



**International
Standard**

ISO 16610-31

**Geometrical product specifications
(GPS) — Filtration —**

**Part 31:
Robust profile filters: Gaussian
regression filters**

Spécification géométrique des produits (GPS) — Filtrage —

*Partie 31: Filtres de profil robustes: Filtres de régression
gaussiens*

**Second edition
2025-02**

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 290, *Dimensional and geometrical product specification and verification*, in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

This second edition cancels and replaces the first edition (ISO 16610-31:2016), which has been technically revised.

The main changes compared to the previous edition are as follows:

- providing continuous Gaussian regression filters for open and for closed profiles;
- providing a normative iterative solution for continuous Gaussian regression filters.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences chain links C and E in the GPS matrix structure.

The ISO GPS matrix model given in ISO 14638 gives an overview of the ISO GPS system of which this document is a part. The fundamental rules of ISO GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to the specifications made in accordance with this document, unless otherwise indicated.

For more information on the relationship of this document to the filtration matrix model, see [Annex C](#).

For more detailed information on the relation of this document to other standards and the GPS matrix model, see [Annex D](#).

This document develops the terminology and concepts of robust Gaussian regression filters for surface profiles. It separates the large- and small-scale lateral components of surface profiles in such a way that the surface profiles can be reconstructed without altering. The robust Gaussian regression filter for surface profiles reduces the influence of protruding dales and hills. Depending on the selected nesting index and regression degree, robust Gaussian regression filters offer one possible method for the F-Operation.

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Geometrical product specifications (GPS) — Filtration —

Part 31:

Robust profile filters: Gaussian regression filters

1 Scope

This document specifies robust Gaussian regression filters for the filtration of surface profiles. It defines, in particular, how to separate large- and small-scale lateral components of surface profiles with protruding dales and hills.

The concept presented for closed profiles are applicable to the case of roundness filtering. Where appropriate, these concept can be extended to generalized closed profiles, especially for surface profiles with re-entrant features.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO 16610-21, *Geometrical product specifications (GPS) — Filtration — Part 21: Linear profile filters: Gaussian filters*

ISO 16610-22, *Geometrical product specifications (GPS) — Filtration — Part 22: Linear profile filters: Spline filters*

ISO 16610-30, *Geometrical product specifications (GPS) — Filtration — Part 30: Robust profile filters: Basic concepts*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1, ISO 16610-20, ISO 16610-21, ISO 16610-22, ISO 16610-30, ISO/IEC Guide 99 and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

3.1

surface profile

line resulting from the intersection between a surface portion and an ideal plane

Note 1 to entry: The orientation of the ideal plane is usually perpendicular to the tangent plane of the surface portion.

Note 2 to entry: See ISO 17450-1:2011, 3.3 and 3.3.1, for the definition of an ideal plane.

[SOURCE: ISO 16610-1:2015, 3.1.2, modified — Note 2 to entry replaced.]

3.1.1

open profile

finite length *surface profile* (3.1) with two ends

Note 1 to entry: An open profile has a compact support, i.e. within a certain interval the height values of an open profile can be equal to any real number. Outside the interval, the height values of an open profile are set to zero.

[SOURCE: ISO 16610-1:2015, 3.7, modified — Note 1 to entry replaced.]

3.1.2

unbounded open profile

infinite length *surface profile* (3.1) without ends

Note 1 to entry: In this document, the term “unbounded” refers to the X-axis.

Note 2 to entry: The concept of the unbounded open profile is ideal and do not apply to real surface profiles.

3.1.3

closed profile

connected finite length *surface profile* (3.1) without ends

Note 1 to entry: A closed profile is a closed curve which is periodic with the finite period length L .

Note 2 to entry: A typical example of a closed profile is one from a roundness measurement.

[SOURCE: ISO 16610-1:2015, 3.8, modified — Note 1 to entry replaced and Note 2 to entry added.]

3.2

linear profile filter

profile filter which separates *surface profiles* (3.1) into large- and small-scale lateral components and is also a linear function

Note 1 to entry: If F is a function and X and Y are surface profiles, and if a and b are independent from X and Y , then F being a linear function implies $F(aX + bY) = aF(X) + bF(Y)$.

[SOURCE: ISO 16610-20:2015, 3.1, modified — In definition “profiles” replaced by “surface profiles” and “long wave” and “short wave” replaced by “large-scale lateral” and “small-scale lateral”; Note 1 to entry replaced.]

3.3

weighting function

function to calculate large-scale lateral components by convolution of the surface profile heights with this function

Note 1 to entry: The convolution (see ISO 16610-20:2015, 4.1) performs a weighted moving average of the surface profile heights. The weighting function, reflected at the X-axis, defines the weighting coefficients for the averaging process.

3.4

transmission characteristic of a filter

characteristic that indicates the amount by which the amplitude of a sinusoidal surface profile is attenuated as a function of its wavelength

Note 1 to entry: The transmission characteristic is the Fourier transformation of the *weighting function* (3.3).

Note 2 to entry: [SOURCE: ISO 16610-20:2015, 3.4]

3.5

cut-off wavelength

λ_c

wavelength of a sinusoidal surface profile of which 50 % of the amplitude is transmitted by the profile

Note 1 to entry: Linear profile filters are identified by the filter type and the cut-off wavelength value.

Note 2 to entry: The cut-off wavelength is the nesting index for linear profile filters.

[SOURCE: ISO 16610-20:2015, 3.5, modified — In Note 2 to entry “recommended” deleted.]

3.6 undulations per revolution

UPR

integer number of sinusoidal undulations contained in a *closed profile* (3.1.3)

Note 1 to entry: In this document, UPR is a frequency and is denoted by f .

3.7 cut-off frequency in undulations per revolution

f_c
frequency in UPR of a sinusoidal *closed profile* (3.1.3) of which 50 % of the amplitude is transmitted by the profile filter

3.8 robust profile filter

profile filter which separates *surface profiles* (3.1) into large- and small-scale lateral components and is insensitive against specific phenomena in the input data

Note 1 to entry: A robust profile filter is a nonlinear filter.

Note 2 to entry: See also ISO 16610-1:2015, 3.9.

Note 3 to entry: Outliers, scratches and steps are examples of specific phenomena. Further details can be found in ISO 16610-30:2015.

Note 4 to entry: In particular, the *robust Gaussian regression filter* (3.11) in accordance with this document reduces the influence of specific phenomena such as protruding dales and hills. Profile examples are given in Annex B.

3.9 biweight function of Beaton and Tukey

function used in M-estimation and defined by Formula (1)

$$\delta(\Delta z(x), c) = \begin{cases} \left[1 - \left(\frac{\Delta z(x)}{c} \right)^2 \right]^2 & \text{for } |\Delta z(x)| \leq c \\ 0 & \text{for } |\Delta z(x)| > c \end{cases} \quad (1)$$

where

x is the given x -coordinate;

$\Delta z(x)$ are heights depending on x ;

c is a scale value.

Note 1 to entry: The biweight function $\delta(\Delta z(x), c)$ of Beaton and Tukey is almost constant and equals nearly 1 for heights $|\Delta z(x)| \ll c$. For increasing heights $|\Delta z(x)|$, the biweight function of Beaton and Tukey approaches zero.

Note 2 to entry: The biweight function of Beaton and Tukey is related to the influence function $\psi(\Delta z(x))$ (see ISO 16610-30:2015) used in M-estimation as follows: $\psi(\Delta z(x)) = \Delta z(x) \delta(\Delta z(x), c)$.

Note 3 to entry: See also ISO 16610-30:2015.

3.10 regression filter

profile filter which based on a local polynomial modelling of the large-scale lateral component of a *surface profile* (3.1)

3.11

robust Gaussian regression filter

regression filter (3.10) based on the Gaussian weighting function and the *biweight function of Beaton and Tukey* (3.9)

4 Characteristics of the robust Gaussian regression filter for open profiles

4.1 General

In this clause, the ideal filtration of open profiles is considered. Since the robust Gaussian regression filter is nonlinear and no transmission characteristic by means of the Fourier transformation can be given, the generic term nesting index N_i is used as the filter parameter instead of cut-off wavelength λ_c . But in many cases of application, values for the cut-off wavelength λ_c used for linear filtration are also suitable as a nesting index N_i for robust filtration.

4.2 Filter equations

4.2.1 Determination of the large-scale lateral component

To determine the large-scale lateral component of an open profile, the robust Gaussian regression filter with degree p is defined by Formula (2):

$$w(x) = (1 \quad 0 \quad \dots \quad 0) \left(\int_{\Omega} \mathbf{v}_p^T(x, u) \mathbf{v}_p(x, u) s(x, u) du \right)^{-1} \int_{\Omega} z(u) \mathbf{v}_p^T(x, u) s(x, u) du \quad (2)$$

where

- Ω is the finite interval, expressed as a set, in which the open profile can be any real number;
- x is the given x -coordinate with $x \in \Omega$;
- u is the integration variable along the X -axis with $u \in \Omega$;
- $z(u)$ is the open profile depending on u ;
- $\mathbf{v}_p(x, u)$ is the vector space of polynomials up to p^{th} degree depending on x and u ;
- $\mathbf{v}_p^T(x, u)$ is the transpose of $\mathbf{v}_p(x, u)$;
- $s(x, u)$ is the modified Gaussian weighting function depending on x and u ;
- $w(x)$ is the large-scale lateral component depending on x .

The vector space $\mathbf{v}_p(x, u)$ is defined by Formula (3):

$$\mathbf{v}_p(x, u) = \begin{bmatrix} 1 & (x-u) & \dots & (x-u)^p \end{bmatrix} \quad (3)$$

The modified Gaussian weighting function $s(x, u)$ is defined by Formula (4):

$$s(x, u) = \delta(z(u) - w(u), c) \frac{1}{\gamma N_i} e^{-\pi \left(\frac{x-u}{\gamma N_i} \right)^2} \quad (4)$$

where

$\delta(\cdot)$ is the biweight function of Beaton and Tukey;

N_i is the nesting index;

γ is the filter constant;

c is a scale value.

The scale value c is defined by [Formula \(5\)](#):

$$c = \frac{3}{\sqrt{2} \operatorname{erf}^{-1}(0,5)} \operatorname{median}_{u \in \Omega} |z(u) - w(u)| \approx 4,447 \ 8 \operatorname{median}_{u \in \Omega} |z(u) - w(u)| \quad (5)$$

The definition for the scale value c is equivalent to three times the standard deviation, if $z(u) - w(u)$ has a Gaussian amplitude distribution.

For $p = 0, 1, 2$, the filter constant γ is defined by [Formula \(6\)](#):

$$\gamma = \begin{cases} \sqrt{\frac{\ln 2}{\pi}} \approx 0,469 \ 7 & \text{for } p = 0, 1 \\ \sqrt{\frac{-1 - W_{-1}\left(-\frac{1}{2e}\right)}{\pi}} \approx 0,730 \ 9 & \text{for } p = 2 \end{cases} \quad (6)$$

NOTE 1 erf^{-1} is the inverse error function.

NOTE 2 W_{-1} is the “Lambert W ” function with branch -1 (see Reference [6]).

NOTE 3 The median of the absolute deviation $|z(u) - w(u)|$ is called MAD (see ISO 16610-30:2015, 3.5.2).

NOTE 4 See [Clause 8](#) for an iterative solution of the robust Gaussian regression filter.

4.2.2 Determination of the small-scale lateral component

The small-scale lateral component of an open profile is determined by subtracting the large-scale lateral component of this open profile, [Formula \(2\)](#), from this open profile according to [Formula \(7\)](#).

$$r(x) = z(x) - w(x) \quad (7)$$

where

x is the given x -coordinate;

$z(x)$ is the open profile depending on x ;

$w(x)$ is the large-scale lateral component of the open profile depending on x ;

$r(x)$ is the small-scale lateral component of the open profile depending on x .

4.3 Transmission characteristics

The modified Gaussian weighting function of the robust Gaussian regression filter depends on the heights of the open profile. Therefore, no transmission characteristics by means of the Fourier transformation can be given.

5 Characteristics of the robust Gaussian regression filter for closed profiles

5.1 General

In this clause, the ideal filtration of closed profiles applied to roundness profiles is considered. Since the robust Gaussian regression filter is nonlinear and no transmission characteristic by means of the Fourier transformation can be given, the generic term nesting index in UPR $\tilde{N}_i$ is used as the filter parameter instead of cut-off frequency in UPR f_c . But in many cases of application, values for the cut-off frequency in UPR f_c used for linear filtration are also suitable as a nesting index in UPR $\tilde{N}_i$ for robust filtration.

5.2 Filter equations

5.2.1 Determination of the large-scale lateral component

To determine the large-scale lateral component of a closed profile, the robust Gaussian regression filter with degree p is defined by [Formula \(8\)](#):

$$\tilde{w}(x) = (1 \quad 0 \quad \dots \quad 0) \begin{pmatrix} \int_{x-\frac{L}{\tilde{N}_i}}^{x+\frac{L}{\tilde{N}_i}} \mathbf{v}_p^T(x,u) \mathbf{v}_p(x,u) \tilde{s}(x,u) du \\ \int_{x-\frac{L}{\tilde{N}_i}}^{x+\frac{L}{\tilde{N}_i}} \tilde{z}(u) \mathbf{v}_p^T(x,u) \tilde{s}(x,u) du \end{pmatrix}^{-1} \begin{pmatrix} \int_{x-\frac{L}{\tilde{N}_i}}^{x+\frac{L}{\tilde{N}_i}} \tilde{z}(u) \mathbf{v}_p^T(x,u) \tilde{s}(x,u) du \\ \int_{x-\frac{L}{\tilde{N}_i}}^{x+\frac{L}{\tilde{N}_i}} \tilde{z}(u) \mathbf{v}_p^T(x,u) \tilde{s}(x,u) du \end{pmatrix} \quad (8)$$

where

- x is the given x -coordinate;
- u is the integration variable along the X -axis;
- L is the period length of the closed profile;
- $\tilde{N}_i$ is the nesting index in UPR;
- $\tilde{z}(u)$ is the closed profile depending on u ;
- $\mathbf{v}_p(x,u)$ is the vector space of polynomials up to p^{th} degree depending on x and u ;
- $\mathbf{v}_p^T(x,u)$ is the transpose of $\mathbf{v}_p(x,u)$;
- $\tilde{s}(x,u)$ is the modified Gaussian weighting function depending on x and u ;
- $\tilde{w}(x)$ is the large-scale lateral component depending on x .

The vector space $\mathbf{v}_p(x,u)$ is defined by [Formula \(9\)](#):

$$\mathbf{v}_p(x,u) = \begin{bmatrix} 1 & (x-u) & \dots & (x-u)^p \end{bmatrix} \quad (9)$$

The modified Gaussian weighting function $\tilde{s}(x,u)$ is defined by [Formula \(10\)](#):

$$\tilde{s}(x,u) = \delta(\tilde{z}(u) - \tilde{w}(u), \tilde{c}) \frac{\tilde{N}_i}{\tilde{\gamma} L} e^{-\pi \left[\frac{(x-u) \tilde{N}_i}{\tilde{\gamma} L} \right]^2} \quad (10)$$

where

- $\delta(\cdot)$ is the biweight function of Beaton and Tukey;

- $\tilde{N}_i$ is the nesting index in UPR;
 $\tilde{\gamma}$ is the filter constant;
 L is the period length of the closed profile;
 $\tilde{c}$ is a scale value.

The scale value $\tilde{c}$ is defined by [Formula \(11\)](#):

$$\tilde{c} = \frac{3}{\sqrt{2} \operatorname{erf}^{-1}(0,5)} \operatorname{median}_{x \leq u < x+L} |\tilde{z}(u) - \tilde{w}(u)| \approx 4,447 \ 8 \operatorname{median}_{x \leq u < x+L} |\tilde{z}(u) - \tilde{w}(u)| \quad (11)$$

The definition for the scale value $\tilde{c}$ is equivalent to three times the standard deviation, if $\tilde{z}(u) - \tilde{w}(u)$ has a Gaussian amplitude distribution.

For $p=0,1,2$, the filter constant $\tilde{\gamma}$ is defined by [Formula \(12\)](#):

$$\tilde{\gamma} = \begin{cases} \sqrt{\frac{\ln 2}{\pi}} \approx 0,469 \ 7 & \text{for } p=0,1 \\ \sqrt{\frac{-1 - W_{-1}\left(-\frac{1}{2e}\right)}{\pi}} \approx 0,730 \ 9 & \text{for } p=2 \end{cases} \quad (12)$$

NOTE 1 erf^{-1} is the inverse error function.

NOTE 2 W_{-1} is the “Lambert W” function with branch -1 (see Reference [6]).

NOTE 3 The median of the absolute deviation $|\tilde{z}(u) - \tilde{w}(u)|$ is called MAD (see ISO 16610-30:2015, 3.5.2).

NOTE 4 The relationship between the nesting index $\tilde{N}_i$ in UPR and the nesting index N_i is given by $N_i = L / \tilde{N}_i$, where L is the period length of the closed profile.

NOTE 5 See [Clause 8](#) for an iterative solution of the robust Gaussian regression filter.

5.2.2 Determination of the small-scale lateral component

The small-scale lateral component of a closed profile is determined by subtracting the large-scale lateral component of this closed profile, [Formula \(8\)](#), from this closed profile according to [Formula \(13\)](#):

$$\tilde{r}(x) = \tilde{z}(x) - \tilde{w}(x) \quad (13)$$

where

- x is the given x -coordinate;
 $\tilde{z}(x)$ is the closed profile depending on x ;
 $\tilde{w}(x)$ is the large-scale lateral component of the closed profile depending on x ;
 $\tilde{r}(x)$ is the small-scale lateral component of the closed profile depending on x .

5.3 Transmission characteristics

The modified Gaussian weighting function of the robust Gaussian regression filter depends on the heights of the closed profile. Therefore, no transmission characteristics by means of the Fourier transformation can be given.

6 Series of nesting index values

For open profiles and if not otherwise specified, the nesting index from the following series of values shall be used:

$$N_i = \dots; 2,5 \mu\text{m}; 8 \mu\text{m}; 25 \mu\text{m}; 80 \mu\text{m}; 250 \mu\text{m}; 0,8 \text{ mm}; 2,5 \text{ mm}; 8 \text{ mm}; 25 \text{ mm}; \dots$$

For roundness profiles exemplarily used as a closed profile in this document and if not otherwise specified, the nesting index in UPR from the following series of values shall be used:

$$\tilde{N}_i = 5; 15; 50; 150; 500; 1\,500; 5\,000; \dots$$

NOTE 1 The nesting index N_i for open profiles should be greater than or equal to three times the scale of the coarsest structure of interest.

NOTE 2 The nesting index in UPR $\tilde{N}_i$ should be lower than or equal to three times the scale of the coarsest structure of interest in UPR.

EXAMPLE A surface profile with grooves of different widths shall be filtered. The coarsest structure of interest is the groove with the largest width. The nesting index to be selected is three times this groove width.

7 Regression degree, p

If not otherwise specified, $p=2$ shall be used. In this case, the filtration locally follows a polynomial of second degree and may be used as an F-operation (depending on the nominal shape, $p=1$ may also be used as an F-operation). The degree $p=0$ may be used for surface profiles with an F-operation before applying the robust Gaussian regression filtration. The degree $p=1$ or $p=0$ increases the robustness of the regression filtration for surface profiles with protruding dales or hills at the surface profile boundaries (see [Figure B.6](#)).

8 Iterative solution

[Formulae \(2\)](#) and [\(8\)](#) are nonlinear and shall be solved iteratively. This iteration procedure is defined exemplarily for the open profile [Formula \(2\)](#). For closed profiles, [Formulae \(2\)](#) and [\(5\)](#) shall be replaced by [Formulae \(8\)](#) and [\(11\)](#), respectively, in the following iteration procedure:

- Set $\delta(\cdot)=1$ and set the number of iterations $n_{\text{iter}}=0$.
- Calculate the large-scale lateral component $w(x)$ according to [Formula \(2\)](#). Increase the number of iterations n_{iter} by 1.
- Calculate the scale value c according to [Formula \(5\)](#) and store c as c_{old} .
- Calculate the weights $\delta(z(u)-w(u), c)$ according to [Formula \(1\)](#).
- Calculate the large-scale lateral component $w(x)$ according to [Formula \(2\)](#). Increase the number of iterations n_{iter} by 1.
- Calculate the scale value c according to [Formula \(5\)](#).
- Stop the iteration if $|c - c_{\text{old}}| \leq c_{\text{old}} \cdot f_{\text{tol}}$ or if $n_{\text{iter}} = n_{\text{max}}$.
- Otherwise, store the scale value as c_{old} and continue with step d).

If not otherwise specified, $f_{\text{tol}} = 10^{-5}$ and $n_{\text{max}} = 12$ shall be used.

NOTE The first calculated large-scale lateral component $w(x)$ with $\delta(\cdot) = 1$ corresponds to a linear profile filter for which a weighting function and a transfer characteristic can be specified. This results in the filter constants γ and $\tilde{\gamma}$ according to [Formulae \(6\)](#) and [\(12\)](#), respectively. See [Annex A](#) for details.

9 Filter designation

Robust Gaussian regression filters in accordance with this document and ISO 16610-1:2015, Clause 5, shall be designated as:

FPRG

NOTE FPRG corresponds to filter for surface profiles of the robust Gauss type.

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Annex A (informative)

Linear Gaussian regression filter for unbounded open profiles

A.1 General

In many applications, the values for the cut-off wavelength used for linear filtration are also suitable as nesting indices for robust filtration. To solve [Formulae \(2\)](#) and [\(8\)](#), the biweight function of Beaton and Tukey is set to $\delta(\cdot)=1$ for the first iteration. This makes it a linear Gaussian regression filter (see Reference [\[5\]](#)). Therefore, in this case, a weighting function and transfer characteristic with a cut-off wavelength according to ISO 16610-20 can be specified for the Gaussian regression filter. This also yields the filter constants γ and $\tilde{\gamma}$ of the modified Gaussian weighting function (see [Formulae \(6\)](#) and [\(12\)](#)). For simplicity, the weighting function and the transfer characteristic are shown as functions of the cut-off wavelength λ_c for an unbounded open profile. For roundness profiles exemplarily used as a closed profile in this document, the relationship between the frequency in UPR f and the wavelength λ is given by $\lambda=L/f$, where L is the period length of the closed profile.

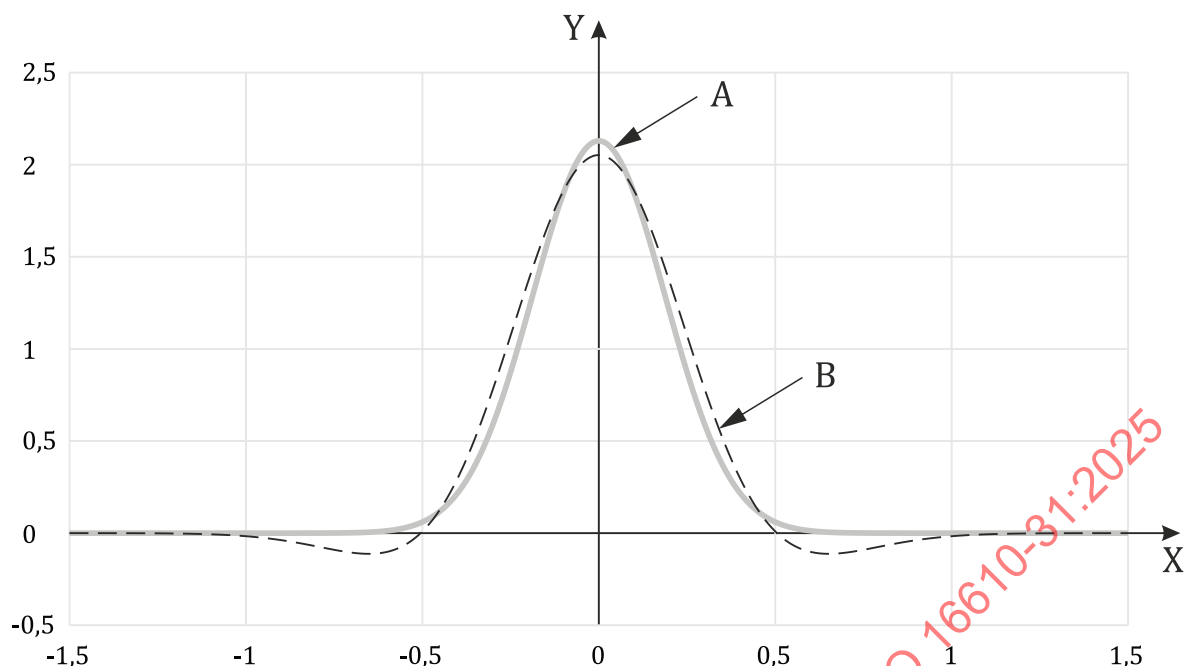
A.2 Weighting function of the linear Gaussian regression filter

The weighting function of the linear Gaussian regression filter with cut-off wavelength λ_c and degree p (see [Figure A.1](#)) for unbounded open profiles is defined by [Formula \(A.1\)](#):

$$s(v) = \begin{cases} \frac{1}{\gamma \lambda_c} e^{-\pi \left(\frac{v}{\gamma \lambda_c} \right)^2} & \text{for } p=0, 1 \\ \left[\frac{3}{2} - \pi \left(\frac{v}{\gamma \lambda_c} \right)^2 \right] \frac{1}{\gamma \lambda_c} e^{-\pi \left(\frac{v}{\gamma \lambda_c} \right)^2} & \text{for } p=2 \end{cases} \quad (\text{A.1})$$

where

- v is the distance from the centre (maximum) of the Gaussian weighting function;
- $s(v)$ is the weighting function of the linear Gaussian regression filter depending on v ;
- λ_c is the cut-off wavelength;
- γ is the constant to provide 50 % transmission characteristic at λ_c and which is given by [Formulae \(6\)](#).



Key

X v / λ_c

Y $s(v) \lambda_c$

A weighting function of the linear Gaussian regression filter with degree $p = 0$ and $p = 1$

B weighting function of the linear Gaussian regression filter with degree $p = 2$

Figure A.1 — Weighting function of the linear Gaussian regression filter with different degrees p for unbounded open profiles

A.3 Transmission characteristic of the linear Gaussian regression filter for the large-scale lateral component

The transmission characteristic for the large-scale lateral component of an unbounded open profile (see [Figure A.2](#)) is determined from the weighting function of the linear Gaussian regression filter, [Formula \(A.1\)](#), by means of the Fourier transformation and is given by [Formula \(A.2\)](#):

$$s(v) = \begin{cases} \frac{a_1}{a_0} = e^{-\pi \left(\frac{\gamma \lambda_c}{\lambda} \right)^2} = 2^{-\left(\frac{\lambda_c}{\lambda} \right)^2} & \text{for } p=0,1 \\ \frac{a_1}{a_0} = \left[1 + \pi \left(\frac{\gamma \lambda_c}{\lambda} \right)^2 \right] e^{-\pi \left(\frac{\gamma \lambda_c}{\lambda} \right)^2} & \text{for } p=2 \end{cases} \quad (\text{A.2})$$

where

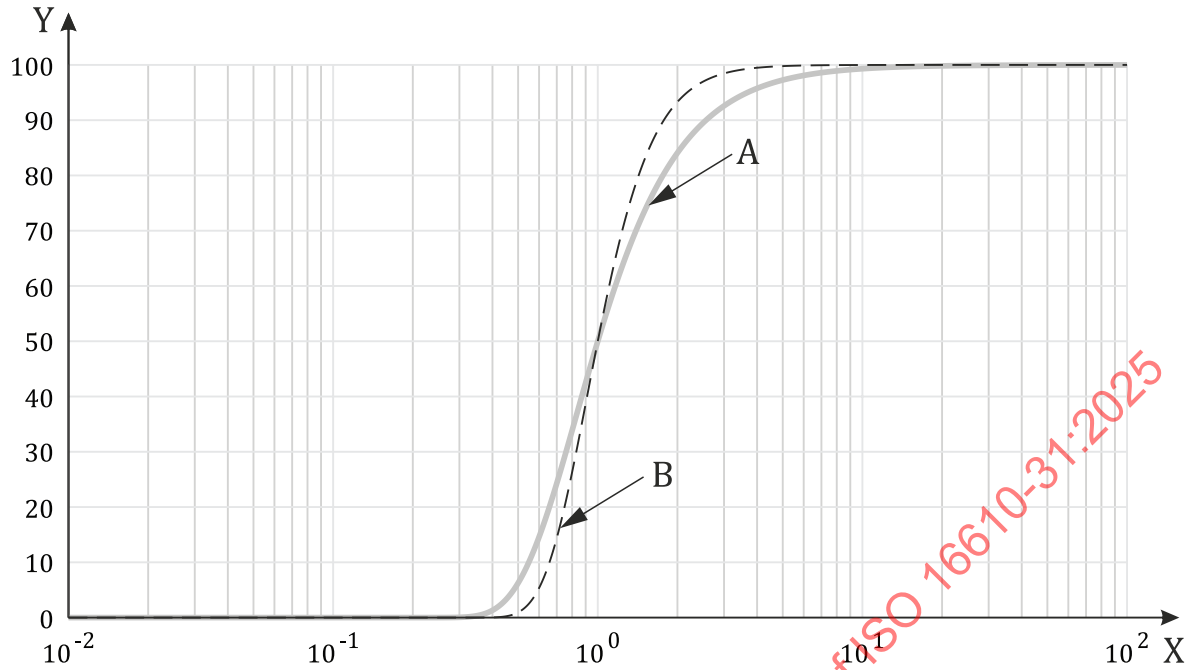
a_0 is the amplitude of a sinusoidal open profile before filtration;

a_1 is the amplitude of this sinusoidal open profile after filtration;

λ is the wavelength of this sinusoidal open profile;

λ_c is the cut-off wavelength;

γ is the constant to provide 50 % transmission characteristic at λ_c and which is given by [Formulae \(6\)](#).



Key

X λ / λ_c

Y amplitude transmission a_1 / a_0 in per cent

A transmission characteristic for the large-scale lateral component for degree $p = 0$ and $p = 1$

B transmission characteristic for the large-scale lateral component for degree $p = 2$

Figure A.2 — Transmission characteristic of the linear Gaussian regression filter for the large-scale lateral component of unbounded open profiles depending on the degree p

Annex B (informative)

Examples for the application of the robust Gaussian regression filter

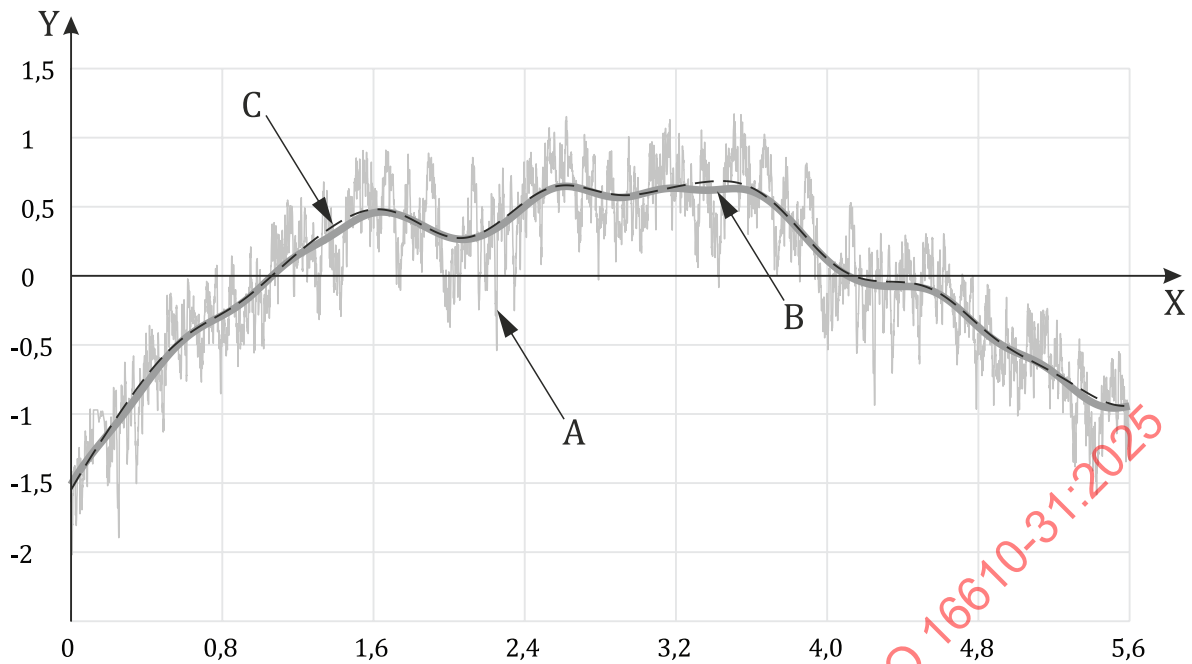
B.1 General

This annex gives examples for the application of the robust Gaussian regression filter to surface profiles of various manufacturing processes. For comparison, the figures in this annex also show the filtration result for the Gaussian filter in accordance with ISO 16610-21 (with treatment of the end effect regions with parameter $p = 1$) or the spline filter in accordance with ISO 16610-22.

B.2 Examples

B.2.1 Robust Gaussian regression filter applied to a ground surface profile

[Figure B.1](#) shows the filter behaviour of the Gaussian filter (key B) and the robust Gaussian regression filter in accordance with this document (key C) applied to a ground surface profile in presence of nominal shape. The amplitude density distribution of this ground surface profile is Gaussian without protruding hills or dales. Therefore, the filter behaviour of the Gaussian regression filter is similar to that of the Gaussian filter, and consequently, for $\lambda_c = N_i$, the large-scale lateral components determined, which are shown in [Figure B.1](#), are almost identical.

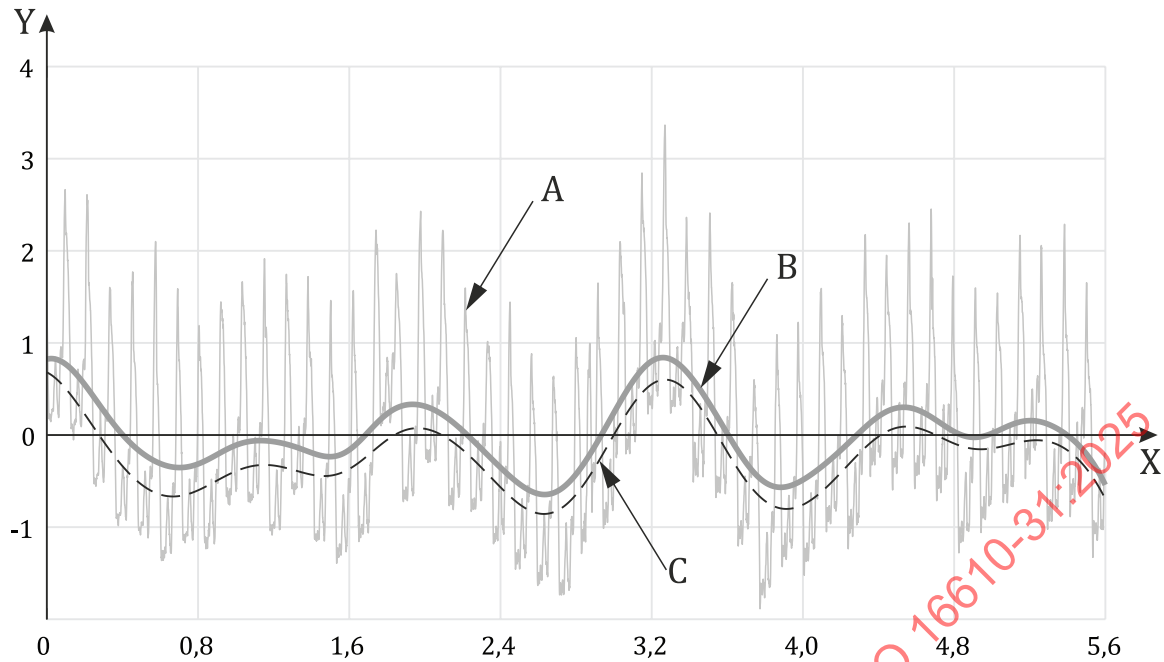
**Key**

- X length in mm
- Y height in μm
- A unfiltered open profile
- B large-scale lateral component of the open profile after filtration with the Gaussian filter: $\lambda_c = 0,8 \text{ mm}$, $p = 1$ and $L_c = 1$ (grey solid line)
- C large-scale lateral component of the open profile after filtration with robust Gaussian regression filter: $N_i = 0,8 \text{ mm}$ and $p = 2$ (black dashed line)

Figure B.1 — Comparison of the large-scale lateral components of a ground surface profile in presence of nominal shape, determined with the Gaussian filter and the robust Gaussian regression filter

B.2.2 Robust Gaussian regression filter applied to turned surface profiles

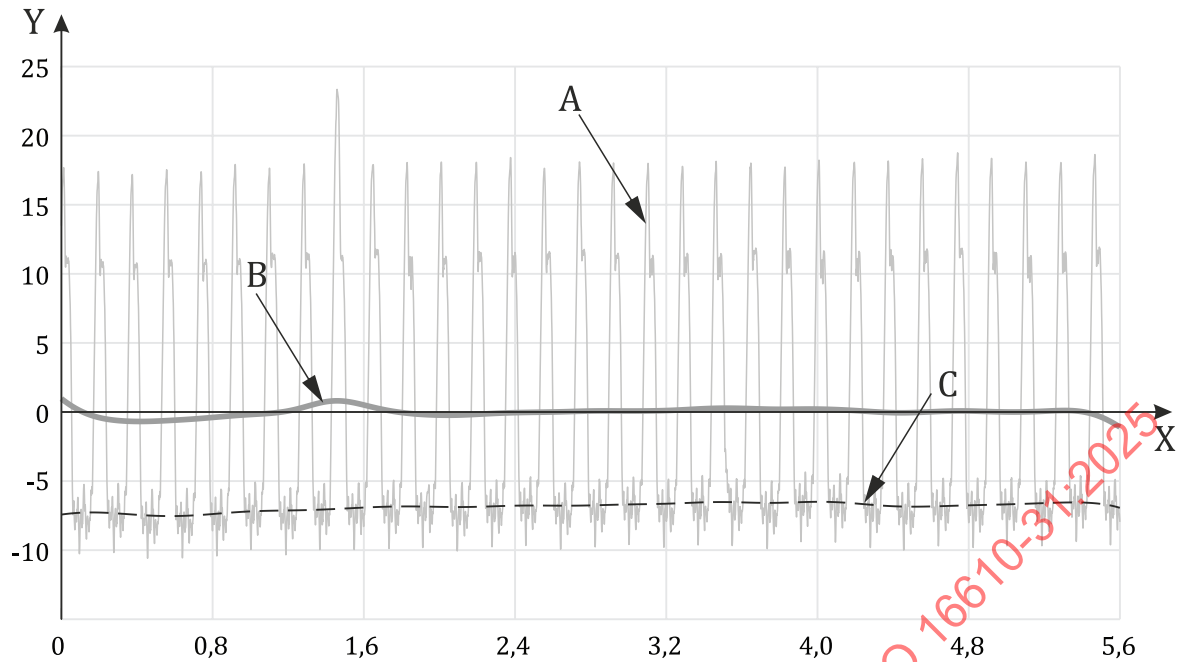
Figure B.2 shows the filter behaviour of the Gaussian filter (key B) and the robust Gaussian regression filter in accordance with this document (key C) applied to a turned surface profile. The turned microstructure is homogeneous and has a positive skewness. The large-scale lateral component of the turned surface profile determined with the Gaussian filter and the robust Gaussian regression filter are similar. Due to the skewness of the turned surface profile, the peaks are weighted less than the pits in robust filtering. Therefore, the large-scale component of the turned surface profile determined with the robust Gaussian regression filter runs below that of the Gaussian filter.

**Key**

- X length in mm
- Y height in μm
- A unfiltered open profile
- B large-scale lateral component of the open profile after filtration with the Gaussian filter: $\lambda_c = 0,8 \text{ mm}$, $p = 1$ and $L_c = 1$ (grey solid line)
- C large-scale lateral component of the open profile after filtration with robust Gaussian regression filter: $N_i = 0,8 \text{ mm}$ and $p = 2$ (black dashed line)

Figure B.2 — Comparison of the large-scale lateral components of a turned surface profile, determined with the Gaussian filter and the robust Gaussian regression filter

[Figure B.3](#) shows the filter behaviour of the Gaussian filter (key B) and the robust Gaussian regression filter in accordance with this document (key C) applied to another turned surface profile. The turned microstructure is homogeneous with a positive skewness, but the peaks of this turned surface profile are more pronounced compared to the peaks of the turned surface profile shown in [Figure B.2](#). Therefore, the large-scale component determined with the robust Gaussian regression filter runs in the area of the pits of the dales. In addition, the large-scale component determined with the Gaussian filter is deflected upwards by the protruding peak at the position $x \approx 1,5 \text{ mm}$.

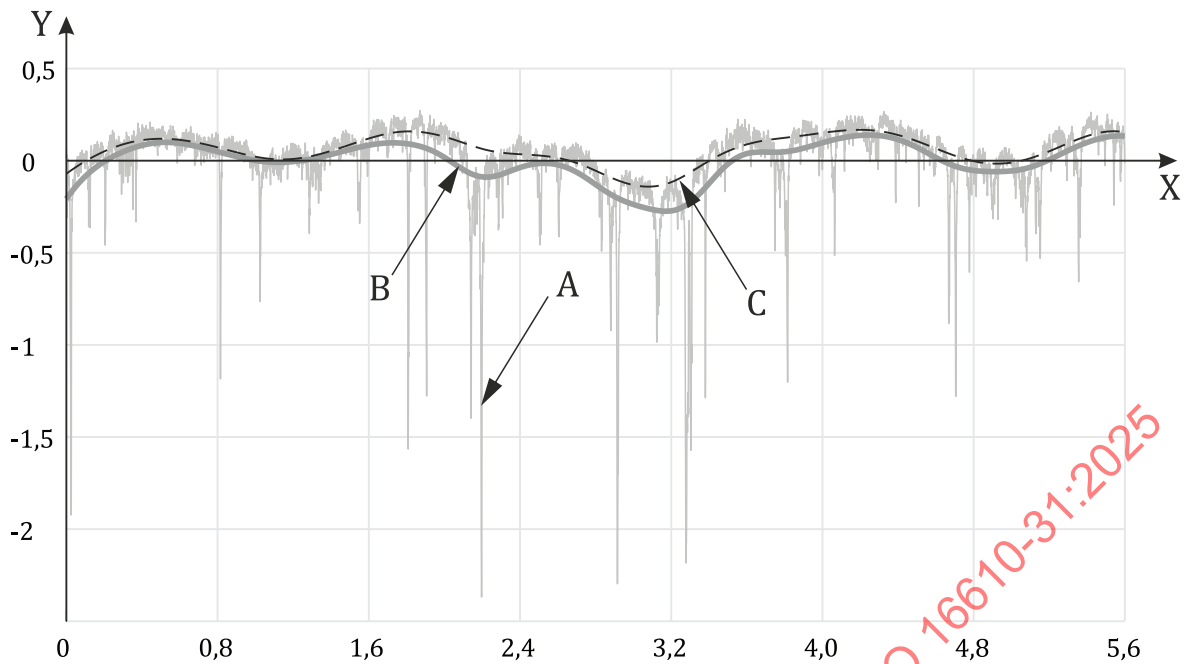
**Key**

- X length in mm
- Y height in μm
- A unfiltered open profile
- B large-scale lateral component of the open profile after filtration with the Gaussian filter: $\lambda_c = 0,8 \text{ mm}$, $p = 1$ and $L_c = 1$ (grey solid line)
- C large-scale lateral component of the open profile after filtration with robust Gaussian regression filter: $N_i = 0,8 \text{ mm}$ and $p = 2$ (black dashed line)

Figure B.3 — Comparison of the large-scale lateral components of a turned surface profile with protruding peaks, determined with the Gaussian filter and the robust Gaussian regression filter

B.2.3 Robust Gaussian regression filter applied to plateau-like surface profiles with grooves

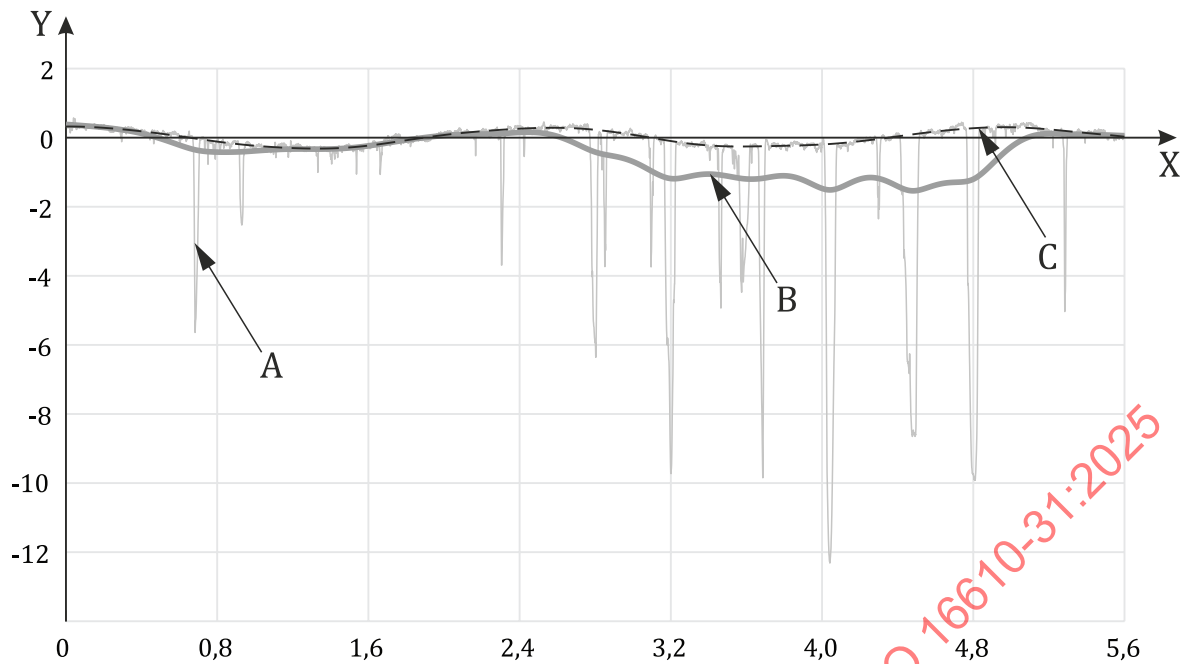
Figure B.4 shows the filter behaviour of the Gaussian filter (key B) and the robust Gaussian regression filter in accordance with this document (key C) applied to a finished iron cast surface profile. The microstructure of this surface profile is plateau-like with grooves and has a negative skewness. While the large-scale component determined with the Gaussian filter is clearly influenced by the grooves, the large-scale component determined with the robust Gaussian regression filter runs in the plateau region of the surface profile.

**Key**

- X length, in mm
- Y height, in μm
- A unfiltered open profile
- B large-scale lateral component of the open profile after filtration with the Gaussian filter: $\lambda_c = 0,8 \text{ mm}$, $p = 1$ and $L_c = 1$ (grey solid line)
- C large-scale lateral component of the open profile after filtration with robust Gaussian regression filter: $N_i = 0,8 \text{ mm}$ and $p = 2$ (black dashed line)

Figure B.4 — Comparison of the large-scale lateral components of a finished iron cast surface profile, determined with the Gaussian filter and the robust Gaussian regression filter

Figure B.5 shows the filter behaviour of the spline filter (key B) and the robust Gaussian regression filter in accordance with this document (key C) applied to a plateau honed surface profile. The microstructure is plateau-like with grooves and has a negative skewness. Due to the distinctive grooves, the large-scale lateral component determined with the spline filter is deflected in the direction of the grooves. In contrast, the large-scale lateral component determined with the Gaussian regression filter is not affected by the grooves.

**Key**

- X length, in mm
- Y height, in μm
- A unfiltered open profile
- B large-scale lateral component of the open profile after filtration with the spline filter: $\lambda_c = 0,8 \text{ mm}$, $\beta = 0$ (grey solid line)
- C large-scale lateral component of the open profile after filtration with robust Gaussian regression filter: $N_i = 0,8 \text{ mm}$ and $p = 2$ (black dashed line)

Figure B.5 — Comparison of the large-scale lateral components of a plateau-honed surface profile, determined with the spline filter and the robust Gaussian regression filter

Figure B.6 shows the filter behaviour of the robust Gaussian regression filter in accordance with this document with regression degree $p=0$ (key B), $p=1$ (key C) and $p=2$ (key D) applied to a sintered surface profile. The microstructure is plateau-like with a pronounced right boundary groove. As the regression degree p increases, the calculated large-scale lateral component follows the groove on the right boundary. Within the surface profile, the different regression degrees p behave almost identically.