

INTERNATIONAL STANDARD

ISO
19111

First edition
2003-02-15

Geographic information — Spatial referencing by coordinates

*Information géographique — Système de références spatiales par
coordonnées*

STANDARDSISO.COM : Click to view the full PDF of ISO 19111:2003



Reference number
ISO 19111:2003(E)

© ISO 2003

PDF disclaimer

This PDF file may contain embedded typefaces. In accordance with Adobe's licensing policy, this file may be printed or viewed but shall not be edited unless the typefaces which are embedded are licensed to and installed on the computer performing the editing. In downloading this file, parties accept therein the responsibility of not infringing Adobe's licensing policy. The ISO Central Secretariat accepts no liability in this area.

Adobe is a trademark of Adobe Systems Incorporated.

Details of the software products used to create this PDF file can be found in the General Info relative to the file; the PDF-creation parameters were optimized for printing. Every care has been taken to ensure that the file is suitable for use by ISO member bodies. In the unlikely event that a problem relating to it is found, please inform the Central Secretariat at the address given below.

STANDARDSISO.COM : Click to view the full PDF of ISO 19111:2003

© ISO 2003

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Published in Switzerland

Contents

Page

Foreword	iv
Introduction	v
1 Scope	1
2 Conformance requirements	1
3 Normative references	1
4 Terms and definitions	1
5 Conventions	5
5.1 Symbols and abbreviated terms	5
5.2 UML notation	6
6 Definition of the conceptual schema for coordinate reference systems	7
6.1 Introduction	7
6.2 Coordinate reference system	7
6.2.1 Type of coordinate reference system	7
6.2.2 Single coordinate reference system	8
6.2.3 Compound coordinate reference system	8
6.3 Datum	9
6.3.1 Types of datums	9
6.3.2 Datum description	10
6.3.3 Prime meridian	10
6.3.4 Ellipsoid	11
6.4 Coordinate system	11
6.5 Coordinate operation — coordinate conversion and coordinate transformation	12
6.5.1 General	12
6.5.2 Coordinate conversion (including map projection)	13
6.5.3 Coordinate transformation	14
6.5.4 Requirements for describing a coordinate operation	14
6.5.5 Concatenated coordinate operation	16
6.6 Citations	17
6.7 Accuracy and precision of coordinates, coordinate operations, and parameters	18
6.8 Attributes to describe a coordinate reference system	19
Annex A (normative) Conformance	22
Annex B (normative) UML schemas	24
Annex C (informative) Decision trees	27
Annex D (informative) Geodetic relationships	29
Annex E (informative) Examples	35

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 19111 was prepared by Technical Committee ISO/TC 211, *Geographic information/Geomatics*.

STANDARDSISO.COM : Click to view the full PDF of ISO 19111:2003

Introduction

Geographic information contains spatial references which relate the features represented in the data to positions in the real world. Spatial references fall into two categories:

- those using coordinates;
- those based on geographic identifiers.

This International Standard deals only with spatial referencing by coordinates. Spatial referencing by geographic identifiers is the subject of ISO 19112, *Geographic information — Spatial referencing by geographic identifiers*.

Coordinates are unambiguous only when the coordinate reference system to which those coordinates are related has been fully defined. A coordinate reference system is a coordinate system which has a reference to the Earth. This International Standard describes the elements that are necessary to define fully various types of coordinate systems and coordinate reference systems applicable to geographic information. The subset of elements required is partially dependent upon the type of coordinates. This International Standard also includes optional fields to allow for the inclusion of non-essential coordinate reference system information. The elements are intended to be both machine and human readable. A set of coordinates on the same coordinate reference system requires one coordinate reference system description.

In addition to describing a coordinate reference system, this International Standard provides for the description of a coordinate transformation or coordinate conversion between two different coordinate reference systems. With such information, geographic data referred to different coordinate reference systems can be merged together for integrated manipulation. Alternatively, an audit trail of coordinate reference system manipulations can be maintained.

Geographic information — Spatial referencing by coordinates

1 Scope

This International Standard defines the conceptual schema for the description of spatial referencing by coordinates. It describes the minimum data required to define one-, two- and three-dimensional coordinate reference systems. It allows additional descriptive information to be provided. It also describes the information required to change coordinate values from one coordinate reference system to another.

This International Standard is applicable to producers and users of geographic information. Although it is applicable to digital geographic data, its principles can be extended to many other forms of geographic data such as maps, charts, and text documents.

2 Conformance requirements

This International Standard defines two classes of conformance, Class A for conformance of coordinate reference systems and Class B for coordinate operations between two coordinate reference systems. Any coordinate reference system claiming conformance to this International Standard shall satisfy the requirements given in Annex A, Clause A.1. Any coordinate operation claiming conformance to this International Standard shall satisfy the requirements given in Annex A, Clause A.2.

3 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1000, *SI units and recommendations for use of their multiples and of certain other units*

ISO/TS 19103:—¹⁾, *Geographic information — Conceptual schema language*

ISO 19113:2002, *Geographic information — Quality principles*

ISO 19114:—¹⁾, *Geographic information — Quality evaluation procedures*

4 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

4.1

Cartesian coordinate system

coordinate system which gives the position of points relative to n mutually perpendicular axes

¹⁾ To be published.

NOTE n is 1, 2 or 3 for the purposes of this International Standard.

4.2

compound coordinate reference system

coordinate reference system using two other independent coordinate reference systems to describe a position

EXAMPLE One coordinate reference system based on a two- or three-dimensional coordinate system and the other coordinate reference system based on a gravity-related height system.

4.3

coordinate

one of a sequence of n numbers designating the position of a point in n -dimensional space

NOTE 1 In a coordinate reference system, the numbers must be qualified by units.

NOTE 2 A coordinate operation is performed on coordinates in a source system resulting in coordinates in a target system.

4.4

coordinate conversion

change of **coordinates**, based on a one-to-one relationship, from one **coordinate system** to another based on the same **datum**

EXAMPLE Between geodetic and Cartesian coordinate systems or between geodetic coordinates and projected coordinates, or change of units such as from radians to degrees or feet to metres.

NOTE A coordinate conversion uses parameters which have constant values.

4.5

coordinate operation

change of **coordinates**, based on a one-to-one relationship, from one **coordinate reference system** to another

NOTE Supertype of coordinate transformation and coordinate conversion.

4.6

coordinate reference system

coordinate system that is related to the real world by a **datum**

NOTE For geodetic and vertical datums, it will be related to the Earth.

4.7

coordinate system

set of mathematical rules for specifying how **coordinates** are to be assigned to points

4.8

coordinate transformation

change of **coordinates** from one **coordinate reference system** to another **coordinate reference system** based on a different **datum** through a one-to-one relationship

NOTE A coordinate transformation uses parameters which are derived empirically by a set of points with known coordinates in both coordinate reference systems.

4.9

datum

parameter or set of parameters that serve as a reference or basis for the calculation of other parameters

NOTE A datum defines the position of the origin, the scale, and the orientation of the axes of a coordinate system.

4.10**easting***E*

distance in a **coordinate system**, eastwards (positive) or westwards (negative) from a north-south reference line

4.11**ellipsoid**

surface formed by the rotation of an ellipse about a main axis

NOTE In this International Standard, ellipsoids are always oblate, meaning that the axis of rotation is always the minor axis.

4.12**ellipsoidal height****geodetic height***h*

distance of a point from the **ellipsoid** measured along the perpendicular from the **ellipsoid** to this point positive if upwards or outside of the **ellipsoid**

NOTE Only used as part of a three-dimensional geodetic coordinate system and never on its own.

4.13**engineering datum****local datum**

datum describing the relationship of a **coordinate system** to a local reference

NOTE Engineering datum excludes both geodetic and vertical datums.

EXAMPLE A system for identifying relative positions within a few kilometres of the reference point.

4.14**flattening***f*

ratio of the difference between the **semi-major** (*a*) and **semi-minor axis** (*b*) of an **ellipsoid** to the **semi-major axis**: $f = (a - b)/a$

NOTE Sometimes inverse flattening $1/f = a/(a - b)$ is given instead; $1/f$ is also known as reciprocal flattening.

4.15**geodetic coordinate system****ellipsoidal coordinate system**

coordinate system in which position is specified by **geodetic latitude**, **geodetic longitude** and (in the three-dimensional case) **ellipsoidal height**

4.16**geodetic datum**

datum describing the relationship of a **coordinate system** to the Earth

NOTE In most cases, the geodetic datum includes an ellipsoid definition.

4.17**geodetic latitude****ellipsoidal latitude***φ*

angle from the equatorial plane to the perpendicular to the **ellipsoid** through a given point, northwards treated as positive

4.18

geodetic longitude
ellipsoidal longitude

λ

angle from the prime **meridian** plane to the **meridian** plane of a given point, eastward treated as positive

4.19

geoid

level surface which best fits mean sea level either locally or globally

NOTE "Level surface" means an equipotential surface of the Earth's gravity field which is everywhere perpendicular to the direction of gravity.

4.20

gravity-related height

H

height dependent on the Earth's gravity field

NOTE In particular, orthometric height or normal height, which are both approximations of the distance of a point above the mean sea level.

4.21

Greenwich meridian

meridian that passes through the position of the Airy Transit Circle at the Royal Observatory Greenwich, United Kingdom

NOTE Most geodetic datums use the Greenwich meridian as the prime meridian. Its precise position differs slightly between different datums.

4.22

height

h, H

distance of a point from a chosen reference surface along a line perpendicular to that surface

NOTE 1 See ellipsoidal height and gravity-related height.

NOTE 2 Height of a point outside the surface treated as positive; negative height is also called depth.

4.23

map projection

coordinate conversion from a **geodetic coordinate system** to a plane

4.24

mean sea level

average level of the surface of the sea over all stages of tide and seasonal variations

NOTE Mean sea level in a local context normally means mean sea level for the region calculated from observations at one or more points over a given period of time. Mean sea level in a global context differs from a global geoid by not more than 2 m.

4.25

meridian

intersection of an **ellipsoid** by a plane containing the **semi-minor axis** of the **ellipsoid**

NOTE This term is often used for the pole-to-pole arc rather than the complete closed figure.

4.26

northing

N

distance in a **coordinate system**, northwards (positive) or southwards (negative) from an east-west reference line

4.27**polar coordinate system**

coordinate system in which position is specified by distance and direction from the origin

NOTE In three dimensions also called spherical coordinate system.

4.28**prime meridian****zero meridian**

meridian from which the longitudes of other **meridians** are quantified

4.29**projected coordinate system**

two-dimensional **coordinate system** resulting from a **map projection**

4.30**semi-major axis**

a

longest radius of an **ellipsoid**

NOTE For an ellipsoid representing the Earth, it is the radius of the equator.

4.31**semi-minor axis**

b

shortest radius of an **ellipsoid**

NOTE For an ellipsoid representing the Earth, it is the distance from the centre of the ellipsoid to either pole.

4.32**spatial reference**

description of position in the real world

NOTE This may take the form of a label, code or set of coordinates.

4.33**vertical datum**

datum describing the relation of gravity-related **heights** to the Earth

NOTE In most cases the vertical datum will be related to a defined mean sea level based on water level observations over a long time period. Ellipsoidal heights are treated as related to a three-dimensional ellipsoidal coordinate system referenced to a geodetic datum. Vertical datums include sounding datums (used for hydrographic purposes), in which case the heights may be negative heights or depths.

5 Conventions**5.1 Symbols and abbreviated terms**

<i>a</i>	semi-major axis
<i>b</i>	semi-minor axis
CCRS	Compound coordinate reference system
<i>E</i>	easting
<i>h</i>	ellipsoidal height
<i>N</i>	northing
SC	Spatial referencing by Coordinates

SI	le Système International d'unités
UML	Unified Modeling Language
λ	geodetic longitude
φ	geodetic latitude
x, y, z	Cartesian coordinates in a geodetic datum
i, j, k	Cartesian coordinates in a engineering datum
r, Ω, θ	spherical polar coordinates

5.2 UML notation

The diagrams that appear in this International Standard are presented using the Unified Modeling Language (UML) static structure diagram with the ISO Interface Definition Language (IDL) basic type definitions and the UML Object Constraint Language (OCL) as the conceptual schema language. The UML notations used in this International Standard are described in Figure 1.

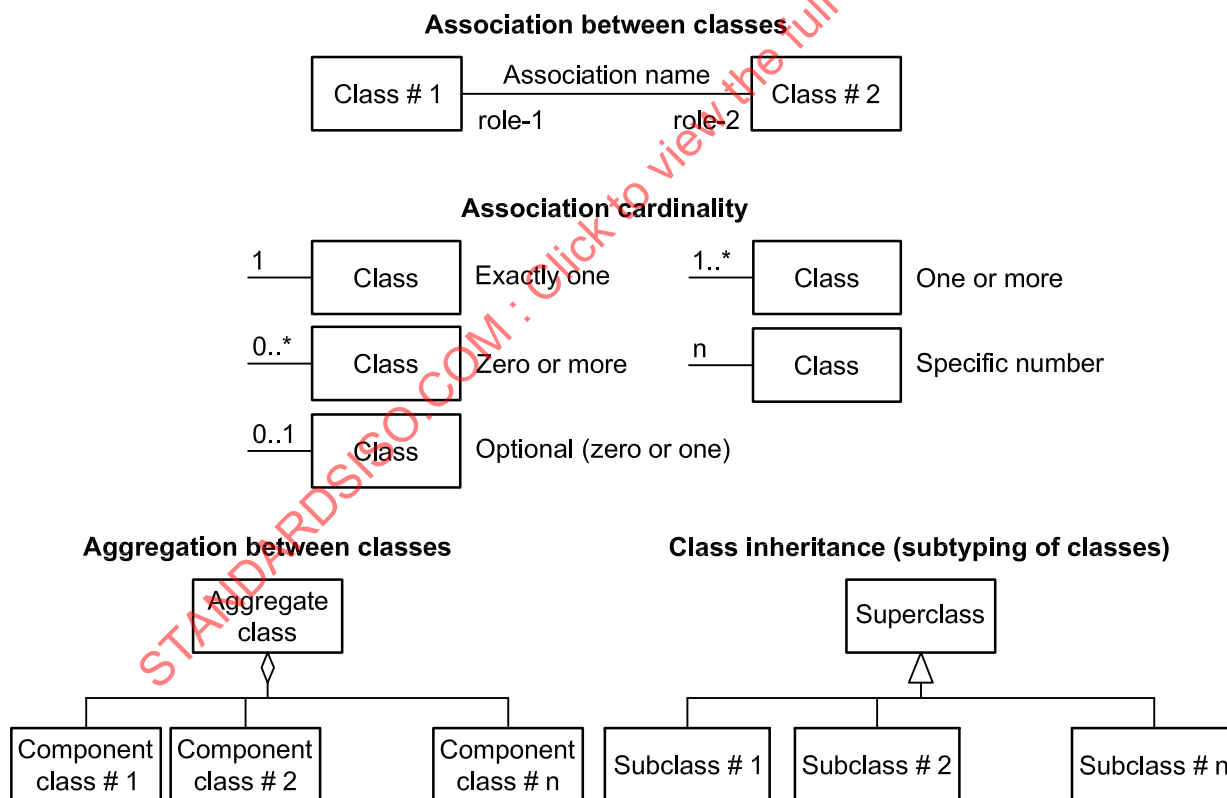


Figure 1 — UML notation

6 Definition of the conceptual schema for coordinate reference systems

6.1 Introduction

Location or position on or near the Earth's surface may be described using coordinates. Coordinates are unambiguous only when the coordinate reference system to which those coordinates are related has been fully defined. Each position shall be described by a set of coordinates in a coordinate reference system.

Coordinates supplied in a dataset shall belong to the same coordinate reference system. A description of this coordinate reference system shall be supplied with the dataset. Coordinate data shall be accompanied by information sufficient to make the coordinates unambiguous. This information varies by coordinate system type and datum type.

In the clauses below, attributes are given a requirement status:

Requirement	Definition	Comment
M	mandatory	This attribute shall be supplied.
C	conditional	This attribute shall be supplied if the condition (given in the attribute description) is true. It may be supplied if the condition is false.
O	optional	This attribute may be supplied.

The Maximum Occurrence column in the following tables indicates the maximum number of occurrences of attribute values that are permissible, with N indicating no upper limit. The conceptual schema for describing coordinate reference systems is modelled with the Unified Modeling Language (UML) in Annex B. In case of inconsistency between the metadata textual description and the UML model (re: Annex B), the textual description shall prevail. The basic data types are defined in ISO/TS 19103.

6.2 Coordinate reference system

6.2.1 Type of coordinate reference system

A coordinate reference system may be either single or compound. A single coordinate reference system is defined in 6.2.2 and a compound coordinate reference system is defined in 6.2.3. The requirements for describing the type of coordinate reference system shall be in accordance with Table 1.

Table 1 — Requirements for describing the type of coordinate reference system

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Coordinate reference system type code	typeCode	SC_TypeCode	M	1	Code denoting the type of coordinate reference system: 1 — a single coordinate reference system 2 — a compound coordinate reference system
Coordinate reference system remarks	remarks	CharacterString	O	1	Comments on the coordinate reference system including source information.

To determine whether the coordinate reference system is compound or single, decision tree 1 in Annex C may be used (see Figure C.1).

6.2.2 Single coordinate reference system

A position of a feature can be given by a set of coordinates. Coordinates are unambiguous if the coordinate reference system to which those coordinates are related has been fully defined.

A coordinate reference system is realized by a set of coordinates. The realization is sometimes known as a reference frame.

A coordinate reference system shall be defined by one datum and by one coordinate system; see Figure 2.

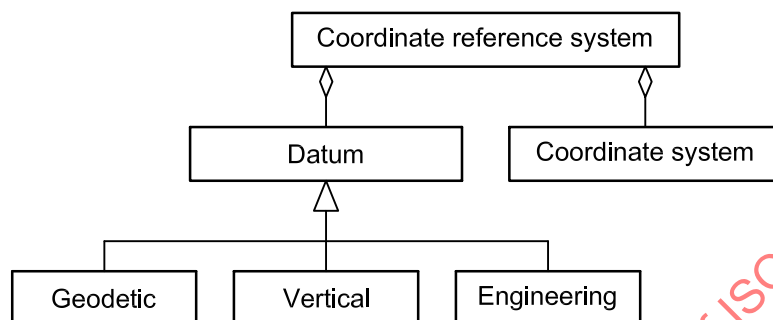


Figure 2 — Coordinate reference system

For the purposes of this International Standard, a coordinate reference system shall not change with time. When a reference frame changes with time, a new datum and coordinate reference system shall be created, with date of realization of the datum and coordinate reference system included in their names or identifiers.

The requirements for describing a coordinate reference system shall be in accordance with Table 2.

Table 2 — Requirements for describing a coordinate reference system

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Coordinate reference system identifier	CRSID	RS_Identifier	M	1	Identifier of the coordinate reference system.
Coordinate reference system alias	alias	CharacterString	O	N	Alternative name or identifier by which this coordinate reference system is known.
Coordinate reference system valid area	validArea	EX_Extent	O	1	Area for which the coordinate reference system is valid.
Coordinate reference system scope	scope	CharacterString	O	N	Application for which the coordinate reference system is valid.

6.2.3 Compound coordinate reference system

The horizontal and vertical components of a description of position in three dimensions may sometimes come from different coordinate reference systems rather than through a single three-dimensional coordinate reference system. This is always the case for positions where vertical coordinates are related to mean sea level. This shall be handled through a compound coordinate reference system (CCRS) which identifies the two coordinate reference systems utilized, see Figure 3. Vertical datum and gravity-related height are an example of a datum and coordinate system for coordinate reference system 2.

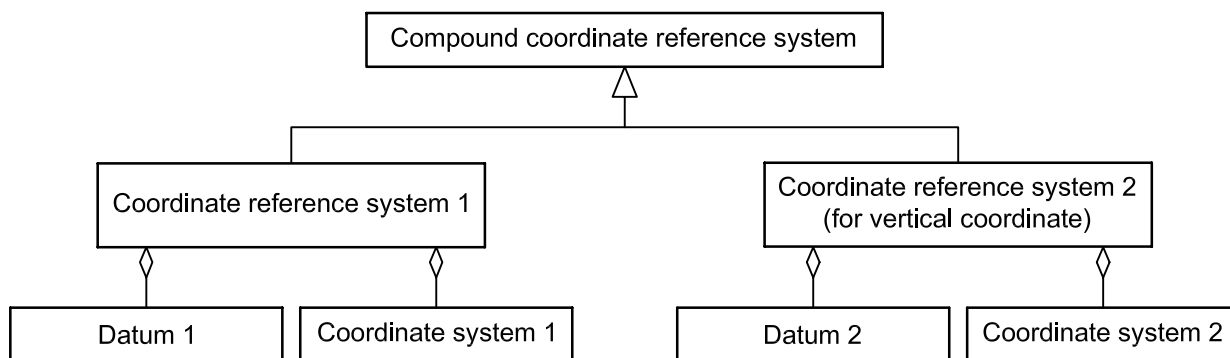


Figure 3 — Compound coordinate reference system

The requirements for describing a compound coordinate reference system shall be in accordance with Table 3. Each of the two coordinate reference systems shall then be described in the normal way.

The compound coordinate reference system identifier may be a concatenation of the coordinate reference system identifiers for the component coordinate reference systems.

Table 3 — Requirements for describing a compound coordinate reference system

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Compound coordinate reference system identifier	CCRSID	RS_Identifier	O	1	Identifier of the compound coordinate reference system.
Compound coordinate reference system alias	alias	CharacterString	O	N	Alternative name or identifier by which this compound coordinate reference system is known.
Compound coordinate reference system valid area	validArea	EX_Extent	O	1	Area for which the compound coordinate reference system is valid.
Compound coordinate reference system scope	scope	CharacterString	O	N	Application for which the compound coordinate reference system is valid.

6.3 Datum

6.3.1 Types of datums

A datum is geodetic, vertical or engineering. A geodetic datum gives the relationship of a coordinate system to the Earth and is used as the basis for two- or three-dimensional systems. In most cases, it shall require an ellipsoid definition. A vertical datum gives the relationship of gravity-related heights to a surface known as the geoid. The geoid is a surface close to mean sea level. In this International Standard, a datum shall be engineering if it is neither geodetic nor vertical.

For geographic information purposes it is necessary to identify a datum, but the definition of the datum itself is optional.

If the type of coordinate reference system is not known, decision tree 2 in Annex C may be used in the determination of the datum type (see Figure C.2).

6.3.2 Datum description

If a coordinate reference system citation is not supplied, then a datum description in accordance with Table 4 shall be supplied.

Table 4 — Requirements for describing a datum

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Datum identifier	datumID	RS_Identifier	M	1	Identifier of the datum.
Datum alias	alias	CharacterString	O	N	Alternative name or names by which this datum is known.
Datum type	type	CharacterString	O	1	Type of datum. Valid values are: — geodetic, — vertical, or — engineering
Datum anchor point	point	CharacterString	O	1	Description including coordinates of the point or points used to anchor the datum to the Earth.
Datum realization epoch	realization Epoch	Date	O	1	Epoch of realization of the datum.
Datum valid area	validArea	EX_Extent	O	1	Area for which the datum is valid.
Datum scope	scope	CharacterString	O	N	Application for which the datum is valid.
Datum remarks	remarks	CharacterString	O	1	Comments on the datum including source information.

When the datum type is geodetic, then certain prime meridian and ellipsoid attributes as described below shall be mandatory regardless of whether a value for datum type has been provided or not.

6.3.3 Prime meridian

A prime meridian defines the origin from which longitude values are specified. Most geodetic datums use Greenwich as their prime meridian.

A prime meridian description shall be mandatory if the datum type is geodetic and its prime meridian is not Greenwich and if neither coordinate reference system citation nor datum citation is supplied.

The requirements for describing a prime meridian shall be in accordance with Table 5.

Table 5 — Requirements for describing a prime meridian

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Prime meridian identifier	meridianID	RS_Identifier	M	1	Identifier of the prime meridian.
Prime meridian Greenwich longitude	Greenwich Longitude	Angle	M	1	Longitude of the prime meridian measured from the Greenwich meridian, positive eastward. If the datum type is geodetic and the prime meridian name is not supplied, then the prime meridian name is taken to be "Greenwich" and the prime meridian Greenwich longitude is taken to be "0".
Prime meridian remarks	remarks	CharacterString	O	1	Comments on the prime meridian including source information.

6.3.4 Ellipsoid

An ellipsoid description is not required if the datum type is

- a) vertical,
- b) engineering, or
- c) geodetic,

and any of the following circumstances apply:

- the coordinate reference system citation is supplied;
- the datum citation is supplied;
- the coordinate system type is Cartesian.

The requirements for describing an ellipsoid shall be in accordance with Table 6.

Table 6 — Requirements for describing an ellipsoid

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Ellipsoid identifier	ellipsoidID	RS_Identifier	M	1	Identifier of the ellipsoid for the datum.
Ellipsoid alias	alias	CharacterString	O	N	Alternative name or names of the ellipsoid.
Ellipsoid semi-major axis	semiMajor Axis	Length	M	1	Length of the semi-major axis of the ellipsoid.
Ellipsoid shape	ellipsoid Shape	boolean	M	1	boolean = TRUE when the reference surface is an ellipsoid, FALSE when the reference surface is a sphere.
Ellipsoid inverse flattening	inverse Flattening	SC_inverse Flattening	C	1	Inverse flattening of the ellipsoid. Unitless. Condition 1 (cd 1): Mandatory if <i>ellipsoid shape</i> is true.
Ellipsoid remarks	remarks	CharacterString	O	1	Comments on or information about the ellipsoid.

6.4 Coordinate system

A coordinate system is described by the name, the units, the direction and sequence of the axes. Coordinates in a set are listed according to this sequence. Coordinates based on a projected coordinate reference system are the result of a coordinate conversion which is described in 6.5.

The coordinate system description shall be mandatory if a coordinate reference system citation is not supplied.

The requirements for describing a coordinate system shall be in accordance with Tables 7 and 8.

Table 7 — Requirements for describing a coordinate system

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Coordinate system identifier	CSID	RS_Identifier	M	1	Identifier of the coordinate system.
Coordinate system type	type	SC_Coordinate SystemType	M	1	Type of the coordinate system. The most commonly used entries are: — Cartesian — geodetic — projected — polar — gravity-related Do not use Cartesian if the system is projected.
Coordinate system dimension	dimension	Integer	M	1	Number of coordinates {3,2,1} in the set.
Coordinate system remarks	remarks	CharacterString	O	1	Comments on or information about the coordinate system.

Each coordinate system axis shall be described, the order of each axis description following the order of the coordinates in the data set. The elements for each coordinate system axis, as described in Table 8, shall be kept together (as in a data block), and the number of data blocks shall be equal to the value provided for coordinate system dimension in Table 7.

Table 8 — Requirements for describing a coordinate system axis

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Coordinate system axis name	axisName	CharacterString	M	1	Name of the coordinate system axis.
Coordinate system axis direction	axisDirection	CharacterString	M	1	Direction of the coordinate system axes (or, in the case of Cartesian or projected coordinates, the direction of the coordinate system axis at the origin). Examples: north, east, up
Coordinate system axis unit identifier	axisUnitID	UnitOf Measure	M	1	Identifier of the unit for the coordinate system axis.

If the type of coordinate reference system is not known, decision tree 2 in Annex C may be used in the determination of the coordinate system type (see Figure C.2).

6.5 Coordinate operation — coordinate conversion and coordinate transformation

6.5.1 General

This subclause describes coordinate operations to change coordinate values from one coordinate reference system to coordinate values based on another coordinate reference system. Coordinate operation information may be given if datasets having coordinates using different coordinate reference systems are to be merged.

Generally the description of a coordinate operation is not required for the unambiguous identification of coordinates. However, projected coordinates are the result of a coordinate conversion applied to geodetic coordinates; in this special case, a coordinate operation description must be part of the coordinate reference system description.

In this International Standard, two types of coordinate operations shall be recognized:

- 1) A *coordinate conversion* changes coordinates from one coordinate system to another based on the same datum. In a coordinate conversion, the parameter values are exact.
- 2) A *coordinate transformation* changes coordinates from a coordinate reference system based on one datum to a coordinate reference system based on a second datum. A coordinate transformation differs from a coordinate conversion in that the coordinate transformation parameter values are derived empirically: therefore there may be several different estimations (or realizations).

Once the parameter values are obtained, both coordinate conversion and coordinate transformation use similar mathematical processes.

6.5.2 Coordinate conversion (including map projection)

A coordinate conversion is a one-to-one mapping of coordinates based on one coordinate reference system to another coordinate reference system on the same datum. These coordinate conversions are widely used to provide mapping projections of ellipsoidal coordinates to two-dimensional Cartesian coordinates. Other coordinate conversions include conversion of the units of measure or shifting the origin of coordinate system. In this International Standard, coordinate conversions (see Figure 4) shall be distinguished from coordinate transformations (see Figure 5). Coordinate conversions do not change the underlying datum since they use analytical mathematical functions which do not alter the fundamental accuracy of the coordinate values.

Coordinate conversions include

- map projections, which is a method using mathematical functions to convert ellipsoidal coordinates (excluding height) to two-dimensional Cartesian coordinates, or vice-versa;
- coordinate conversions of ellipsoidal coordinates (including ellipsoidal height) to three-dimensional Cartesian coordinates, or vice-versa;
- unit changes by application of a multiplication factor (for example, metres to feet) or an algorithm (for example, radians to degrees, minutes and seconds);
- shifting the origin of a plane to make a local grid.

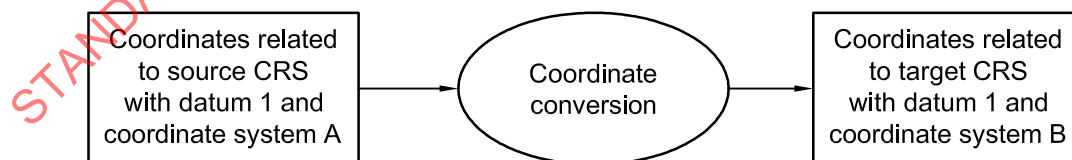


Figure 4 — Coordinate conversion

The map projection is a special coordinate conversion of coordinate systems from the ellipsoid to the plane. For the description of coordinates belonging to a projected coordinate system, the provision of a coordinate operation description shall be mandatory.

6.5.3 Coordinate transformation

Coordinates may be transformed by changing them to another datum. The coordinate systems shall be of the same type (for example, both geodetic or both Cartesian). A coordinate transformation is performed through a method which has an algorithm. Each algorithm has a set of related parameters. Because their values are empirically derived, they depend upon the measurements used and include measurement errors. Different sets of measurements will result in multiple sets of parameter values of a coordinate transformation between two datums.



Figure 5 — Coordinate transformation

A coordinate transformation description is not necessary for describing a coordinate reference system. However, it may sometimes be useful to describe a coordinate transformation that has already been applied to the coordinates, or a coordinate transformation from that system to a user-defined coordinate reference system.

6.5.4 Requirements for describing a coordinate operation

The requirements for describing a coordinate operation and connected terms shall be in accordance with Tables 9 and 10.

A coordinate operation description shall also be given if the coordinate system type is projected and neither a coordinate reference system citation nor a coordinate system citation has been supplied.

Table 9 — Requirements for describing a coordinate operation and connected terms

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Coordinate operation identifier	Coordinate OperationID	RS_Identifier	M	1	Identifier of the coordinate operation.
Coordinate operation valid area	validArea	EX_Extent	O	1	Area for which the coordinate operation is valid.
Coordinate operation scope	scope	CharacterString	O	N	Application for which the coordinate operation is valid.
Source coordinate reference system identifier	sourceID	RS_Identifier	C	1	Identifier of the source coordinate reference system. Condition 2 (cd 2): Mandatory if describing a coordinate transformation.
Target coordinate reference system identifier	targetID	RS_Identifier	C	1	Identifier of the target coordinate reference system. cd 2
Coordinate operation version	version	CharacterString	C	1	Version of the coordinate operation between the source coordinate reference system and the target coordinate reference system. cd 2
Coordinate operation method name	methodName	CharacterString	C	1	Name of the algorithm used for the coordinate operation. Example (in case of coordinate transformation): — Abridged Molodenski — Similarity transformation Example (in case of coordinate conversion): — Cartesian into ellipsoidal — Universal Transverse Mercator — Mercator — Lambert Conformal Conic — Albers equal area — Stereographic — metres to feet — radians to degrees Condition 3 (cd 3): Mandatory either (i) if describing a projected coordinate system and none of <i>coordinate reference system citation</i> , <i>coordinate system citation</i> , or <i>coordinate operation citation</i> is supplied, or (ii) if describing a single coordinate conversion or a coordinate transformation.
Coordinate operation method name alias	methodName Alias	CharacterString	O	N	Alternative name or names of the coordinate operation method identifier.
Coordinate operation method formula(s)	formula	CharacterString	M	1	Formula(s) used by the coordinate operation method. This may be a reference to a publication.
Coordinate operation method number of parameters	numberOfParameters	Integer	M	1	Number of parameters required by this coordinate operation method.
Coordinate operation method remarks	remarks	CharacterString	O	1	Comments on or information about the coordinate operation method. It is often useful to include an example. This may define a time dependent parameter such as epoch.

Coordinate operation parameters shall be described following the order of coordinate operation parameters in the data set.

When several coordinate operation parameters are being described, the elements for each parameter as detailed in Table 10 shall be kept together in a data block, and the number of data blocks shall be the same as the value given by coordinate operation method parameters number.

Table 10 — Requirements for describing coordinate operation parameters

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Coordinate operation parameter name	name	CharacterString	M	1	Identifier of the coordinate operation parameter that is defined or used with this coordinate operation method. The parameters differ among coordinate operation methods. Example (in case of coordinate transformation): — geocentric x translation — geocentric y translation — geocentric z translation Example (in case of coordinate conversion): — latitude of origin — longitude of origin — scale factor — false easting — false northing
Coordinate operation parameter value	value	Measure	M	1	Value of the coordinate operation parameter.
Coordinate operation parameter remarks	remarks	CharacterString	O	1	Comments on or information about the coordinate operation parameter.

6.5.5 Concatenated coordinate operation

The change of coordinates from one coordinate reference system to another coordinate reference system may follow from a series of coordinate operations consisting of one or more coordinate transformations and/or one or more coordinate conversions. This is called a concatenated coordinate operation. Figure 6 shows a two-step concatenated coordinate operation. There is no upper limit to the number of steps a concatenated coordinate operation may have. Each step is a coordinate operation described in the normal way.

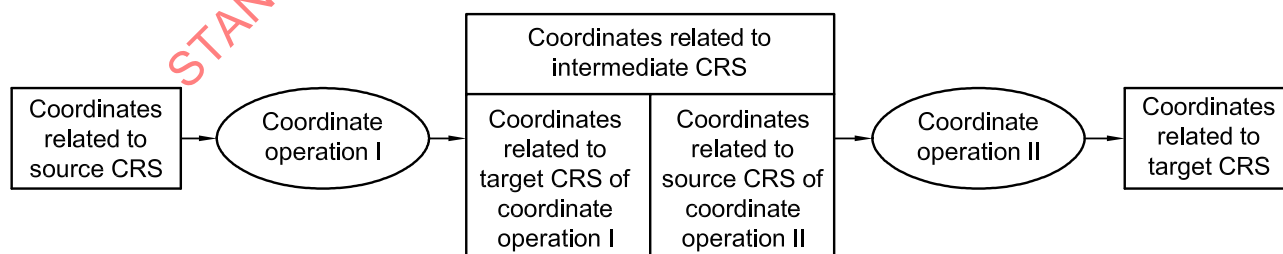


Figure 6 — Concatenated coordinate operation

The requirements for describing a concatenated coordinate operation are given in Table 11. Individual coordinate operations shall be described according to Table 9 and coordinate operation parameters for individual coordinate operations shall be described according to Table 10. The order of individual coordinate operations is significant and follows the order in which the steps are performed. The number of individual coordinate operations described shall be the same as the value for concatenated coordinate operation step number.

Table 11 — Requirements for describing a concatenated coordinate operation

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Concatenated coordinate operation identifier	concatOpID	RS_Identifier	M	1	Identifier of the concatenated coordinate operation.
Concatenated coordinate operation number of steps	numberOfSteps	Integer	M	1	Number of steps in the concatenated coordinate operation.
Concatenated coordinate operation step sequence	stepID	Sequence <RS_Identifier>	M	1	Identifier for each of the steps in this concatenated coordinate operation. The order is significant and shall reflect the order in which the steps shall be performed. The number provided shall be consistent with the value for <i>concatenated coordinate operation number of steps</i> .
Concatenated coordinate operation valid area	validArea	EX_Extent	O	1	Area for which the coordinate operation is valid.
Concatenated coordinate operation scope	scope	CharacterString	O	N	Application for which the coordinate operation is valid.
Concatenated coordinate operation remarks	remarks	CharacterString	O	1	Comments on or information about the concatenated coordinate operation.

6.6 Citations

The description of class attributes can be done in two manners, the direct description or citation. The direct description is performed according to the requirements in the tables of 6.2 to 6.5. The citation is realized by using recognized sources. The class citation is described in ISO/TS 19103.

The requirements for describing identifiers are given in Table 12.

Table 12 — Requirements for describing identifiers

Element name	UML identifier	Data type	Obligation	Maximum Occurrence	Description
Identifier	identifier	CharacterString	M	1	Name or identifier of the class or attribute.
Citation	citation	CI_Citation	O	1	Citation.

The requirements for describing a citation are given in Table 13.

Table 13 — Requirements for describing a citation

Element name	UML identifier	Data type	Obligation	Maximum occurrence	Description
Citation title	title	CharacterString	M	1	Name by which the cited information is known, e.g. the author or authors of the cited source, the title of the cited source or publication.
Citation alternative title	alternativeTitle	CharacterString	O	N	Alternative title or subtitle of cited source.
Citation date	data	Date	M	1	Date of cited source or publication.
Citation edition	edition	CharacterString	O	1	The number of edition.
Citation edition date	editionDate	Date	O	1	Date of the edition.
Citation identifier	identifier	CharacterString	O	N	Place of publication, publishing house.
Citation identifier type	identifierType	CharacterString	O	N	Reference form of the identifier.
Citation cited responsible party	citedResponsibleParty	CI_ResponsibleParty	O	N	Responsible party for citation.
Citation presentation form code	presentationFormCode	CI_PresentationFormCode	O	N	Mode in which the data is presented.
Citation series name	seriesName	CharacterString	O	1	Name of a series of the cited source.
Citation issue identification	issueIdentification	CharacterString	O	1	Issue number of periodical.
Citation collective title	collectiveTitle	CharacterString	O	1	Common title with holdings note.
Citation page	page	CharacterString	O	1	Details, on which pages of the periodical the article was published.
Citation ISBN	ISBN	CharacterString	O	1	International Standard Book Number.
Citation ISSN	ISSN	CharacterString	O	1	International Standard Serial Number.
Citation other citation details	otherCitationDetails	CharacterString	O	1	Comments on the cited source.

If the identifier is a citation and the citation describes all or parts of mandatory attributes, the described attributes of the class are optional.

6.7 Accuracy and precision of coordinates, coordinate operations, and parameters

From the viewpoint of accuracy, the definition of a coordinate reference system is error-free. A coordinate reference system is realized as the coordinates of a set of fundamental points. The accuracy of spatial referencing depends on the accuracy of the realization of the coordinate reference system as well as the accuracy of the further coordinate densifications.

The accuracy of the coordinate transformation between two datums depends on the accuracy of the measurements in both coordinate reference systems. Coordinate conversions are not influenced by any errors of measuring systems or datum parameters.

Information about the accuracy of coordinates and coordinate operation parameters and the precision of coordinate operations is quality information and shall be reported in conformance to ISO 19113 and ISO 19114.

6.8 Attributes to describe a coordinate reference system

This clause summarizes the elements that must be provided for descriptions of datasets containing coordinate reference systems to meet the requirements of this International Standard. In Table 14, the attributes needed to make coordinates unambiguous are tabulated for the most frequently encountered coordinate reference system types. Mandatory attributes are shown by a tick (✓). In most cases, it is necessary to identify the coordinate system type and datum type before the relevant attribute set can be distinguished. A series of decision trees are provided in Annex C to assist users of this International Standard.

Such information may be supplied directly or, either in part or in whole, through one or more citations of an external reference. These citations shall conform to the requirements given in 6.1 to 6.5. Some citations may not contain all components. Where a citation contains several related components, a user shall refer only to the highest level related component available. For example, if a citation includes both geodetic datums and their related ellipsoids, a user shall refer only to geodetic datum entity, as ellipsoid information is obtained through the geodetic datum information. Any citation shall include the mandatory elements for each of the components described in Table 14.

Table 14 — Attributes to be provided to describe a coordinate reference system

Element name	obligation ^a	$E\ N$	$\varphi\ \lambda\ h$ $\varphi\ \lambda$	$x\ y\ z$	$r\ \Omega\ \Theta$ $r\ \Omega$	$i\ j\ k$ $i\ j$ k	$r\ \Omega\ \Theta$ $r\ \Omega$	H
		Datum type						
		geodetic	geodetic	geodetic	geodetic	engineering	engineering	vertical
		Coordinate system type						
		projected	geodetic	Cartesian	spherical polar	Cartesian	spherical polar	gravity-related height
Coordinate reference system identifier	M	✓	✓	✓	✓	✓	✓	✓
Coordinate reference system alias	O							
Coordinate reference system valid area	O							
Coordinate reference system scope	O							
Datum identifier	M	✓	✓	✓	✓	✓	✓	✓
Datum alias	O							
Datum type	O							
Datum anchor point	O							
Datum realization epoch	O							
Datum valid area	O							
Datum scope	O							
Datum remarks	O							
Prime meridian identifier	M	✓	✓	✓	✓			
Prime meridian Greenwich longitude	M	✓	✓	✓	✓			
Prime meridian remarks	O							
Ellipsoid identifier	M	✓	✓		✓			
Ellipsoid alias	O							
Ellipsoid semi-major axis	M	✓	✓		✓			
Ellipsoid shape	M	✓	✓		✓			
Ellipsoid inverse flattening	cd 1	✓	✓		✓			
Ellipsoid remarks	O							
Coordinate system identifier	M	✓	✓	✓	✓	✓	✓	✓
Coordinate system type	M	✓	✓	✓	✓	✓	✓	✓
Coordinate system dimension	M	✓	✓	✓	✓	✓	✓	✓
Coordinate system remarks	O							
Coordinate system axis name (NOTE)	M	✓	✓	✓	✓	✓	✓	✓
Coordinate system axis direction (NOTE)	M	✓	✓	✓	✓	✓	✓	✓
Coordinate system axis unit identifier (NOTE)	M	✓	✓	✓	✓	✓	✓	✓
Coordinate operation identifier	M	✓						
Coordinate operation valid area	O							
Coordinate operation scope	O							
Source coordinate reference system identifier	cd 2							

Table 14 (continued)

Element name	obligation ^a	$E\ N$	$\varphi\ \lambda\ h$ $\varphi\ \lambda$	$x\ y\ z$	$r\ \Omega\ \Theta$ $r\ \Omega$	$i\ j\ k$ $i\ j$ k	$r\ \Omega\ \Theta$ $r\ \Omega$	H
		Datum type						
		geodetic	geodetic	geodetic	geodetic	engineering	engineering	vertical
		Coordinate system type						
		projected	geodetic	Cartesian	spherical polar	Cartesian	spherical polar	gravity-related height
Target coordinate reference system identifier	cd 2							
Coordinate operation version	cd 2							
Coordinate operation method name	cd 3	✓						
Coordinate operation method name alias	O							
Coordinate operation method formula(s)	M	✓						
Coordinate operation method parameters number	M	✓						
Coordinate operation method remarks	O							
Coordinate operation parameter name	M	✓						
Coordinate operation parameter value	M	✓						
Coordinate operation parameter remarks	O							
Each of the three elements coordinate system axis name, coordinate system axis direction and coordinate system unit identifier shall be repeated for each axis.								
NOTE The coordinate operation requirements listed here refer to a description of a projected coordinate reference system.								
^a Conditions are as follows: cd1 — Mandatory if an ellipsoid shape is true. cd2 — Mandatory if describing a coordinate transformation. cd3 — Mandatory if describing either (i) a projected coordinate system and none of coordinate reference system citation, coordinate system citation, or coordinate operation citation is supplied, or (ii) a single coordinate conversion or a coordinate transformation								

Annex A (normative)

Conformance

A.1 Class A: Conformance of a coordinate reference system

A.1.1 Abstract test suite

To check that a coordinate reference system is in accordance with this International Standard, check that it satisfies the requirements given in A.1.2 to A.1.5. For coordinate reference system descriptions, conformance shall be tested against the mandatory and conditional elements (where the condition is true) that are listed in 6.2, 6.3, 6.4 and 6.8. If the type of coordinate system is projected, the test shall be extended to the mandatory elements and conditional element attributes (where the condition is true), as required by 6.5.

A.1.2 Test case identifier: Completeness test

- a) Test purpose: To determine whether all of the relevant entities and elements which are specified to be mandatory or mandatory under the conditions specified have been provided in the description.
- b) Test method: Check the coordinate reference system to ensure that the coordinate reference system description includes as a minimum all the elements indicated as mandatory for that type of system in Tables 1-8 and, in the case of projected coordinate reference systems, additionally in Tables 9 and 10.
- c) Reference: 6.2 to 6.4 and, in the case of projected coordinate reference systems, also 6.5.
- d) Test type: Basic.

A.1.3 Test case identifier: Maximum occurrence test

- a) Test purpose: To ensure each coordinate reference system element occurs not more than the number of times specified in the standard.
- b) Test method: Examine the subject coordinate reference system for the number of occurrences of each entity and element provided to ensure that the number of occurrences for each shall be not more than the 'Maximum Occurrences' attribute specified in 6.2 to 6.4 and, in the case of projected coordinate reference systems, additionally in 6.5.
- c) Reference: 6.2 to 6.4 and, in the case of projected coordinate reference systems, also 6.5.
- d) Test type: Basic.

A.1.4 Test case identifier: Data type test

- a) Test purpose: To determine if each coordinate reference system in the dataset uses the specified data type.
- b) Test method: Check the data type of each element of the description of a coordinate reference system to ensure that it is of the data type specified in 6.2 to 6.4 and, in the case of projected coordinate reference systems, additionally in 6.5.
- c) Reference: 6.2 to 6.4 and, in the case of projected coordinate reference systems, also 6.5.
- d) Test type: Basic.

A.1.5 Test case identifier: Unit test

- a) Test purpose: To ensure that the units are in accordance with ISO 1000.
- b) Test method: Check that any units of elements are in accordance with ISO 1000.
- c) Reference: 6.2 to 6.4 and, in the case of projected coordinate reference systems, also 6.5.
- d) Test type: Basic.

A.2 Class B: Conformance of a coordinate operation**A.2.1 Abstract test suite**

To check that a coordinate operation is in accordance with this International Standard, check that it satisfies the requirements given in A.2.2 to A.2.5.

A.2.2 Test case identifier: Completeness test

- a) Test purpose: To determine whether all of the relevant, entities and elements which are specified to be mandatory or mandatory under the conditions specified have been provided in the description.
- b) Test method: Check that the coordinate operation description includes all of the elements indicated as mandatory in Tables 9 to 11.
- c) Reference: 6.5.
- d) Test type: Basic.

A.2.3 Test case identifier: Maximum occurrence test

- a) Test purpose: To ensure that each coordinate operation element occurs not more than the number of times specified in the standard.
- b) Test method: Examine the coordinate operation dataset for the number of occurrences of each entity and element provided to ensure that the number of occurrences for each shall be not more than the "Maximum Occurrences" attribute specified in Tables 9 to 11.
- c) Reference: 6.5.
- d) Test type: Basic.

A.2.4 Test case identifier: Data type test

- a) Test purpose: To determine if each coordinate operation element in the dataset uses the specified data type.
- b) Test method: Check the data type of each element of the description of a coordinate operation to ensure that it is of the data type specified in Tables 9 to 11.
- c) Reference: 6.5.
- d) Test type: Basic.

A.2.5 Test case identifier: Unit test

- a) Test purpose: To ensure that the units are in accordance with ISO 1000.
- b) Test method: Check that any units of elements conform to ISO 1000.
- c) Reference: 6.5.
- d) Test type: Basic.

Annex B
(normative)

UML schemas

B.1 UML schema for describing coordinate reference systems

STANDARDSISO.COM : Click to view the full PDF of ISO 19111:2003

© ISO 2003 — All rights reserved

B.2 UML schema for describing coordinate operations

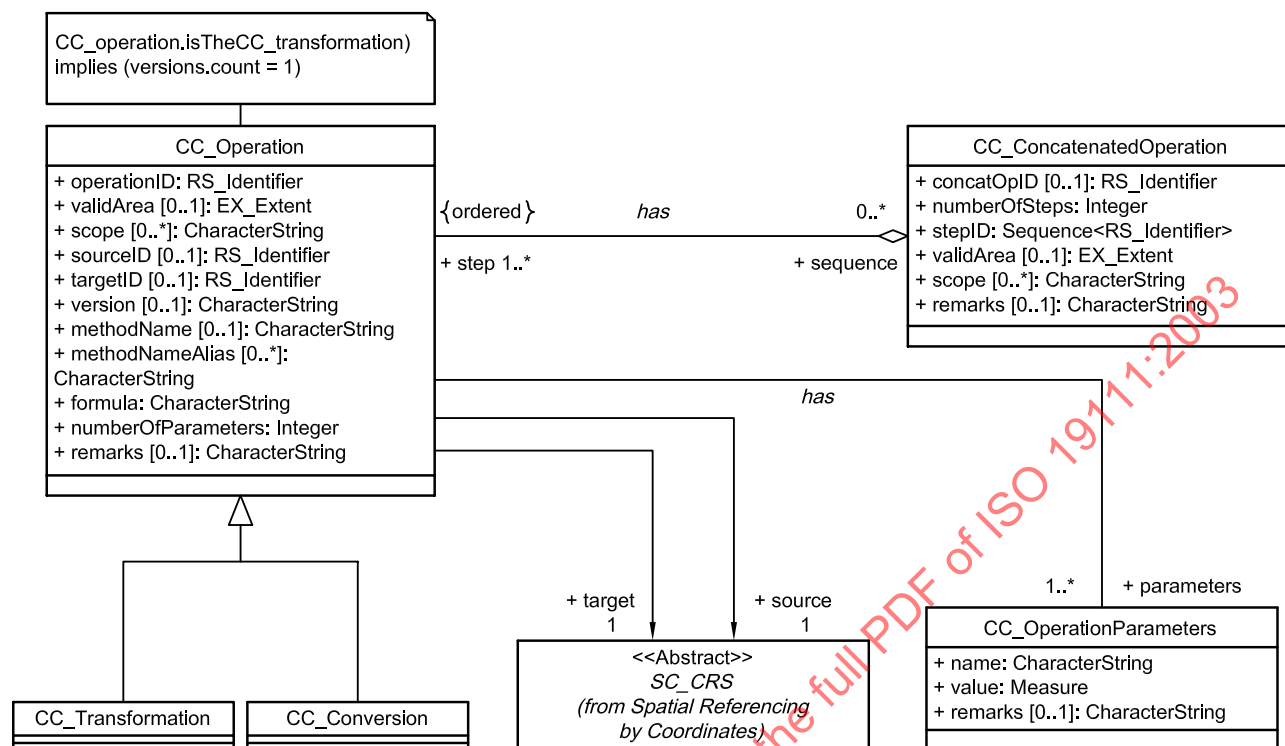


Figure B.2 — UML schema for describing coordinate operations

Annex C (informative)

Decision trees

C.1 Decision tree 1 (coordinate reference system kind)

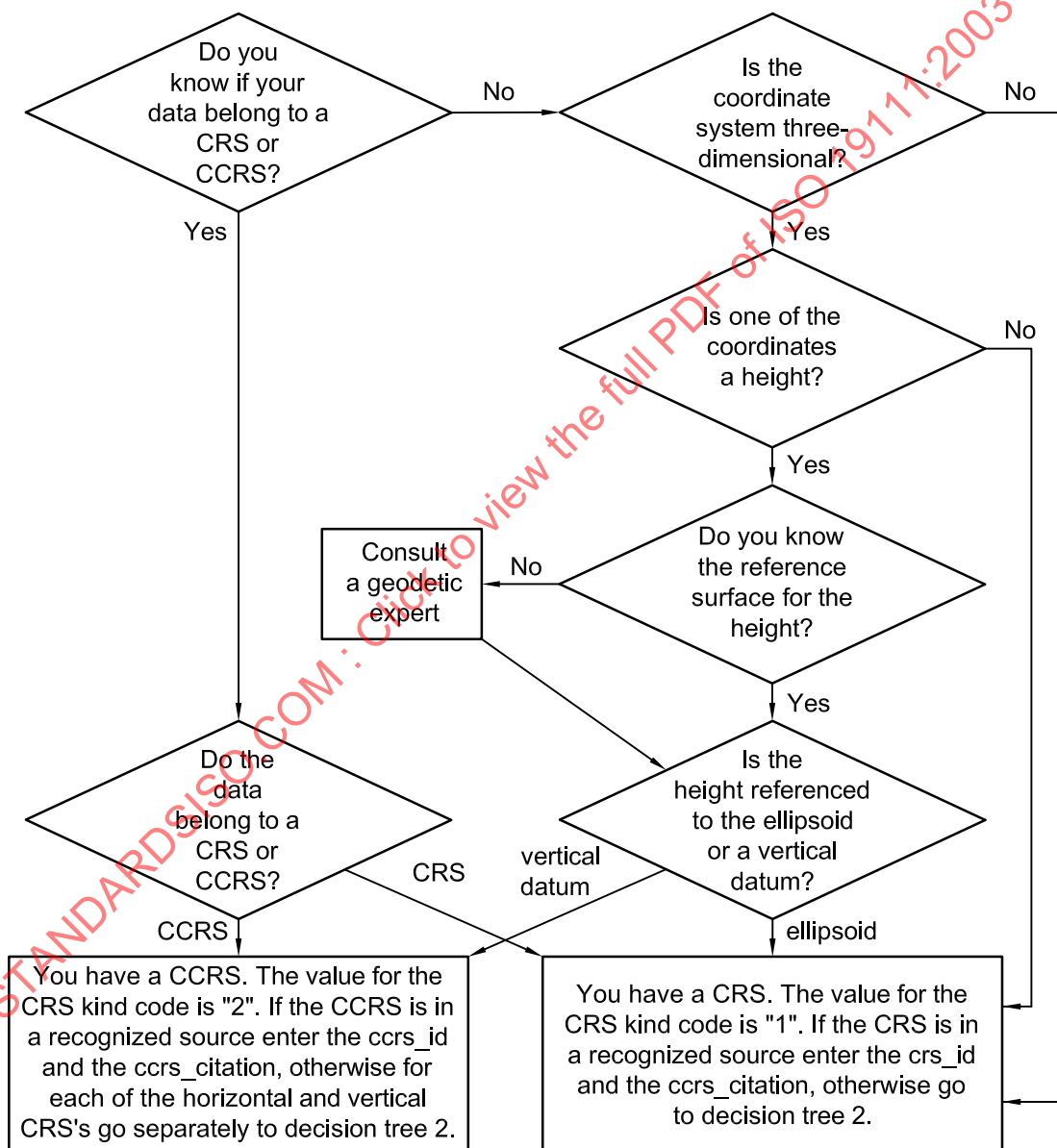
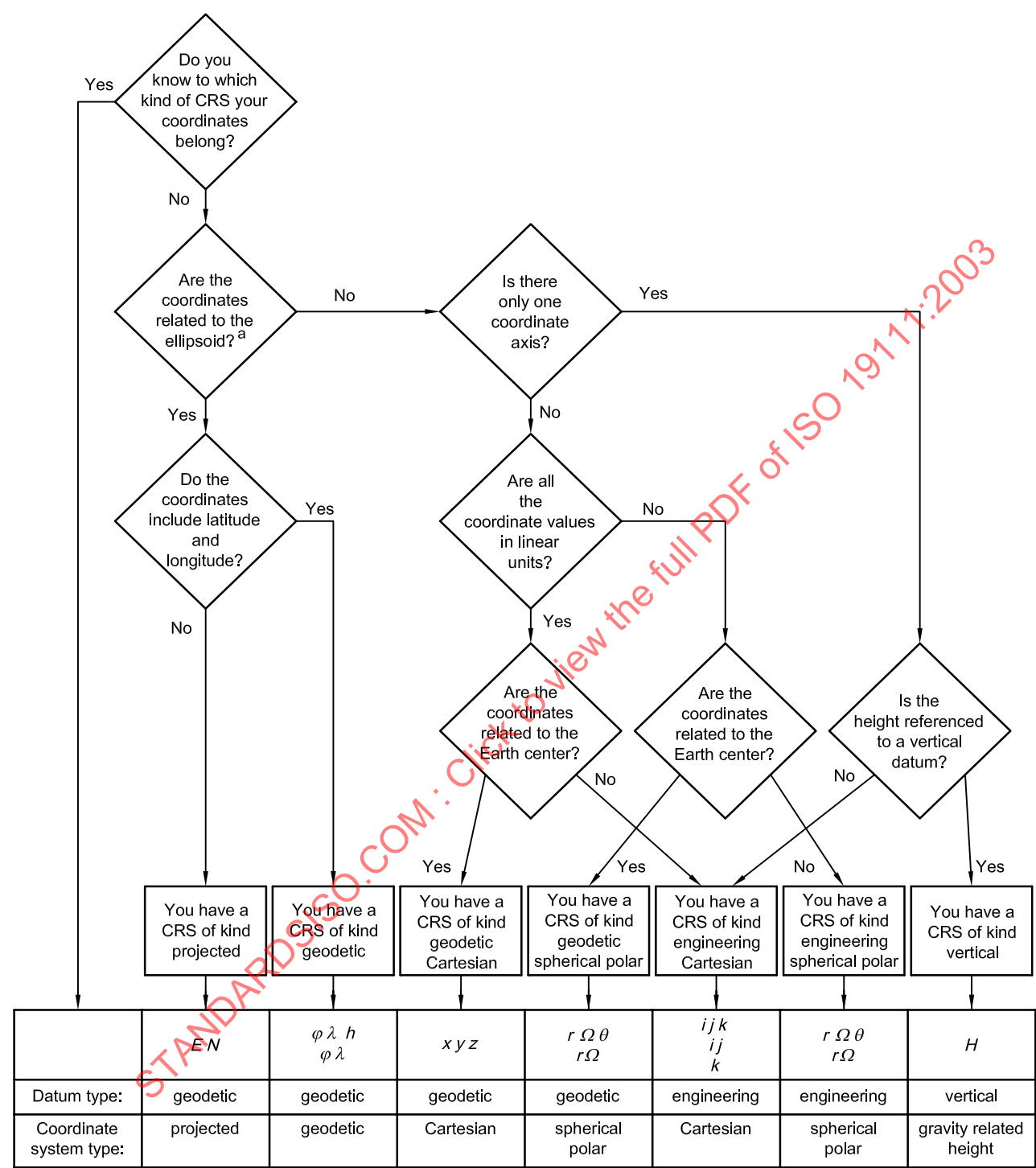


Figure C.1 — Decision tree 1 — kind of coordinate reference system

C.2 Decision tree 2 (coordinate reference system type)



^a NOTE This includes projected coordinates, for example UTM coordinates.

Figure C.2 — Decision tree 2 — type of coordinate reference system

Annex D (informative)

Geodetic relationships

D.1 Coordinate reference system — coordinate conversion and coordinate transformation

Figure D.1 shows three different coordinate reference systems:

- Coordinate reference system 1 (datum 1, coordinate system A),
- Coordinate reference system 2 (datum 1, coordinate system B),
- Coordinate reference system 3 (datum 2, coordinate system A).

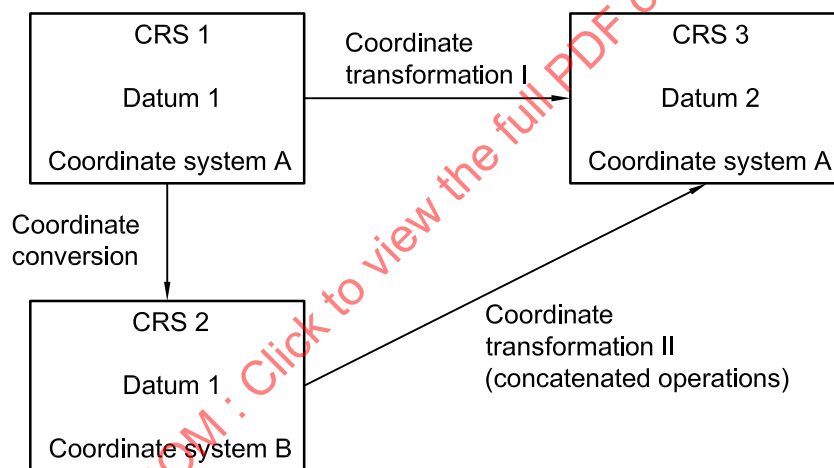


Figure D.1 — Coordinate conversion and coordinate transformation

A coordinate conversion is used to change from coordinate reference system 1 to coordinate reference system 2, both of which are based on datum 1. A coordinate transformation is used to change from coordinate reference system 1 to coordinate reference system 3 which both use the same coordinate system but are based on different datums. A different coordinate transformation is used to change values directly from coordinate reference system 2 to coordinate reference system 3 which again are both based on different datums. The change from coordinate reference system 2 to coordinate reference system 3 may also be described as a concatenation coordinate operation consisting of the coordinate conversion plus coordinate transformation 1.

D.2 Coordinate conversion — relationship between ellipsoidal and Cartesian coordinates

To determine the position of a point in space, a three-dimensional coordinate reference system is necessary. Each reference system can be mapped into infinitely many curvilinear coordinate systems. Coordinate systems establish the ordered relation between physical points in space and real numbers (coordinates).

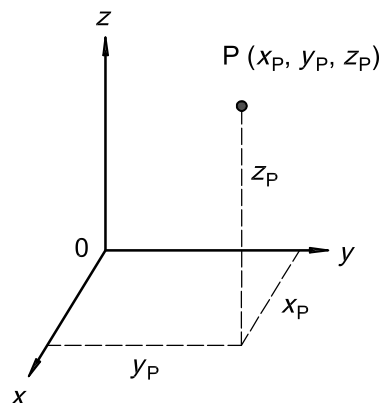
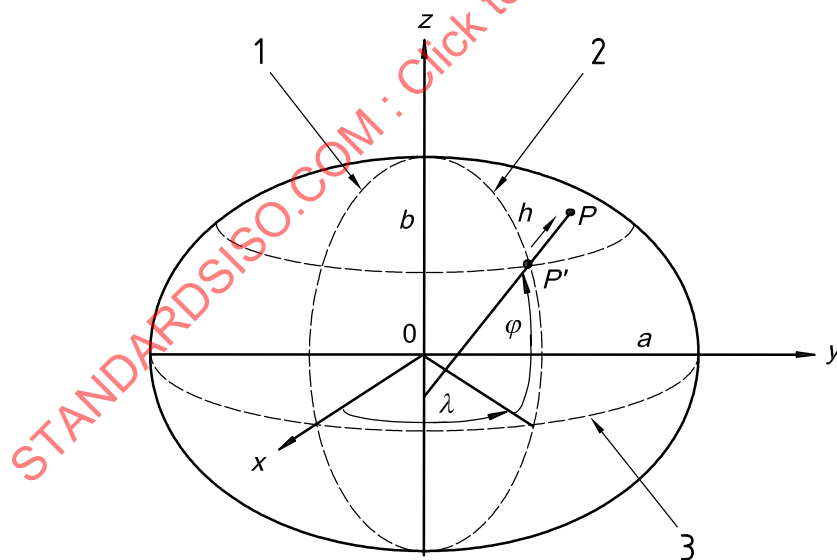


Figure D.2 — Cartesian coordinates

In modern three-dimensional geodesy, the three-dimensional Cartesian coordinate system is applied for global tasks. It is defined by three orthogonal coordinate axes which form a right-handed system. The x , y , z coordinate axes intersect each other at the origin of the coordinate system, see Figure D.2.

The coordinate lines of the ellipsoidal coordinate system are curvilinear lines on the surface of the ellipsoid, see Figure D.3. They are called parallels at constant latitude (φ) and meridians for constant longitude (λ).

When the ellipsoid is related to the shape of the Earth, the ellipsoidal coordinates are named geodetic coordinates. Traditionally an alternative to geodetic coordinates have been astronomical latitude and astronomical longitude. Geographic coordinates is the generic term including astronomical latitude and astronomical longitude as well as geodetic latitude and geodetic longitude. Since astronomical observations have been taken over by satellite based methods, in most cases today the term geographic coordinates implies geodetic coordinates.

**Key**

- 1 zero meridian
- 2 local meridian
- 3 equator

Figure D.3 — Cartesian coordinates and ellipsoidal coordinates

If the origin of a right-handed Cartesian coordinate system coincides with the centre of the ellipsoid, the Cartesian z -axis coincides with the axis of rotation of the ellipsoid and the positive x -axis passes through the point $\varphi = 0$, $\lambda = 0$, then the following formula converts ellipsoidal coordinates to geocentric Cartesian coordinates:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} [N + h] \cos \varphi \cos \lambda \\ [N + h] \cos \varphi \sin \lambda \\ [N(1 - e^2) + h] \sin \varphi \end{bmatrix}$$

with the radius of curvature in the prime vertical (perpendicular to the meridian)

$$N = a(1 - e^2 \sin^2 \varphi)^{-1/2}$$

and the first numerical eccentricity of the ellipsoid

$$e = (2f - f^2)^{1/2} \text{ where } f = (a - b) / b \text{ is flattening of the ellipsoid.}$$

The following method converts geocentric Cartesian coordinates to ellipsoidal coordinates:

$$\lambda = \arctan \frac{y}{x}$$

(λ is outside the range -90 degrees to $+90$ degrees if x is negative)

and

$$\varphi_0 = \arctan \frac{z}{(1 - e^2)(x^2 + y^2)^{1/2}}$$

Solve for φ and h by iteration through

$$N_i = a(1 - e^2 \sin^2 \varphi_{i-1})^{-1/2}$$

$$h_i = \frac{(x^2 + y^2)^{1/2}}{\cos \varphi_{i-1}} - N_i \text{ for } |\varphi_0| < 45^\circ \text{ and } h_i = \frac{z}{\sin \varphi_{i-1}} - (1 - e^2)N_i \text{ for } |\varphi_0| \geq 45^\circ$$

$$\varphi_i = \arctan \left[\frac{z}{(x^2 + y^2)^{1/2}} \cdot \frac{1}{1 - \frac{e^2 N_i}{N_i + h_i}} \right]$$

D.3 Coordinate transformation

In general, the parameters of coordinate transformations are determined from coordinates of sets of points which belong to the realization of both coordinate reference systems. In general, they are approximate, and their validity may be restricted to a particular region.

The most widely used coordinate transformations are similarity transformations, where the two coordinate reference systems differ only by their position and orientation in space and by their scale.

The similarity transformation is conformal. It can be performed on Cartesian coordinates as well as on ellipsoidal coordinates.

Although some similarity transformations from source datum S to target datum T use only three parameters (T_1, T_2, T_3), the generic seven-parameter formula takes the following form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{(T)} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{(S)} + \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} 0 & -R_3 & R_2 \\ R_3 & 0 & -R_1 \\ -R_2 & R_1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{(S)} + D \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{(S)} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} 1+D & -R_3 & R_2 \\ R_3 & 1+D & -R_1 \\ -R_2 & R_1 & 1+D \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{(S)}$$

As the coordinate transformation equation is the result of an approximation of a strict formula, the rotations R_1, R_2, R_3 must be small.

The value of the three translations T_1, T_2, T_3 along the coordinates axes x, y, z , the three rotations R_1, R_2, R_3 , and the scale correction D are established by minimizing the residuals between the coordinates of identical points in coordinate reference systems (S) and (T). Identical points means identical in space and time. Within the similarity transformation, movements of points are neglected; these movements are reflected in the residual deviations after the adjustment. The relation of Cartesian coordinates used in the coordinate transformation to ellipsoidal coordinates is given by formulas in Clause D.2.

A special case of a coordinate transformation is a longitude shift. This occurs between datums which are identical except for their prime meridians. One example is the longitude shift between the Carthage datum with prime meridian Paris and the Carthage datum with prime meridian Greenwich.

D.4 Coordinate conversion — map projection

The geodetic coordinates (φ, λ, h) refer to the curved surface of the ellipsoid. For mapping purposes, the necessary projection onto the plane (see Figure D.4) can be performed by different projection methods. Doubly curved surfaces are always distorted when mapped onto a plane. Mappings of ellipsoid or sphere into the plane can be isogonal (conformal) or area-preserving. Length-preserving mapping, where scale is the same in all directions, is not possible. A map projection is a mathematical mapping of an ellipsoid, or part of an ellipsoid, to a plane. A projection will have an associated reference point called the projection origin for the region being mapped. This will be the origin "O" of the reference frame in the plane.

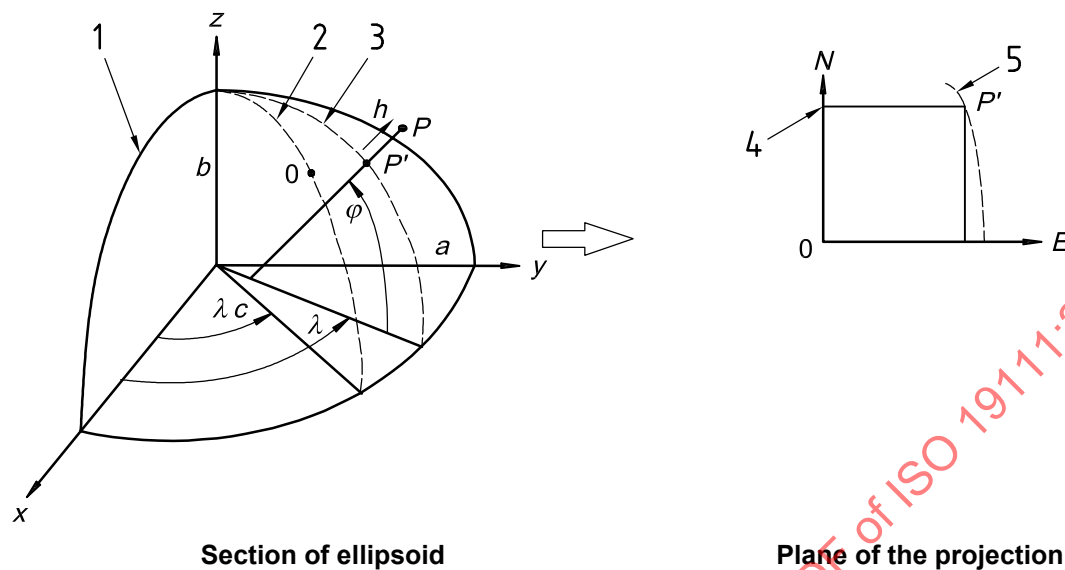
Mathematically, if a point P' is identified by its geographic coordinates (φ, λ) on the ellipsoid and by its Cartesian coordinates (N, E) in an orthogonal reference frame (O, N, E) of this plane, a map projection is defined by two functions f and g such that

$$N = f(\varphi, \lambda)$$

$$E = g(\varphi, \lambda)$$

It can be deduced that the position of any point P in space can be represented by the three-dimensional coordinates (N, E, h) , based on a datum and a map projection.

Let us consider N as northing and E as easting, which are directly available only for either direct or transverse map projections. In the special case of an oblique map projection, there are initial plane coordinates (x, y) which are related to (E, N) by rotation.



Key

- 1 zero meridian
- 2 central meridian
- 3 local meridian
- 4 mapping of the central meridian
- 5 mapping of the local meridian

Figure D.4 — Map projection

D.5 The geoid and heights

The Earth's gravity field is expressed through its gravity potential W which includes both gravitational effect and centrifugal effect. The gravity vector is defined by

$$\vec{s} = \text{grad} W$$

The modulus of $\vec{g}$ is the gravity, g , and its spherical directions, φ (astronomical latitude), and λ (astronomical longitude), are the astronomical coordinates with respect to the conventional terrestrial reference system (CTS).

The geoid is an equipotential surface of the Earth's gravity field which globally approximates mean sea level. In practice, the expression "closely approximates mean sea level" allows consideration of slightly different equipotential surfaces as geoid models.

Due to multiple geophysical phenomena, the geoid is not an ellipsoid. It cannot be expressed by any simple mathematical description.

Let the geopotential number c_P be the difference in gravity potential between the geoid and the equipotential surface of the point P:

$$c_P = W_{\text{geoid}} - W_P$$

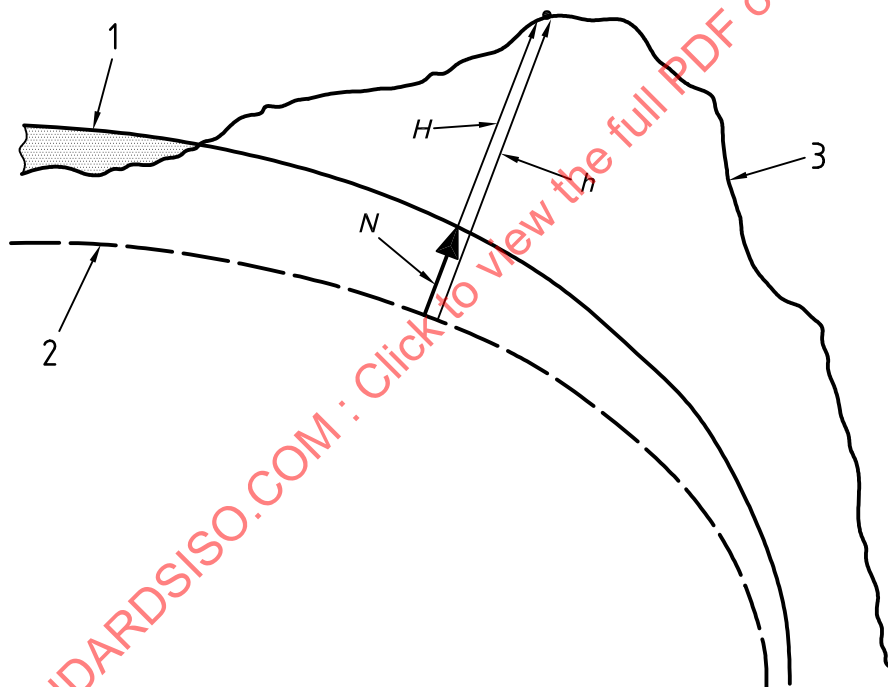
where c_p is expressed in square metres per square second.

Height is a number expressing the distance between a point P and a horizontal reference surface, see Figure D.5. In the definition of the ellipsoidal height h , the reference surface is an ellipsoid. But the heights in geographic information systems over continents uses the geoid as reference surface. More precisely, heights are defined using the geopotential number c_P according to the three following choices:

- orthometric height $H_o = c_P / \bar{g}$, where $\bar{g}$ is the mean gravity value along the plumb line of the Earth gravity field between the point and the geoid;
- normal height $H_n = c_P / \bar{\gamma}$, where $\bar{\gamma}$ is the mean gravity value along the plumb line of the normal gravity field between the ellipsoid and the point where the normal potential is equal to the actual potential at the calculation point;
- dynamic height $H_d = c_P / g_o$, where g_o is an arbitrarily-agreed gravity value. g_o can be a normal gravity value in some standard latitude (in general, it is 45° or the average latitude of the surveying area). The dynamic height is closely related to the geopotential number.

Orthometric heights and normal heights are gravity-related heights.

NOTE In some countries where sufficient gravity data are not available, normal orthometric height is used as an approximation of orthometric height.



Key

- 1 geoid
- 2 ellipsoid
- 3 surface of the Earth

h = ellipsoidal height, measured from ellipsoid along perpendicular passing through point; $h = H + N$

H = gravity-related height, measured along direction of gravity from vertical datum plane at geoid

N = geoid height, height of geoid above ellipsoid

Figure D.5 — Ellipsoidal and gravity-related heights