
**Railway applications — Calculation
of braking performance (stopping,
slowing and stationary braking) —**

**Part 2:
General algorithms utilizing step by
step calculation**

*Applications ferroviaires — Calcul des performances de freinage
(freinage d'arrêt, de ralentissement et d'immobilisation) —*

Partie 2: Algorithmes généraux utilisant le calcul pas à pas



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 269, *Railway applications*, Subcommittee SC 2, *Rolling stock*.

A list of all parts in the ISO 20138 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document describes methodologies for calculation of braking performance such as stopping distance, deceleration, power and energy for railway rolling stock.

The objective of this document is to enable the railway industry and operators to work with common calculation methods.

The ISO 20138 series consists of two parts (ISO 20138-1 and this document) which complement each other.

This document describes the step by step calculation methods for railway applications applicable to all countries. In addition, the algorithms provide a means of comparing the results of other braking performance calculation methods.

The methodology of step by step calculation is based on numerical time integration.

The step by step calculation method cannot be used for stationary braking. This document considers an example for stationary braking of a multiple unit in accordance with ISO 20138-1.

When calculating stopping and slowing distances using the step by step calculation method, it is intended that both ISO 20138-1 and this document be considered.

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Railway applications — Calculation of braking performance (stopping, slowing and stationary braking) —

Part 2: General algorithms utilizing step by step calculation

1 Scope

This document specifies the methodologies for calculation of braking performance for railway rolling stock.

This document describes the general algorithms/formulae using instantaneous value inputs to perform calculations of brake equipment and braking performance, in terms of stopping/slowing distances, braking power and energy for all types of rolling stock, either as vehicles or units.

The calculations can be performed at any stage of the assessment process (design, manufacture, testing, verification, investigation, etc.) of railway rolling stock. This document does not set out specific acceptance criteria (pass/fail).

This document is not intended to be used as a design guide for the selection of brake systems and does not specify performance requirements. This document does not provide a method to calculate the extension of stopping distances when the level of demanded adhesion exceeds the available adhesion (wheel slide activity).

This document contains examples of the calculation of brake forces for different brake equipment types and examples of the calculation of stopping distance for vehicles or units.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 20138-1:2018, *Railway applications — Calculation of braking performance (stopping, slowing and stationary braking) — Part 1: General algorithms utilizing mean value calculation*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 20138-1 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

3.1

slowing distance

s

distance run between the initial brake demand and achieving the final speed v_{fin}

Note 1 to entry: When the final speed $v_{fin} = 0$ m/s, slowing distance is also known as stopping distance.

3.2 slowing time

t

elapsed time from the initial brake demand until the final speed v_{fin} is reached

Note 1 to entry: When the final speed $v_{\text{fin}} = 0$ m/s, slowing time is also known as stopping time.

4 Symbols

For the purposes of this document, the general symbols given in [Table 1](#) and ISO 20138-1:2018, Table 1 apply.

Table 1 — Symbols

Symbol	Definition	Unit
a	Instantaneous deceleration of the vehicle/unit	m/s ²
$a_{f(t) = 100 \%}$	Deceleration during each chosen time step	m/s ²
a_j	Constant deceleration during iteration step j	m/s ²
D_{max}	Wheel diameter max.	m
D_{min}	Wheel diameter min.	m
$F_{\text{B,ax,st}}$	Stationary brake force acting on that wheelset	N
$F_{\text{pad},i}$	Force acting on single disc surface (i is an index used for sorting)	N
$F_{\text{r},n}$	Instantaneous retarding force of brake equipment type n	N
$F_{\text{r},n,j}$	Instantaneous retarding force for brake equipment type n during iteration step j	N
$F_{\text{r,nom}}$	Nominal retarding force	N
$f(t)$	Factor dependent on time	—
$f(t) = 100 \%$	Index for 100 % applied braking force without consideration of any time characteristics	—
$f(v)$	Factor dependent on speed	—
$f(x)$	Factor (common characteristic) dependent on another variable x	—
i_{tra}	Transmission ratio	—
j	Iteration step number	—
P_n	Instantaneous braking power of brake equipment type n	W
s_j	Distance travelled from brake command at time t_0 to time t_j	m
$s_{n,j}$	Distance travelled during iteration step j whilst the brake equipment type n is applied	m
$s_{\text{ref}}(\Delta t)$	Stopping/slowng distance, calculated with time step Δt	m
$s_{\text{comp}}(2 \cdot \Delta t)$	Stopping/slowng distance, calculated with doubled time step ($2 \cdot \Delta t$)	m
$s_{f(t) = 100 \%}$	Braking distance without consideration of any time characteristics from initial speed v_0 to final speed v_{fin}	m
t	Slowing time/stopping time	s
t_j	Elapsed time from brake command to iteration step j	s
Δt	Time step	s
v	Current speed	m/s
v_j	Speed at time t_j	m/s
$v_{1,\text{ECB}}$	Deactivating speed of eddy current brake	m/s
ε	Speed deviation from v_{fin}	m/s
μ	Coefficient of friction (brake pad or block)	—
ξ	Relative distance deviation	%

Table 1 (continued)

Symbol	Definition	Unit
τ_{ax}	Value of the instantaneous adhesion required between wheel and rail for the braked wheelset	—
τ_{req}	Required wheel/rail adhesion	—

5 General explanation of step-by-step calculation

5.1 Method

The step-by-step method is used when it is not appropriate or desirable to represent the non-constant retarding and braking forces by mean values. Further details of when the use of mean value calculations is appropriate are given in ISO 20138-1.

Time steps are defined in such a way that the braking forces can be considered as constant throughout each step's duration. The duration of each step can depend on changes in the braking force and is not necessarily fixed (i.e. algorithms can be based either on constant or adaptive time step integration schemes). Each time step is characterised by an initial state and a final state, e.g. an initial and a final speed.

For each time step, the distance travelled during that time step as well as the final speed are calculated and the deceleration at the end of that time step is calculated according to Newton's laws. The outputs of the calculations for each time step are used as inputs to the calculations for each subsequent time step.

The calculation shall be done in accordance with the workflow as shown in [Annex A](#).

5.2 Retarding force models

Mathematical models for common brake systems (e.g. magnetic track brakes, electrodynamic brakes, etc.) are described in [Annex B](#). The mathematical models for disc brakes, tread brakes and external deceleration forces (e.g. wind forces, running resistance) are described in ISO 20138-1.

The impact of time, speed, load, temperature, etc. on the nominal retarding force can also be expressed in terms of dimensionless factors (functions), e.g. time dependency $f(t)$, speed dependency $f(v)$, etc. Thus, any deceleration force characteristics due to brake system applications or acting external forces (e.g. wind forces) can be modelled.

These dimensionless factors can take effect at the same time and are thus superposed by multiplication as set out in [Formula \(1\)](#).

$$F_r = F_{r,nom} \cdot f(t) \cdot f(v) \cdot \dots \cdot f(x) \quad (1)$$

where

- F_r is the instantaneous retarding force acting at the rail generated by the brake equipment, expressed in N;
- $F_{r,nom}$ is the nominal retarding force, expressed in N;
- $f(t)$ is the factor dependent on time;
- $f(v)$ is the factor dependent on speed;
- $f(x)$ is the factor (common characteristic) dependent on another variable x .

NOTE For nominal retarding forces $F_{r,nom}$, the factors $f(t)$, $f(v)$ and $f(x)$ are equal to 1.

5.3 Algorithm

5.3.1 General description

Instantaneous values are the input data for step-by-step (iterative) calculation. The workflow of [Figure A.1](#) shall be used for performing stopping and slowing calculations.

The numerical integration is time-based.

Every calculation begins with the initial brake demand and the initial vehicle/unit speed.

The initial time step begins at time $t_0 = 0$ s simultaneously with the start of the braking demand. The braking forces which are acting in the initial time step are calculated.

The result of the first iteration step refers to $j = 1$, i.e. $v_1 = v_0 - a_0 \Delta t$, whereas initial values (e.g. initial speed) refer to index $j = 0$.

The vehicle/unit speed at the end of the time step and the distance travelled during this time step are calculated.

The value of the selected parameter (e.g. speed, distance) at the end of the time step is compared with its target value.

If the target value has not been reached, the calculations are repeated for the next time step.

The time step calculation continues until the target value is reached.

5.3.2 Time integration

The time integration should continue until the calculated value of the selected parameter (e.g. speed) is considered equal to the target value of that parameter, i.e. when the condition given in [Formula \(2\)](#) is achieved (where speed is used as an example selected parameter):

$$|v_j - v_{fin}| < \varepsilon \quad (2)$$

where

- v_j is the speed at time t_j , expressed in m/s;
- v_{fin} is the final speed, expressed in m/s;
- ε is the speed deviation from v_{fin} , expressed in m/s.

A speed deviation not greater than 10^{-3} m/s is considered as suitable for high speed train calculations. For lower speeds or slowing calculations, other values may be used.

Based on the calculation of retarding forces and external forces, the constant deceleration a_j during iteration step j can be calculated as set out in [Formula \(3\)](#):

$$a_j = \frac{(\sum F_{r,n} + \sum F_{\text{ext}})_j}{m_{\text{dyn}}} \quad (3)$$

where

j is the iteration step number;

a_j is the constant deceleration during iteration step j , expressed in m/s^2 ;

$F_{r,n}$ is the instantaneous retarding force of brake equipment type n , expressed in N;

F_{ext} is the external force, expressed in N;

m_{dyn} is the dynamic mass, expressed in kg.

If the target value of the selected parameter has not been achieved, the next time step integration is conducted, utilising the outputs of the preceding step, as shown in [Formulae \(4\) to \(8\)](#):

$$\text{Speed at start of step } t_{j+1} \quad v_{j+1} = v_j - a_j \cdot \Delta t \quad (4)$$

$$\text{Distance at start of step } t_{j+1} \quad s_{j+1} = s_j + v_j \cdot \Delta t - \frac{1}{2} a_j \cdot \Delta t^2 \quad (5)$$

$$\text{Deceleration during step } t_{j+1} \quad a_{j+1} = \frac{(\sum F_{r,n} + \sum F_{\text{ext}})_{j+1}}{m_{\text{dyn}}} \quad (6)$$

$$\text{Next time step} \quad t_{j+1} = t_j + \Delta t \quad (7)$$

$$\text{Next time increment} \quad j \rightarrow j+1 \quad (8)$$

where

a_j is the constant deceleration during iteration step j , expressed in m/s^2 ;

$F_{r,n}$ is the instantaneous retarding force of brake equipment type n , expressed in N;

F_{ext} is the external force, expressed in N (for decelerating force **positive** value, for accelerating force **negative** value);

j is the iteration step number;

m_{dyn} is the dynamic mass, expressed in kg;

s_j is the distance travelled from brake command at time t_0 to time t_j , expressed in m;

t_j is the elapsed time from brake command to iteration step j , expressed in s;

Δt is the time step, expressed in s.

The final time step sometimes needs to be adjusted, if necessary, to meet the target value of the selected parameter (see [5.3.1](#)).

Other more detailed algorithms may be used if considered necessary.

5.3.3 Determination of time step/relative distance deviation ξ

The relative distance deviation ξ has to be calculated if the applied integration procedure imposes constant time steps. If an adaptive time integration is used, the requirements in this clause are not applicable.

The time step Δt shall be chosen in such a way that the relative distance deviation is not greater than the minimum precision required. The relative distance deviation ξ is obtained by two separate integrations. The original calculation with time step Δt determines the reference stopping/slowness distance $s_{\text{ref}(\Delta t)}$ and the second integration with doubled time step $2 \cdot \Delta t$ determines a new stopping/slowness distance $s_{\text{comp}(2 \cdot \Delta t)}$ for comparison. The relative distance deviation ξ is calculated in accordance with [Formula \(9\)](#) and shall not be greater than the minimum precision required.

The value of the relative distance deviation ξ shall not exceed a predefined limit value and can be calculated as set out in [Formula \(9\)](#):

$$\xi = \left| \frac{s_{\text{comp}(2 \cdot \Delta t)} - s_{\text{ref}(\Delta t)}}{s_{\text{ref}(\Delta t)}} \right| \cdot 100 \quad (9)$$

where

- ξ is the relative distance deviation, expressed in %;
- $s_{\text{ref}(\Delta t)}$ is the stopping/slowness distance, calculated with time step Δt , expressed in m;
- $s_{\text{comp}(2 \cdot \Delta t)}$ is the stopping/slowness distance, calculated with doubled time step $(2 \cdot \Delta t)$, expressed in m.

Usually, a relative distance deviation of $\xi \leq 0,1\%$ is considered as acceptable. For low speeds and slowing calculations, greater values of deviation ratio may be used.

NOTE The definition of validation requirements of any numerical integration procedure is outside the scope of this document.

5.3.4 Equivalent system response time t_e

The calculation of equivalent system response time allows the assumption that braking consists first of a "free running time" with braking force equal to zero, followed by a braking time with fully applied braking force. ISO 20138-1 describes the equivalent response time when considering the free running time.

The equivalent system response time t_e based on stopping and braking distance shall be calculated with two separate time integrations:

- a) the stopping/slowness distance calculated taking into account the time characteristics of each acting brake equipment type starting at time $t_0 = 0$ s simultaneously with the start of the braking demand until achieving the final speed v_{fin} ;
- b) the stopping/slowness distance calculated assuming each acting brake equipment type fully applies (100 %) at time $t_0 = 0$ s simultaneously with the start of the braking demand until achieving the final speed v_{fin} .

The equivalent system response time can be calculated as set out in [Formula \(10\)](#).

$$t_e = \frac{s - s_{f(t)=100\%}}{v_0} \quad (10)$$

where

- v_0 is initial speed, in m/s;
- s is the stopping/slowng distance with all time characteristics taken into account, expressed in m;
- $S_{f(t) = 100 \%}$ is the braking distance without consideration of any time characteristics from initial speed v_0 to final speed v_{fin} .

5.4 Supplementary dynamic calculations

5.4.1 Energy dissipated by each brake equipment type

ISO 20138-1 describes the calculation of energy dissipated during braking based on mean retarding forces.

The total energy dissipated by each brake equipment type during iteration steps $j = 0$ to $j = J$ can be calculated based on instantaneous values as set out in [Formula \(11\)](#).

$$W_{B,n} = \sum_{j=0}^J (F_{r,n,j} \cdot s_{n,j}) \quad (11)$$

where

- $W_{B,n}$ is the energy dissipated by brake equipment type n , expressed in J;
- $F_{r,n,j}$ is the instantaneous retarding force for brake equipment type n during iteration step j , expressed in N;
- $s_{n,j}$ is the distance travelled during iteration step j whilst the brake equipment type n is applied, expressed in m.

5.4.2 Value of the instantaneous adhesion required between wheel and rail for the braked wheelset (τ_{ax})

The value of the instantaneous adhesion required between wheel and rail for the braked wheelset can be calculated as set out in [Formula \(12\)](#).

$$\tau_{ax} = \frac{\sum_{n=1}^N F_{r,n} - m_{rot,ax} \cdot a}{m_{st,ax} \cdot g} \cdot \sqrt{i^2 + 1} \quad (12)$$

where

- τ_{ax} is the value of the instantaneous adhesion required between wheel and rail for the braked wheelset;
- N is the number of brake equipment types;
- $\sum_{n=1}^N F_{r,n}$ is the sum of all adhesion dependent retarding forces from all brake equipment types per wheelset, expressed in N;
- $F_{r,n}$ is the instantaneous retarding force of brake equipment type n , expressed in N;
- a is the instantaneous deceleration of the vehicle/unit, expressed in m/s²;
- g is the standard acceleration due to gravity, expressed in m/s²;

- i is the gradient of the track (positive rising/negative falling);
- $m_{\text{rot,ax}}$ is the equivalent rotating mass per wheelset, expressed in kg;
- $m_{\text{st,ax}}$ is the static mass per wheelset, expressed in kg.

5.4.3 Maximum braking power of each brake equipment type

The step by step calculation can be used to determine the speed v when the maximum braking power $P_{\text{max},n}$ for each brake equipment type n is reached.

The instantaneous braking power P_n can be calculated as set out in [Formula \(13\)](#).

$$P_n = F_{r,n} \cdot v \quad (13)$$

The maximum braking power can be calculated as set out in [Formula \(14\)](#).

$$P_{\text{max},n} = \max(P_n) \quad (14)$$

where

- P_n is the instantaneous braking power of brake equipment type n , expressed in W;
- $P_{\text{max},n}$ is the maximum braking power of brake equipment type n , expressed in W;
- $F_{r,n}$ is the instantaneous retarding force of brake equipment type n , expressed in N;
- v is the current speed, expressed in m/s.

The maximum braking power of a disc brake can also be calculated using the pad forces and the peripheral speed at the effective point of contact on the brake disc.

6 Considerations for stopping/slowing distances and deceleration calculations

6.1 Accuracy of input values

The accuracy of the calculation described here depends directly on the accuracy of the input data.

The accuracy of the input data values shall be relevant to the purpose of the calculation and shall be traceable as to how these values were established, e.g. engineer's estimation, test results, manufacturer's data. Supporting calculations or test reports (or extracts of these documents) should be attached with the performance calculation where applicable.

Representative curves of the performance of a type of brake equipment, e.g. electrodynamic brake, can be determined by numerical or practical methods. The values can be given as a table.

6.2 Distance calculations

The calculated stopping or slowing distance s is obtained by conducting the time step calculation (see [5.3](#)).

6.3 General characteristics

Descriptions of general characteristics, e.g. train formation, train mass, static and equivalent rotating mass, dynamic mass and wheel diameter, are given in ISO 20138-1, if not otherwise specified in [6.4](#) and [6.5](#).

6.4 Brake equipment type characteristics

6.4.1 General

The brake equipment types are described in ISO 20138-1.

Except where alternative formulae are set out in this document, the formulae in ISO 20138-1 can be used for mean value and step by step calculation.

6.4.2 Input data

The values of the input data can be given as a function, table, single value, chart (see [Annex C](#)).

6.5 Initial and operating characteristics

6.5.1 Nominal conditions

In general, braking performance calculations are based on the assumption of a straight and level track, and dry and nominal vehicle/unit and rail conditions.

6.5.2 Wheel diameter

The maximum wheel diameter shall be used for all calculations, except for calculations of maximum wheel and rail adhesion utilization.

For calculation of maximum wheel and rail adhesion utilization, the minimum wheel diameter and — if no load correction is acting — the minimum vehicle loading condition shall be used.

For vehicles equipped only with brake equipment of the type tread brake, the variance of the wheel diameter is not relevant for the calculation of maximum wheel and rail adhesion utilization.

6.5.3 Initial speed

The initial speed shall be set to the maximum design speed. Calculations using other operational speeds shall be performed as required.

6.5.4 Gradient

The formulae for calculation of the downhill force due to gravity depending on the gradient (ISO 20138-1:2018, 5.6.2) shall be used replacing the mean value-forces by instantaneous forces.

The effect of the gradient has influence on the downhill force. The gradient can change during the brake application. In such cases, the downhill force shall be recalculated.

As an alternative, the influence of the gradient on the braking force may be calculated using a factor as described in [5.2](#).

6.5.5 Level of the brake demand

If step by step calculations are performed, typically the brake demand level used shall be the emergency level. This method can also be used for other levels of brake demand, e.g. full service brake, service brake.

6.5.6 Degraded mode

Degraded mode means operation with specified quantities of isolated, non-active or non-functional brake equipment.

This calculation method can be used to calculate brake performance when operating in degraded modes.

6.5.7 Degraded condition

Degraded condition means reduced friction coefficient due to environmental conditions, pollution and/or lower wheel and rail adhesion.

Generally, brake calculations are performed with nominal parameters of the brake equipment in use. The influence of degraded conditions on brake performance can be calculated with this step by step calculation method.

NOTE The determination of degraded conditions is outside the scope of this document.

6.5.8 Available coefficient of wheel and rail adhesion

If the required wheel and rail adhesion exceeds the available adhesion, it can lead to an increase of the stopping distance compared to a theoretical calculation as a consequence of a sliding wheelset or regulation by the wheel slide protection device.

The required wheel and rail adhesion of each wheelset, calculated as set out in ISO 20138-1:2018, 5.6.4, shall be lower than the assumed or specified available wheel and rail adhesion. This available coefficient of wheel and rail adhesion is dependent on the conditions prevalent at the time of braking e.g. sanding, speed, environmental conditions, number of axles, etc.

6.6 Other deceleration calculations

6.6.1 General

By convention, deceleration is considered as a positive value.

6.6.2 Decelerations resulting from the force generated by each brake equipment type ($a_{j,n}$)

Refer to ISO 20138-1 for a general description of this calculation.

For a step-by-step calculation, average values of braking forces during the time step j should be used to obtain the average deceleration $a_{j,n}$ provided by equipment n .

6.6.3 Equivalent (mean) deceleration (a_e) based on distance

The equivalent deceleration is equal to a mean deceleration with respect to the distance during braking over a specific speed range. The deceleration a_e is based on a calculation with a fully applied braking force, as given in [Formula \(15\)](#):

$$a_e = \frac{1}{s_{f(t)=100\%}} \int_0^{s_{f(t)=100\%}} a_{f(t)=100\%} ds = \frac{v_0^2 - v_{fin}^2}{2 \cdot s_{f(t)=100\%}} \quad (15)$$

where

a_e is the equivalent (mean) deceleration, expressed in m/s^2 ;

$s_{f(t)=100\%}$ is the braking distance without consideration of any time characteristics from initial speed v_0 to final speed v_{fin} , expressed in m ;

$a_{f(t)=100\%}$ is the deceleration during a time step without consideration of any time characteristics, expressed in m/s^2 ;

v_0	is the initial speed, expressed in m/s;
v_{fin}	is the final speed, expressed in m/s;
$f(t)=100\%$	is the index for 100 % applied braking force without consideration of any time characteristics.

7 Immobilization brake calculation

The step-by-step method is not appropriate for immobilization brake calculations. Refer to ISO 20138-1 for immobilization brake calculations.

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Annex A (normative)

Workflow of kinetic calculations

Figure A.1 describes the workflow for conducting stopping and slowing calculations.

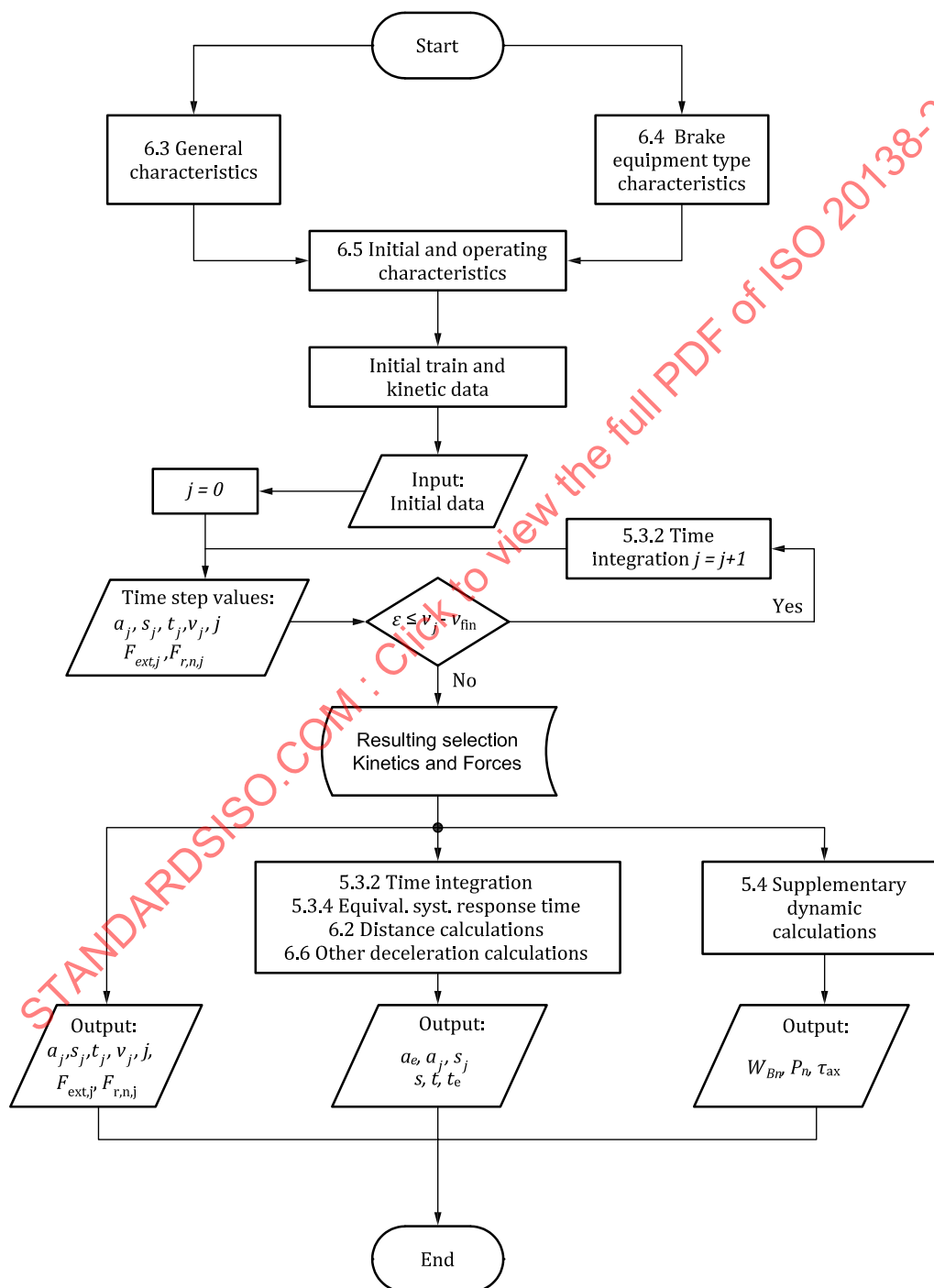


Figure A.1 — Calculation flow diagram for stopping and slowing

Annex B (informative)

Calculation of retarding forces (non-stationary)

B.1 Retarding force of magnetic track brake

The retarding force generated by one magnet of a magnetic track brake F_{rMg} is represented by a characteristic curve showing the retarding force vs. the speed (see [Figure B.1](#)).

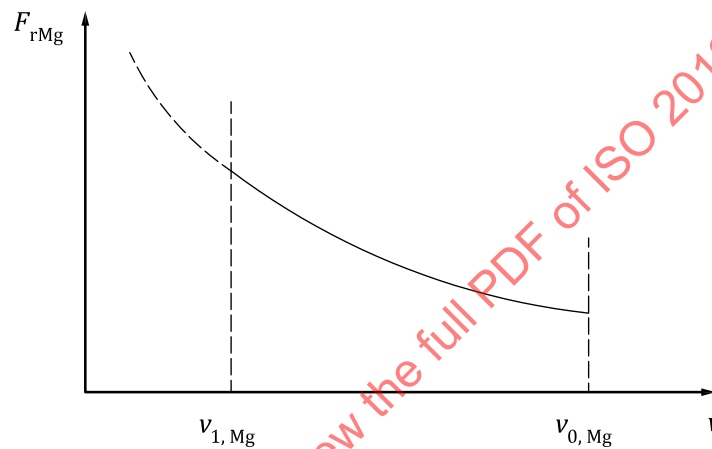


Figure B.1 — Characteristics of the retarding force of one magnet of a magnetic track brake vs. speed

The instantaneous friction coefficient of one magnet (pole shoe) μ_{Mg} can be calculated as set out in [Formula \(B.1\)](#).

$$\mu_{Mg} = \frac{1}{k_1 \cdot v + k_0} \quad (B.1)$$

The retarding force of one magnet F_{rMg} can be calculated as set out in [Formula \(B.2\)](#).

$$F_{rMg} = F_{AMg} \cdot \mu_{Mg} \quad (B.2)$$

where

- F_{rMg} is the retarding force of one magnet, expressed in N;
- F_{AMg} is the attraction force of one magnet, expressed in N;
- μ_{Mg} is the instantaneous friction coefficient of magnet (pole shoe);
- v is the current speed, expressed in m/s;
- $v_{0, Mg}$ is the activating speed of magnetic track brake, expressed in m/s;

$v_{1,Mg}$ is the deactivating speed of magnetic track brake, expressed in m/s;

k_0 is the coefficient (provided by the supplier);

k_1 is the coefficient (provided by the supplier), expressed in s/m.

NOTE The total retarding force of all magnets in a vehicle $F_{rMg,tot}$ is calculated as set out in ISO 20138-1.

B.2 Retarding force of linear eddy current brake

The retarding force generated by eddy current brake F_{rECB} is represented by a characteristic curve showing the retarding force vs. the speed (see [Figure B.2](#)).

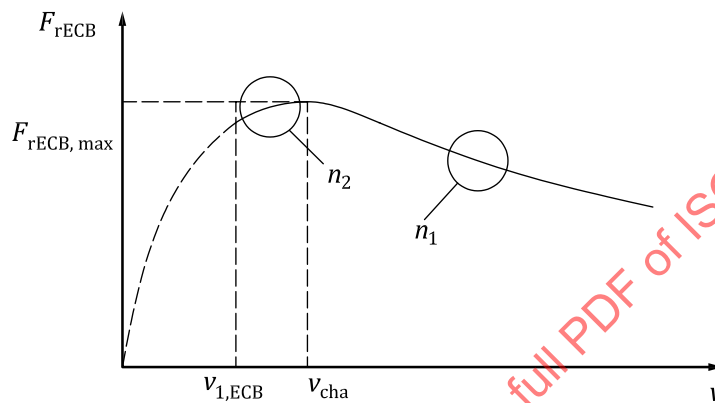


Figure B.2 — Characteristics of the eddy current retarding force

The eddy current brake is used for speeds higher than the deactivating speed $v_{1,ECB}$ (see [Figure B.2](#)).

The characteristic speed v_{cha} is the speed at which the eddy current brake generates the maximum retarding force. The instantaneous retarding force of eddy current brake can be calculated as set out in [Formula \(B.3\)](#).

$$F_{rECB} = F_{rECB,max} \cdot [1 - k_2 \cdot (v - v_{cha})] \quad (B.3)$$

where

F_{rECB} is the instantaneous retarding force of linear eddy current brake, expressed in N;

$F_{rECB,max}$ is the maximum retarding force of linear eddy current brake, expressed in N;

k_2 is the coefficient (provided by the supplier), expressed in s/m;

v is the current speed, expressed in m/s;

v_{cha} is the characteristic speed (corresponding to maximum retarding force), expressed in m/s.

As an alternative, the instantaneous retarding force of an eddy current brake can be calculated as set out in [Formula \(B.4\)](#).

$$F_{\text{rECB}} = F_{\text{rECB,max}} \cdot \frac{2}{\left(\frac{v}{v_{\text{cha}}}\right)^n + \left(\frac{v_{\text{cha}}}{v}\right)^n} \quad (\text{B.4})$$

with

$$n = n_1 \text{ for } v \geq v_{\text{cha}}$$

$$n = n_2 \text{ for } v < v_{\text{cha}}$$

where

- F_{rECB} is the retarding force of linear eddy current brake, expressed in N;
- $F_{\text{rECB,max}}$ is the maximum retarding force of linear eddy current brake, expressed in N;
- v_0 is the initial speed, expressed in m/s;
- $v_{1,\text{ECB}}$ is the deactivating speed of eddy current brake, expressed in m/s;
- v_{cha} is the characteristic speed (corresponding to maximum retarding force), expressed in m/s;
- v is the current speed, expressed in m/s;
- n_1 is the value of power in speed range above v_{cha} (provided by the supplier);
- n_2 is the value of power in speed range below v_{cha} (provided by the supplier).

B.3 Retarding force of electro-dynamic brake

The retarding force of electro-dynamic brake F_{BED} is represented by a characteristic curve shown in [Figure B.3](#).

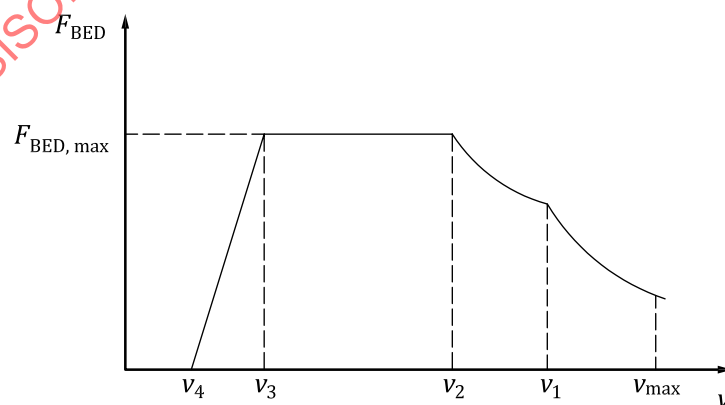


Figure B.3 — Characteristic of an electro-dynamic retarding force

The instantaneous values of the retarding forces F_{BED} can be calculated in different speed ranges as set out in [Formulae \(B.5\) to \(B.9\)](#):

If $v < v_4$:

$$F_{\text{BED}} = 0 \quad (\text{B.5})$$

If $v_4 \leq v < v_3$:

$$F_{\text{BED}} = F_{\text{BED,max}} \cdot \frac{v - v_4}{v_3 - v_4} \quad (\text{B.6})$$

If $v_3 \leq v < v_2$:

$$F_{\text{BED}} = F_{\text{BED,max}} \quad (\text{B.7})$$

If $v_2 \leq v < v_1$, the retarding force is governed by the power hyperbolics:

$$F_{\text{BED}} = F_{\text{BED,max}} \cdot \left[\frac{v_2}{v} \right] \quad (\text{B.8})$$

If $v \geq v_1$:

$$F_{\text{BED}} = F_{\text{BED,max}} \cdot \left[\frac{v_2 \cdot v_1}{v^2} \right] \quad (\text{B.9})$$

where

- F_{BED} is the retarding force of electro-dynamic brake, expressed in N;
- $F_{\text{BED,max}}$ is the maximum retarding force of electro-dynamic brake, expressed in N;
- v is the current speed, expressed in m/s;
- $v_1 \dots v_4$ are the particular speeds, expressed in m/s.

B.4 Retarding force of fluid retarder

The retarding force of a fluid retarder F_{BFR} is represented by a characteristic curve shown in [Figure B.4](#).

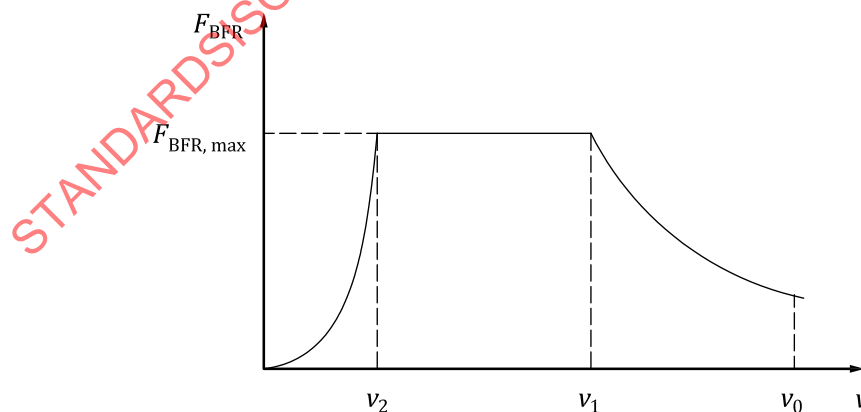


Figure B.4 — Indicative characteristic retarding force of a fluid retarder

The instantaneous retarding force can be calculated in different speed ranges as set out in [Formulae \(B.10\) to \(B.12\)](#):

Speed range $v < v_2$:

$$F_{\text{BFR}} = F_{\text{BFR,max}} \cdot \frac{v^2}{v_2^2} \quad (\text{B.10})$$

Speed range $v_2 \leq v < v_1$:

$$F_{\text{BFR}} = F_{\text{BFR,max}} \quad (\text{B.11})$$

Speed range $v \geq v_1$:

$$F_{\text{BFR}} = F_{\text{BFR,max}} \cdot \frac{v_1}{v} \quad (\text{B.12})$$

where

F_{BFR} is the instantaneous retarding force of fluid retarder, expressed in N;

$F_{\text{BFR,max}}$ is the maximum retarding force of fluid retarder, expressed in N;

v_0 is the initial speed, expressed in m/s;

v is the current speed, expressed in m/s;

$v_1 \dots v_2$ are the particular speeds, expressed in m/s.

Annex C

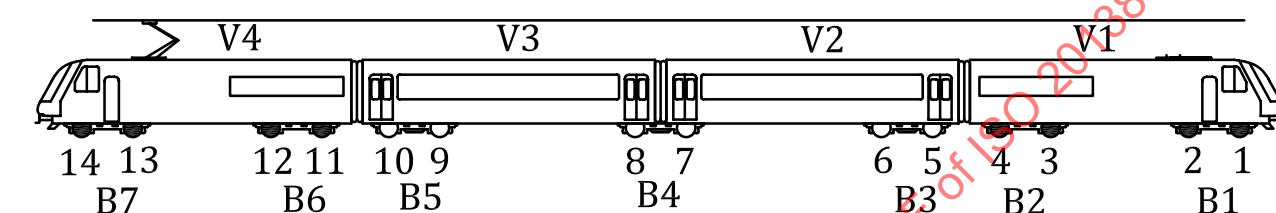
(informative)

Examples for brake calculation

C.1 Multiple unit

C.1.1 General

Figure C.1 shows a schematic layout of a multiple unit.



Key

1 to 14 wheelset number

B1 to B7 bogie number

V1 to V4 vehicle number

Figure C.1 — Schematic layout of a multiple unit

Table C.1 describes the location of different types of brake equipment within the multiple unit shown in Figure C.1.

Table C.1 — Multiple unit — Brake equipment type

Bogie	Wheelset	Brake equipment types and clause reference
B1, B7	1, 2, 13, 14	Main brake system motor vehicle (electric drive): — Wheel mounted disc brake (sintered brake pads) — Electro-dynamic brake
B2, B6	3, 4, 11, 12	Main brake system motor vehicle (electric drive): — Wheel mounted disc brake (sintered brake pads) — Electro-dynamic brake — Magnetic track brake (per bogie)
B3, B4, B5	5, 6, 7, 8, 9, 10	Main brake system trailer vehicle: — Axle mounted disc brake (two per wheelset, sinter pad) — Magnetic track brake (per bogie)
B1, B3, B5, B7	1, 2, 5, 6, 9, 10, 13, 14	Parking brake: Spring applied brake (one per wheelset)

C.1.2 Example for brake calculation of multiple unit

Table C.2 shows the input data.

Table C.2 — Input data

	Characteristic description	Symbol	Example value	Unit
Vehicle data				
	Characteristic coefficient of the train independent of speed	C_1	1 500	N
	Characteristic coefficient of the train proportional to the speed	C_2	50	N/(m/s)
	Characteristic coefficient of aerodynamic resistance due to pressure drag and skin friction drag	C_3	3,5	N/(m/s) ²
	Initial speed	v_0	200	km/h
	Final speed	v_{fin}	0	km/h
Vehicle/Unit/ Axle no.	Characteristic description	Symbol	Example value	Unit
Empty: Design mass in working order				
V1 and V4	Static mass	m_{st}	67 400	kg
V1 and V4	Equivalent rotating mass	m_{rot}	5 392	kg
V2 and V3	Static mass	m_{st}	56 301	kg
V2 and V3	Equivalent rotating mass	m_{rot}	2 250	kg
1, 2, 13, 14	Static mass per wheelset	$m_{st,ax}$	16 100	kg
3, 4, 11, 12	Static mass per wheelset	$m_{st,ax}$	17 600	kg
5, 6, 7, 8, 9, 10,	Static mass per wheelset	$m_{st,ax}$	18 767	kg
Laden: Design mass in working order plus payload (maximum braking load)				
V1 and V4	Static mass	m_{st}	70 400	kg
V1 and V4	Equivalent rotating mass	m_{rot}	5 392	kg
V2 and V3	Static mass	m_{st}	60 300	kg
V2 and V3	Equivalent rotating mass	m_{rot}	2 250	kg
1, 2, 13, 14	Static mass per wheelset	$m_{st,ax}$	16 850	kg
3, 4, 11, 12	Static mass per wheelset	$m_{st,ax}$	18 350	kg
5, 6, 7, 8, 9, 10	Static mass per wheelset	$m_{st,ax}$	20 100	kg
Wheel diameter				
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	Wheel diameter max.	D_{max}	0,92	m
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	Wheel diameter min.	D_{min}	0,84	m
One electro-dynamic brake per wheelset				
1, 2, 3, 4, 11, 12, 13, 14	Maximum retarding force of electro-dynamic brake	$F_{BED,max}$	17 000	N
1, 2, 3, 4, 11, 12, 13, 14	Particular speed	v_1	200	km/h
1, 2, 3, 4, 11, 12, 13, 14	Particular speed	v_2	120	km/h
1, 2, 3, 4, 11, 12, 13, 14	Particular speed	v_3	15	km/h
1, 2, 3, 4, 11, 12, 13, 14	Particular speed	v_4	10	km/h

Table C.2 (continued)

	Characteristic description	Symbol	Example value	Unit
Time characteristic of electro-dynamic brake				
	Retarding force from 0 s to 0,5 s		0	%
	Retarding force from 0,5 s to 2 s		Linear increase	%
	Retarding force from ≥ 2 s		100	%
Two disc brakes per wheelset				
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	Brake cylinder area	A_c	176,7	cm ²
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	Efficiency of brake rigging	η_{rig}	0,95	—
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	Mean friction coefficient of brake pad	μ_m	0,35	—
1, 2, 13, 14	Restoring force of brake unit or spring applied force	$F_{S,C}$	-0,63	kN
5, 6, 9, 10	Restoring force of brake unit or spring applied force	$F_{S,C}$	-0,5	kN
1, 2, 3, 4, 11, 12, 13, 14	Rigging ratio	i_{rig}	8,78	—
5, 6, 7, 8, 9, 10	Rigging ratio	i_{rig}	10,96	—
1, 2, 3, 4, 11, 12, 13, 14	Mean swept radius of the brake pad on the disc face	r_m	305	mm
5, 6, 7, 8, 9, 10	Mean swept radius of the brake pad on the disc face	r_m	251	mm
Brake cylinder pressure in speed interval				
V1 to V4	Brake cylinder pressure in speed interval 200 km/h to 170 km/h	p_c	2,3	bar
V1 to V4	Brake cylinder pressure in speed interval 170 km/h to 0 km/h	p_c	3,8	bar
Time characteristic of disc brake				
	Retarding force from 0 s to 0,5 s		0	%
	Retarding force from 0,5 s to 3,5 s		Linear increase	%
	Retarding force from $\geq 3,5$ s		100	%
Magnetic track brake				
Two magnets per bogie				
B2, B3, B4, B5, B6	Attraction force of one magnet	F_{AMg}	84 000	N
	Activating speed of magnetic track brake	$v_{0,Mg}$	200	km/h
	Deactivating speed of magnetic track brake	$v_{1,Mg}$	50	km/h
	Coefficient (provided by supplier)	k_1	0,068	h/km
	Coefficient (provided by supplier)	k_0	5	—
Time characteristic of magnetic track brake				
	Retarding force from 0 s to 2 s		0	%
	Retarding force from 2 s to 2,5 s		Linear increase	%
	Retarding force from $\geq 2,5$ s		100	%

Table C.2 (continued)

	Characteristic description	Symbol	Example value	Unit
Stationary brake (spring applied parking brake) wheel mounted				
	Force acting on single disc surface (i is an index used for sorting)	$F_{\text{pad},i}$	27 500	N
	Number of disc faces	n_{face}	2	—
	Static friction coefficient of brake pad	μ_{st}	0,3	—
	Angle of slope (26 ‰)	α	$1,5/90 \cdot \pi/2$	°
1, 2, 13, 14	Number of spring brake units per wheelset	n_{SP}	1	—
	Maximum permitted wheel and rail adhesion	τ_{max}	0,12	—
	External forces (e.g. wind force)	F_{ext}	0	N
	Mean swept radius of the brake pad on the disc face	r_{m}	305	mm
Stationary brake (spring applied parking brake) axle mounted				
	Force acting on single disc surface (i is an index used for sorting)	$F_{\text{pad},i}$	27 500	N
	Number of disc faces	n_{face}	2	—
	Static friction coefficient of brake pad	μ_{st}	0,3	—
	Angle of slope (26 ‰)	α	$1,5/90 \cdot \pi/2$	°
5, 6, 9, 10	Number of spring brake units per wheelset	n_{SP}	1	—
	Maximum permitted wheel and rail adhesion	τ_{max}	0,12	—
	External forces (e.g. wind force)	F_{ext}	0	N
	Mean swept radius of the brake pad on the disc face	r_{m}	251	mm

The maximum braking load is based on the maximum expected density of standing passengers on board in addition to the normal load and should be specified and agreed for each project. For guidance, the following categories can be considered:

- 0 kg/m² in the standing area for trains with restricted seat reservation system and no standing passengers at all;
- 160 kg/m² in the standing area for regional trains and long-distance trains;
- 300 kg/m² in the standing area for trains that are worked intensely with a medium volume of passengers;
- 500 kg/m² in the standing area for trains that are worked intensely with high volumes of passengers such as found in inner cities and suburbs.

C.1.3 Brake system design

The brake system should be designed under the following conditions in all required brake modes:

Step 1:

- Emergency brake condition;
- Maximum initial speed;
- Design mass in working order plus payload (maximum payload).

Step 2:

- Emergency brake condition;

- Maximum initial speed;
- Design mass in working order.

Step 3:

- Stationary brake.

Step 4 (if required):

- Degraded modes/degraded conditions;
- Service brake.

The purpose of the blending concept as described in [C.2.7.5](#) is e.g. prioritization of wear-free brake systems, optimisation of wheel and rail contact, reduction of longitudinal forces.

Requirements for step 4 should not be more extensive than requirements for emergency brake.

The stopping distance of a service brake should never be less than the stopping distance of an emergency brake.

NOTE The definition of a blending concept is outside the scope of this document (see ISO 20138-1:2018, 5.6.3).

C.2 Results

C.2.1 Retarding force of single brake equipment types

[Figure C.2](#) shows the sum of different retarding forces versus speed related to the rail of each brake equipment type applying high and low brake cylinder pressure.

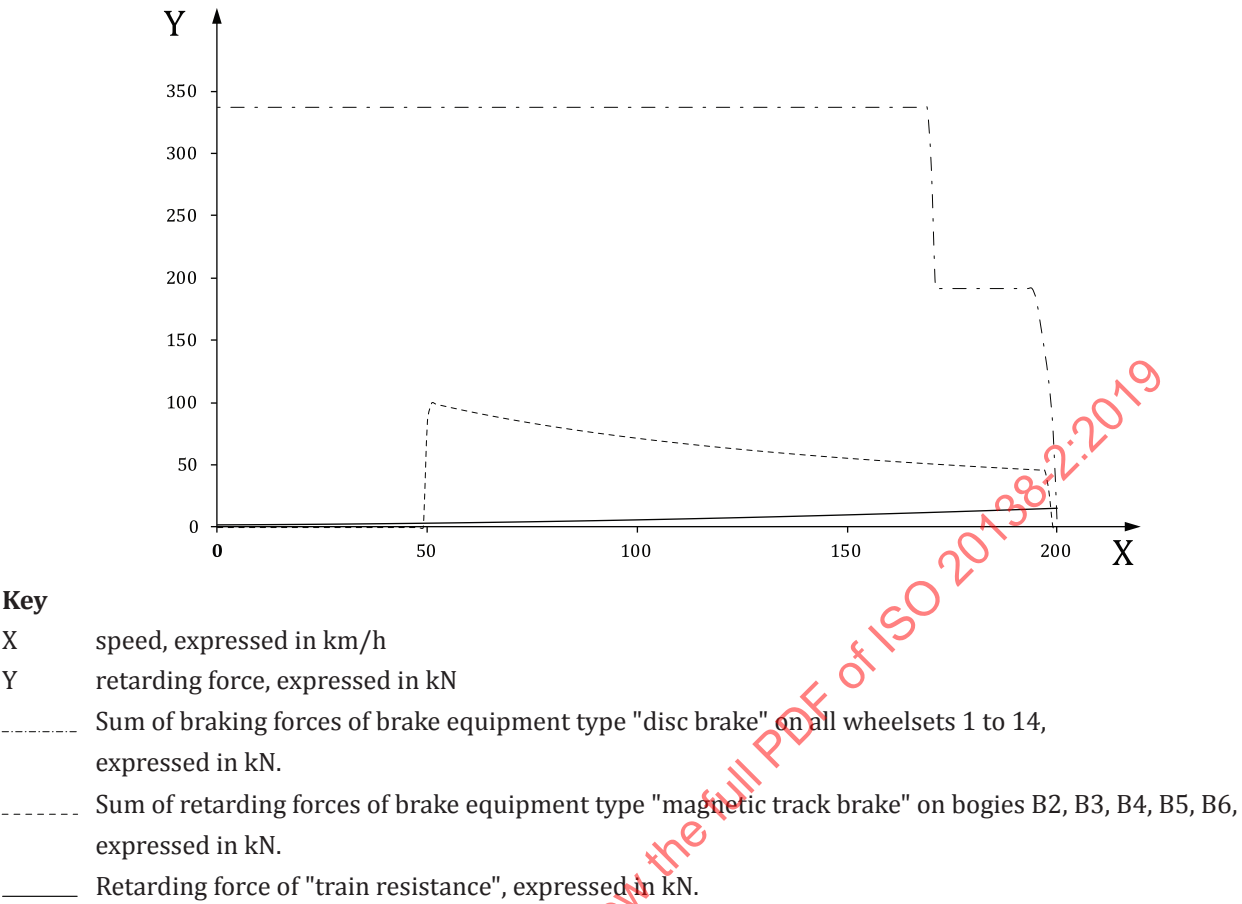


Figure C.2 — Retarding forces of brake equipment types versus speed

C.2.2 Stopping distance

The stopping distance is calculated in design mass in working order plus payload (maximum payload), straight level track and maximum wheel diameter for the different initial speeds of 200 km/h, 160 km/h, 140 km/h and 120 km/h.

The relative distance deviation ξ is calculated as set out in [Formula \(9\)](#) and should not exceed the predefined limit value.

The results of the distance calculations are shown in [Table C.3](#).

Table C.3 — Stopping distances

$\Delta t = 0,01 \text{ s}$					
Initial speed	v_0 km/h	200	160	140	120
Stopping distance	s m	1 333	761	592	446
Stopping distance for calculation of ξ	s m	1 332,69	760,69	592,02	445,52
$2 \times \Delta t = 0,02 \text{ s}$					
Initial speed	v_0 km/h	200	160	140	120
Stopping distance	s m	1 333	761	592	446
Stopping distance for calculation of ξ	s m	1 332,77	760,67	592	445,54
Δs					
Initial speed	v_0 km/h	200	160	140	120
Δs	s m	0,08	0,02	0,02	0,02
Relative distance deviation	ξ %	0,005 48	0,002 69	0,003 45	0,004 59

C.2.3 Equivalent (mean) deceleration based on distance

The equivalent deceleration is calculated as set out in [Formula \(15\)](#) for design mass in working order (empty) and design mass in working order plus payload (maximum payload) (laden) and with and without response time (see [Table C.4](#)).

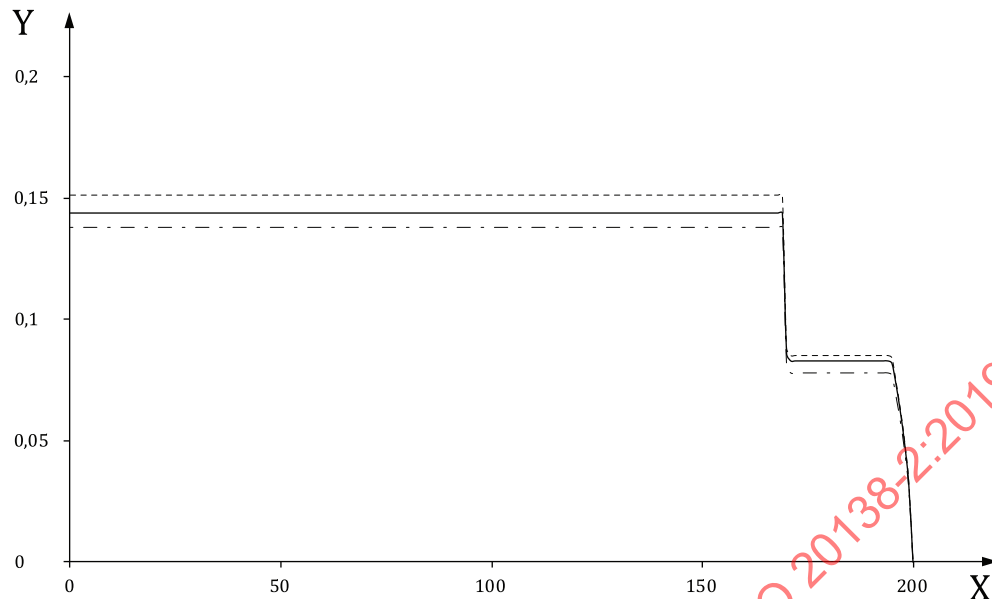
Table C.4 — Equivalent deceleration

$\Delta t = 0,01$			
Speed range	v_0 km/h	200 – 170	170 – 0
Equivalent deceleration laden	a_e m/s ²	0,92	1,47
Equivalent deceleration empty	a_e m/s ²	0,97	1,54

C.2.4 Required wheel and rail adhesion τ_{req}

The required wheel and rail adhesion τ_{req} is calculated as set out in in ISO 20138-1:2018, 5.6.4, Formula (42) applying the instantaneous values.

[Figure C.3](#) shows the calculated required adhesion per wheelset versus train speed.



Key

X speed, expressed in km/h

Y wheel and rail adhesion τ_{req}

----- Required wheel and rail adhesion τ_{req} on wheelsets 1, 2, 13, 14.

----- Required wheel and rail adhesion τ_{req} on wheelsets 3, 4, 11, 12.

—— Required wheel and rail adhesion τ_{req} on wheelsets 5, 6, 7, 8, 9, 10.

Figure C.3 — Wheel and rail adhesion versus speed and wheelset type

C.2.5 Calculation of the mass to be held M

The mass to be held M can be calculated for the (single) vehicle. In that case, it is equal to the static mass M_{st} .

The mass to be held M for the unit is calculated according to the procedure set out in ISO 20138-1:2018, B.3.

C.2.6 Stationary brake calculation

C.2.6.1 General

The safety against rolling, safety against sliding and safety retention are calculated as set out in ISO 20138-1:2018, 6.14, applying the maximum wheel diameter. Dependent on the vehicle type and the normal load case when the vehicle is parked indefinitely, the safety is calculated for the "empty" or "laden" load case. Formally, a safe situation is given when the safety factor is higher than "1".

Normally, a passenger vehicle is parked in empty condition.

NOTE For safety calculation, the following values are used:

- in laden situation, the safety factor is $\geq 1,1$;
- in empty situation, the safety factor is $\geq 1,4$.

C.2.6.2 Safety against rolling

The result of the calculation of the safety against rolling factor is shown in [Table C.5](#).

Table C.5 — Safety against rolling for empty vehicle/unit

	Empty
Max wheel diameter	1,27

C.2.6.3 Safety against sliding

The intermediate results used in the calculation of safety against sliding are shown in [Table C.6](#) and [Table C.7](#).

Table C.6 — Iterative calculation according to ISO 20138-1:2018, 6.14, Empty

Empty: Design mass in working order								
		m_{st}	$F_{Perp,ax}$	$F_{B,ax,st}$	$\tau_{req,max,ax}$	$F_{B,\tau,req,rem}$	$F_{N,rem}$	$\overline{\tau_{ax,i}}$
Multiple unit		247 400	2 425 365			63 059	2 425 365	0,026
Bogie	Wheelset							
B2	3	17 600	172 539	0	0	63 059	2 252 826	0,028
	4	17 600	172 539	0	0	63 059	2 080 288	0,030
B4	7	18 767	183 979	0	0	63 059	1 896 308	0,033
	8	18 767	183 979	0	0	63 059	1 712 329	0,037
B6	11	17 600	172 539	0	0	63 059	1 539 791	0,041
	12	17 600	172 539	0	0	63 059	1 367 252	0,046
B3	5	18 767	183 979	9 003	0,049	54 574	1 183 273	0,046
	6	18 767	183 979	9 003	0,049	46 089	999 293	0,046
B5	9	18 767	183 979	9 003	0,049	37 603	815 314	0,046
	10	18 767	183 979	9 003	0,049	29 118	631 335	0,046
B1	1	16 100	157 834	10 940	0,069	21 839	473 501	0,046
	2	16 100	157 834	10 940	0,069	14 559	315 667	0,046
B7	13	16 100	157 834	10 940	0,069	7 280	157 834	0,046
	14	16 100	157 834	10 940	0,069	0	0	0,046

NOTE Splitting of total vehicle mass into single mass of each wheelset is sufficient without position after decimal place.

Table C.7 — Iterative calculation according to ISO 20138-1:2018, 6.14, Laden

Laden: Design mass in working order plus payload (maximum braking load)								
		m_{st}	$F_{Perp,ax}$	$F_{B,ax,st}$	$\tau_{req,max,ax}$	$F_{B,\tau,req,rem}$	$F_{N,rem}$	$\overline{\tau_{ax,i}}$
Multiple unit		261 400	2 562 579			67 139	2 562 579	0,026 2
Bogie	Wheelset							
B2	3	18 350	179 891	0	0,000	66 627	2 382 701	0,028 2
	4	18 350	179 891	0	0,000	66 627	2 202 810	0,030 2
B4	7	20 100	197 047	0	0,000	66 627	2 005 763	0,033 2
	8	20 100	197 047	0	0,000	66 627	1 808 716	0,036 6
B6	11	18 350	179 891	0	0,000	66 627	1 628 824	0,040 9
	12	18 350	179 891	0	0,000	66 627	1 448 933	0,046 0
B3	5	20 100	197 047	9 003	0,046	57 566	1 251 886	0,046 0
	6	20 100	197 047	9 003	0,046	48 563	1 054 839	0,046 0