17                                                 |
| 0,233            | 0,584 6              | 0,361 93                                                 | 0,271            | 0,584 6              | 0,528 02                                                 |
| 0,234            | 0,584 6              | 0,365 82                                                 | 0,272            | 0,584 6              | 0,532 91                                                 |
| 0,235            | 0,584 6              | 0,369 74                                                 | 0,273            | 0,584 6              | 0,537 82                                                 |
| 0,236            | 0,584 6              | 0,373 69                                                 | 0,274            | 0,584 6              | 0,542 76                                                 |
| 0,237            | 0,584 6              | 0,377 66                                                 | 0,275            | 0,584 6              | 0,547 72                                                 |
| 0,238            | 0,584 6              | 0,381 66                                                 | 0,276            | 0,584 6              | 0,552 72                                                 |

Table E.1 (continued)

| Head<br>$h$<br>m | Coefficient<br>$C_d$ | Discharge<br>$Q$<br>$\text{m}^3/\text{s} \times 10^{-1}$ | Head<br>$h$<br>m | Coefficient<br>$C_d$ | Discharge<br>$Q$<br>$\text{m}^3/\text{s} \times 10^{-1}$ |
|------------------|----------------------|----------------------------------------------------------|------------------|----------------------|----------------------------------------------------------|
| 0,277            | 0,584 6              | 0,557 74                                                 | 0,316            | 0,584 9              | 0,775 66                                                 |
| 0,278            | 0,584 6              | 0,562 82                                                 | 0,317            | 0,584 9              | 0,781 81                                                 |
| 0,279            | 0,584 7              | 0,567 94                                                 | 0,318            | 0,584 9              | 0,788 02                                                 |
| 0,280            | 0,584 7              | 0,573 06                                                 | 0,319            | 0,585 0              | 0,794 28                                                 |
| 0,281            | 0,584 7              | 0,578 19                                                 |                  |                      |                                                          |
| 0,282            | 0,584 7              | 0,583 35                                                 | 0,320            | 0,585 0              | 0,800 57                                                 |
| 0,283            | 0,584 7              | 0,588 53                                                 | 0,321            | 0,585 0              | 0,806 85                                                 |
| 0,284            | 0,584 7              | 0,593 75                                                 | 0,322            | 0,585 0              | 0,813 14                                                 |
| 0,285            | 0,584 7              | 0,598 99                                                 | 0,323            | 0,585 0              | 0,819 47                                                 |
| 0,286            | 0,584 7              | 0,604 25                                                 | 0,324            | 0,585 0              | 0,825 83                                                 |
| 0,287            | 0,584 7              | 0,609 55                                                 | 0,325            | 0,585 0              | 0,832 22                                                 |
| 0,288            | 0,584 7              | 0,614 87                                                 | 0,326            | 0,585 0              | 0,838 63                                                 |
| 0,289            | 0,584 7              | 0,620 23                                                 | 0,327            | 0,585 0              | 0,845 08                                                 |
|                  |                      |                                                          | 0,328            | 0,585 0              | 0,851 55                                                 |
| 0,290            | 0,584 7              | 0,625 60                                                 | 0,329            | 0,585 0              | 0,858 06                                                 |
| 0,291            | 0,584 7              | 0,631 01                                                 |                  |                      |                                                          |
| 0,292            | 0,584 7              | 0,636 45                                                 | 0,330            | 0,585 0              | 0,864 59                                                 |
| 0,293            | 0,584 7              | 0,664 95                                                 | 0,331            | 0,585 0              | 0,871 16                                                 |
| 0,294            | 0,584 8              | 0,647 48                                                 | 0,332            | 0,585 0              | 0,877 75                                                 |
| 0,295            | 0,584 8              | 0,653 03                                                 | 0,333            | 0,585 0              | 0,884 38                                                 |
| 0,296            | 0,584 8              | 0,658 58                                                 | 0,334            | 0,585 0              | 0,891 03                                                 |
| 0,297            | 0,584 8              | 0,664 16                                                 | 0,335            | 0,585 0              | 0,897 72                                                 |
| 0,298            | 0,584 8              | 0,669 76                                                 | 0,336            | 0,585 0              | 0,904 48                                                 |
| 0,299            | 0,584 8              | 0,675 39                                                 | 0,337            | 0,585 1              | 0,911 28                                                 |
|                  |                      |                                                          | 0,338            | 0,585 1              | 0,918 11                                                 |
| 0,300            | 0,584 8              | 0,681 06                                                 | 0,339            | 0,585 1              | 0,924 91                                                 |
| 0,301            | 0,584 8              | 0,686 75                                                 |                  |                      |                                                          |
| 0,302            | 0,584 8              | 0,692 46                                                 | 0,340            | 0,585 1              | 0,931 75                                                 |
| 0,303            | 0,584 8              | 0,698 21                                                 | 0,341            | 0,585 1              | 0,938 62                                                 |
| 0,304            | 0,584 8              | 0,703 98                                                 | 0,342            | 0,585 1              | 0,945 51                                                 |
| 0,305            | 0,584 8              | 0,709 80                                                 | 0,343            | 0,585 1              | 0,952 44                                                 |
| 0,306            | 0,584 8              | 0,715 68                                                 | 0,344            | 0,585 1              | 0,959 40                                                 |
| 0,307            | 0,584 9              | 0,721 59                                                 | 0,345            | 0,585 1              | 0,966 38                                                 |
| 0,308            | 0,584 9              | 0,727 50                                                 | 0,346            | 0,585 1              | 0,973 40                                                 |
| 0,309            | 0,584 9              | 0,733 41                                                 | 0,347            | 0,585 1              | 0,980 45                                                 |
|                  |                      |                                                          | 0,348            | 0,585 1              | 0,987 53                                                 |
| 0,310            | 0,584 9              | 0,739 36                                                 | 0,349            | 0,585 1              | 0,994 71                                                 |
| 0,311            | 0,584 9              | 0,745 34                                                 | 0,350            | 0,585 2              | 1,001 92                                                 |
| 0,312            | 0,584 9              | 0,751 35                                                 | 0,351            | 0,585 2              | 1,009 12                                                 |
| 0,313            | 0,584 9              | 0,757 38                                                 | 0,352            | 0,585 2              | 1,016 33                                                 |
| 0,314            | 0,584 9              | 0,763 44                                                 | 0,353            | 0,585 2              | 1,023 56                                                 |
| 0,315            | 0,584 9              | 0,769 54                                                 | 0,354            | 0,585 2              | 1,030 82                                                 |