
**Geometrical product specifications
(GPS) — Surface texture: Profile —**

**Part 2:
Terms, definitions and surface texture
parameters**

*Spécification géométrique des produits (GPS) — État de surface:
Méthode du profil*

Partie 2: Termes, définitions et paramètres d'état de surface



STANDARDSISO.COM : Click to view the full PDF of ISO 21920-2:2021



COPYRIGHT PROTECTED DOCUMENT

© ISO 2021

All rights reserved. Unless otherwise specified, or required in the context of its implementation, no part of this publication may be reproduced or utilized otherwise in any form or by any means, electronic or mechanical, including photocopying, or posting on the internet or an intranet, without prior written permission. Permission can be requested from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office
CP 401 • Ch. de Blandonnet 8
CH-1214 Vernier, Geneva
Phone: +41 22 749 01 11
Email: copyright@iso.org
Website: www.iso.org

Published in Switzerland

Contents

Page

Foreword	v
Introduction	vi
1 Scope	1
2 Normative references	1
3 Terms and definitions	1
3.1 General terms	1
3.2 Geometrical parameter terms	10
3.3 Geometrical feature terms	14
4 Field parameters	22
4.1 General	22
4.2 Height parameters	22
4.2.1 General	22
4.2.2 Arithmetic mean height	22
4.2.3 Root mean square height	22
4.2.4 Skewness	22
4.2.5 Kurtosis	22
4.2.6 Total height	23
4.2.7 Maximum height per section	23
4.3 Spatial parameters	24
4.3.1 General	24
4.3.2 Autocorrelation length	24
4.3.3 Dominant spatial wavelength	24
4.4 Hybrid parameters	25
4.4.1 General	25
4.4.2 Root mean square gradient	25
4.4.3 Arithmetic mean of absolute gradient	25
4.4.4 Maximum absolute gradient	25
4.4.5 Developed length	25
4.4.6 Developed length ratio	26
4.5 Material ratio functions and related parameters	26
4.5.1 Material ratio functions	26
4.5.2 Material ratio parameters	31
4.5.3 Parameters for stratified surfaces using the material ratio curve	33
4.5.4 Parameters for stratified surfaces using the material probability curve	35
4.5.5 Volume parameters	36
5 Feature parameters	38
5.1 Parameters based on peak heights and pit depths	38
5.1.1 General	38
5.1.2 Maximum peak height	39
5.1.3 Mean peak height	39
5.1.4 Maximum pit depth	39
5.1.5 Mean pit depth	40
5.1.6 Maximum height	40
5.2 Parameters based on profile elements	40
5.2.1 General	40
5.2.2 Mean profile element spacing	42
5.2.3 Maximum profile element spacing	42
5.2.4 Standard deviation of profile element spacings	42
5.2.5 Mean profile element height	42
5.2.6 Maximum profile element height	42
5.2.7 Standard deviation of profile element heights	42
5.2.8 Peak count parameter	43

5.3	Parameters based on feature characterization.....	43
5.3.1	General.....	43
5.3.2	Named feature parameters.....	43
Annex A	(informative) Determination of the first and second derivative	45
Annex B	(informative) Determination of the local curvature.....	48
Annex C	(normative) Determination of the material ratio curve.....	49
Annex D	(normative) Determination of profile parameters for stratified surfaces	50
Annex E	(normative) Crossing-the-line segmentation to determine profile elements.....	59
Annex F	(normative) Feature characterization.....	65
Annex G	(informative) Summary of profile surface texture parameters and functions.....	69
Annex H	(informative) Specification analysis workflow	72
Annex I	(informative) Changes to previous ISO profile documents.....	74
Annex J	(informative) Overview of profile and areal standards in the GPS matrix model.....	75
Annex K	(informative) Relation to the GPS matrix model.....	76
Bibliography		77

STANDARDSISO.COM : Click to view the full PDF of ISO 21920-2:2021

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 290, *Dimensional and geometrical product specification and verification*, in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

This first edition of ISO 21920-2 cancels and replaces ISO 4287:1997, ISO 12085:1996, ISO 13565-2:1996 and ISO 13565-3:1998, which have been technically revised.

It also incorporates the Amendment ISO 4287:1997/Amd 1:2009 and the Technical Corrigenda ISO 4287:1997/Cor 1:1998, ISO 4287:1997/Cor 2:2005, ISO 12085:1996/Cor 1:1998 and ISO 13565-2:1996/Cor 1:1998.

The main changes are related to ISO 4287 and are as follows:

- all field parameters are now related to the evaluation length;
- unambiguous evaluation of profile elements;
- definition of new parameters, in particular parameters based on the watershed transformation.

A list of all parts in the ISO 21920 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences chain link B of the chains of standards on profile surface texture.

The ISO GPS matrix model given in ISO 14638 gives an overview of the ISO GPS system of which this document is a part. The fundamental rules of ISO GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to the specifications made in accordance with this document, unless otherwise indicated.

For more detailed information of the relation of this document to other standards and the GPS matrix model, see [Annex K](#).

This document develops the terminology, concepts and parameters for profile surface texture.

Throughout this document, parameters are written as abbreviated terms with lower-case suffixes (as in R_q) when used in a sentence, and are written as symbols with subscripts (as in R_q) when used in formulae, to avoid misinterpretations of compound letters as an indication of multiplication between quantities in formulae. The parameters with lower-case suffixes are used in product documentation, drawings and data sheets.

STANDARDSISO.COM : Click to view the full PDF of ISO 21920-2:2021

Geometrical product specifications (GPS) — Surface texture: Profile —

Part 2: Terms, definitions and surface texture parameters

1 Scope

This document specifies terms, definitions and parameters for the determination of surface texture by profile methods.

NOTE 1 The main changes to previous ISO profile documents are described in [Annex I](#).

NOTE 2 An overview of profile and areal standards in the GPS matrix model is given in [Annex J](#).

NOTE 3 The relation of this document to the GPS matrix model is given in [Annex K](#).

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1:2015, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1 and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

3.1 General terms

3.1.1

skin model

non-ideal surface model

<of a workpiece> model of the physical interface of the workpiece with its environment

[SOURCE: ISO 17450-1:2011, 3.2.2]

3.1.2

surface texture

geometrical irregularities contained in a scale-limited profile

Note 1 to entry: Surface texture does not include geometrical irregularities contributing to the form or shape of the profile.

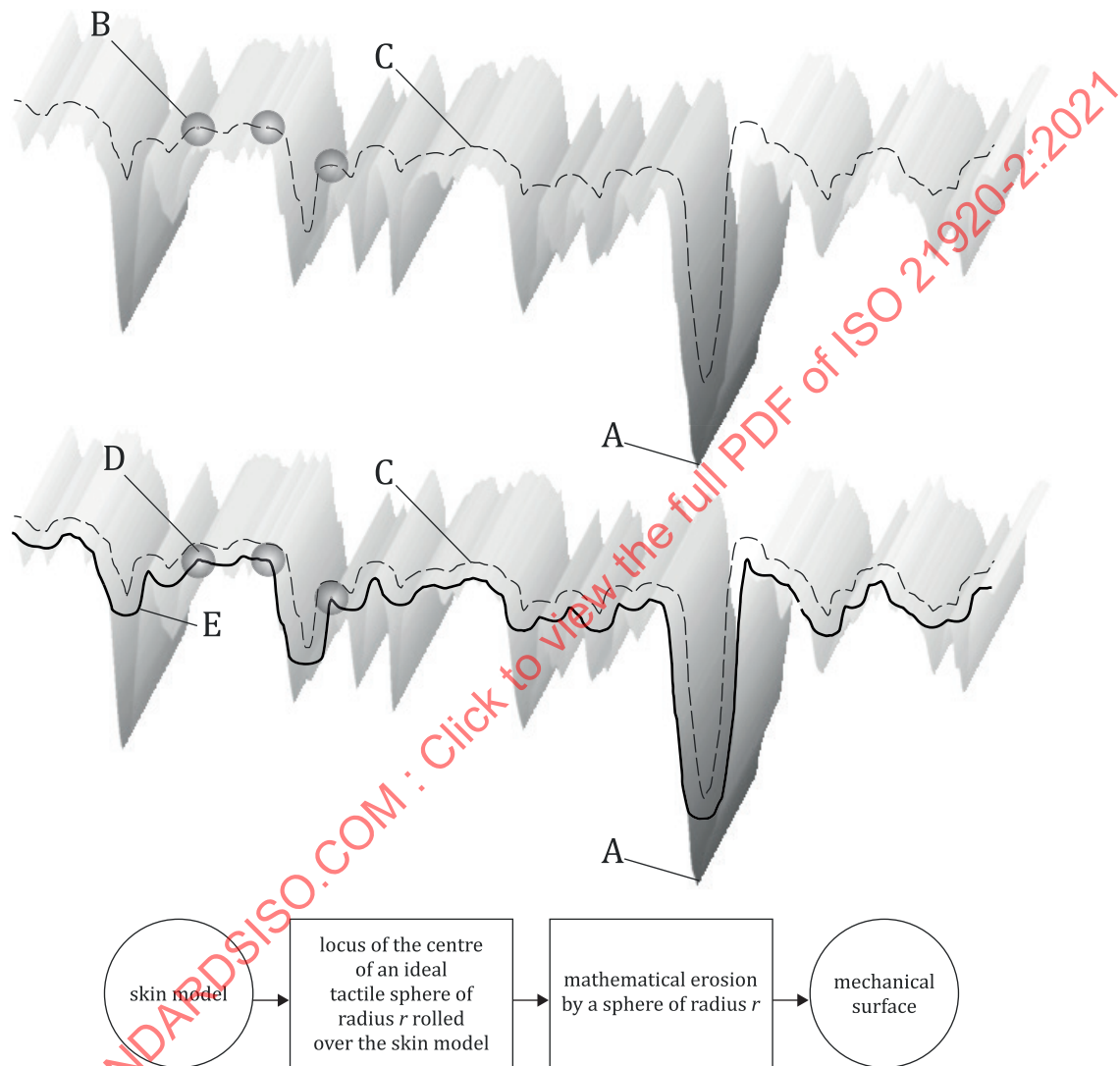
3.1.3

mechanical surface

boundary of the mathematical erosion, by a sphere of radius r , of the locus of the centre of an ideal tactile sphere, also with radius r , rolled over the skin model of a workpiece

Note 1 to entry: [Figure 1](#) is an example to show the effect of mechanical filtering and is not related to a real measured surface.

[SOURCE: ISO 14406:2010, 3.1.1, modified — Notes to entry replaced.]



Key

- A skin model
- B ideal tactile sphere of radius r
- C envelope curve of the locus of the centre of an ideal tactile sphere B rolled over the skin model
- D sphere of radius r
- E mechanical surface: boundary of the mathematical erosion, by the sphere D, of the envelope curve C

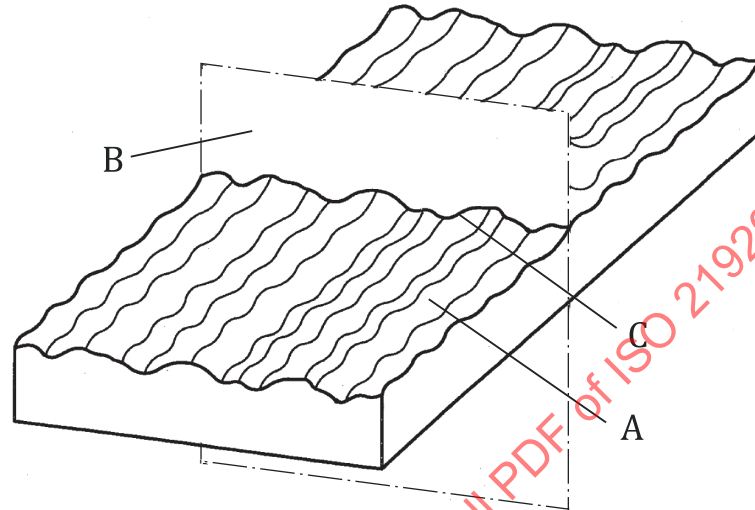
Figure 1 — Mechanical surface

3.1.4**profile trace**

intersection of the skin model by an intersection plane perpendicular to the skin model and in a specified direction

Note 1 to entry: See [Figure 2](#).

Note 2 to entry: See ISO 21920-3:2021, 4.3.

**Key**

- A skin model
- B intersection plane
- C profile trace

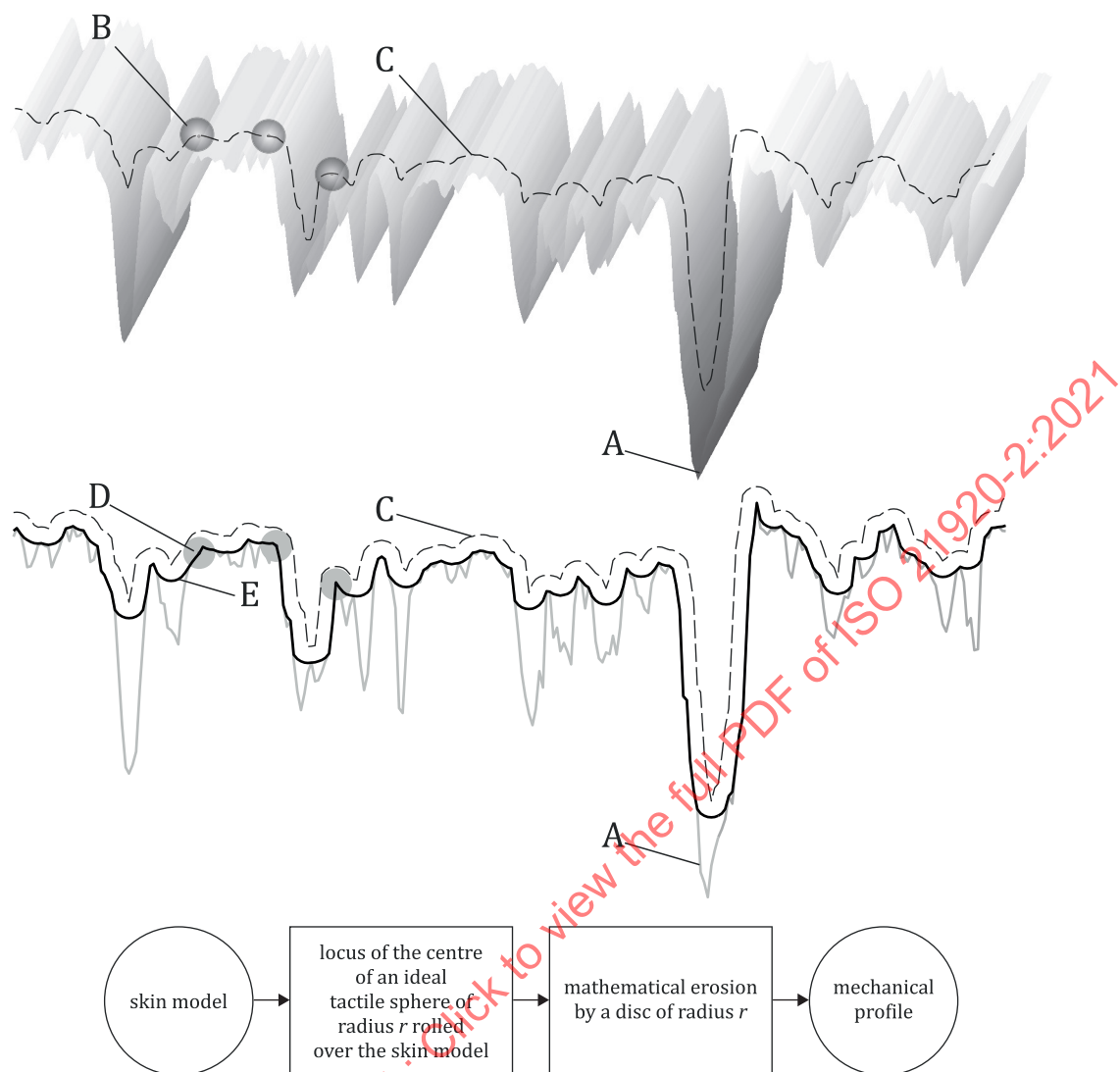
Figure 2 — Profile trace

3.1.5**mechanical profile**

boundary of the mathematical erosion, by a circular disc of radius r , of the locus of the centre of an ideal tactile sphere, also with radius r , rolled along a trace over the skin model of a workpiece

Note 1 to entry: [Figure 3](#) is an example to show the effect of mechanical filtering and is not related to a real measured profile.

Note 2 to entry: The treatment of non-measured points and spurious points is part of the extraction process (see ISO 17450-1:2011, 8.1.3) and is not considered in this document.



Key

- A skin model
- B ideal tactile sphere of radius r
- C envelope curve of the planar locus of the centre of an ideal tactile sphere rolled over the skin model
- D circular disc of radius r
- E mechanical profile: boundary of the mathematical erosion, by the circular disc D, of the envelope curve C

Figure 3 — Mechanical profile

3.1.6

electromagnetic surface

surface obtained by the electromagnetic interaction with the skin model of a workpiece

Note 1 to entry: See [Figure 4](#).

Note 2 to entry: The electromagnetic surface is an inherent characteristic of a skin model of a workpiece.

Note 3 to entry: Electromagnetic surfaces depend on the optical measurement principle used for extraction.

[SOURCE: ISO 14406:2010, 3.1.2, modified — Notes to entry replaced.]

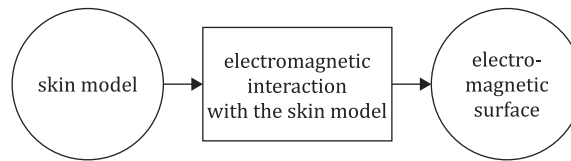


Figure 4 — Electromagnetic surface

3.1.7**electromagnetic profile**

profile obtained by the electromagnetic interaction with the skin model of a workpiece

Note 1 to entry: See [Figure 5](#).

Note 2 to entry: The electromagnetic profile is an inherent characteristic of a skin model of a workpiece.

Note 3 to entry: Electromagnetic profiles depend on the optical measurement principle used for extraction.

Note 4 to entry: In most cases, the profile trace results from the intersection of the skin model by an intersection plane perpendicular to the *skin model* ([3.1.1](#)) and in a specified direction (see ISO 21920-3).

Note 5 to entry: The treatment of non-measured points and spurious points is part of the extraction process and is not considered in this document.

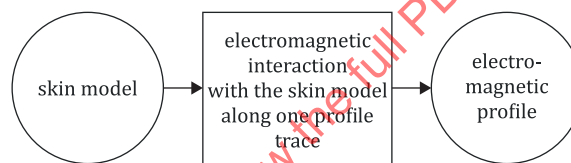


Figure 5 — Electromagnetic profile

3.1.8**auxiliary surface**

surface obtained by an interaction, other than mechanical or electromagnetic, with the *skin model* ([3.1.1](#)) of a workpiece

Note 1 to entry: A software measurement standard is an example of an auxiliary surface. Other physical measurement principles which differ from a mechanical or electromagnetic surface, such as scanning tunnelling microscopy or atomic force microscopy, can also serve as an auxiliary surface. See [Figure 6](#).

3.1.9**auxiliary profile**

profile obtained by an interaction, other than mechanical or electromagnetic, with the *skin model* ([3.1.1](#)) of a workpiece

Note 1 to entry: A software measurement standard is an example of an auxiliary profile. Other physical measurement principles which differ from a mechanical or electromagnetic profile, such as scanning tunnelling microscopy or atomic force microscopy, can also serve as an auxiliary profile. See [Figure 6](#) and [Annex H](#).

3.1.10**specification coordinate system**

system of coordinates in which surface texture parameters are specified

Note 1 to entry: If the nominal surface is a plane (or portion of a plane), it is common practice to use a rectangular coordinate system in which the axes form a right-handed Cartesian set, the *x*-axis and the *y*-axis also lying on the nominal surface, and the *z*-axis being in an outward direction (from the material to the surrounding medium). This convention is adopted throughout the rest of this document.

3.1.11

nesting index

N_{is} , N_{ic} , N_{if}

number or set of numbers indicating the relative level of nesting for a particular primary mathematical model

Note 1 to entry: The cut-off wavelength for the Gaussian filter is an example of a nesting index.

Note 2 to entry: Using the different nesting indices, specific lateral scale components of a scale-limited profile are extracted.

[SOURCE: ISO 16610-1:2015, 3.2.1, modified — definition and notes to entry revised.]

3.1.12

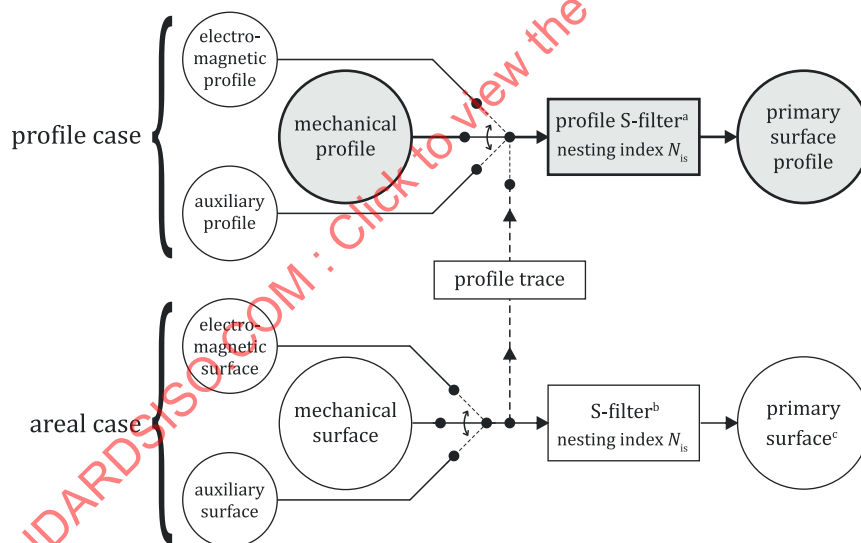
primary surface profile

surface profile trace obtained when a surface profile trace is represented as a specified primary mathematical model with specified nesting index N_{is}

Note 1 to entry: In the ISO 21920 series, a profile S-filter is used to derive the primary surface profile from a profile trace (e.g. mechanical profile). See [Figure 6](#) and [Annex H](#).

Note 2 to entry: For some applications, the profile S-filter is not used. In such cases, for example for multi-scale analysis, the nesting index is equal to “zero”.

Note 3 to entry: In most situations, the primary surface profile can be derived with sufficient accuracy from either the mechanical surface (the default choice), the electromagnetic surface or the auxiliary surface, using an intersection plane perpendicular to the chosen type of surface and in a specified direction. See [Figure 6](#).



NOTE The evaluation chain for the default case is indicated by the grey fill colour.

^a See [3.1.13.1](#) for profile S-filter.

^b See ISO 25178-2:2021, 3.1.6.1, for S-filter.

^c See ISO 25178-2:2021, 3.1.5, for primary surface.

Figure 6 — Definition of the primary surface and primary surface profile

3.1.13

profile filter

filtration operator applied to a profile

3.1.13.1**profile S-filter**

profile filter which removes small lateral scale components from a profile

Note 1 to entry: See [Figure 7](#).

3.1.13.2**profile L-filter**

profile filter which removes large lateral scale components from a profile

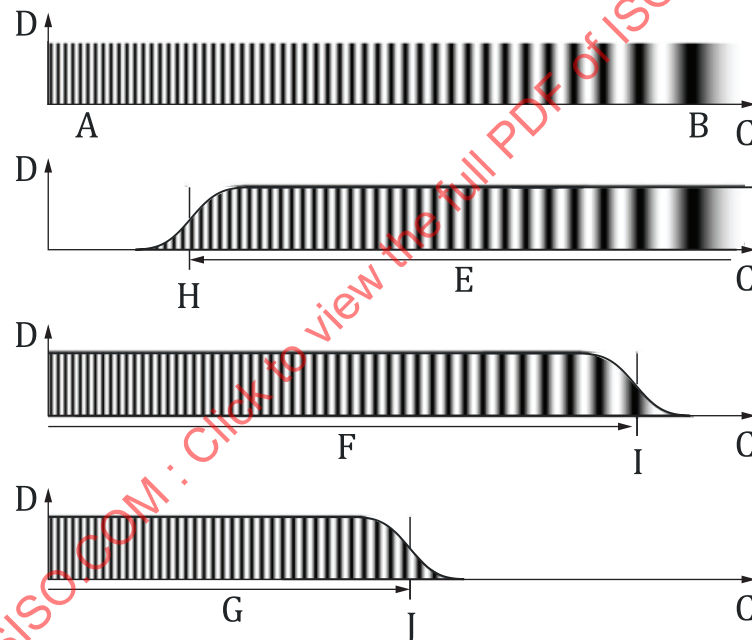
Note 1 to entry: Some profile L-filters are sensitive to form and require the profile F-operation first as a prefilter before being applied.

Note 2 to entry: See [Figure 7](#).

3.1.13.3**profile F-operation**

operation which removes form from a profile

Note 1 to entry: See [Figure 7](#).

**Key**

- A small lateral scale (e.g. short wavelengths)
- B large lateral scale (e.g. long wavelengths)
- C scale axis
- D amplitude axis
- E lateral scale component extracted by the profile S-filter
- F lateral scale component extracted by the profile F-operation
- G lateral scale component extracted by the profile L-filter
- H profile S-filter nesting index N_{is}
- I profile F-operation nesting index N_{if}
- J profile L-filter nesting index N_{ic}

Figure 7 — Relationships between the S-filter, L-filter and F-operation

3.1.14

scale-limited profile

profile structure scale components between specified nesting indices

EXAMPLE A profile is scale-limited after applying a profile filter with a specified nesting index.

3.1.14.1

primary profile

P-profile

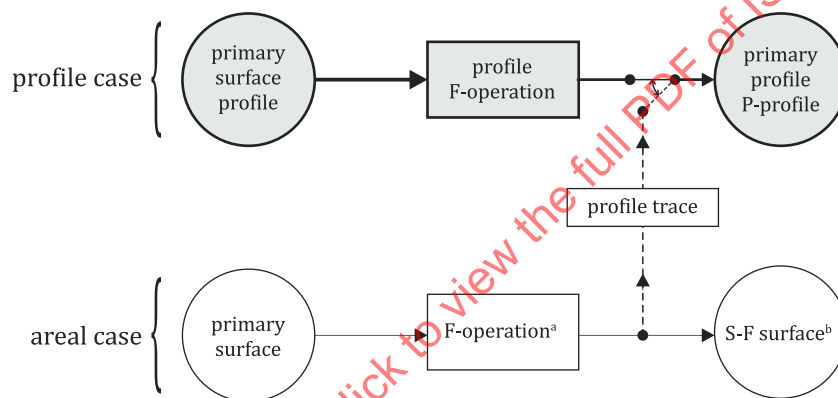
scale-limited profile at any position x derived from the primary surface profile by removing the form using a profile F-operation with nesting index N_{if}

Note 1 to entry: In most cases, the primary profile can be derived with sufficient accuracy from the S-F surface using an intersection plane perpendicular to the S-F surface and in a specified direction. See [Figure 8](#).

Note 2 to entry: The primary profile is the basis for evaluation of the *P-parameters* (3.2.5). See [Figures 9](#) and [10](#).

Note 3 to entry: The profile F-operation can be performed as a multi-stage operation, for example a combination of a total least square fit and a profile L-filter.

Note 4 to entry: See [Annex H](#) for additional information.



NOTE The evaluation chain for the default case is indicated by the grey fill colour.

^a See ISO 25178-2:2021, 3.1.6.3, for F-operation.

^b See ISO 25178-2:2021, 3.1.7, for S-F surface.

Figure 8 — Primary profile derived from the primary surface profile (default) or S-F surface

3.1.14.2

waviness profile

W-profile

scale-limited profile at any position x derived from the primary profile by removing small-scale lateral components by a specific type of profile S-filter with a nesting index N_{ic}

Note 1 to entry: The waviness profile is the basis for evaluation of the *W-parameters* (3.2.6). See [Figures 9](#) and [10](#).

Note 2 to entry: The choice of filter settings for W-parameters is highly dependent on the functional requirements. This is why no default tables for W-parameters are found in ISO 21920-3.

Note 3 to entry: See [Annex H](#) for additional information.

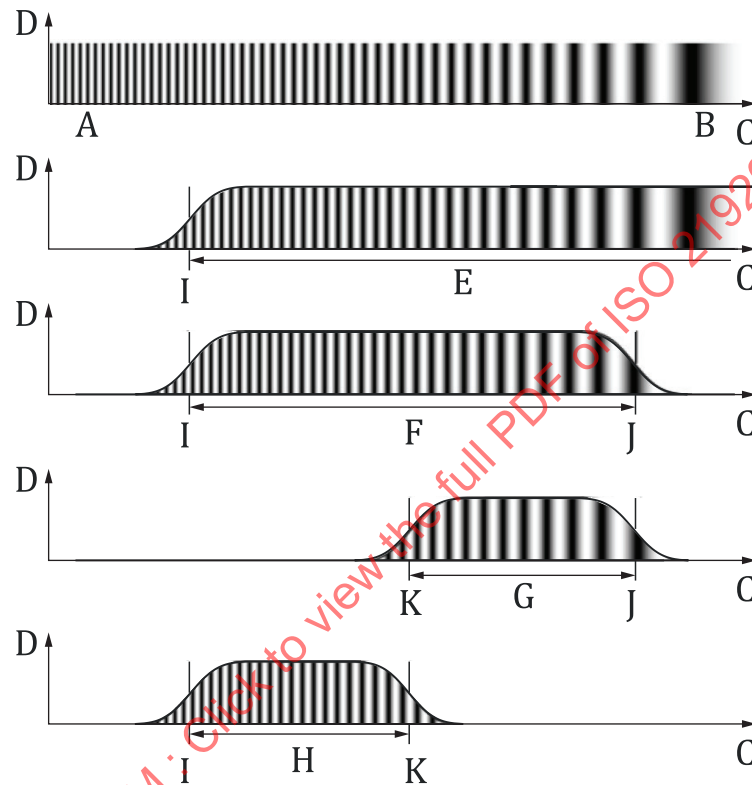
3.1.14.3 roughness profile

R-profile

scale-limited profile at any position x derived from the primary profile by removing large-scale lateral components by a specific type of profile L-filter with a nesting index N_{ic}

Note 1 to entry: The roughness profile is the basis for evaluation of the R -parameters (3.2.7). See Figures 9 and 10.

Note 2 to entry: See Annex H for additional information.



Key

- A small lateral scale
- B large lateral scale
- C scale axis
- D amplitude axis
- E lateral scale component of primary surface profile
- F lateral scale component of P-profile
- G lateral scale component of W-profile
- H lateral scale component of R-profile
- I profile S-filter nesting index N_{is}
- J profile F-operation nesting index N_{if}
- K profile L-filter nesting index N_{ic}

Figure 9 — Relationship between the primary surface profile, P-profile, W-profile and R-profile

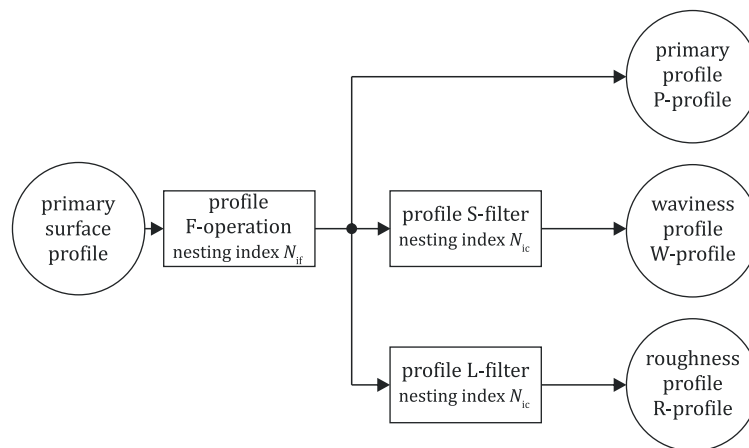


Figure 10 — Measuring chain to determine the P-profile, W-profile and R-profile

3.1.15

reference line

line corresponding to a specific large lateral scale component

Note 1 to entry: The x -axis of the *specification coordinate system* (3.1.10) coincides with the reference line of the assessed profile and the z -axis is oriented in an outward direction (from the material to the surrounding medium). This convention is adopted throughout the rest of this document.

Note 2 to entry: The reference line for the primary profile and waviness profile are the large lateral scale component of primary surface profile removed by the profile F-operation.

Note 3 to entry: The reference line for the roughness profile is the component of the primary profile removed by the profile L-filter.

3.1.16

evaluation length

l_e

length in the direction of the x -axis used for identifying the geometrical structures characterizing the scale-limited profile

Note 1 to entry: The traverse length is longer than the evaluation length.

Note 2 to entry: See 3.2.3 for evaluation length parameters.

Note 3 to entry: In ISO 4287, the evaluation length was given by l_n .

3.1.17

section length

l_{sc}

length in the direction of the x -axis used to obtain *section length parameters* (3.2.4)

Note 1 to entry: Default values of l_{sc} are found in ISO 21920-3.

3.1.18

number of sections

n_{sc}

integer number used to obtain *section length parameters* (3.2.4)

Note 1 to entry: Default values of n_{sc} are found in ISO 21920-3.

3.2 Geometrical parameter terms

NOTE Parameter symbols are written with subscripts (e.g. R_q) when used in formulae to avoid misinterpretations of compound letters as an indication of multiplication between quantities. Parameter symbols are written with lower-case suffixes (e.g. R_q) when used in product documentation, drawings and data sheets.

3.2.1

field parameter

parameter defined from all the points on a scale-limited profile

3.2.2

feature parameter

parameter defined from a subset of predefined topographic features from the scale-limited profile

Note 1 to entry: For feature parameters, see [Clause 5](#).

3.2.3

evaluation length parameter

parameter defined on the evaluation length

Note 1 to entry: See [Clause 4](#), [5.2](#) and [5.3](#) for evaluation length parameters.

3.2.4

section length parameter

parameter defined on a set of section length

Note 1 to entry: See [5.1](#) for section length parameters.

3.2.5

P-parameter

parameter determined from the primary profile

3.2.6

W-parameter

parameter determined from the waviness profile

3.2.7

R-parameter

parameter determined from the roughness profile

Note 1 to entry: Formulae for parameter definitions are exemplarily given for R-parameters. P- and W-parameters are defined in a similar manner, replacing the parameters related to the R-profile with those related to the P-profile or W-profile. Default specification operators for the different types of parameter definitions can be found in ISO 21920-3.

3.2.8

height

signed normal distance from the reference line to the scale-limited profile

Note 1 to entry: Where the scale-limited profile is below the reference line, the height has a negative value.

Note 2 to entry: This definition as an absolute coordinate applies when the term 'height' is used alone. Later terms in this document include the word 'height' or 'depth' in their name, such as the maximum height R_z (see [5.1.6](#)) or *local depth* ([3.3.18](#)). The definitions of some of those later terms use an alternative reference point and/or refer to an unsigned distance in a specified direction from the reference point. See those definitions for details.

3.2.9

depth

height multiplied by minus one

Note 1 to entry: Where the scale-limited profile is above the reference line, the depth has a negative value.

Note 2 to entry: This definition as an absolute coordinate applies when the term 'depth' is used alone. Later terms in this document include the word 'height' or 'depth' in their name, such as the maximum height R_z (see 5.1.6) or *dale local depth* (3.3.18). The definitions of some of those later terms use an alternative reference point and/or refer to an unsigned distance in a specified direction from the reference point. See those definitions for details.

3.2.10

ordinate value

$z(x)$

height of the assessed scale-limited profile

3.2.11

local gradient

$dz(x)/dx$

first derivative of the scale-limited profile z with respect to the position x

Note 1 to entry: See [Annex A](#) for the determination of the gradient.

Note 2 to entry: The local gradient is also called slope.

3.2.12

local curvature

$\kappa(x)$

curvature of the scale-limited profile z with respect to the position x

$$\kappa(x) = \frac{d^2 z(x)/dx^2}{\left(1 + (dz(x)/dx)^2\right)^{\frac{3}{2}}} \quad (1)$$

Note 1 to entry: See [Annex A](#) and [Annex B](#) for the determination of the curvature.

Note 2 to entry: For most engineering surfaces, the local gradient (slope) is small, enabling a good approximation of local curvature by the local second derivative $\kappa(x) \cong (d^2 z(x))/dx^2$.

3.2.13

autocorrelation function

$f_{ACF}(t_x)$

function which describes the correlation between a scale-limited profile z and the same profile spatial shifted by t_x

$$f_{ACF}(t_x) = \frac{\frac{1}{l_e - |t_x|} \int_{l_0}^{l_e} (z(x) - \bar{z})(z(x + t_x) - \bar{z}) dx}{\frac{1}{l_e} \int_0^{l_e} (z(x) - \bar{z})^2 dx} \quad (2)$$

where

$\bar{z}$ is the arithmetic mean of the profile $z(x)$ over the evaluation length l_e ;

$l_0 = \{x \in \mathbf{R} \mid \max(0, -t_x) \leq x \leq \min(l_e, l_e - t_x)\}$ is the overlap interval;

$|t_x| < l_e$ is the spatial shift.

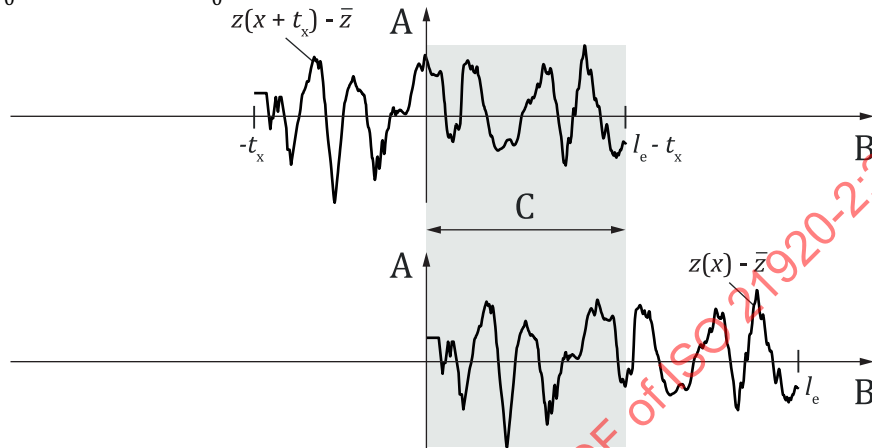
Note 1 to entry: See [Figure 11](#) for an illustration of the overlap interval.

Note 2 to entry: The autocorrelation function is symmetrical in t_x , i.e. $f_{ACF}(t_x) = f_{ACF}(-t_x)$.

Note 3 to entry: [Formula \(2\)](#) is an unbiased estimator for the autocorrelation function.

Note 4 to entry: Some disciplines use a shift-dependent Pearson correlation coefficient instead of the autocorrelation function. It is defined by [Formula \(3\)](#).

$$\rho_{XX}(t_x) = \frac{\int_{l_0} (z(x) - \bar{z})(z(x+t_x) - \bar{z}) dx}{\sqrt{\int_{l_0} (z(x) - \bar{z})^2 dx \cdot \int_{l_0} (z(x+t_x) - \bar{z})^2 dx}} \quad \text{with } -1 \leq \rho_{XX}(t_x) \leq 1 \quad (3)$$



Key

A height

B x-axis (reference line)

C overlap interval l_0

Figure 11 — Illustration of the overlap interval l_0

3.2.14

Fourier transformation

$F(p)$

operator which transforms the scale-limited profile z into Fourier domain

$$F(p) = \int_0^{l_e} z(x) e^{-i2\pi p x} dx \quad (4)$$

where

i is the imaginary unit $i^2 = -1$;

p is the spatial frequency.

3.2.15

amplitude spectral density

$f_{ASD}(p)$

absolute value of the Fourier transformation of the scale-limited profile z

$$f_{ASD}(p) = |F(p)| \quad (5)$$

where

$F(p)$ is the Fourier transformation of the scale-limited profile z ;

p is the spatial frequency.

3.2.16

power spectral density

$f_{\text{PSD}}(p)$

function which describes the power of a scale-limited profile z in the Fourier domain

$$f_{\text{PSD}}(p) = \frac{|F(p)|^2}{l_e} \quad (6)$$

where

$F(p)$ is the Fourier transformation of the scale-limited profile z ;

p is the spatial frequency.

Note 1 to entry: The power spectral density fulfils [Formula \(7\)](#):

$$\frac{1}{l_e} \int_0^{l_e} z^2(x) dx = \int_{-\infty}^{\infty} f_{\text{PSD}}(p) dp \quad (7)$$

3.3 Geometrical feature terms

3.3.1

segmentation

method which partitions a scale-limited profile into distinct features

Note 1 to entry: There are three types of segmentation:

- for the parameters based on peak heights and pit depths (see [5.1](#)), the segmentation is realized by identification of the *hills* ([3.3.11](#)) and *dales* ([3.3.17](#)) by determination of the positions where the ordinate values change their sign or are equal to zero;
- for the parameters based on profile elements (see [5.2](#)), the segmentation is realized by the *crossing-the-line segmentation* ([3.3.2](#));
- for the parameters based on feature characterization (see [5.3](#)), the segmentation is realized by the *watershed segmentation* ([3.3.3](#)).

3.3.2

crossing-the-line segmentation

operation based on crossings of the reference line by a scale-limited profile in conjunction with a combination algorithm to leave a set of significant segments

Note 1 to entry: See [Annex E](#) for determining crossing-the-line segmentation.

Note 2 to entry: The crossing-the-line segmentation requires *height discrimination* ([3.3.31](#)).

3.3.3

watershed segmentation

filtration operation that spatially decomposes a profile into mutually exclusive portions of that profile

3.3.4

peak

<watershed segmentation> point on the profile which is higher than all other points within a neighbourhood of that point

Note 1 to entry: There is a theoretical possibility of a plateau. In this case, the peak is the middle single point on the plateau.

3.3.5**peak**

<reference line> highest point of a *hill* ([3.3.11](#))

Note 1 to entry: There is a theoretical possibility of a plateau. In this case, the peak is the middle single point on the plateau.

3.3.6**number of peaks**

n_p

integer number representing the number of significant peaks within the evaluation length

Note 1 to entry: The number of significant peaks depends on the segmentation method.

3.3.7**pit**

<watershed segmentation> point on the profile which is lower than all other points within the neighbourhood of that point

Note 1 to entry: There is a theoretical possibility of a plateau. In this case, the pit is the middle single point on the plateau.

3.3.8**pit**

<reference line> lowest point of a *dale* ([3.3.17](#))

Note 1 to entry: There is a theoretical possibility of a plateau. In this case, the peak is the middle single point on the plateau.

3.3.9**number of pits**

n_v

integer number representing the number of significant pits within the evaluation length

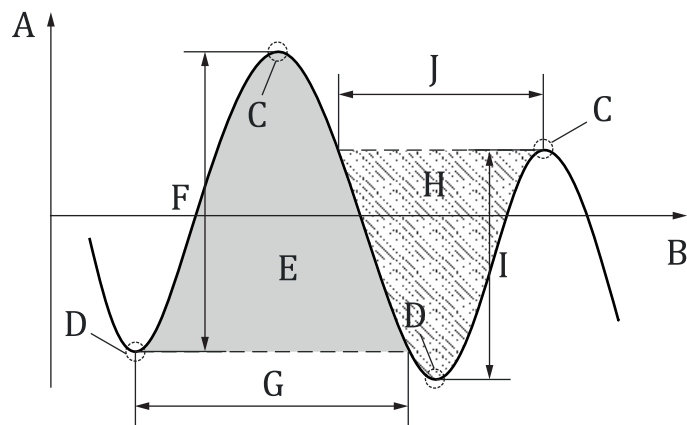
Note 1 to entry: The number of significant pits depends on the segmentation method.

3.3.10**hill**

<watershed segmentation> region around a peak such that all maximal upward paths end at the peak

Note 1 to entry: This definition is used for parameters based on feature characterization (see [5.3](#)).

Note 2 to entry: See [Figure 12](#).



Key

A	height	F	hill local height
B	x -axis (reference line)	G	hill local width
C	peak	H	dale/dale local volume (hatched fill area)
D	pit	I	dale local depth
E	hill/hill local volume (grey fill colour)	J	dale local width

Figure 12 — Hill local height, dale local depth, hill local width, dale local width, hill local volume and dale local volume (watershed segmentation)

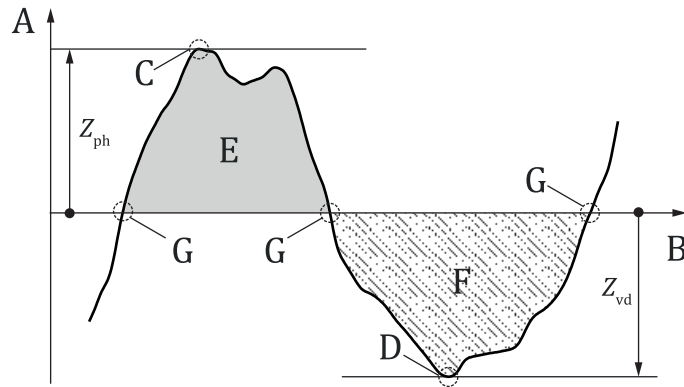
3.3.11

hill

<reference line> outwardly directed (from material to surrounding medium) contiguous portion of the scale-limited profile above the reference line bounded by the two adjacent points where the ordinate values change their sign

Note 1 to entry: This definition is used for parameters based on peak heights and pit depths (see 5.1) and parameters based on profile elements (see 5.2).

Note 2 to entry: See Figure 13.

**Key**

A	height	F	dale
B	x -axis (reference line)	G	change of sign of the ordinate values
C	peak	Z_{ph}	peak height
D	pit	Z_{vd}	pit depth
E	hill		

Figure 13 — Peak height and pit depth (reference line)**3.3.12****hill local height**

height difference between a peak and the highest pit connected to that peak

Note 1 to entry: See [Figure 12](#).

3.3.13**hill local width**

length of the line intersecting a hill at a height associated to the highest pit connected to that hill

Note 1 to entry: See [Figure 12](#).

3.3.14**hill local volume**

ratio of the hill area above the highest pit connected to that hill to the evaluation length

Note 1 to entry: See grey fill colour in [Figure 12](#).

Note 2 to entry: The volume is expressed in millilitres per metre square (ml / m^2).

Note 3 to entry: For anisotropic surfaces, the areal and profile volume parameters are highly correlated.

3.3.15**peak height**

Z_{ph}

height difference between a peak and the reference line

Note 1 to entry: See [Figure 13](#).

3.3.16**dale**

<watershed segmentation> region around a pit such that all maximal downward paths end at the pit

Note 1 to entry: This definition is used for parameters based on feature characterization (see [5.3](#)).

Note 2 to entry: See [Figure 12](#).

3.3.17

dale

<reference line> inwardly directed (from surrounding medium to material) contiguous portion of the scale-limited profile below the reference line bounded by the two adjacent points where the ordinate values change their sign

Note 1 to entry: This definition is used for parameters based on peak heights and pit depths (see 5.1) and parameters based on profile elements (see 5.2).

Note 2 to entry: See Figure 13.

3.3.18

dale local depth

height difference between a pit and the lowest peak connected to that pit

Note 1 to entry: See Figure 12.

3.3.19

dale local width

length of the line intersecting a dale at a height associated to the lowest peak connected to that dale

Note 1 to entry: See Figure 12.

3.3.20

dale local volume

ratio of the dale area below the lowest peak connected to that dale to the evaluation length

Note 1 to entry: See hatched filled area in Figure 12.

Note 2 to entry: The volume is expressed in millilitres per metre square (ml/m^2).

Note 3 to entry: For anisotropic surfaces, the areal and profile volume parameters are highly correlated.

3.3.21

pit depth

Z_{vd}

depth difference between a pit and the reference line

Note 1 to entry: See Figure 13.

3.3.22

motif

hill or dale defined with watershed segmentation

Note 1 to entry: The term motif is used to designate a feature obtained by segmentation.

Note 2 to entry: On a profile, a hill (or dale) is enclosed between two pits (or peaks).

3.3.23

topographic feature

line feature or point feature on a scale-limited profile

3.3.24

line feature

hill or dale

3.3.25

point feature

peak or pit

3.3.26
hill change tree

graph which describes the relationships between peaks and pits sorted by their hill local heights

Note 1 to entry: Peaks are represented on a change tree by the ends of lines. Pits are represented on a change tree by joining lines.

Note 2 to entry: See [Figure 14](#).

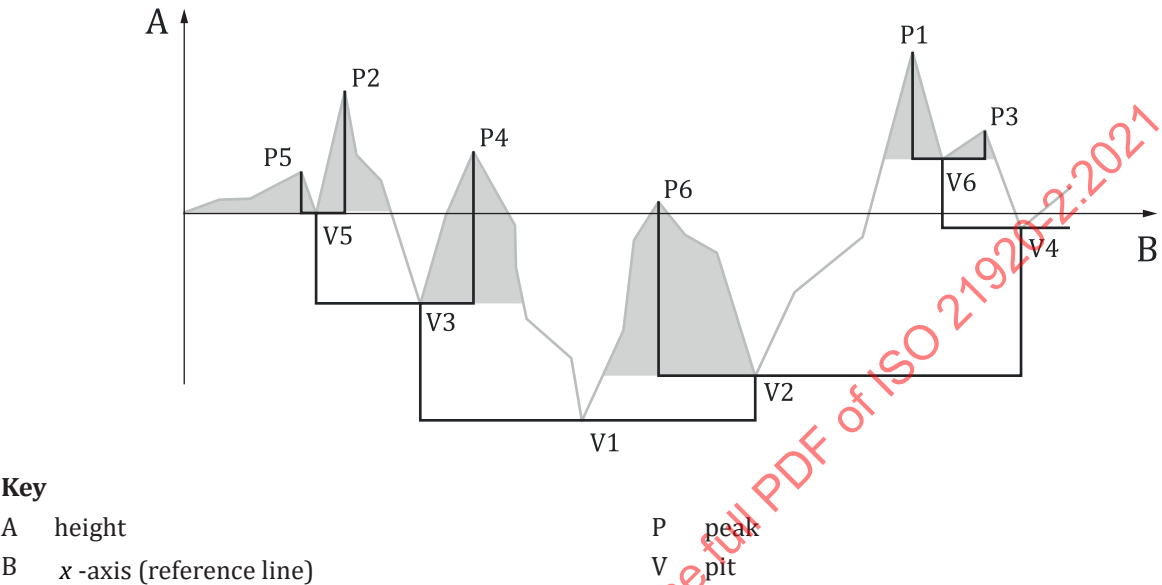


Figure 14 — Hill change tree

3.3.27
dale change tree

graph which describes the relationships between pits and peaks sorted by their dale local depths

Note 1 to entry: Pits are represented on a change tree by the end of lines. Peaks are represented on a change tree by joining lines.

Note 2 to entry: See [Figure 15](#).

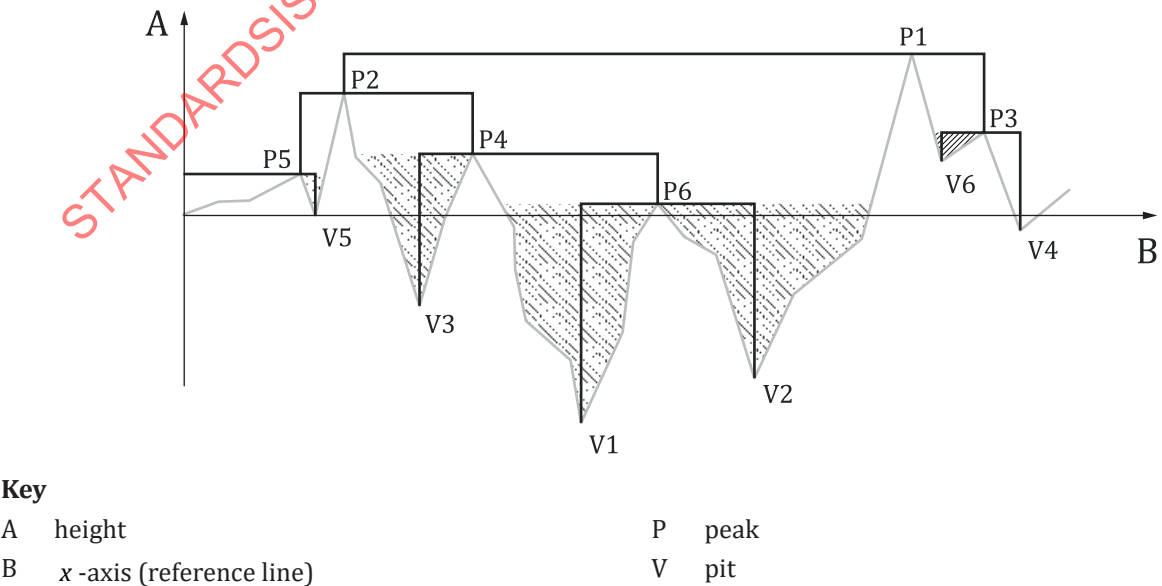


Figure 15 — Dale change tree

3.3.28

pruning

method to simplify a change tree in which lines from peaks (or pits) to their connected pits (or peaks) are removed

3.3.29

Wolf pruning

pruning where lines are removed in order from a peak (or pit) with the smallest hill local height (or dale local depth) up to the peak (or pit) with a specified hill local height (or dale local depth)

Note 1 to entry: The peak local heights and pit local depths will change during Wolf pruning as removing lines from a change tree will also remove the associated pits and peaks, respectively.

3.3.30

height discrimination

<watershed segmentation> minimum hill local height or dale local depth of the scale-limited profile

Note 1 to entry: This definition is used for parameters based on feature characterization (see 5.3).

3.3.31

height discrimination

<reference line> minimum peak height or pit depth of the scale-limited profile

Note 1 to entry: This definition is used for parameters based on profile elements (see 5.2).

3.3.31.1

peak height discrimination

minimum peak height used as a threshold during segmentation

Note 1 to entry: This definition is used for parameters based on profile elements (see 5.2).

3.3.31.2

pit depth discrimination

minimum pit depth used as a threshold during segmentation

Note 1 to entry: This definition is used for parameters based on profile elements (see 5.2).

3.3.32

profile element

<reference line> hill followed by a dale or dale followed by a hill

Note 1 to entry: This definition is used for parameters based on profile elements (see 5.2).

Note 2 to entry: See Figures 16 and 17.

3.3.33

profile element height

Z_t

sum of the *peak height* (3.3.15) (Z_{ph}) and the *pit depth* (3.3.21) (Z_{vd}) of a profile element

$$Z_t = Z_{ph} + Z_{vd} \quad (8)$$

Note 1 to entry: See Figures 16 and 17.

3.3.34

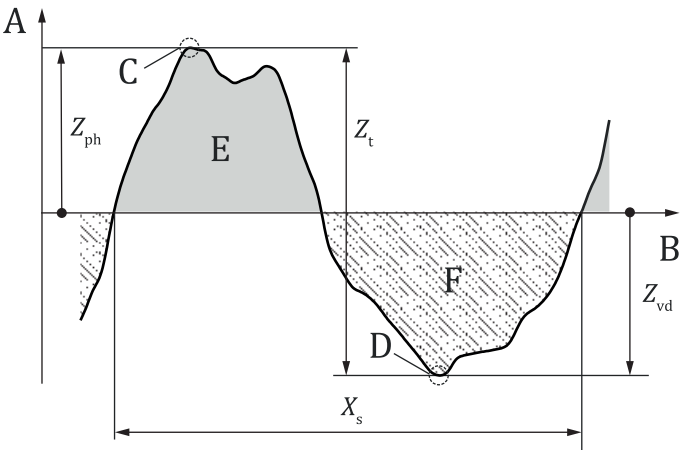
profile element spacing

X_s

distance on the reference line between the beginning of two adjacent profile elements

Note 1 to entry: This definition is used for parameters based on profile elements (see 5.2).

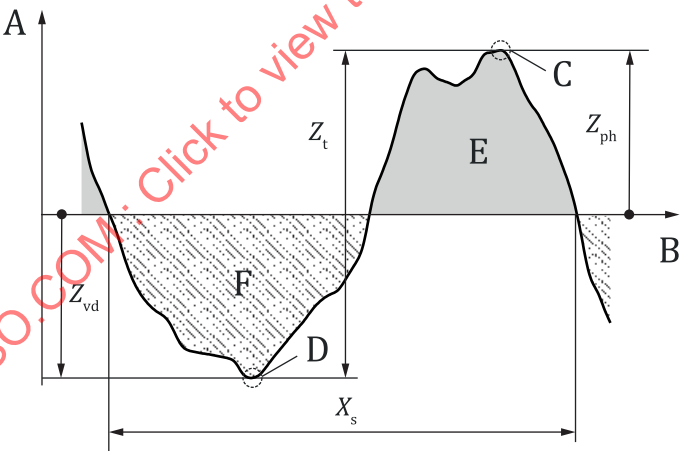
Note 2 to entry: See Figures 16 and 17.



Key

- | | | | |
|---|-------------------------|----------|-------------------------|
| A | height | F | dale |
| B | x-axis (reference line) | X_s | profile element spacing |
| C | peak | Z_{ph} | peak height |
| D | pit | Z_t | profile element height |
| E | hill | Z_{vd} | pit depth |

Figure 16 — Hill followed by a dale



Key

- | | | | |
|---|-------------------------|----------|-------------------------|
| A | height | F | dale |
| B | x-axis (reference line) | X_s | profile element spacing |
| C | peak | Z_{ph} | peak height |
| D | pit | Z_t | profile element height |
| E | hill | Z_{vd} | pit depth |

Figure 17 — Dale followed by a hill

4 Field parameters

4.1 General

Field parameters are related to the evaluation length l_e on the scale-limited profile. Therefore, all field parameters are categorized as evaluation length parameters. A summary of all field parameters is given in [Annex G](#).

NOTE 1 The following parameter definitions assume a continuous representation of the profile. Most instruments use discrete approximations to the given ideal operators.

NOTE 2 On some small, rough surfaces, such as for additive manufacturing, it can be impossible to plan an inspection trace that will have its entire evaluation length inside the available physical surface area. In this case, the possible evaluation length differs from the default length. This can lead to an increase in parameter uncertainty.

4.2 Height parameters

4.2.1 General

Height parameters are a set of parameters based on ordinate values.

4.2.2 Arithmetic mean height

P_a , W_a , R_a

The arithmetic mean height parameter is the arithmetic mean of the absolute values of the ordinate values. It is calculated according to [Formula \(9\)](#).

$$R_a = \frac{1}{l_e} \int_0^{l_e} |z(x)| dx \quad (9)$$

4.2.3 Root mean square height

P_q , W_q , R_q

The root mean square height parameter is the square root of the mean square of the ordinate values. It is calculated according to [Formula \(10\)](#).

$$R_q = \sqrt{\frac{1}{l_e} \int_0^{l_e} z^2(x) dx} \quad (10)$$

4.2.4 Skewness

P_{sk} , W_{sk} , R_{sk}

The skewness parameter is the quotient of the mean cube value of the ordinate values and the cube of R_q . It is calculated according to [Formula \(11\)](#).

$$R_{sk} = \frac{1}{R_q^3} \frac{1}{l_e} \int_0^{l_e} z^3(x) dx \quad (11)$$

4.2.5 Kurtosis

P_{ku} , W_{ku} , R_{ku}

The kurtosis parameter is the quotient of the mean quartic value of the ordinate values and the fourth power of R_q . It is calculated according to [Formula \(12\)](#).

$$R_{ku} = \frac{1}{R_q^4} \frac{1}{l_e} \int_0^{l_e} z^4(x) dx \quad (12)$$

4.2.6 Total height

Pt, Wt, Rt

The total height parameter is the sum of the largest height and the largest depth. It is calculated according to [Formula \(13\)](#).

$$R_t = \max_{x \in X} (z(x)) - \min_{x \in X} (z(x)) \quad (13)$$

where

$$X = \{x \in \mathbf{R} \mid 0 \leq x \leq l_e\}.$$

4.2.7 Maximum height per section

Pzx(l), Wzx(l), Rzx(l)

The maximum height per section parameter is the maximum value of the difference between the highest ordinate value and the lowest ordinate value calculated within a section of length l moving over the evaluation length l_e . It is calculated according to [Formula \(14\)](#). The specification of the value l is not required if the default value is used.

$$R_{zx}(l) = \max_{l \leq x_0 \leq l_e} (\Delta z(x_0)) \text{ with } \Delta z(x_0) = \max_{x \in X} (z(x)) - \min_{x \in X} (z(x)) \quad (14)$$

where

x_0 is the moving position of the right bound of the regarded section.

$\Delta z(x_0)$ is the maximum value of the difference between the highest ordinate value and the lowest ordinate value within the regarded section at the moving position x_0 .

$X = \{x \in \mathbf{R} \mid x_0 - l \leq x \leq x_0\}$ is the set of x -values of the regarded section at the moving position x_0 .

NOTE 1 See [Figure 18](#).

NOTE 2 The default value for l for the moving section is found in ISO 21920-3:2021, Table 7.

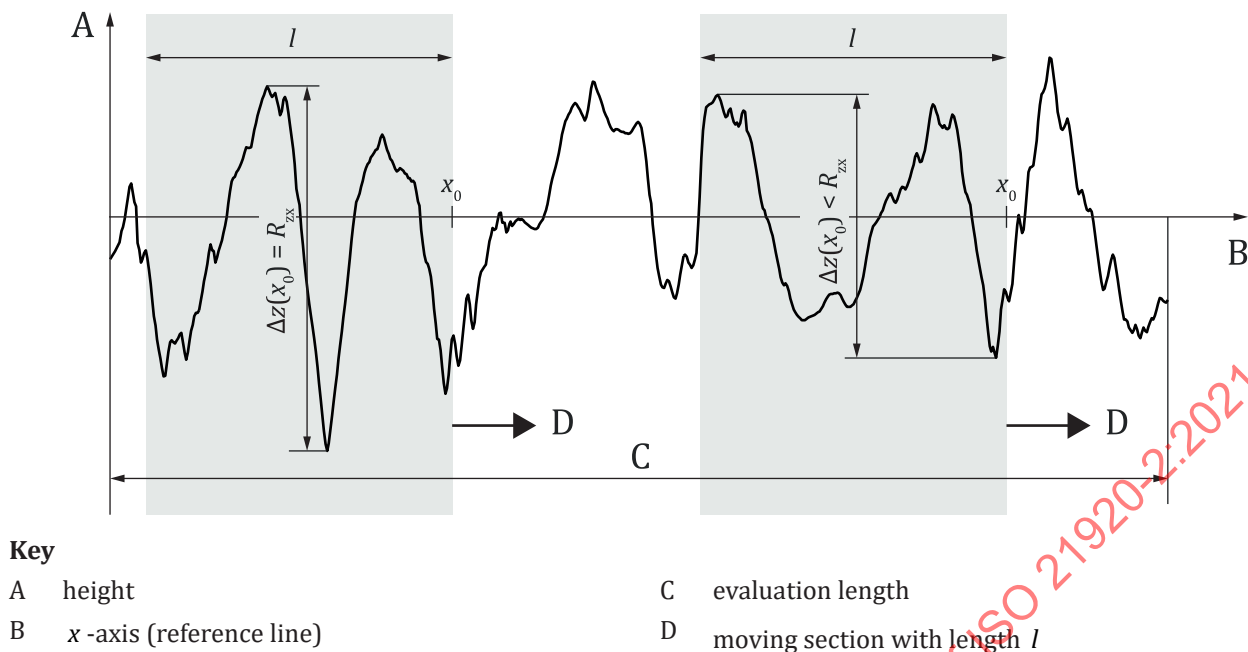


Figure 18 — Maximum height per section

4.3 Spatial parameters

4.3.1 General

Spatial parameters are a set of parameters based on the spatial relationship between geometrical irregularities.

4.3.2 Autocorrelation length

$Pal(s)$, $Wal(s)$, $Ral(s)$

The autocorrelation length parameter is the horizontal distance where the autocorrelation function $f_{ACF}(t_x)$ decays to a specified value s , with $0 \leq s < 1$. It is calculated according to [Formula \(15\)](#). The specification of the value s is not required if the default value is used.

$$R_{al}(s) = \min_{t_x \in T} (|t_x|) \quad (15)$$

where

$$T = \{t_x \in \mathbf{R} \mid f_{ACF}(t_x) \leq s\}$$

NOTE 1 See [3.2.13](#) for the autocorrelation function.

NOTE 2 The default value for the decay s is found in ISO 21920-3.

4.3.3 Dominant spatial wavelength

Psw , Wsw , Rsw

The dominant spatial wavelength parameter is the wavelength which corresponds to the largest absolute value of amplitude spectral density. It is calculated according to [Formula \(16\)](#).

$$R_{sw} = \frac{1}{\arg \max_p (f_{ASD}(p))} \quad (16)$$

NOTE 1 See [3.2.15](#) for the amplitude spectral density.

NOTE 2 The dominant spatial wavelength might not be applicable to profiles lacking strong periodicity.

4.4 Hybrid parameters

4.4.1 General

Hybrid parameters are a set of parameters based on the local gradient.

4.4.2 Root mean square gradient

Pdq, Wdq, Rdq

The root mean square gradient parameter is the root mean square of the local gradient of the ordinate values. It is calculated according to [Formula \(17\)](#).

$$R_{dq} = \sqrt{\frac{1}{l_e} \int_0^{l_e} \left(\frac{dz(x)}{dx} \right)^2 dx} \quad (17)$$

4.4.3 Arithmetic mean of absolute gradient

Pda, Wda, Rda

The arithmetic mean of the absolute gradient parameter is the arithmetic mean of the absolute values of the local gradient of the ordinate values. It is calculated according to [Formula \(18\)](#).

$$R_{da} = \frac{1}{l_e} \int_0^{l_e} \left| \frac{dz(x)}{dx} \right| dx \quad (18)$$

4.4.4 Maximum absolute gradient

Pdt, Wdt, Rdt

The maximum absolute gradient parameter is the maximum of the absolute values of the local gradient of the ordinate values. It is calculated according to [Formula \(19\)](#).

$$R_{dt} = \max_{x \in X} \left(\left| \frac{dz(x)}{dx} \right| \right) \quad (19)$$

where

$$X = \{x \in \mathbf{R} \mid 0 \leq x \leq l_e\}.$$

4.4.5 Developed length

Pdl, Wdl, Rdl

The developed length parameter is the path length of the profile. It is calculated according to [Formula \(20\)](#).

$$R_{dl} = \int_0^{l_e} \sqrt{1 + \left(\frac{dz(x)}{dx} \right)^2} dx \quad (20)$$

4.4.6 Developed length ratio

Pdr, Wdr, Rdr

The developed length ratio parameter is the fractional increment of the path length of the profile. It is calculated according to [Formula \(21\)](#).

$$R_{dr} = \frac{R_{dl} - l_e}{l_e} \quad (21)$$

4.5 Material ratio functions and related parameters

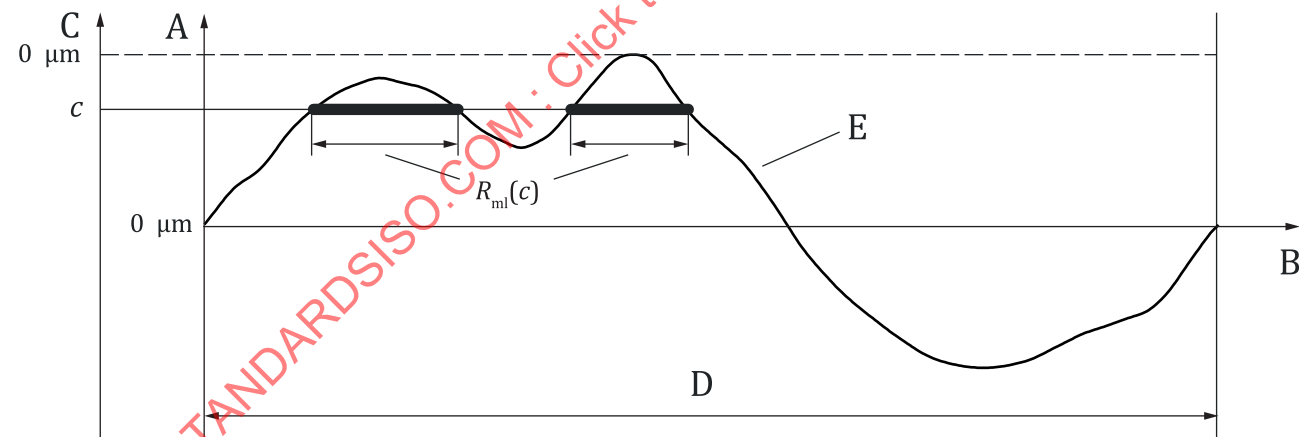
4.5.1 Material ratio functions

4.5.1.1 Material length

Pml(*c*), Wml(*c*), Rml(*c*)

The material length function is the cumulated length of the profile portions intersected by a line at level *c*. The reference height for the level *c* = 0 µm is the maximum height of the scale-limited profile within the evaluation length *l_e*.

NOTE See [Figure 19](#).



Key

- | | | | |
|---|---|---|-------------------|
| A | height | D | evaluation length |
| B | x-axis (reference line) | E | profile |
| C | axis for the level <i>c</i> of intersection | | |

Figure 19 — Material length

4.5.1.2 Material ratio

Pmc(*c*), Wmc(*c*), Rmc(*c*)

The material ratio function is the ratio of the material length parameter at a given level c of intersection to the evaluation length l_e . It is calculated according to [Formula \(22\)](#).

$$R_{mc}(c) = \frac{R_{ml}(c)}{l_e} \tag{22}$$

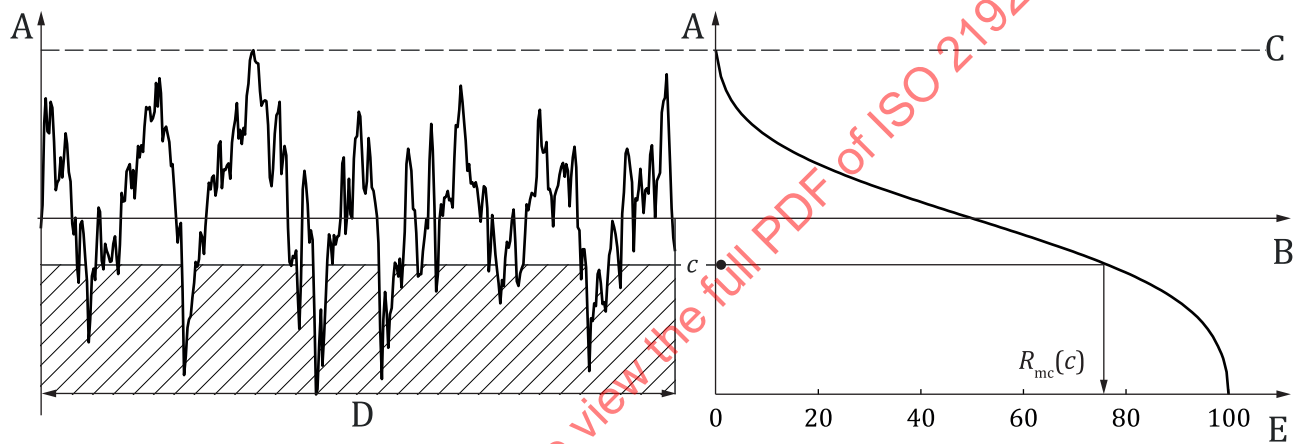
NOTE The material ratio is usually expressed as a percentage.

4.5.1.3 Material ratio curve

The material ratio curve is representing the material ratio of the scale-limited profile as a function of the level c of intersection. The material ratio curve shall be determined as specified in [Annex C](#).

NOTE 1 See [Figure 20](#).

NOTE 2 The material ratio curve is also called the Abbott Firestone curve.



Key

- | | | | |
|---|---------------------------|---|--|
| A | height | D | evaluation length |
| B | x-axis (reference line) | E | material ratio expressed as a percentage |
| C | level $c = 0 \mu\text{m}$ | | |

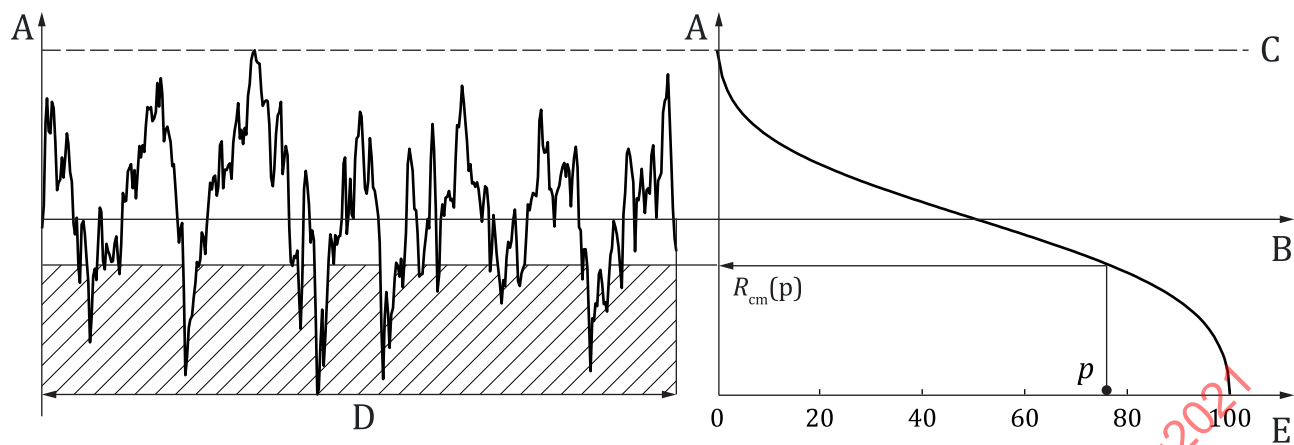
Figure 20 — Material ratio curve of an R-profile

4.5.1.4 Inverse material ratio

$P_{cm}(p)$, $W_{cm}(p)$, $R_{cm}(p)$

The inverse material ratio function is the level of intersection at which a given material ratio p is satisfied. The inverse material ratio function is $0 \mu\text{m}$ for a material ratio of $p = 0 \%$. The level $0 \mu\text{m}$ is the maximum height of the scale-limited profile within the evaluation length l_e .

NOTE See [Figure 21](#).



Key

- | | | | |
|---|---|---|--|
| A | height | D | evaluation length |
| B | x-axis (reference line) | E | material ratio expressed as a percentage |
| C | level 0 μm for a material ratio of $p = 0 \%$ | | |

Figure 21 — Inverse material ratio curve of an R-profile

4.5.1.5 S-shaped material ratio curve

A material ratio curve with one inflection point with a negative third derivative is called S-shaped.

NOTE See [Figure 20](#) for an example of an S-shaped material ratio curve.

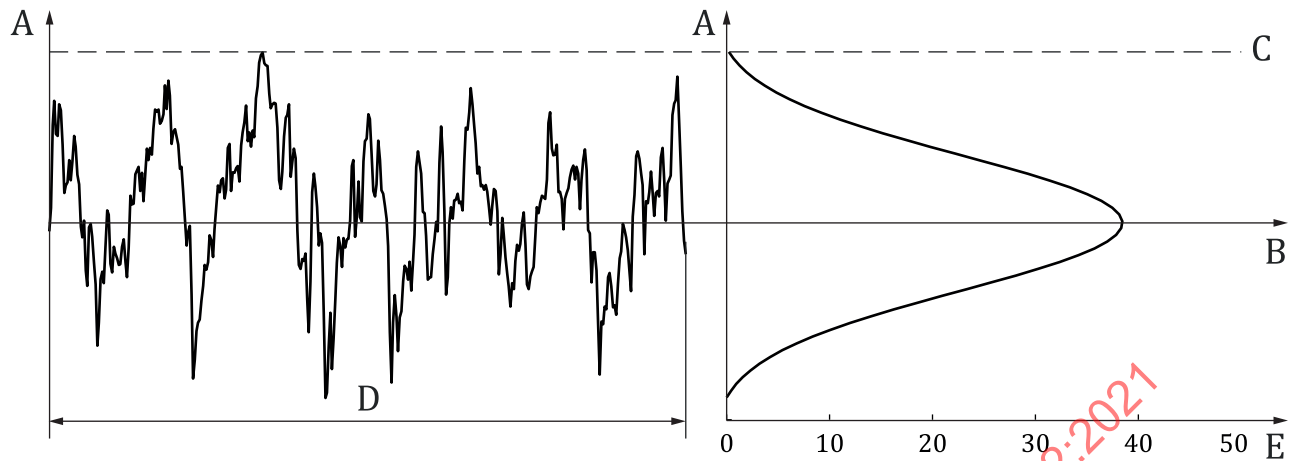
4.5.1.6 Height density

$Phd(c)$, $Whd(c)$, $Rhd(c)$

The height density function is the first derivative of the inverse material ratio $R_{mc}(c)$ at a given level c of intersection. It is calculated according to [Formula \(23\)](#).

$$R_{hd}(c) = - \frac{dR_{mc}(c)}{dc} \tag{23}$$

NOTE See [Figure 22](#).

**Key**

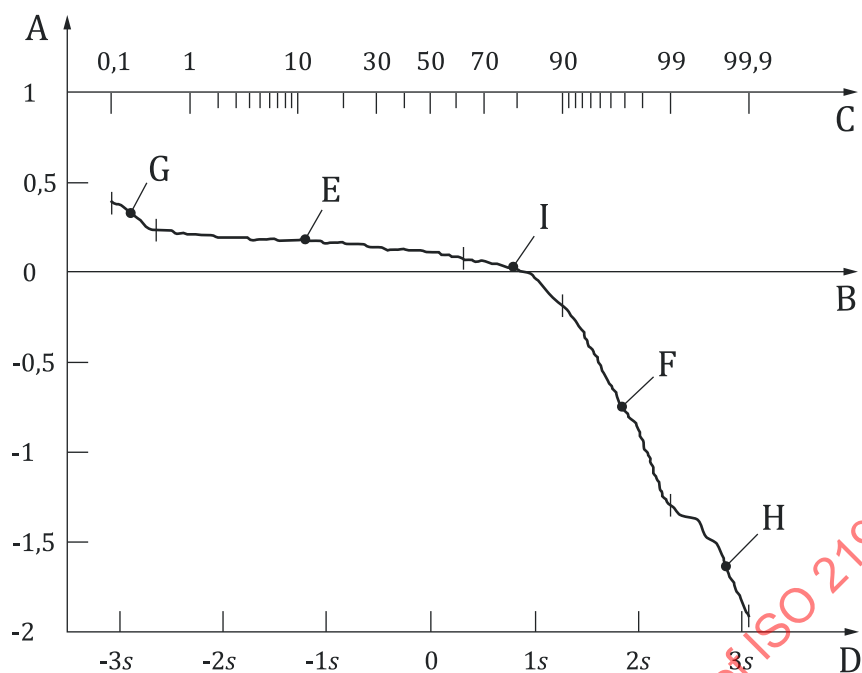
- | | | | |
|---|---------------------------|---|--|
| A | height | D | evaluation length |
| B | x-axis (reference line) | E | height density expressed as a percentage per unit of amplitude |
| C | level $c = 0 \mu\text{m}$ | | |

Figure 22 — Height density of an R-profile**4.5.1.7 Material probability curve**

The material probability curve is a representation of the material ratio curve in which the material ratio is expressed as a Gaussian probability in standard deviation values, plotted linearly on the horizontal axis.

NOTE 1 See [Figure 23](#).

NOTE 2 This scale is expressed linearly in standard deviations according to the Gaussian distribution. In this scale, the material ratio curve of a Gaussian distribution becomes a straight line. For stratified surfaces composed of two Gaussian distributions, for example plateau honed surfaces such as the one shown in [Figure 28](#), the material probability curve will exhibit two linear regions (see keys E and F in [Figure 23](#)).



Key

- A height
- B reference line
- C material ratio expressed as a Gaussian probability in percentage
- D material ratio expressed as a Gaussian probability in standard deviation values s
- E plateau region
- F dale region
- G debris or outlying hills in the data
- H deep scratches or outlying dales in the data
- I unstable region (curvature) introduced at the plateau-to-dale transition point based on the combination of two distributions

Figure 23 — Material probability curve

4.5.1.8 Material volume

$P_{vm}(p)$, $W_{vm}(p)$, $R_{vm}(p)$

The material volume function is the volume of the material at a given material ratio p determined from the material ratio curve. It is calculated according to [Formula \(24\)](#).

$$R_{vm}(p) = K \int_0^p (R_{cm}(q) - R_{cm}(p)) dq \quad (24)$$

where

K is a constant to convert to millilitres per square metre (ml/m^2).

NOTE See [Figure 24](#).

EXAMPLE $K = 1/\mu\text{m} \cdot \text{ml}/\text{m}^2$ if the unit of the scale-limited profile is μm .

4.5.1.9 Void volume

$P_{vv}(p)$, $W_{vv}(p)$, $R_{vv}(p)$

The void volume function is the volume of the voids at a given material ratio p determined from the material ratio curve. It is calculated according to [Formula \(25\)](#).

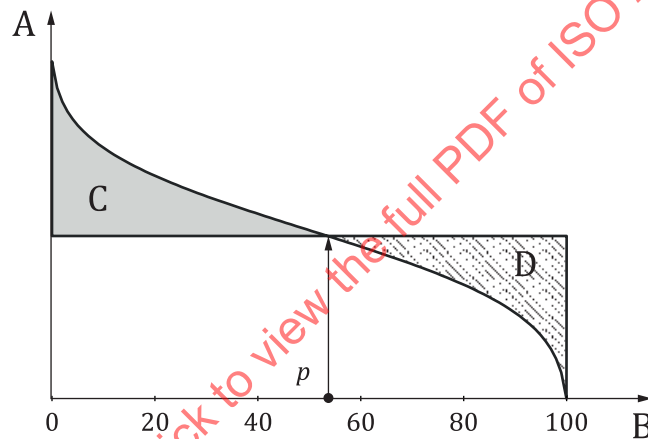
$$R_{vv}(p) = K \int_p^1 (R_{cm}(p) - R_{cm}(q)) dq \quad (25)$$

where

K is a constant to convert to millilitres per square metre (ml/m^2).

NOTE See [Figure 24](#).

EXAMPLE $K = 1/\mu\text{m} \cdot \text{ml}/\text{m}^2$ if the unit of the scale-limited profile is μm .



Key

A	height	C	material volume
B	material ratio expressed as a percentage	D	void volume

Figure 24 — Material volume and void volume

4.5.2 Material ratio parameters

4.5.2.1 Relative material ratio

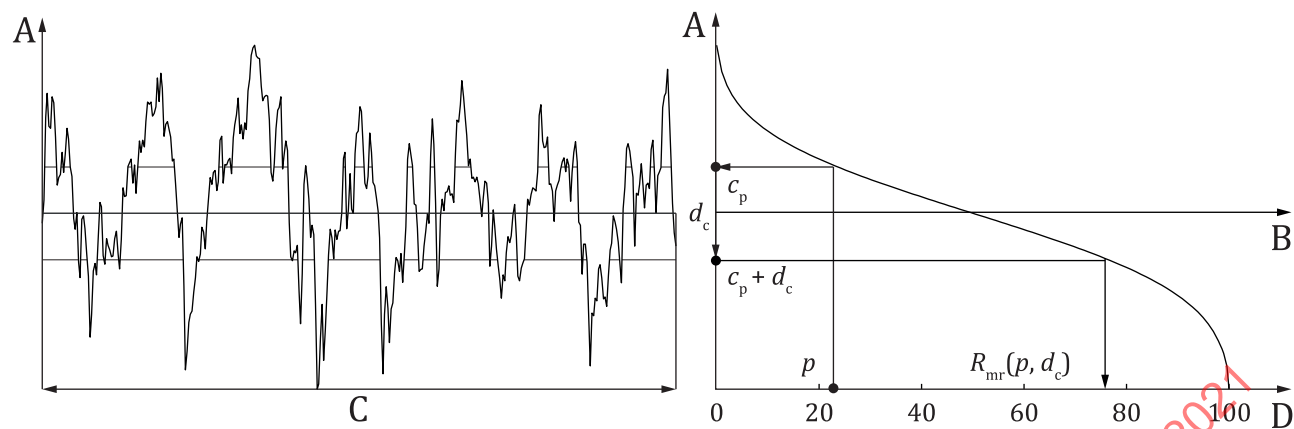
$P_{mr}(p, d_c)$, $W_{mr}(p, d_c)$, $R_{mr}(p, d_c)$

The relative material ratio parameter is the material ratio determined at a level $c_p + d_c$ of intersection where c_p is the inverse material ratio at a material ratio p and d_c is a relative level of intersection. It is calculated according to [Formula \(26\)](#). The specification of the material ratio p is not required if the default value $p = 0\%$ is used.

$$R_{mr}(p, d_c) = R_{mc}(c_p + d_c) \text{ with } c_p = R_{cm}(p) \quad (26)$$

NOTE 1 See [Figure 25](#).

NOTE 2 Usually $d_c < 0$ applies.



Key

A	height	C	evaluation length
B	x-axis (reference line)	D	material ratio expressed as a percentage

Figure 25 — Relative material ratio of an R-profile ($d_c < 0$)

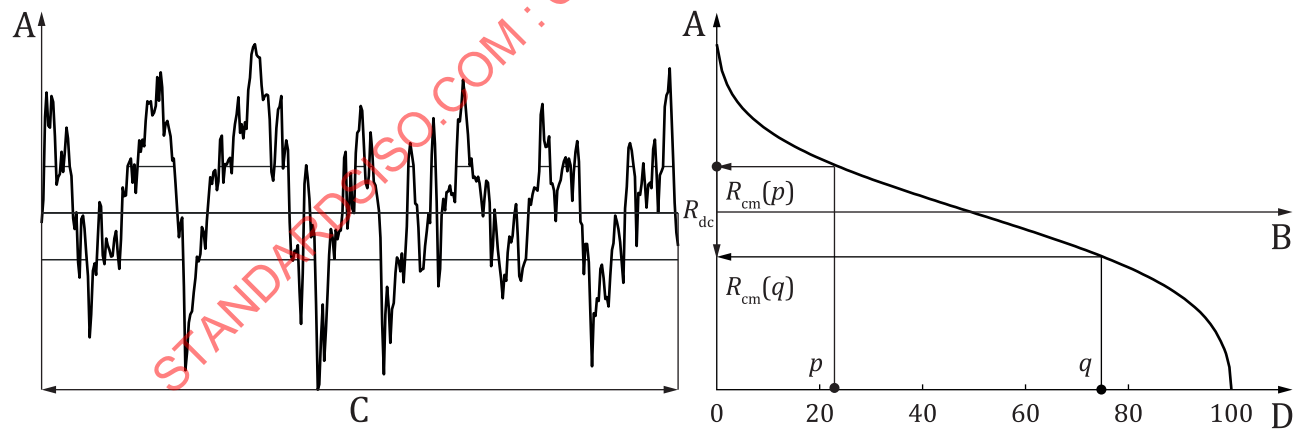
4.5.2.2 Material ratio height difference

$Pdc(p,q)$, $Wdc(p,q)$, $Rdc(p,q)$

The material ratio height difference parameter is the difference between two intersection levels of given material ratio p and q . It is calculated according to [Formula \(27\)](#). The specification of the material ratio p is not required if the default value $p=0\%$ is used.

$$R_{dc}(p,q) = R_{cm}(q) - R_{cm}(p), \quad p \leq q \tag{27}$$

NOTE See [Figure 26](#).



Key

A	height	C	evaluation length
B	x-axis (reference line)	D	material ratio expressed as a percentage

Figure 26 — Material ratio height difference of an R-profile

4.5.3 Parameters for stratified surfaces using the material ratio curve

4.5.3.1 General

Parameters defined in this clause represent the material ratio of stratified surfaces as a function of height of the scale-limited profile.

NOTE 1 A stratified surface is characterized by the fact that it has different statistical properties depending on the profile height. For example, two sequential different machining processes can lead to a stratified surface^[15].

NOTE 2 In most cases, the material ratio curve for W-profile doesn't fulfil the S-shape requirement, i.e. parameters are only defined for P- and R-profiles.

4.5.3.2 Core profile

The core profile is a scale-limited profile excluding protruding hills and dales.

NOTE See [Figure 27](#).

4.5.3.3 Core height

P_k, R_k

The core height parameter is the distance between the highest and lowest levels of the core profile. The core height parameter shall be determined as specified in [Annex D](#).

NOTE See [Figure 27](#) for a visualization of R_k .

4.5.3.4 Reduced peak height

P_{pk}, R_{pk}

The reduced peak height parameter is the height of the protruding peaks above the core profile after the reduction process. The reduced peak height parameter shall be determined as specified in [Annex D](#).

NOTE 1 See [Figure 27](#) for a visualization of R_{pk} .

NOTE 2 The reduction process defined in [Annex D](#) reduces the effect of outlier values on this parameter.

4.5.3.5 Reduced pit depth

P_{vk}, R_{vk}

The reduced pit depth parameter is the depth of the protruding pits below the core profile after the reduction process. The reduced pit depth parameter shall be determined as specified in [Annex D](#).

NOTE 1 See [Figure 27](#) for a visualization of R_{vk} .

NOTE 2 The reduction process defined in [Annex D](#) reduces the effect of outlier values on this parameter.

4.5.3.6 Maximum peak height

P_{pkx}, R_{pkx}

<material ratio>

The maximum peak height parameter is the maximum height of the protruding peaks above the core profile before the reduction process. The maximum peak height parameter shall be determined as specified in [Annex D](#).

NOTE See [Figure 27](#) for a visualization of R_{pkx} .

4.5.3.7 Maximum pit depth

Pvkx, Rvkx

<material ratio>

The maximum pit depth parameter is the maximum depth of the protruding pits below the core profile before the reduction process. The maximum pit depth parameter shall be determined as specified in [Annex D](#).

NOTE See [Figure 27](#) for a visualization of Rvkx.

4.5.3.8 Material ratio of hills

Pmrk1, Rmrk1

The material ratio of hills parameter is the material ratio at the intersection height which separates the protruding hills from the core profile. The material ratio of hills parameter shall be determined as specified in [Annex D](#).

NOTE Pmrk1 and Rmrk1 are usually expressed as a percentage.

4.5.3.9 Material ratio of dales

Pmrk2, Rmrk2

The material ratio of dales parameter is the material ratio at the intersection height which separates the protruding dales from the core profile. The material ratio of dales parameter shall be determined as specified in [Annex D](#).

NOTE Pmrk2 and Rmrk2 are usually expressed as a percentage.

4.5.3.10 Area of hills

Pak1, Rak1

The area of hills parameter is the material volume according to [4.5.1.8](#) determined at material ratio Rmrk1 and for $K = 1$. The area of hills parameter shall be determined as specified in [Annex D](#).

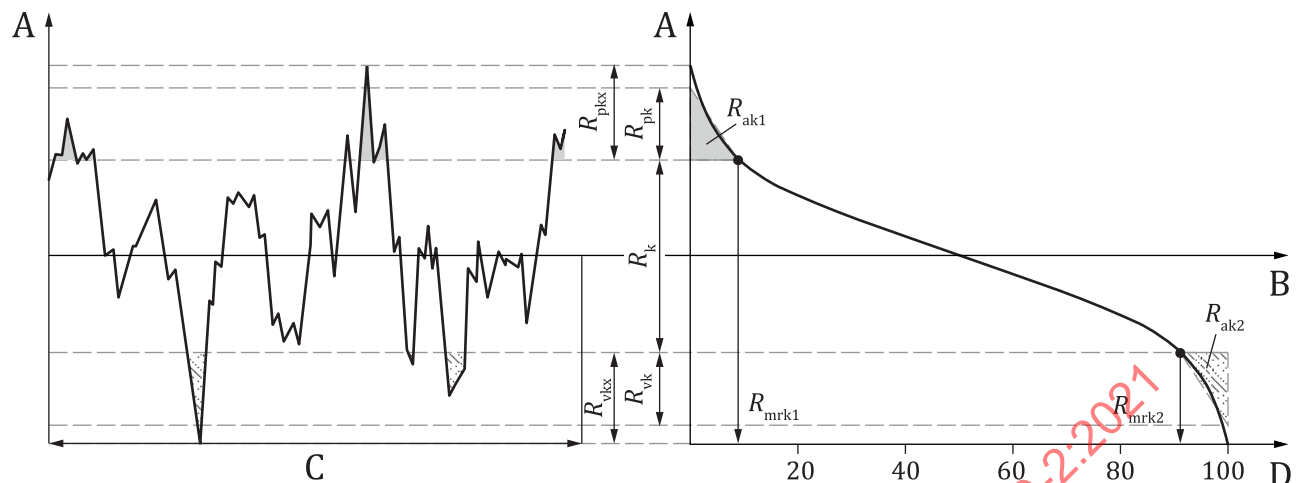
NOTE Pak1 and Rak1 are equal to the area of the triangle obtained during the reduction process of the protruding hills. The height of the triangle is Ppk or Rpk and its base is Pmrk1 or Rmrk1, respectively.

4.5.3.11 Area of dales

Pak2, Rak2

The area of dales parameter is the void volume according to [4.5.1.9](#) determined at material ratio Rmrk2 and for $K = 1$. The area of dales parameter shall be determined as specified in [Annex D](#).

NOTE Pak2 and Rak2 are equal to the area of the triangle obtained during the reduction process of the protruding dales. The height of the triangle is Pvk or Rvk and its base is $100 \% - \text{Pmrk2}$ or $100 \% - \text{Rmrk2}$, respectively.



Key

A	height	C	evaluation length
B	x-axis (reference line)	D	material ratio expressed as a percentage

Figure 27 — Parameters R_k , R_{pk} , R_{vk} , R_{pkx} , R_{vtx} , R_{mrk1} , R_{mrk2} , R_{ak1} and R_{ak2}

4.5.4 Parameters for stratified surfaces using the material probability curve

4.5.4.1 General

Parameters defined in this clause represent the material probability of stratified surfaces as a function of height of the scale-limited profile.

NOTE 1 A stratified surface is characterized by the fact that it has different statistical properties depending on the profile height. For example, two sequential different machining processes can lead to a stratified surface^[15].

NOTE 2 Parameters in this clause are only defined if the stratified surface is characterized by two vertical statistical components. The W-profile often cannot be characterized by two vertical statistical components due to its long scale component, i.e. parameters in this clause are only defined for P- and R-profiles.

4.5.4.2 Plateau root mean square deviation

P_{pq} , R_{pq}

The plateau root mean square deviation parameter is the slope of a linear regression performed to the plateau region. The plateau root mean square deviation parameter shall be determined as specified in [Annex D](#).

NOTE 1 See [Figure 28](#).

NOTE 2 P_{pq} and R_{pq} can thus be interpreted as the P_q -value and the R_q -value, respectively, of the random process that generated the plateau component of the profile.

4.5.4.3 Dale root mean square deviation

P_{vq} , R_{vq}

The dale root mean square deviation parameter is the slope of a linear regression performed to the dale region. The dale root mean square deviation parameter shall be determined as specified in [Annex D](#).

NOTE 1 See [Figure 28](#).

NOTE 2 P_{vq} and R_{vq} can thus be interpreted as the P_q -value and the R_q -value, respectively, of the random process that generated the dale component of the profile.

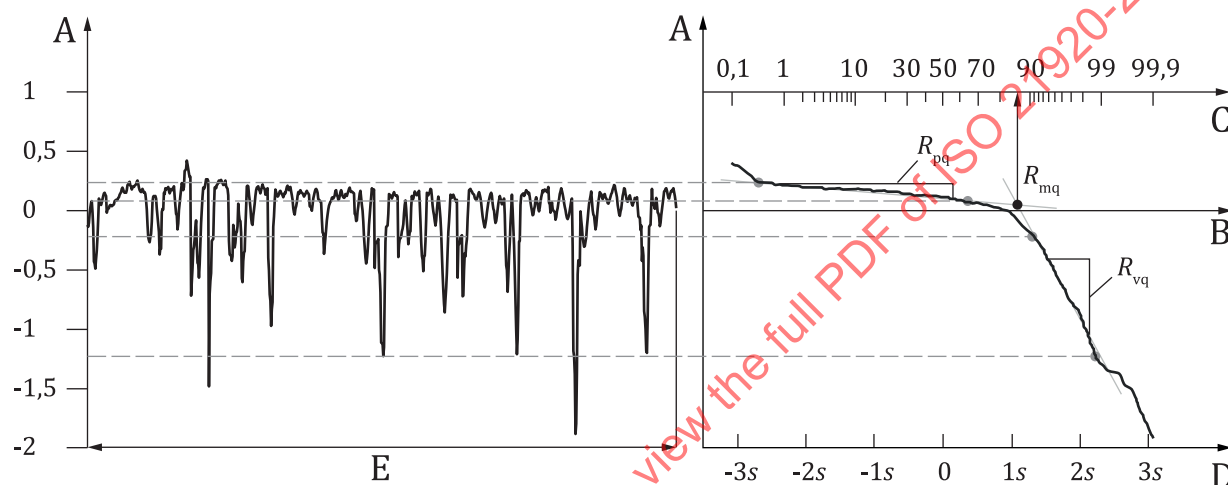
4.5.4.4 Material ratio at plateau-to-dale transition

P_{mq} , R_{mq}

The material ratio at plateau-to-dale transition parameter is the material ratio at the intersection point of the linear regression performed to the hill region and the linear regression performed to the dale region. The material ratio at plateau-to-dale transition parameter shall be determined as specified in [Annex D](#).

NOTE 1 See [Figure 28](#).

NOTE 2 Material ratio expressed as a percentage.



Key

- A height
- B x-axis (reference line)
- C material ratio expressed as a Gaussian probability in percentage
- D material ratio expressed as a Gaussian probability in standard deviation values s
- E evaluation length

Figure 28 — Profile with its corresponding material probability curve and the regions used in the definitions of the parameters R_{mq} , R_{pq} and R_{vq}

4.5.5 Volume parameters

4.5.5.1 General

For anisotropic surfaces, the material volume or void volume of a surface at a given material ratio p can be estimated using a profile trace of the surface. For example, the areal and profile volume parameters of a turned surface are highly correlated.

4.5.5.2 Hill material volume

$P_{vmp}(p)$, $W_{vmp}(p)$, $R_{vmp}(p)$

The hill material volume parameter is the material volume at p material ratio. It is calculated according to [Formula \(28\)](#). The specification of the material ratio p is not required if the default value is used.

$$R_{\text{vmp}}(p) = R_{\text{vm}}(p) \quad (28)$$

NOTE 1 See [Figure 29](#).

NOTE 2 The default value for p is found in ISO 21920-3.

4.5.5.3 Core material volume

$P_{\text{vmc}}(p, q)$, $W_{\text{vmc}}(p, q)$, $R_{\text{vmc}}(p, q)$

The core material volume parameter is the difference in material volume between p and q material ratio. It is calculated according to [Formula \(29\)](#). The specification of the material ratios p and q is not required if the default values are used.

$$R_{\text{vmc}}(p, q) = R_{\text{vm}}(q) - R_{\text{vm}}(p), \quad p \leq q \quad (29)$$

NOTE 1 See [Figure 29](#).

NOTE 2 The default values for p and q are found in ISO 21920-3.

4.5.5.4 Core void volume

$P_{\text{vvc}}(p, q)$, $W_{\text{vvc}}(p, q)$, $R_{\text{vvc}}(p, q)$

The core void volume parameter is the difference in void volume between p and q material ratios. It is calculated according to [Formula \(30\)](#). The specification of the material ratios p and q is not required if the default values are used.

$$R_{\text{vvc}}(p, q) = R_{\text{vv}}(p) - R_{\text{vv}}(q), \quad p \leq q \quad (30)$$

NOTE 1 See [Figure 29](#).

NOTE 2 The default values for p and q are found in ISO 21920-3.

4.5.5.5 Dale void volume

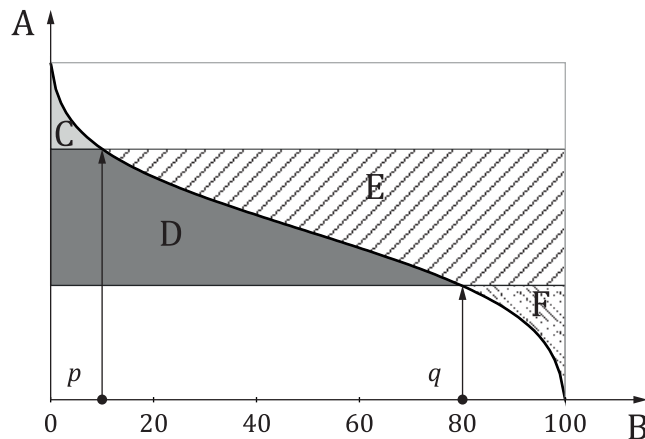
$P_{\text{vvv}}(q)$, $W_{\text{vvv}}(q)$, $R_{\text{vvv}}(q)$

The dale void volume parameter is the dale volume at q material ratio. It is calculated according to [Formula \(31\)](#). The specification of the material ratio q is not required if the default value is used.

$$R_{\text{vvv}}(q) = R_{\text{vv}}(q) \quad (31)$$

NOTE 1 See [Figure 29](#).

NOTE 2 The default value for q is found in ISO 21920-3.



Key

A	height	D	core material volume
B	material ratio expressed as a percentage	E	core void volume
C	hill material volume	F	dale void volume

Figure 29 — Void volume and material volume parameters

5 Feature parameters

5.1 Parameters based on peak heights and pit depths

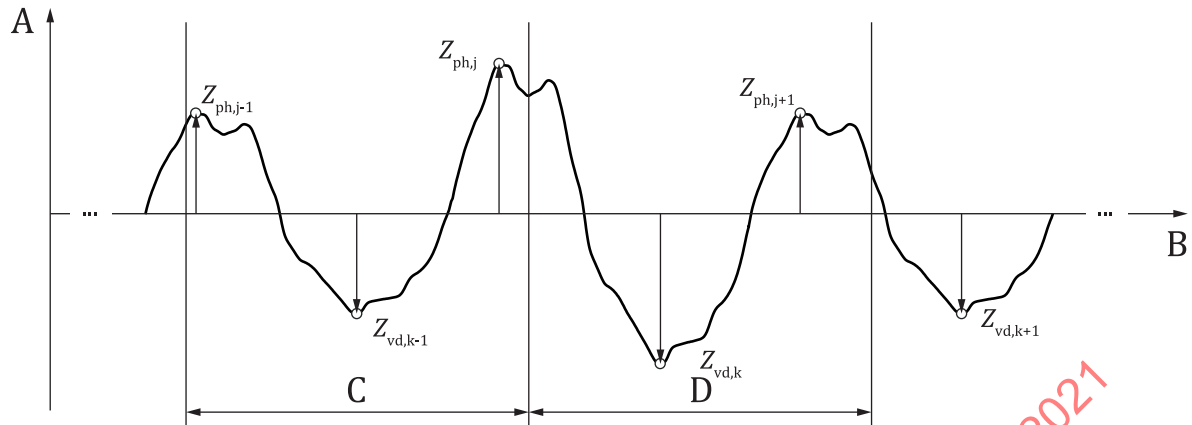
5.1.1 General

Parameters based on peak heights and pit depths are related to the section length l_{sc} of a scale-limited profile. Therefore, feature parameters in 5.1 are categorized as section length parameters. The calculation of the associated hills and dales is carried out by simply regarding the sign changes of the ordinate values (see 3.3.11 and 3.3.17) without any height discrimination (see 3.3.31). For parameters defined in this clause, the following definitions are used:

- n_p the number of peaks within the evaluation length (see 3.3.6);
- n_v the number of pits within the evaluation length (see 3.3.9);
- n_{sc} the number of sections within the evaluation length (see 3.1.18);
- l_{sc} the section length (see 3.1.17);
- $Z_{ph,j}$ the peak height (see 3.3.15) of the j -th peak;
- $Z_{vd,j}$ the pit depth (see 3.3.21) of the j -th pit.

The positive or negative portion of the assessed profile at the beginning or end of the evaluation length shall always be considered as a hill or as a dale.

NOTE The parameter definitions in 5.1.3, 5.1.5 and 5.1.6 contain mathematical expressions of the form $N_i = \{j = 1, 2, \dots, n_p \mid (i-1)l_{sc} \leq x_j < il_{sc}\}$ with $i = 1, 2, \dots, n_{sc}$. This example expression for N_i can be understood as follows: The peaks of the hills found within the evaluation length are assigned a unique ID from 1 to n_p . That list of peak IDs is $j = 1, 2, \dots, n_p$. Each peak has an associated lateral position x_j . N_i is a subset of the list of peak IDs retaining only the peaks with a lateral position x_j that falls within the i -th section length from the left end of the evaluation length (see Figure 30). The same notation applies to the pits of the dales.

**Key**

A height

B x-axis (reference line)

C (i -th -1) section length

D i -th section length

Figure 30 — Allocation of peaks and pits to the section lengths**5.1.2 Maximum peak height**

Ppt, Wpt, Rpt

The maximum peak height parameter is the largest peak height of all section lengths. It is calculated according to [Formula \(32\)](#).

$$R_{pt} = \max_{j=1, \dots, n_p} (Z_{ph,j}) \quad (32)$$

NOTE Ppt, Wpt or Rpt is zero if no profile hills exist.

5.1.3 Mean peak height

Pp, Wp, Rp

The mean peak height parameter is the mean value, from all section lengths, of the largest peak height of each section length. It is calculated according to [Formula \(33\)](#).

$$R_p = \frac{1}{n_{sc}} \sum_{i=1}^{n_{sc}} \max_{j \in N_i} (Z_{ph,j}) \quad (33)$$

where

$$N_i = \{j = 1, 2, \dots, n_p \mid (i-1) l_{sc} \leq x_j < i l_{sc}\};$$

x_j is the position of the j -th profile peak on the x -axis.

NOTE Pp, Wp or Rp is zero if no profile hills exist.

5.1.4 Maximum pit depth

Pvt, Wvt, Rvt

The maximum pit depth parameter is the largest pit depth of all section lengths. It is calculated according to [Formula \(34\)](#).

$$R_{vt} = \max_{j=1, \dots, n_v} (Z_{vd,j}) \quad (34)$$

NOTE P_{vt} , W_{vt} or R_{vt} is zero if no profile dales exist.

5.1.5 Mean pit depth

P_v , W_v , R_v

The mean pit depth parameter is the mean value, from all section lengths, of the largest pit depth of each section length. It is calculated according to [Formula \(35\)](#).

$$R_v = \frac{1}{n_{sc}} \sum_{i=1}^{n_{sc}} \max_{j \in N_i} (Z_{vd,j}) \quad (35)$$

where

$$N_i = \{j = 1, 2, \dots, n_v \mid (i-1) l_{sc} \leq x_j < i l_{sc}\};$$

x_j is the position of the j -th profile peak on the x -axis.

NOTE P_v , W_v or R_v is zero if no profile dales exist.

5.1.6 Maximum height

P_z , W_z , R_z

The maximum height parameter is the mean value, from all section lengths, of the per section sum of the largest peak height and largest pit depth. It is calculated according to [Formula \(36\)](#).

$$R_z = \frac{1}{n_{sc}} \sum_{i=1}^{n_{sc}} \left(\max_{j \in N_{p,i}} (Z_{ph,j}) + \max_{k \in N_{v,i}} (Z_{vd,k}) \right) \quad (36)$$

where

$$N_{p,i} = \{j = 1, 2, \dots, n_p \mid (i-1) l_{sc} \leq x_j < i l_{sc}\};$$

$$N_{v,i} = \{k = 1, 2, \dots, n_v \mid (i-1) l_{sc} \leq x_k < i l_{sc}\};$$

x_j is the position of the j -th profile peak on the x -axis;

x_k is the position of the k -th profile pit on the x -axis.

NOTE P_z , W_z or R_z is zero if no profile hills and profile dales exist.

5.2 Parameters based on profile elements

5.2.1 General

Profile elements are determined using the crossing-the-line segmentation (see [Annex E](#)) in conjunction with height discrimination. Profile elements are evaluated from the beginning to the end of the evaluation length and vice versa, as shown in [Figures 31](#) and [32](#), following by an averaging. For parameters defined in this clause, the following definitions are used:

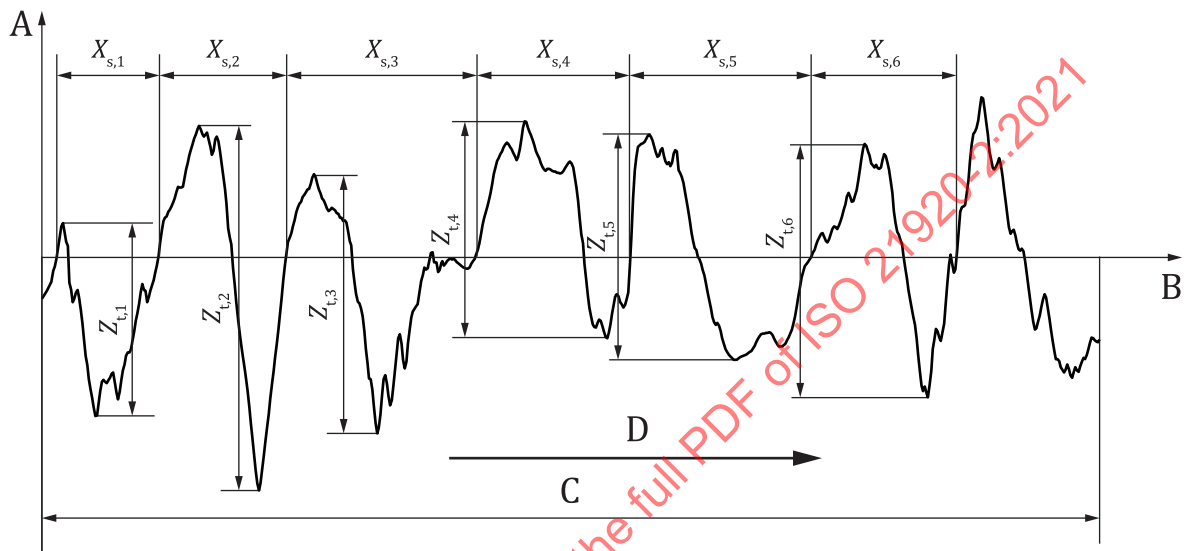
— n_{pe} the total number of profile elements;

- $Z_{t,i}$ the profile element height (see 3.3.33) of the i -th profile element;
- $X_{s,i}$ the profile element spacing (see 3.3.34) of the i -th profile element.

The profile element height Z_t and profile element spacing X_s shall be determined as specified in Annex E.

NOTE 1 If not otherwise specified, the default values for the peak height discrimination and for the pit depth discrimination are found in ISO 21920-3:2021, 5.5 (see also Annex E).

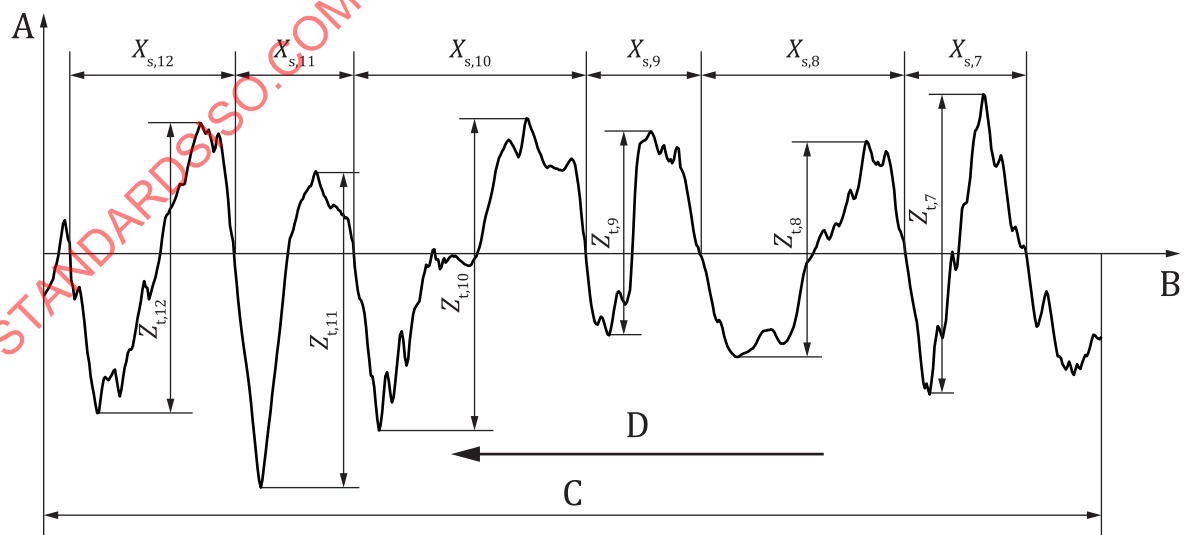
NOTE 2 Parameters are determined without spacing discrimination. See Reference [20] for more details.



Key

- | | | | |
|---|-------------------------|---|-------------------------|
| A | height | C | evaluation length |
| B | x-axis (reference line) | D | direction of evaluation |

Figure 31 — Determination of profile elements from the beginning to the end



Key

- | | | | |
|---|-------------------------|---|-------------------------|
| A | height | C | evaluation length |
| B | x-axis (reference line) | D | direction of evaluation |

Figure 32 — Determination of profile elements from the end to the beginning

5.2.2 Mean profile element spacing

Psm, Wsm, Rsm

The mean profile elements spacing parameter is the mean value of the profile element spacings X_s . It is calculated according to [Formula \(37\)](#).

$$R_{sm} = \frac{1}{n_{pe}} \sum_{i=1}^{n_{pe}} X_{s,i} \quad (37)$$

5.2.3 Maximum profile element spacing

Psmx, Wsmx, Rsmx

The maximum profile element spacing parameter is the maximum of the profile element spacings X_s . It is calculated according to [Formula \(38\)](#).

$$R_{smx} = \max_{i=1, \dots, n_{pe}} X_{s,i} \quad (38)$$

5.2.4 Standard deviation of profile element spacings

Psmq, Wsmq, Rsmq

The standard deviation of profile element spacings parameter is the standard deviation of the profile element spacings X_s . It is calculated according to [Formula \(39\)](#).

$$R_{smq} = \sqrt{\frac{1}{n_{pe}-1} \cdot \sum_{i=1}^{n_{pe}} (X_{s,i} - R_{sm})^2}, \quad n_{pe} > 1 \quad (39)$$

5.2.5 Mean profile element height

Pc, Wc, Rc

The mean profile element height parameter is the mean value of the profile element heights Z_t . It is calculated according to [Formula \(40\)](#).

$$R_c = \frac{1}{n_{pe}} \cdot \sum_{i=1}^{n_{pe}} Z_{t,i} \quad (40)$$

5.2.6 Maximum profile element height

Pcx, Wcx, Rcx

The maximum profile element height parameter is the maximum of the profile element heights Z_t . It is calculated according to [Formula \(41\)](#).

$$R_{cx} = \max_{i=1, \dots, n_{pe}} Z_{t,i} \quad (41)$$

5.2.7 Standard deviation of profile element heights

Pcq, Wcq, Rcq

The standard deviation of the profile element heights parameter is the standard deviation of the profile element heights Z_t . It is calculated according to [Formula \(42\)](#).

$$R_{cq} = \sqrt{\frac{1}{n_{pe} - 1} \sum_{i=1}^{n_{pe}} (Z_{t,i} - R_c)^2}, \quad n_{pe} > 1 \quad (42)$$

5.2.8 Peak count parameter

Ppc, Wpc, Rpc

The peak count parameter is the number of mean spacings of profile elements per unit length L . It is calculated according to [Formula \(43\)](#) with the mean profile element spacing Rsm (see [5.2.2](#)).

$$R_{pc} = \frac{L}{R_{sm}} \quad (43)$$

NOTE The default unit length L and the height discrimination level to determine Psm, Wsm or Rsm can be found in ISO 21920-3:2021, 5.5.

5.3 Parameters based on feature characterization

5.3.1 General

Parameters defined in this clause are determined using the watershed segmentation over the evaluation length l_e . The feature characterization method shall be carried out as specified in [Annex F](#).

5.3.2 Named feature parameters

5.3.2.1 Density of peaks

Ppd, Wpd, Rpd

The density of peaks parameter is the number of peaks per unit length. It corresponds to the following feature specification:

Rpd = FC; P; Wolfprune X ; All; Count; Density

NOTE If not otherwise specified, the default value of X expressed as a percentage is found in ISO 21920-3:2021, 5.5.

5.3.2.2 Density of pits

Pvd, Wvd, Rvd

The density of pits parameter is the number of peaks per unit length. It corresponds to the following feature specification:

Rvd = FC; V; Wolfprune X ; All; Count; Density

NOTE If not otherwise specified, the default value of X expressed as a percentage is found in ISO 21920-3:2021, 5.5.

5.3.2.3 Arithmetic mean peak curvature

Pmpc, Wmpc, Rmpc

The arithmetic mean peak curvature parameter is the arithmetic mean of the local mean curvature of the peaks. It corresponds to the following feature specification:

Rmpc = FC; P; Wolfprune X; All; Curvature; Mean

NOTE If not otherwise specified, the default value of X expressed as a percentage is found in ISO 21920-3:2021, 5.5.

5.3.2.4 Arithmetic mean pit curvature

Pmvc, Wmvc, Rmvc

The arithmetic mean pit curvature parameter is the arithmetic mean of the local mean curvature of the peaks. It corresponds to the following feature specification:

Rmvc = FC; V; Wolfprune X; All; Curvature; Mean

NOTE If not otherwise specified, the default value of X expressed as a percentage is found in ISO 21920-3:2021, 5.5.

5.3.2.5 Five-point peak height

P5p, W5p, R5p

The five-point peak height parameter is the arithmetic mean of the five largest peak heights. It corresponds to the following feature specification:

R5p = FC; P; Wolfprune X; Top 5; PVh; Mean

NOTE If not otherwise specified, the default value of X expressed as a percentage is found in ISO 21920-3:2021, 5.5.

5.3.2.6 Five-point pit depth

P5v, W5v, R5v

The five-point pit depth parameter is the arithmetic mean of the five largest pit depths. It corresponds to the following feature specification:

R5v = FC; V; Wolfprune X; Bot 5; PVh; Mean

NOTE If not otherwise specified, the default value of X expressed as a percentage is found in ISO 21920-3:2021, 5.5.

5.3.2.7 Ten-point height

P10z, W10z, R10z

The 10-point height parameter is the sum of the five-point peak height and the five-point pit depth. It is calculated according to [Formula \(44\)](#).

$$R_{10z} = R_{5p} + R_{5v} \quad (44)$$

Annex A (informative)

Determination of the first and second derivative

A.1 General

The determination of the first and second derivative will be applied to a uniformly sampled profile with the sampling distance $\Delta x = l/n$, where l is a given length and n the number of samples. Each sample is expressed by $z_i = z(x_i)$ with $x_i = i \Delta x$ and $i = 0, 1, 2, \dots, n-1$.

A.2 Estimation of the first derivative (local gradient)

A.2.1 Polynomial of sixth degree

Inner profile region $3 \leq i \leq n-4$, see [Formula \(A.1\)](#):

$$\left. \frac{dz(x)}{dx} \right|_{x=x_i} \approx \frac{(-z_{i-3} + 9z_{i-2} - 45z_{i-1} + 45z_{i+1} - 9z_{i+2} + z_{i+3})}{60\Delta x} \quad (\text{A.1})$$

Left profile boundary ($i = 0, 1, 2$), see [Formulae \(A.2\) to \(A.4\)](#):

$$\left. \frac{dz(x)}{dx} \right|_{x=x_0} \approx \frac{(-147z_0 + 360z_1 - 450z_2 + 400z_3 - 225z_4 + 72z_5 - 10z_6)}{60\Delta x} \quad (\text{A.2})$$

$$\left. \frac{dz(x)}{dx} \right|_{x=x_1} \approx \frac{(-10z_0 - 77z_1 + 150z_2 - 100z_3 + 50z_4 - 15z_5 + 2z_6)}{60\Delta x} \quad (\text{A.3})$$

$$\left. \frac{dz(x)}{dx} \right|_{x=x_2} \approx \frac{(2z_0 - 24z_1 - 35z_2 + 80z_3 - 30z_4 + 8z_5 - z_6)}{60\Delta x} \quad (\text{A.4})$$

Right profile boundary ($i = n-3, n-2, n-1$), see [Formulae \(A.5\) to \(A.7\)](#):

$$\left. \frac{dz(x)}{dx} \right|_{x=x_{n-3}} \approx \frac{(z_{n-7} - 8z_{n-6} + 30z_{n-5} - 80z_{n-4} + 35z_{n-3} + 24z_{n-2} - 2z_{n-1})}{60\Delta x} \quad (\text{A.5})$$

$$\left. \frac{dz(x)}{dx} \right|_{x=x_{n-2}} \approx \frac{(-2z_{n-7} + 15z_{n-6} - 50z_{n-5} + 100z_{n-4} - 150z_{n-3} + 77z_{n-2} + 10z_{n-1})}{60\Delta x} \quad (\text{A.6})$$

$$\left. \frac{dz(x)}{dx} \right|_{x=x_{n-1}} \approx \frac{(10z_{n-7} - 72z_{n-6} + 225z_{n-5} - 400z_{n-4} + 450z_{n-3} - 360z_{n-2} + 147z_{n-1})}{60\Delta x} \quad (\text{A.7})$$

NOTE This definition of the local gradient is given in ISO 4287 and is used by the majority of commercial software packages.

A.2.2 Cubic spline with natural end conditions

The vector c of coefficients proportionally to the second derivatives at position x_i of a cubic spline $s(x)$ with natural end conditions is solved by the matrix in [Formula \(A.8\)](#):

$$c = Q^{-1} \cdot P \cdot z \quad (\text{A.8})$$

with the matrix in [Formula \(A.9\)](#):

$$Q^{(n-2) \times (n-2)} = \begin{pmatrix} 4 & 1 & & & \\ 1 & 4 & & & \\ & & \ddots & \ddots & \ddots \\ & & & 1 & 4 & 1 \\ & & & & 1 & 4 \end{pmatrix}, \quad P^{(n-2) \times n} = \begin{pmatrix} 3 & -6 & 3 & & & \\ & 3 & -6 & 3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & 3 & -6 & 3 \\ & & & & 3 & -6 & 3 \end{pmatrix} \quad (\text{A.9})$$

and [Formula \(A.10\)](#):

$$c^{(n-2) \times 1} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n-2} \end{pmatrix}, \quad z^{N \times 1} = \begin{pmatrix} z_0 \\ z_1 \\ \vdots \\ z_{n-1} \end{pmatrix} \quad (\text{A.10})$$

where

n is the number of samples;

z is the vector of the sampled profile values z_i ;

c is the vector of spline coefficients proportionally to the second derivatives at position x_i of a cubic spline $s(x)$ with natural end conditions ($c_0 = 0$ and $c_{n-1} = 0$).

The first derivatives at position x_i are given by [Formula \(A.11\)](#):

$$\left. \frac{dz(x)}{dx} \right|_{x=x_i} \approx \left. \frac{ds(x)}{dx} \right|_{x=x_i} = \frac{1}{\Delta x} \begin{cases} z_1 - z_0 - \frac{1}{3}c_1 & i=0 \\ z_{i+1} - z_i - \frac{1}{3}(c_{i+1} + 2c_i) & i=1, \dots, n-2 \\ z_{N-1} - z_{N-2} + \frac{1}{3}c_{N-2} & i=n-1 \end{cases} \quad (\text{A.11})$$

A.3 Estimation of the second derivative

A.3.1 Polynomial of sixth degree

Inner profile region $3 \leq i \leq n-4$, see [Formula \(A.12\)](#):

$$\left. \frac{d^2 z(x)}{dx^2} \right|_{x=x_i} \approx \frac{(2z_{i-3} - 27z_{i-2} + 270z_{i-1} - 490z_i + 270z_{i+1} - 27z_{i+2} + 2z_{i+3})}{180\Delta x^2} \quad (\text{A.12})$$

Left profile boundary ($i=0, 1, 2$), see [Formulae \(A.13\)](#) to [\(A.15\)](#):

$$\left. \frac{d^2 z(x)}{dx^2} \right|_{x=x_0} \approx \frac{(812z_0 - 3132z_1 + 5265z_2 - 5080z_3 + 2970z_4 - 972z_5 + 137z_6)}{180\Delta x^2} \quad (\text{A.13})$$

$$\left. \frac{d^2 z(x)}{dx^2} \right|_{x=x_1} \approx \frac{(137z_0 - 147z_1 - 255z_2 + 470z_3 - 285z_4 + 93z_5 - 13z_6)}{180\Delta x^2} \quad (\text{A.14})$$

$$\left. \frac{d^2 z(x)}{dx^2} \right|_{x=x_2} \approx \frac{(-13z_0 + 228z_1 - 420z_2 + 200z_3 + 15z_4 - 12z_5 + 2z_6)}{180\Delta x^2} \quad (\text{A.15})$$

Right profile boundary ($i = n-3, n-2, n-1$), see [Formulae \(A.16\)](#) to [\(A.18\)](#):

$$\left. \frac{d^2 z(x)}{dx^2} \right|_{x=x_{n-3}} \approx \frac{(2z_{n-7} - 12z_{n-6} + 15z_{n-5} + 200z_{n-4} - 420z_{n-3} + 228z_{n-2} - 13z_{n-1})}{180\Delta x^2} \quad (\text{A.16})$$

$$\left. \frac{d^2 z(x)}{dx^2} \right|_{x=x_{n-2}} \approx \quad (\text{A.17})$$

$$\frac{(-13z_{n-7} + 93z_{n-6} - 285z_{n-5} + 470z_{n-4} - 255z_{n-3} - 147z_{n-2} + 137z_{n-1})}{180\Delta x^2}$$

$$\left. \frac{d^2 z(x)}{dx^2} \right|_{x=x_{n-1}} \approx \quad (\text{A.18})$$

$$\frac{(137z_{n-7} - 972z_{n-6} + 2970z_{n-5} - 5080z_{n-4} + 5265z_{n-3} - 3132z_{n-2} + 812z_{n-1})}{180\Delta x^2}$$

A.3.2 Cubic spline with natural end conditions

The second derivatives at position x_i of a cubic spline $s(x)$ with natural end conditions is given by [Formula \(A.19\)](#):

$$\left. \frac{d^2 s(x)}{dx^2} \right|_{x=x_i} = \frac{1}{\Delta x^2} \begin{cases} 0 & i=0 \\ 2c_i & i=1, \dots, n-2 \\ 0 & i=n-1 \end{cases} \quad (\text{A.19})$$

where

c_i is the i -th spline coefficient calculated according to [Formula \(A.8\)](#).

Annex B (informative)

Determination of the local curvature

The local curvature of the scale-limited profile is defined according to [Formula \(B.1\)](#):

$$\kappa(x)|_{x=x_i} = \frac{\left. \frac{d^2 z(x)}{dx^2} \right|_{x=x_i}}{\left(1 + \left(\left. \frac{dz(x)}{dx} \right|_{x=x_i} \right)^2 \right)^{\frac{3}{2}}} \quad (\text{B.1})$$

The first and second derivatives can be estimated by a local polynomial of sixth degree according to [A.2.1](#) and [A.3.1](#) or by a cubic spline interpolation according to [A.2.2](#) and [A.3.2](#), respectively.

Annex C (normative)

Determination of the material ratio curve

If not otherwise specified, the local material ratio curve shall be estimated by sorting the uniformly sampled profile in descending order. Let $\Delta x = l/n$ the sampling distance, where l is a given length and n the number of samples. Each sampled profile value is expressed by $z_i = z(x_i)$ with $x_i = i \Delta x$ and $i = 0, 1, 2, \dots, n-1$. The material ratio curve is given by the value pairs (interim values are determined by interpolation), as shown in [Formula \(C.1\)](#):

$$\left(\frac{k}{n}, c_k \right), \quad k = 1, 2, \dots, n \quad (\text{C.1})$$

where

n is the number of samples;

c_k are the uniformly sampled profile values sorted in descending order $c_1 \geq c_2 \geq \dots \geq c_n$.

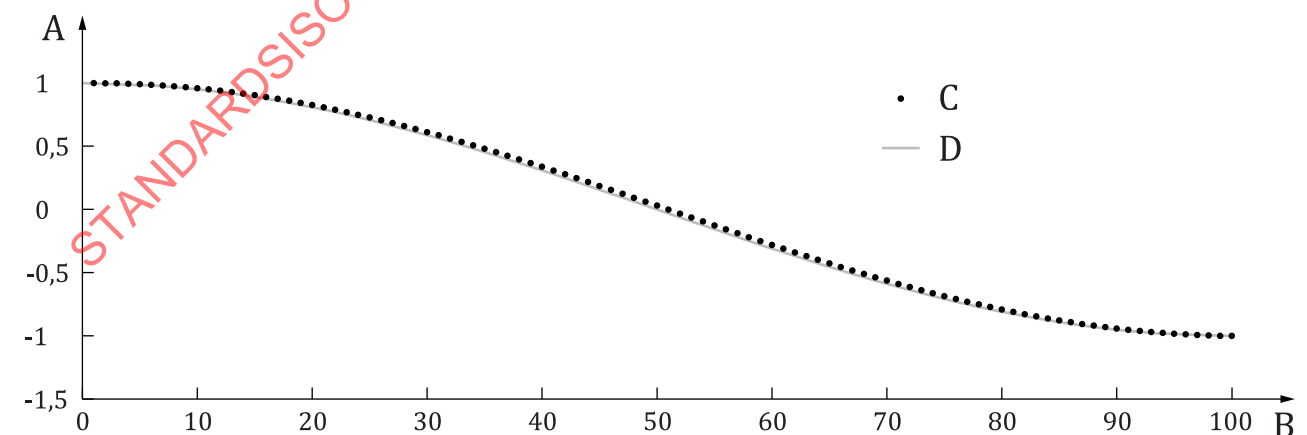
EXAMPLE Suppose a profile $z(x) = \cos(2\pi/(1,6 \text{ mm})x) \mu\text{m}$ with wavelength $\lambda = 1,6 \text{ mm}$. The profile is extracted over a length of $l = 0,8 \text{ mm}$ with $n = 100$ samples and a sampling distance of $\Delta x = (0,8 \text{ mm})/100$. The sorted values z_i are given by [Formula \(C.2\)](#):

$$c_k = \{z(0 \Delta x), z(1 \Delta x), \dots, z(98 \Delta x), z(99 \Delta x)\}, \quad k = 1, \dots, n \quad (\text{C.2})$$

For an infinitely small sampling distance Δx , the material ratio curve is given by [Formula \(C.3\)](#):

$$p = \frac{1}{\pi} \arccos\left(\frac{z}{\mu\text{m}}\right), \quad -1 \mu\text{m} \leq z \leq 1 \mu\text{m} \quad (\text{C.3})$$

In [Figure C.1](#), the material ratio is shown depending on the height c_k and z , respectively.



Key

A	height	C	estimation ($k/n, c_k$)
B	material ratio expressed as a percentage	D	theoretical value (p, z)

Figure C.1 — Estimation of the material ratio curve of a sinusoidal profile

Annex D (normative)

Determination of profile parameters for stratified surfaces

D.1 General

In [Annex D](#), formulae for the determination of parameters are exemplarily given for R-parameters. P-parameters are defined in a similar manner replacing the parameters related to the R-profile by those related to the P-profile.

D.2 Determination of the equivalent best-fit straight line

The equivalent best-fit straight line is determined for the central region of the material ratio curve, which includes 40 % of the measured surface points. This central region lies where the secant of the material ratio curve over 40 % of the material ratio shows the smallest gradient (see [Figure D.1](#)). This is determined by moving a secant line for $\Delta p = 40\%$ along the material ratio curve, starting at the $p=0$ position. The secant line for $\Delta p = 40\%$ which has the smallest gradient establishes the central region of the material ratio curve for the equivalence determination. If there are multiple regions which have equivalent minimum gradients, then the region that is first encountered is the region of choice. A straight line is then determined for this central region, which gives the least square deviation in the direction of the surface ordinates.

To ensure the validity of the material ratio curve, the material ratio curve shall be determined as described in [Annex C](#).

NOTE Optionally, the search of the central region can be done by fitting a least square line instead of a secant line.

D.3 Determination of the parameters R_k , R_{mrk1} and R_{mrk2}

The equivalent best-fit straight line intersects the 0 % and 100 % lines on the material ratio axis (see [Figure D.1](#)). From these points, two lines are plotted parallel to the x -axis; these determine the core profile by separating the protruding hills and dales.

The vertical distance between these intersection heights is the core height R_k . Their intersections with the material ratio curve define the material ratios R_{mrk1} and R_{mrk2} .

D.4 Determination of the parameters R_{pk} , R_{vk} , R_{pkx} and R_{vkx}

The areas above and below the region of the material ratio curve which delimits the core height R_k are shown with grey fill colour and hatched fill area in [Figure D.1](#). These correspond to the cross-sectional area of the profile hills and dales which protrude out of the core surface.

The parameters R_{pk} and R_{vk} are each determined as the height of the right-angle triangle which is constructed to have the same area as the “hill area” or “dale area”, respectively (see [Figure D.1](#)). The right-angle triangle corresponding to the hill area R_{ak1} has R_{mrk1} as its base, and that corresponding to dale area R_{ak2} has $100\% - R_{mrk2}$ as its base.

The parameters R_k , R_{pk} , R_{vk} , R_{mrk1} and R_{mrk2} shall only be determined if the material ratio curve is S-shaped (see [4.5.1.5](#)) as shown in [Figure D.1](#), and thus has only one single point of inflection. Experience has shown that this is always the case for lapped, ground or honed surfaces.

The parameter R_{pkx} is determined as the difference in height between 0 % and R_{mrk1} material ratio. The parameter R_{vkx} is determined as the difference in height between R_{mrk2} and 100 % material ratio.

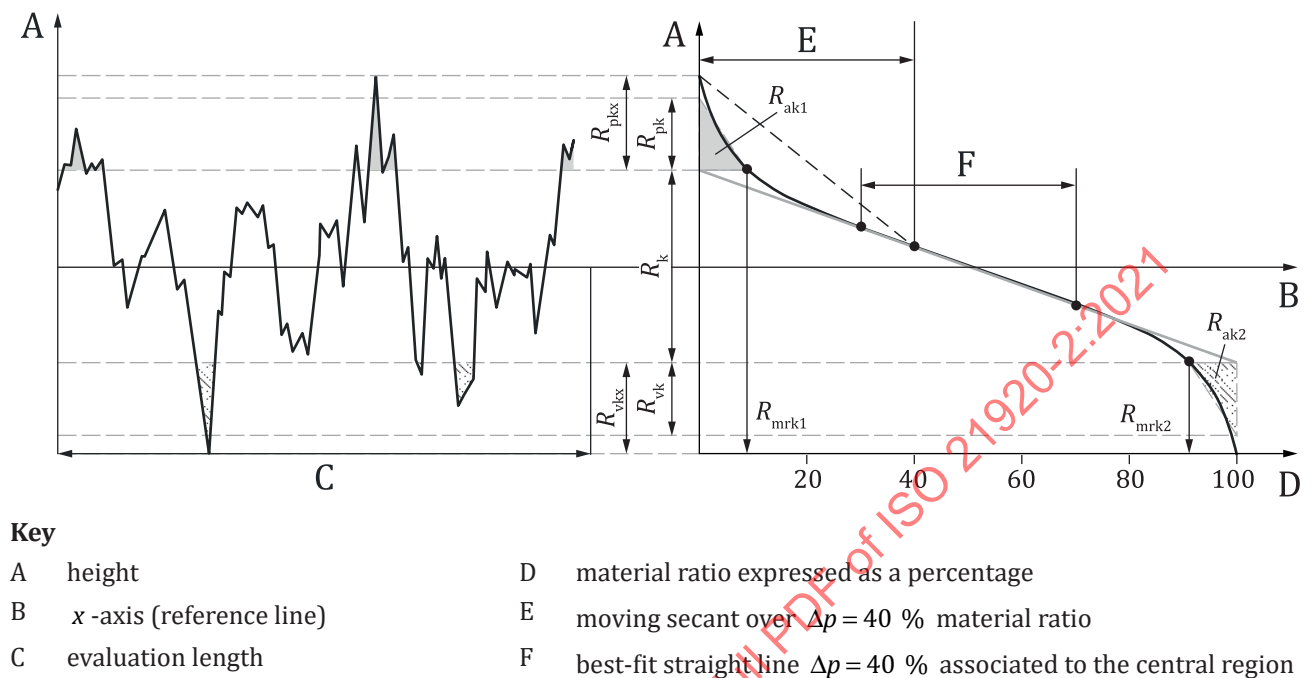


Figure D.1 — Determination of R_k , R_{pk} , R_{vk} , R_{pkx} , R_{vkx} , R_{mrk1} , R_{mrk2} , R_{ak1} and R_{ak2}

D.5 Determination of the parameters R_{pq} , R_{vq} and R_{mq}

Three nonlinear effects can be present in the material probability curve shown in Figure 23 (see 4.5.1.7) for measured profile data from a two-process surface. These effects shall be eliminated by limiting the fitted portions of the material probability curve, using only the statistically sound, Gaussian portions of the material probability curve, excluding a number of influences.

In Figure 23 (see 4.5.1.7), the nonlinear effects originate from:

- debris or outlying hills in the data (scale-limited profile) (key G);
- deep scratches or outlying dales in the data (scale-limited profile) (key H);
- an unstable region (curvature) introduced at the plateau-to-dale transition point based on the combination of two distributions (key I).

These exclusions are intended to keep the parameters more stable for repeated measurements of a given surface.

Figure 28 (see 4.5.4.4) shows a profile with its corresponding material probability curve and its plateau and dale regions and the parts of the profile that define the two regions. The profile has a hill that is outlying and the figure shows how it does not influence the parameters. Figure 28 (see 4.5.4.4) also shows how the bottom parts of the deepest dales, which will vary significantly depending on where the measurements are made on a surface, are disregarded when determining the parameters.

D.6 Procedure for determining the limits of the linear regions of the material probability curve

D.6.1 General

[D.6.2](#) to [D.6.4](#) specify the procedures for determining the upper plateau limit, UPL, and the lower plateau limit, LPL. [D.6.5](#) to [D.6.7](#) specify the procedures for determining the lower plateau limit, LPL, and the upper plateau limit, UPL. [D.6.8](#) specifies the procedure for determining the determination of parameters.

D.6.2 Initial conic fit

A conic section is initially fitted to the material probability curve since it is a very good approximation of the expected form of the material probability curve of surfaces consisting of two vertical random components. This initial conic fit provides a framework for subsequent operations on the material probability curve.

Fit a conic section into the material probability curve (see [Figure D.2](#)). The implicit representation of the conic section is given by [Formula \(D.1\)](#):

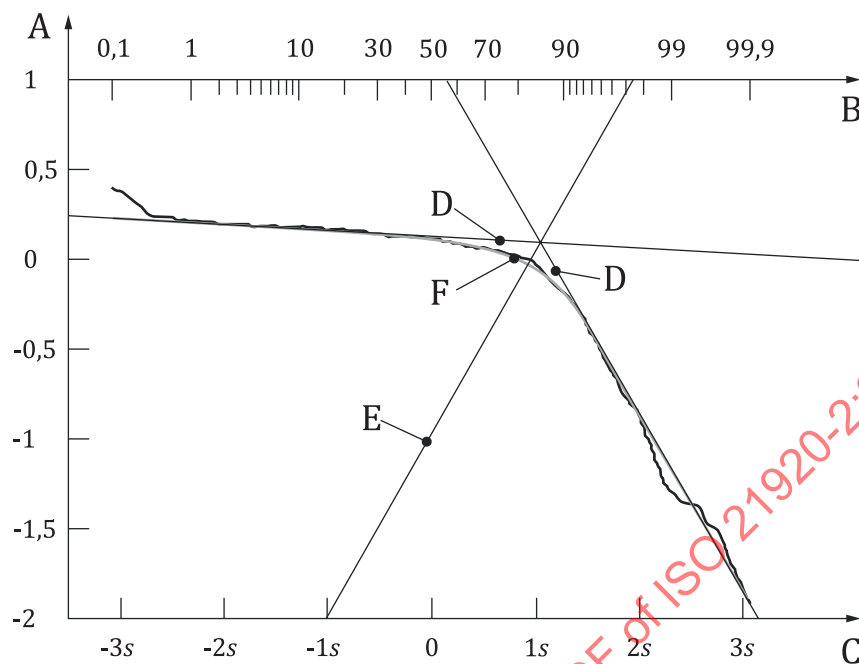
$$a \cdot x^2 + b \cdot x \cdot z_{cs} + c \cdot z_{cs}^2 + d \cdot x + e \cdot z_{cs} + f = 0 \quad (D.1)$$

where

x is the material probability expressed in standard deviation s ;

z_{cs} is the height value of the fitted conic section;

a, b, c, d, e and f are the parameters of the fitted conic section.



Key

- A height
- B material ratio expressed as a Gaussian probability as a percentage
- C material ratio expressed as a Gaussian probability in standard deviation values s
- D asymptotes of the conic section
- E bisector of the asymptotes
- F fitted conic section

Figure D.2 — Conic section based on the entire material probability curve

D.6.3 Estimation of plateau to dale transition

Determine the asymptotes of the conic section (key reference D in [Figure D.2](#)). Bisect the asymptotes with a line (key reference E in [Figure D.2](#)). The intersection of this line with the conic section serves as an initial estimate of the plateau to dale transition (see [Figure D.2](#)).

NOTE Graphically the bisector line (key reference E) might appear to be at an improper angle (see [Figure D.2](#)). This is because of the different scaling of the two axes in [Figure D.2](#).

D.6.4 Determination of UPL and LVL

The second derivative is computed at each point of the material probability curve, starting at the transition point key F and working upward to the plateau region and downward to the dale region.

The second derivative at each point is computed using a “window” of 0,05 standard deviations ($\pm 0,025 s$ around the point at which the derivative is to be recorded). See the middle of [Figure D.3](#).

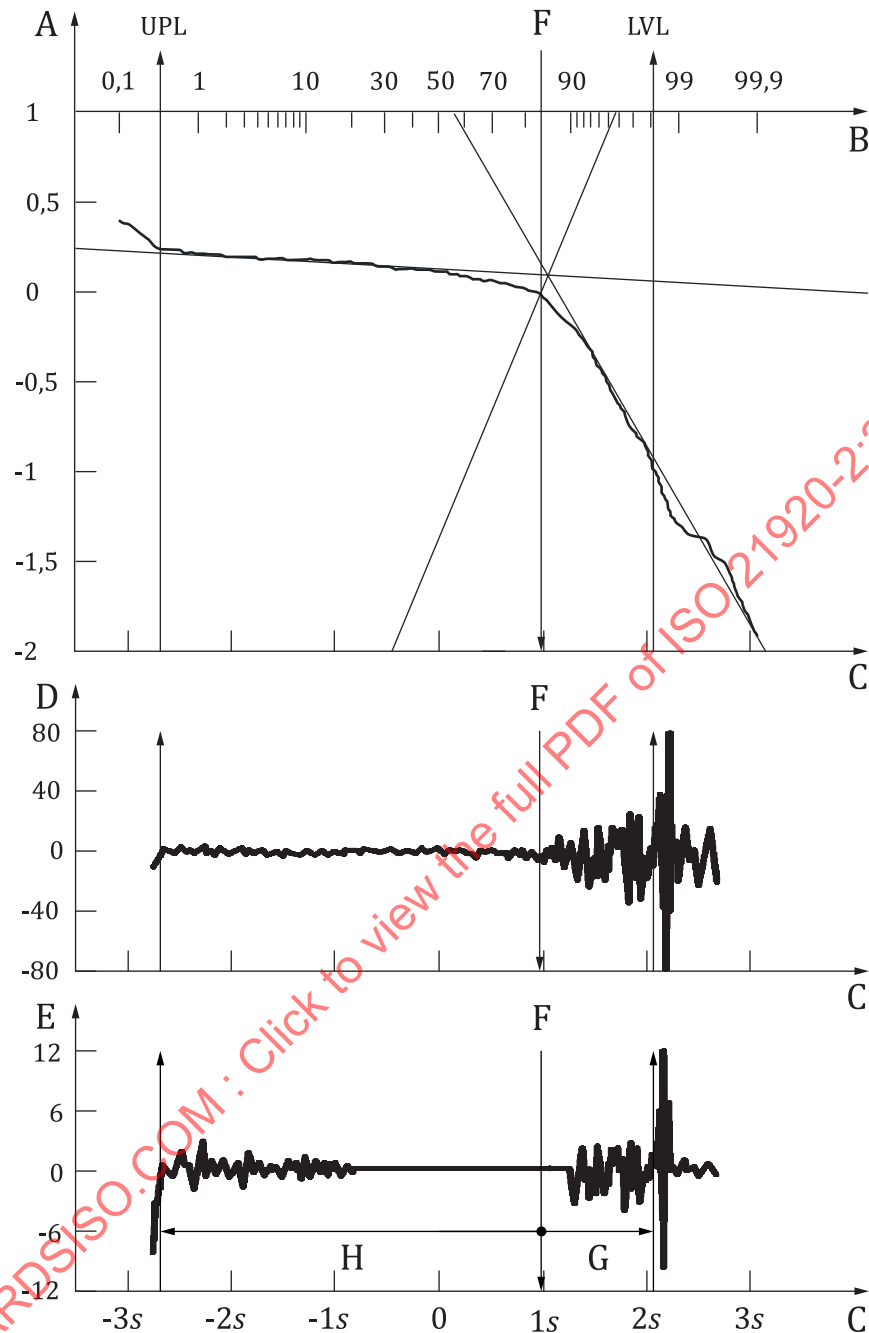
For the dale region and the plateau region individually:

- find 25 % of the number of points to one side of the transition point (key F) and call this value i ;
- working out from transition point (key F), the standard deviation s_i is computed for the second derivative values using i points on one side;

- the value of the second derivative at the next point d_{i+1}^2 is divided by the standard deviation s_i and results in $T = d_{i+1}^2 / s_i$;
- if $T \leq 6$, increment i by 1, recompute s_i and T ;
- if $T > 6$, data point i is the limit of that region (UPL for the plateau region and LVL for the dale region, respectively). See also [Figure D.3](#).

NOTE The number of points within the window will vary as it is passed to the curve.

STANDARDSISO.COM : Click to view the full PDF of ISO 21920-2:2021

**Key**

A	height	F	transition point
B	material ratio expressed as a percentage	G	dale search
C	material ratio expressed as standard deviation values s	H	plateau search
D	height per standard deviation square	UPL	upper plateau limit
E	ratio T	LVL	lower dale limit

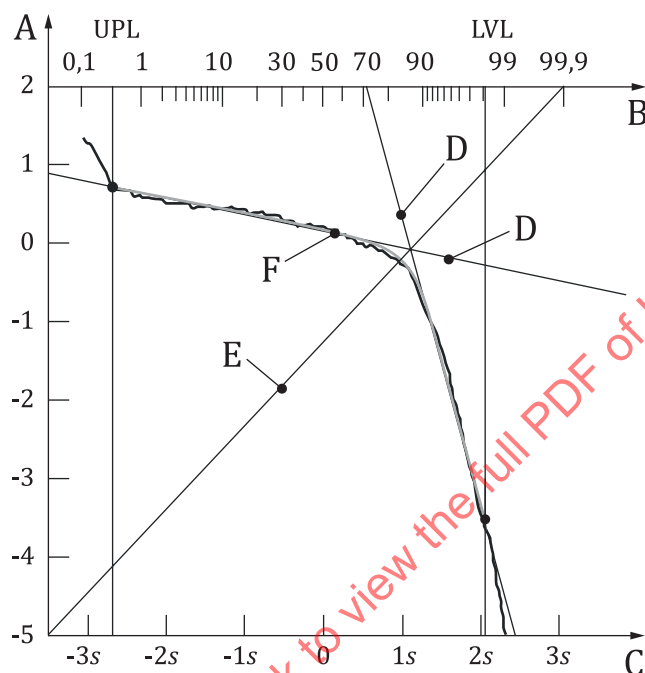
Figure D.3 — Bisection of the asymptotes is the initial transition point between the two regions of the material probability curve and the corresponding second derivatives

D.6.5 Normalization of the bounded region

The z -axis of the material probability curve is normalized such that the bounded region (region between UPL and LVL) is “square”. This ensures consistent bisection of the conic section asymptotes (see [Figure D.4](#)).

D.6.6 Second conic section fit

The conic section is now regressed to the region within UPL and LVL. The asymptotes are constructed (see [Figure D.4](#)).



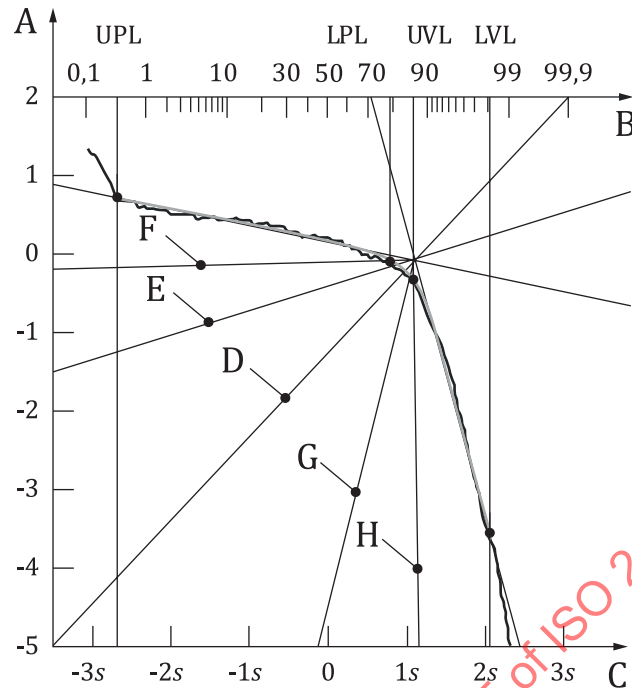
Key

- A height with normalized axis (the length of the horizontal axis and vertical axis is the same)
- B material ratio expressed as a Gaussian probability as a percentage
- C material ratio expressed as a Gaussian probability in standard deviation values s
- D asymptotes of the conic section
- E bisector of the asymptotes
- F fitted conic section
- UPL upper plateau limit
- LVL lower plateau limit

Figure D.4 — Conic section determined within the upper plateau limit, UPL, and the lower plateau limit, LVL — Normalized material probability curve

D.6.7 Determination of LPL and UPL

To determine the lower plateau limit, LPL, and the upper plateau limit, UPL, the asymptotes are bisected three times (D: first time; P2 and V2: second time; P3 and V3: third time). The intersection of these lines (P3 and V3) with the conic section of the material probability curve determines the LPL and UPL (see [Figure D.5](#)).

**Key**

- | | | | |
|-----|---|-----|--------------------------|
| A | height with normalized axis | G,H | bisected asymptotes dale |
| B | material ratio expressed as a percentage | UPL | upper plateau limit |
| C | material ratio expressed as standard deviation values s | LPL | lower plateau limit |
| D | bisector of the asymptotes | UVL | upper dale limit |
| E,F | bisected asymptotes plateau | LVL | lower dale limit |

Figure D.5 — Determination of the lower plateau limit, LPL, and the upper dale limit, UVL — Normalized material probability curve

D.6.8 Determination of parameters R_{pq} , R_{vq} and R_{mq}

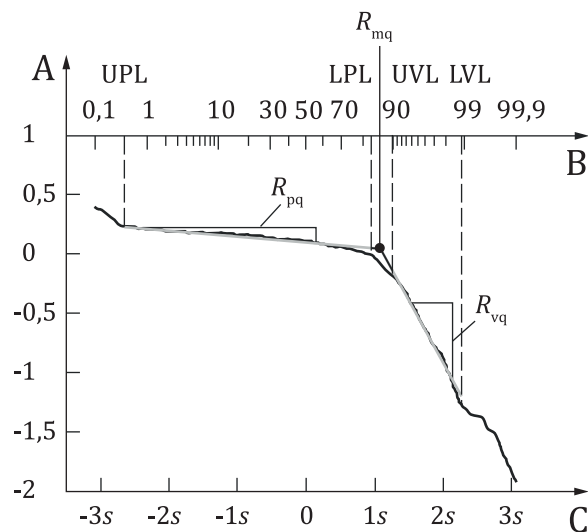
A linear regression is then performed within each region of the original, non-normalized material probability curve (see [Figure D.6](#)).

R_{pq} is the slope of a linear regression ($z = m_p s + t_p$) performed to the plateau region. R_{pq} can thus be interpreted as the R_q -value of the random process that generated the plateau component of the profile.

R_{vq} is the slope of a linear regression ($z = m_v s + t_v$) performed to the dale region. R_{vq} can thus be interpreted as the R_q -value of the random process that generated the dale component of the profile.

R_{mq} is the bearing ratio at the plateau to dale intersection, as shown in [Formula \(D.2\)](#):

$$R_{mq} = \frac{t_v - t_p}{m_p - m_v} \quad (D.2)$$



Key

- | | | | |
|-------------------------|---|-----|---------------------|
| A | height | LVL | lower dale limit |
| B | material ratio expressed as a percentage | UPL | upper plateau limit |
| C | material ratio expressed as standard deviation values s | UVL | upper dale limit |
| LPL lower plateau limit | | | |

Figure D.6 — Plateau and dale regions for the linearization of the material probability with its parameters

Annex E (normative)

Crossing-the-line segmentation to determine profile elements

E.1 General

[Annex E](#) defines the determination of profile elements for feature parameters using crossing-the-line segmentation. Crossing-the-line segmentation is separated into four steps:

- Step 1: Determination of hills and dales within the evaluation length.
- Step 2: Determination of significant profile hills and profile dales.
- Step 3: Merging of adjacent significant profile hills or adjacent significant profile dales.
- Step 4: Determination of profile elements.

NOTE The following variables are used:

- n number of profile values;
- x_k position on the x -axis of profile value z_k with $k=1,2,\dots,n$;
- z_k profile value with $k=1,2,\dots,n$;
- n_{HD} total number of profile hills and profile dales;
- HD_k profile hill or profile dale with $k=1,2,\dots,n_{\text{HD}}$
 - HD_k has four members:
 - $HD.t \in -1,0,1$ to indicate a dale, a zero element or a hill;
 - $HD.h$ peak height or pit depth;
 - $HD.i_l$ index of the left boundary $x_{HD.i_l}$ of a profile hill or a profile dale;
 - $HD.i_r$ index of the right boundary $x_{HD.i_r}$ of a profile hill or a profile dale;
- x_i, x_j intersection points with the reference line;
- H_v peak height discrimination $\in \mathbb{R}_0^+$;
- H_l pit depth discrimination $\in \mathbb{R}_0^+$;
- O_H threshold to suppress numerical noise $\in \mathbb{R}_0^+$ (outwardly directed);
- O_D threshold to suppress numerical noise $\in \mathbb{R}_0^+$ (inwardly directed);
- n_{pe} total number of profile elements;
- $X_{s,m}$ spacing of a profile element with $m=1,2,\dots,n_{\text{pe}}$;
- $Z_{t,m}$ height of a profile element with $m=1,2,\dots,n_{\text{pe}}$.

E.2 Modified signum function

Function to determine the sign of a real number z depending on the positive real numbers u and l , see [Formula \(E.1\)](#):

$$\text{sgm}(z, l, u) = \begin{cases} 1 & \text{if } z \geq u \\ -1 & \text{if } z \leq -l \\ 0 & \text{otherwise} \end{cases} \quad (\text{E.1})$$

E.3 Root function

Function to determine the intersection of the assessed profile with the reference line by linear interpolation, see [Formula \(E.2\)](#):

$$\text{root}(x_a, z_a, x_b, z_b) = \begin{cases} (x_a + x_b)/2 & \text{if } z_a = z_b \\ \min(\max((x_a z_b - x_b z_a)/(z_b - z_a), x_a), x_b) & \text{otherwise} \end{cases} \quad (\text{E.2})$$

where $x_a, z_a \in \mathbb{R}$ and $x_b, z_b \in \mathbb{R}$ are the coordinates of the profile whose linear connection intersects the reference line.

NOTE If the intersection point lies outside the interval $[x_a, x_b]$, then the associated interval limit is used instead of the intersection point.

E.4 Step 1 — Determination of hills and dales within the evaluation length

This subclause defines an algorithm in order to detect hills and dales with arbitrary height and depth within the evaluation length. The result is a sequence HD of hills and dales.

NOTE 1 See [Figure E.1](#).

NOTE 2 If not otherwise specified, the default values of O_H and O_D for the thresholds to suppress numerical noise are found in ISO 21920-3:2021, 5.5.

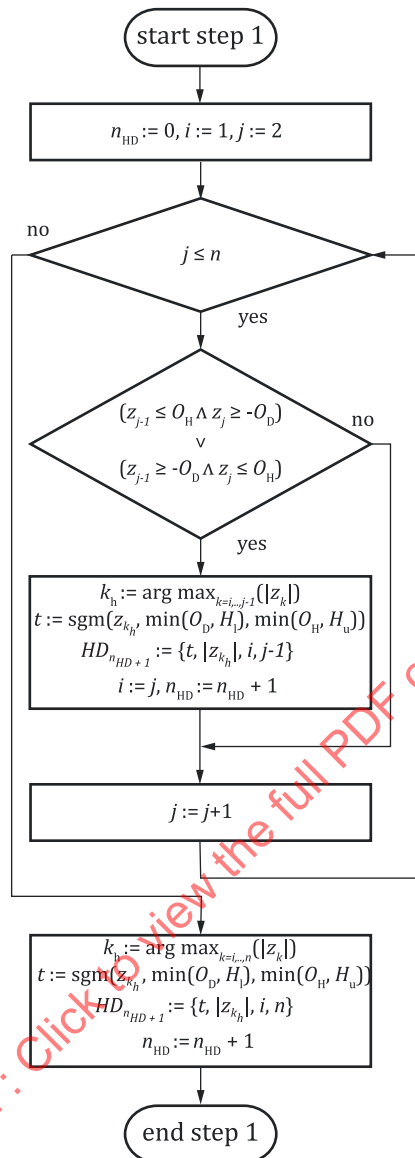


Figure E.1 — Step 1: Determination of hills and dales within the evaluation length

E.5 Step 2 — Determination of significant profile hills and profile dales

This subclause defines an algorithm in order to delete insignificant hills and dales applying the peak height discrimination and the pit depth discrimination.

NOTE 1 See [Figure E.2](#).

NOTE 2 If not otherwise specified, the default values for the peak height discrimination H_u and for the pit depth discrimination H_l are found in ISO 21920-3:2021, 5.5.

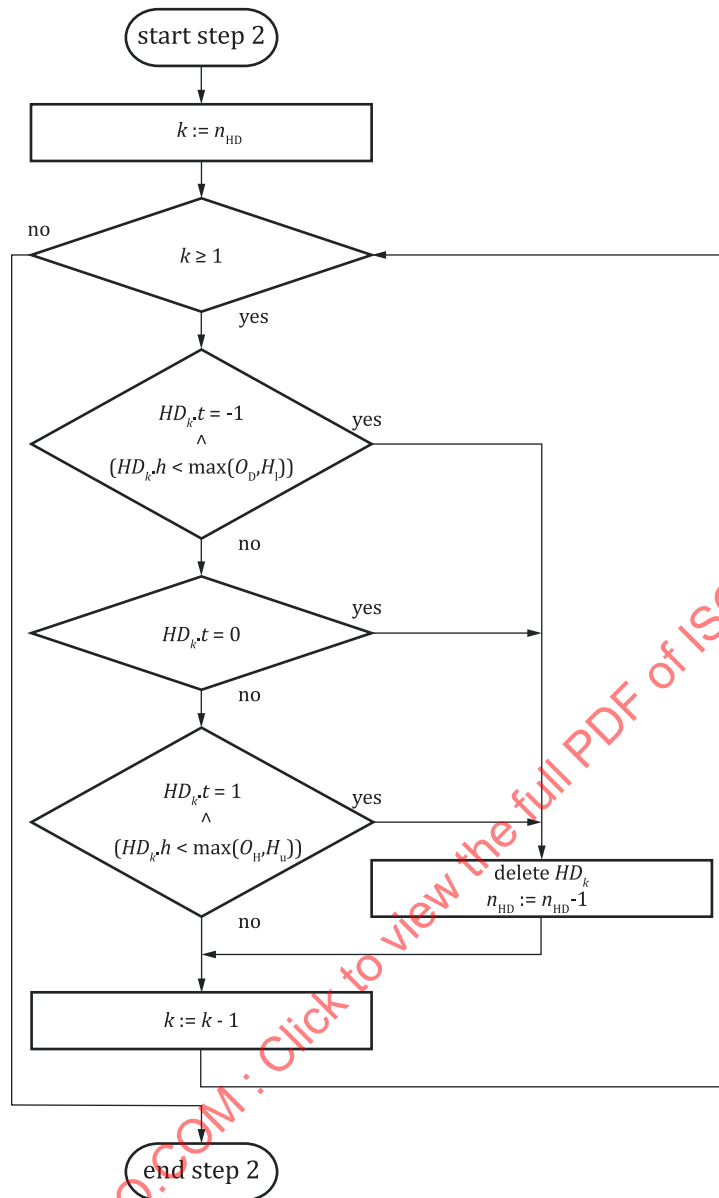


Figure E.2 — Step 2: Determination of significant profile hills and profile dales

E.6 Step 3 — Merging of adjacent significant profile hills or adjacent significant profile dales

This subclause defines an algorithm to merge adjacent hills or adjacent dales after applying the peak height discrimination for profile hills and the pit depth discrimination for profile dales. The result is a sequence HD of adjacent hills and dales or adjacent dales and hills.

NOTE 1 See [Figure E.3](#).

NOTE 2 The maximum operator between H_u and O_H as well as H_l and O_D is needed if O_H and O_D are constants.