

International Standard



3951

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Sampling procedures and charts for inspection by variables for percent defective

Règles et tables d'échantillonnage pour les contrôles par mesures des pourcentages de defectueux

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Foreword

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Sampling procedures and charts for inspection by variables for percent defective

Section one : General

1 Scope and field of application

1.1 Scope

1.1.1 This International Standard establishes sampling plans and procedures for inspection by variables. It is complementary to ISO 2859. When specified by the responsible authority, both this International Standard and ISO 2859 may be referenced in a product or process specification, contract, inspection instructions, or other documents, and the provisions set forth therein shall govern. The "responsible authority" shall be designated in one of the above documents.

1.1.2 The object of the methods laid down in this International Standard is to ensure that lots of an acceptable quality have a high probability of acceptance and that the probability of rejection of inferior lots is as high as possible.

1.1.3 In common with ISO 2859, the percentage of non-conforming products, or the **percent defective**, in the lots is used to define the quality of these lots and of the production process in question.

1.2 Field of application

This International Standard is primarily designed for use under the following conditions :

- a) where the inspection procedure is to be applied to a **continuous series of lots** of discrete products all supplied by one producer using one production process. If there are different producers, this International Standard shall be applied to each one separately;
- b) where only a **single quality characteristic** x of these products is taken into consideration, which must be **measurable on a continuous scale**. If several such characteristics are of importance, this International Standard shall be applied to each separately;
- c) where production is stable (under statistical control) and the quality characteristic x is distributed according to a

normal distribution or a close approximation to the normal distribution;

d) where a contract or standard defines an **upper specification limit** U , a **lower specification limit** L , or both; a product is qualified as non-conforming, or **defective**, when its measured quality characteristic x satisfies one of the following inequalities :

$$x > U \quad \dots(1)$$

$$x < L \quad \dots(2)$$

$$\text{either } x > U \text{ or } x < L \quad \dots(3)$$

Inequalities (1) and (2) are called cases with a **single specification limit**, and (3) a case with **double specification limits**. In this last situation a further distinction is made between separate or combined double limits according to whether the AQL is applied to each limit separately or to both limits combined (see clause 4).

2 References

ISO 2854, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances*.

ISO 2859, *Sampling procedures and tables for inspection by attributes*.

ISO 3534, *Statistics — Vocabulary and symbols*.

ISO 5725, *Precision of test methods — Determination of repeatability and reproducibility by inter-laboratory tests*.

3 Definitions and symbols

3.1 Definitions

For the purpose of this International Standard, the definitions given in ISO 3534 and ISO 2859 apply. The following additional terms also apply.

3.1.1 inspection by variables (contrôle par mesures) : A method which consists in measuring a quantitative characteristic for each item of a population or a sample taken from this population.

3.1.2 acceptance sampling by variables (échantillonnage par mesures en vue d'acceptation) : An acceptance procedure wherein a specified characteristic is measured to establish statistically the acceptability of a lot from the result obtained from the items in a sample.

3.1.3 acceptable quality level (AQL) [niveau de qualité acceptable (NOA)] : The maximum percent defective that, for purposes of sampling inspection, can be considered satisfactory as a process average. (See clause 4.)

3.1.4 limiting quality (qualité limite) : In a sampling plan, a quality level which corresponds to a specified and relatively low probability of acceptance (in this International Standard : 10 %). (See 12.1.)

3.1.5 defective (défectueux) : Any item that does not conform to the specification.

3.1.6 "s" method (méthode «s») : A method of assessing the acceptability of a lot by using the estimate of the standard deviation of the lot based on measurements of all the items in a sample. (See clause 14.)

3.1.7 "σ" method (méthode «σ») : A method of assessing the acceptability of a lot using previous knowledge of its standard deviation. (See clause 15.)

3.1.8 "R" method (méthode «R») : A method of assessing the acceptability of a lot by using an estimate of the standard deviation of the lot based on the average range of the measurements of the items in sub-groups of a sample. (See annex C.)

3.1.9 specification limit (limite de spécification) : The specified maximum or minimum acceptable value of the characteristic.

3.1.10 lower specification limit (L) [limite inférieure de spécification (L_l)] : The specified minimum acceptable value of the characteristic.

3.1.11 upper specification limit (U) [limite supérieure de spécification (L_s)] : The specified maximum acceptable value of the characteristic.

3.1.12 single specification limit (limite unique de spécification) : The term used when one limit only is specified.

3.1.13 separate double specification limits (limites de spécifications doubles séparées) : The term used when both upper and lower limits are specified and separate AQLs are applied to each limit individually. (See 4.3.)

3.1.14 combined double specification limit (limite de spécification double combinée) : The term used when both upper and lower limits are specified and an AQL is given which applies to the combined percent defective at both the limits. (See 4.3.)

3.1.15 acceptability constant (K) (constante d'acceptabilité) : A constant dependent on the specified value of the acceptable quality level and the sample size. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.16 quality parameter (q) (paramètre de qualité) : A function of the specification limit, the mean and the standard deviation of the lot. (See clause B.3 in annex B.)

3.1.17 quality statistic (Q) (statistique de qualité) : A function of the specification limit, the sample mean, and the estimate of the standard deviation of the lot. The lot is sentenced on the result of comparing Q with the acceptability constant K . (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.18 lower quality statistic (Q_L) [statistique de qualité correspondant à la limite inférieure (Q_l)] : A function of the lower specification limit, the sample mean, and the estimate of the standard deviation of the lot. The lot is sentenced on the result of comparing Q_L with the acceptability constant K . (See 14.2.)

3.1.19 upper quality statistic (Q_U) [statistique de qualité correspondant à la limite supérieure (Q_s)] : A function of the upper specification limit, the sample mean, and the estimate of the standard deviation of the lot. The lot is sentenced on the result of comparing Q_U with the acceptability constant K . (See 14.2.)

3.1.20 maximum standard deviation (MSD) [écart-type maximal (ETM)] : Under given conditions, the largest acceptable standard deviation. (See 14.4 and sub-clause B.5.2 in annex B.)

3.1.21 switching rules (règles de modification du contrôle) : Rules that govern the decision to increase or decrease the severity of inspection. (See clause 19.)

3.2 Symbols

The symbols used are as follows :

f A factor, given in table IV, that relates the maximum standard deviation to the difference between U and L .

K The (general) acceptability constant.

k The acceptability constant when using the "s" method, "σ" method or "R" method.

L Lower specification limit. (As a suffix to a variable, denotes its value at L .)

U Upper specification limit. (As a suffix to a variable, denotes its value at U .)

n Number of units in a sample.

N Number of units in a lot.

P The cumulative probability function (distribution function).

p The estimate of the total lot percent or fraction defective

$$p = p_U + p_L$$

p_L Estimate of fraction defective below L .

p_U Estimate of fraction defective above U .

q The quality parameter : equal to z_U or $-z_L$.

Q The quality statistic.

Q_L Lower quality statistic.

Q_U Upper quality statistic.

s Estimate from the sample of the standard deviation of the lot

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

(See also annex A.)

x Measured value of a characteristic in the sample.

$\bar{x}$ Mean value of x for the sample of n items.

z Value of the standardized variate

$$z = \frac{x - \mu}{\sigma}$$

z_L Value of z at the lower specification limit

$$z_L = \frac{L - \mu}{\sigma}$$

z_U Value of z at the upper specification limit

$$z_U = \frac{U - \mu}{\sigma}$$

μ Mean of the lot.

σ Standard deviation of the process (or of a lot) (σ^2 = variance).

Σ "The sum of" (for example, Σx = the sum of the x values).

$$\sum_{i=1}^n x_i \text{ The sum of all the } x \text{ values when } i \text{ takes integral values from 1 to } n.$$

$>$ "Greater than" (for example, $a > b$ means a is greater than b).

$\geq$ "Greater than or equal to" (for example, $a \geq b$ means a is greater than or equal to b).

$<$ "Less than" (for example, $a < b$ means a is less than b).

$\leq$ "Less than or equal to" (for example, $a \leq b$ means a is less than or equal to b).

3.3 Bibliography

A bibliography of documents utilized in the development of this International Standard is given in annex D.

4 Acceptable Quality Level (AQL)

4.1 Definition

The maximum percent defective that, for the purposes of sampling inspection, can be considered satisfactory as a process average.

4.2 Use

The AQL, together with the code letter, is used to index the sampling plans provided herein.

4.3 Specifying AQLs

The AQL to be used will be designated in the product specification contract or by the responsible authority. Where both upper and lower specifications limits are given, separate AQLs may be given to the individual limits, which are then known as "separate double specification limits". Alternatively, an overall AQL may be given which applies to the combined percent defective at both the upper and lower limits; this is then known as a "combined double specification limit".

4.4 Preferred AQLs

The values of AQLs given in this International Standard are preferred AQLs. If, for any product, an AQL is designated other than a preferred AQL, then this International Standard is not applicable. (See 12.2.)

4.5 Caution

From the above definition of the AQL, it follows that desired protection can only be obtained when a continuous series of lots is provided for inspection.

4.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any defective unit of product.

5 Switching rules for normal, tightened and reduced inspection

5.1 In order to discourage the process average exceeding the AQL, this International Standard prescribes a switch to tightened inspection when inspection results indicate that the process average exceeds the AQL, and stopping sampling inspection altogether when tightened inspection does not in time stimulate the producer to improve his production process.

5.2 Hence, tightened inspection and the stopping rule are integral, and therefore obligatory, procedures of this International Standard if the protection implied by the AQL is to be maintained.

5.3 This International Standard also provides the possibility of switching to reduced inspection when inspection results indicate that the process average is stable and reliable at a level below the AQL. This practice is, however, optional (at the discretion of the responsible authority).

5.4 When there is sufficient evidence from the control charts (see 18.1) that the variability is in statistical control, consideration should be given to switching to the " σ " method. If this appears advantageous, the consistent value of s shall be taken as σ .

5.5 When it has been necessary to discontinue sampling inspection, tightened inspection may not be resumed until action is taken by the producer to improve the quality of the submitted product.

5.6 Details of the operation of the switching rules are given in clause 19.

6 Relation to ISO 2859

6.1 Similarities

a) This International Standard is a complement to ISO 2859; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.

b) Both use the AQL to index the sampling plans and the preferred values used in this document are identical with those given in ISO 2859 for the same range of values (i.e. from 0,1 % to 10 %).

c) In the two documents, lot size and inspection level (inspection level II being preferred in default of other instructions) determine a code letter. Then general tables give the sample size to be taken and the acceptability criterion in terms of the code letter and the AQL according to the method chosen (" s ", " σ " or, contingently, " R "). Separate tables are given for normal, tightened and reduced inspection.

d) The switching rules are almost identical.

e) The definitions of critical, major and minor defects remain unchanged, but the concept is less useful as characteristics cannot be grouped together for sentencing in inspection by variables. It does assist, however, in specifying the AQL for the various characteristics.

6.2 Differences

a) **Determination of acceptability.** The acceptability of an attributes sampling plan, taken from ISO 2859, is determined by the number of defectives found in the sample; the acceptability criterion in inspection by variables is based on estimates of the location and variability of the distributed measurements of the lot, in relation to the specification limits, that is in terms of the mean and standard deviation. In this International Standard two ways of estimating the standard deviation are considered: the " s " method and the " σ " method (a third method, " R " method, is given in annex C). In the case of a single specification limit or of two separate limits, the acceptability may be calculated from a formula (see 14.2 and 15.2), but is more easily established by a graphical method (see 14.3). In the case of a combined double limit, this International Standard provides for a graphical method (see 14.4 and 15.3).

b) **Normality.** In ISO 2859 there is no requirement relating to the distribution of the characteristics, but in this International Standard it is necessary to the efficient operation of a plan that the measurements should be distributed according to a normal distribution or a close approximation of the normal distribution.

c) **Operating Characteristics curves (OC curves).** While an individual variables plan may be devised whose OC curve corresponds closely to that of a given attributes plan, it would not be possible to make all the OC curves in this International Standard identical with the corresponding OC curves in ISO 2859 (which are indexed with the same code letter and AQL), as the sample size for a given code letter would then have to increase with the AQL; this would be undesirable for the practical application of this sampling scheme.

d) **Probability of acceptance at the AQL.** The probability that a lot, whose quality is precisely at the AQL, will be accepted increases with the sample size and follows a similar, but not identical, scale to that used in ISO 2859.

e) **Sample sizes.** The variables sample sizes corresponding to given code letters are usually smaller than the attributes sample sizes for the same letters.

f) **Double sampling plans.** No double sampling plans are given in this International Standard.

g) **Average Outgoing Quality Limit (AOQL).** Since it is anticipated that variables plans will mainly be used with destructive testing, where 100 % inspection and rectification of rejected lots is not possible, the AOQL concept can not be applied and therefore these tables have not been included.

7 Non-continuous production

7.1 The sampling scheme contained in this International Standard was not designed to be applied under circumstances different from those specified in 1.2, for example, to an isolated lot or limited number of lots, where tightened inspection and the stopping rules cannot be applied.

7.2 Under such conditions, the concept of AQL as a maximum for the process average of accepted lots no longer holds, and the AQL can no longer be interpreted as a measure for the degree of protection of the consumer against receiving lots of poor average quality. The AQL then only indicates the percent defective that has a high probability of acceptance and acts as an index to a sampling plan.

7.3 The degree of protection of the consumer provided by the individual sampling plans of this International Standard

can, however, be judged from their Operating Characteristic (OC) curves as given in charts V-B to V-P and tables V-B-1 to V-P-1, and these should be consulted in choosing a sampling plan.

The OC curves calculated for the "s" method are applicable to the "σ" method. They are identified by code letters and AQL values.

NOTE — The OC curves given in charts V-B to V-P are in terms of normal inspection. Separate curves are not given for tightened or reduced inspection.

Diagram A will assist in finding a suitable OC curve. On this diagram, the intersection of a vertical line through the chosen value for the limiting quality and a horizontal line through the acceptable quality with a 95 % probability of acceptance (approximately equal to AQL) will lie on, or under, a sloping line indexed with the code letter of a standard plan which meets the specified requirements approximately.

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Section two : Choice of sampling plan

8 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement and some knowledge of both statistics and the product to be inspected. This section of this International Standard is intended to suggest to those responsible for specifying sampling plans the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

9 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account :

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more expensive in time and money per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables as the more precise information obtained indicates how good the product is; earlier warning will be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any defectives. (See the example in 14.4.)

d) A comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes (i.e. from ISO 2859) and the standard plans in this International Standard, is given in table I-B. It will be seen that the smallest samples are required by the " σ " method, used when the standard deviation of the lot is known.

e) Inspection by variables is appropriate particularly in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.

g) A variables scheme becomes less suitable as the number of measurements to be taken on one item increases, as each characteristic has to be considered separately. It may be advantageous to apply "attributes" to the majority of the characteristics and "variables" to one or two of the more important requirements, for example, proof load tests, safety and reliability requirements.

h) The use of this International Standard is only applicable when there is reason to believe that the distribution of

measurements is normal. In case of any doubt, the responsible authority should be consulted.

NOTES

1 Tests for departure from normality are dealt with in section two of ISO 2854, which provides examples of graphical methods which can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

2 More comprehensive documentation on this subject is currently in course of preparation within ISO/TC 69/SC 2 which will additionally give guidance on various numerical tests which might also be utilized. Subsequently, it is envisaged that tests for departure from normality will form the subject of a separate International Standard.

10 Choice of method

If it is desired to apply inspection by variables, the next question is which method should be used, the " s " method or the " σ " method (or the " R " method).

The " σ " method is the most economical in sample size, but before this method may be employed, the value of σ has to be established.

In terms of sample size, the " s " method has a slight advantage over the " R " method, but the calculation of s does involve more computation; the extent and difficulty of this is more apparent than real, especially if an electronic calculator is available. Methods of calculating s are given in annex A.

The " R " method (given in annex C) is simple to calculate, but requires a somewhat larger sample size for the same AQL.

Initially, it will be necessary to begin with the " s " (or the " R " method), but if the quality is satisfactory, the standard switching rules will permit the responsible authority to commence reduced inspection and use a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, will it be economical to change to the " σ " method? It should be noted that the size of the sample will not necessarily be smaller if the AQL is large, but the acceptability criteria become simpler. (See clauses 15.2 and 15.3.) On the other hand, it will still be necessary to calculate s for record purposes and to keep the control charts up to date. (See clause 18.)

11 Choice of inspection level and AQL

In standard sampling plans, the inspection level in conjunction with the AQL determines the size of the sample to be taken, and governs the severity of the inspection. The appropriate OC curve given in one of the tables V-B to V-P shows the extent of the risk that is involved in such a plan.

The choice of the inspection level and AQL is governed by a number of factors, but is mainly a balance between the total

cost of inspection and the consequences of defective items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

12 Choice of a sampling plan

12.1 Standard plans

The standard procedure can be used only when the production of lots is continuous.

The standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the "s" method, has been found in practice to produce workable sampling plans; but it assumes that the order of priority is first the AQL, second the sample size and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see clause 19), which quickly increase the severity of inspection and finally terminate it, if the quality of the process is worse than the AQL.

NOTE — It should also be remembered that the limiting quality is the quality which if offered for inspection would have a 10 % probability of acceptance. The actual risk taken by the consumer therefore also depends on the probability of goods of this low quality being offered for inspection.

However, if, in certain circumstances, the limiting quality has a higher priority than the sample size (for example, when only a limited number of lots are being produced), a suitable plan in this International Standard may be selected by using diagram A. The intersection of a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (approximately equal to AQL) will lie on, or under, a sloping line indexed with the code letter of a standard plan which meets the specified requirements. This should be verified by inspecting the OC curve given in table V for this code letter and AQL.

If the lines intersect at a point above the line marked P (see diagram A), this implies that a sample of over 200 would be necessary and the specification cannot be met by the plans in this International Standard.

12.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. The choice is then to decide which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent, for, when any two have been chosen, the third follows.

NOTE — This choice is not completely unfettered: the "non-central t " tables referred to in annex D have only been computed for a limited, though generally adequate, range of variables and the fact that the size of the sample is necessarily a whole number imposes some constraints. If a special scheme is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

Section three : Operation of a variables sampling plan

13 Preliminary operations

Before starting inspection by variables, check

- that the distribution can be considered to be normal and that production is considered to be continuous;
- whether the "s" (or "R") method is to be used initially or whether the standard deviation is stable and known and the "σ" method is to be used;
- that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;
- that the AQL has been designated and that it is one of the preferred AQLs for use with this International Standard. If it is not, the tables are not applicable;
- if a double specification limit has to be met, whether the limits are separate or combined and, if the limits are separate, whether AQLs are determined for each limit.

14 Standard procedure for "s" method

14.1 Obtaining a plan

The procedure for obtaining a plan is as follows :

- With the inspection level given (normally this will be II) and with the lot size, obtain the code letter using table I-A.
- With this code letter and the AQL, enter table II-A and obtain the sample size n and acceptability constant k .
- Taking a random sample of this size, measure the characteristic x in each item and then calculate $\bar{x}$, the sample mean, and s , the estimated standard deviation (see annex A). If $\bar{x}$ is outside the specification limit, the lot can be rejected without calculating s . It may, however, be necessary to calculate s for record purposes.

14.2 Acceptability criteria for single or separate specification limits

If single or separate specification limits are given, calculate the quality statistic

$$Q_U = \frac{U - \bar{x}}{s}$$

and/or

$$Q_L = \frac{\bar{x} - L}{s}$$

as appropriate,

then compare the quality statistic (Q_U and/or Q_L) with the acceptability constant k obtained from table II-A for normal inspection. If the appropriate quality statistic is greater than or equal to the acceptability constant, accept the lot; if less, reject.

Thus, if only the upper specification limit U is given,

accept if $Q_U \geq k$

reject if $Q_U < k$

Or, if only the lower specification limit L is given,

accept if $Q_L \geq k$

reject if $Q_L < k$

When both U and L are given (k values are different if the AQL are different for the upper limit and the lower limit),

accept if both $Q_L \geq k_L$ and $Q_U \geq k_U$

reject if either $Q_L < k_L$ or $Q_U < k_U$

Example

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items. Inspection level II, normal inspection with AQL = 2,5 % is to be used. From table I-A, the letter code is F; from table II-A it is seen that a sample size of 10 is required and that the acceptability constant k is 1,41. Suppose the measurements are as follows : 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 55 °C; 50 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Values obtained
Sample size : n	10
Sample mean $\bar{x} : \Sigma x/n$	54,9
Standard deviation $s : \sqrt{\Sigma (x - \bar{x})^2 / (n - 1)}$	3,414
Specification limit (upper) : U	60
$Q_U = (U - \bar{x})/s$	1,494
Acceptability constant : k (see table II-A)	1,41
Acceptability criterion : compare Q_U with k	1,494 > 1,41

The lot meets the acceptability criterion, since Q_U is greater than k .

14.3 Graphical method for single or separate specification limits

Write the acceptance conditions

$$\frac{U - \bar{x}}{s} \geq k \text{ and/or } \frac{\bar{x} - L}{s} \geq k$$

as $x \leq U - ks$ and/or $\bar{x} \geq L + ks$.

It can be seen on a graph with s values as the horizontal axis and $\bar{x}$ values as the vertical axis, that the line of general equation $\bar{x} = U - ks$ [straight line through the point ($s = 0, \bar{x} = U$) with a slope $-k$] and/or line $\bar{x} = L + ks$ [straight line through the point ($s = 0, \bar{x} = L$) with a slope k] divides the plan in two zones : accept zone and reject zone according to figure 1.

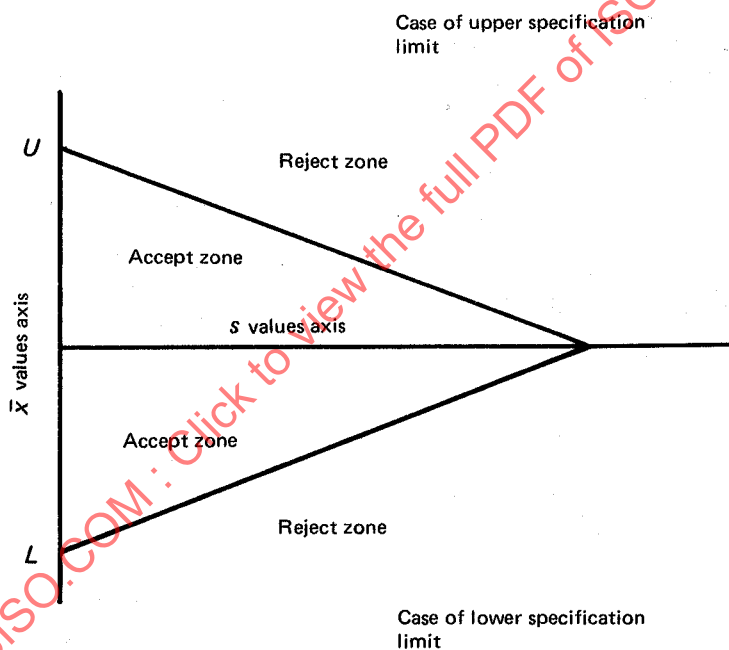


Figure 1

In case of double separate specification limits, the accept zone is the inside area limited by the two lines. Figure 1 assumes that a single value of AQL has been determined for each limit (single k value). The lines intersect at the point whose abscissa is

$$\frac{U - L}{2k}$$

The graph can be prepared before beginning the inspection of a series of lots. Then, for each lot plot the point $(s, \bar{x})$ and decide to accept or reject the lot.

Example

Using the figures given in the example in 14.2, mark the point $U = 60$ on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $-k$ [as $k = 1,41$, this means the line passes through points $(s = 1, \bar{x} = 58,59)$, $(s = 2, \bar{x} = 57,18)$, $(s = 3, \bar{x} = 55,77)$, etc.]. Select a suitable point and draw a straight line through it and $(s = 0, \bar{x} = 60)$, i.e. U . The accept zone is then the area under this line. The calculated values of s and $\bar{x}$ are 3,414 and 54,9. Plotting the point $(s, \bar{x})$, it will be seen from figure 2 that it lies just inside the accept zone and the lot is acceptable.

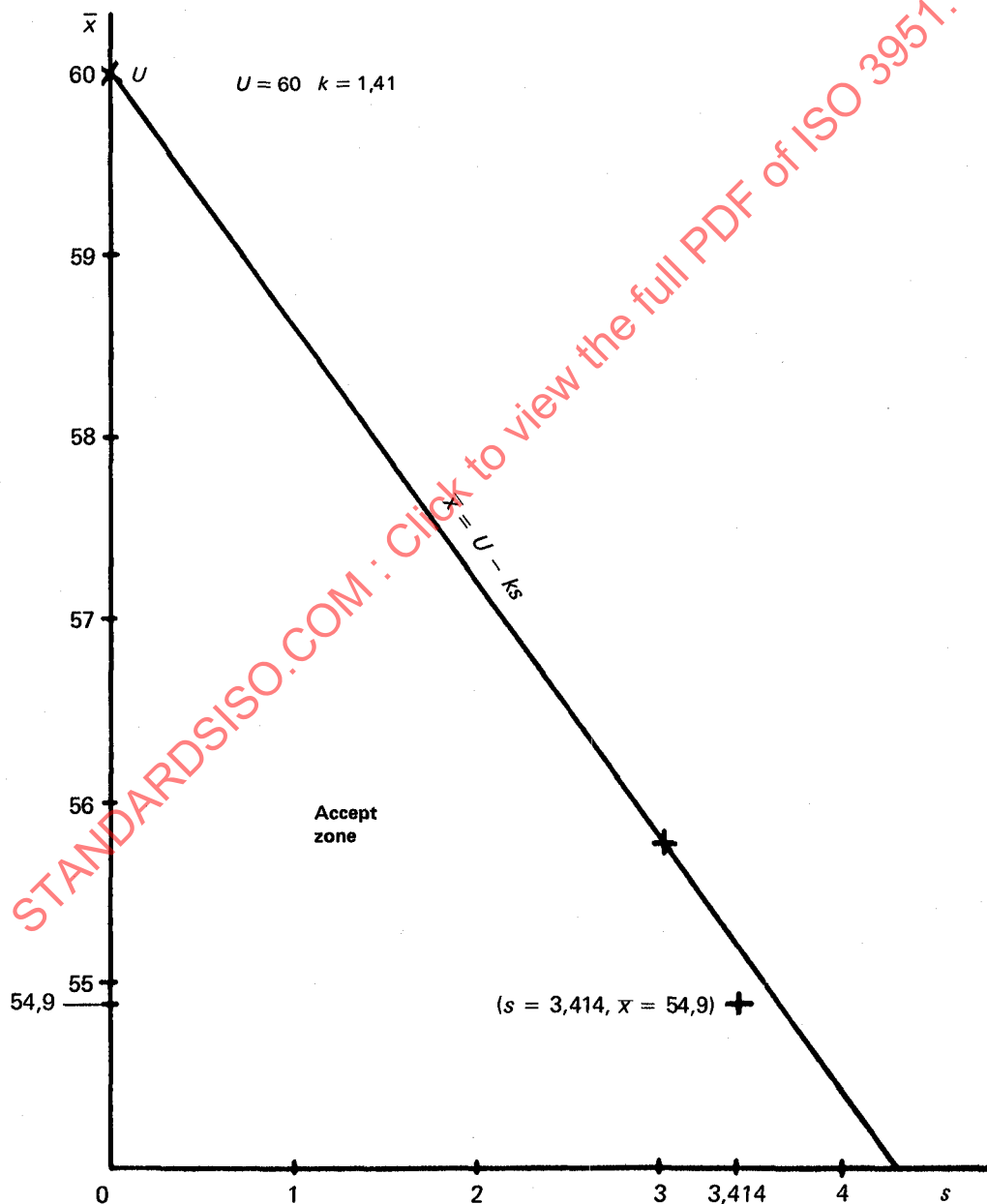


Figure 2

14.4 Acceptability criterion for a combined double specification limit

If a combined upper and lower specification limit has been given, it will be necessary to use a graphical method, unless s is greater than the value of the MSD obtained from table IV, when the lot has to be rejected at once.

Consult the chart in the "s" series which is labelled with the appropriate code letter and select the acceptance curve with the AQL specified for the two limits.

Then calculate the values of

$$\frac{s}{U - L} \text{ and } \frac{\bar{x} - L}{U - L}$$

and plot a point representing these values on the graph (or a copy of it). If the point lies outside the accept zone defined by the curve, reject the lot, otherwise accept.

For greater convenience, it is recommended that before the inspection operations begin the acceptance curve be copied (or traced) onto graph paper and that the scales be adjusted so that s and $\bar{x}$ can be plotted directly (the upper limit is given instead of 1,0 and the lower limit instead of 0 on the $\bar{x}$ scale).

Then plot on the chart the values of s and $\bar{x}$ found from the sample and, if the point lies outside the accept zone, reject the lot, otherwise accept.

NOTES

1 For code letters B and C (i.e. sample sizes 3 and 4), the drawing of the accept zone is bounded by 4 straight lines: the $\bar{x}$ axis, the line $\bar{x} = U - ks$, a line parallel to the $\bar{x}$ axis through the MSD (see table IV) and the line $\bar{x} = L + ks$. The value of k is obtained from the table II-A, II-B or II-C.

2 In order to provide a reasonable scale for the most frequently used values, the AQLs 6,5 % and 10 % have had to be omitted in certain places (for example, diagram s-D). However, the procedure for their construction can be found in annex B.

Example

The minimum temperature of operation for a certain device is specified as 60,0 °C and the maximum temperature as 70,0 °C. Production is in inspection lots of 96 items. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. From

table I-A, the code letter is F; from the table I-B it is seen that a sample size of 10 is required and from table IV that the value of f for the MSD is 0,276. Suppose the measurements obtained are as follows: 63,5 °C; 62,0 °C; 65,2 °C; 61,7 °C; 69,0 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Value obtained
Sample size : n	10
Sample mean : $\bar{x} : \Sigma x/n$	64,57
Standard deviation $s : \sqrt{\Sigma (x - \bar{x})^2 / (n - 1)}$ (See clause A.2, annex A.)	3,01
Standardized mean : $(\bar{x} - L) / (U - L)$	0,457
Standardized standard deviation : $s / (U - L)$	0,301
Value of f for MSD (table IV)	0,276
MSD = $f(U - L)$	2,76

The appropriate acceptance curve is taken from diagram s-F.

If, as on figure 3, the scales have been adjusted to the real measurements, plot the point ($s = 3,01$, $\bar{x} = 64,57$). This lies outside the acceptance curve and the lot is rejected. It could have been rejected as soon as it was seen that s was greater than the MSD.

If the scale of the chart has not been adjusted to the values of s and $\bar{x}$, the following additional calculations are required:

$$\text{Standardized mean : } (\bar{x} - L) / (U - L) = (64,57 - 60) / (70 - 60) = 0,457$$

$$\text{Standardized } s : s / (U - L) = 3,01 / (70 - 60) = 0,301$$

The point (0,301, 0,457) is plotted on figure 3.

As it lies outside the acceptance curve for AQL = 1,5 %, the lot is rejected.

NOTE — This lot is rejected even though all tested items in the sample are within specification limits.

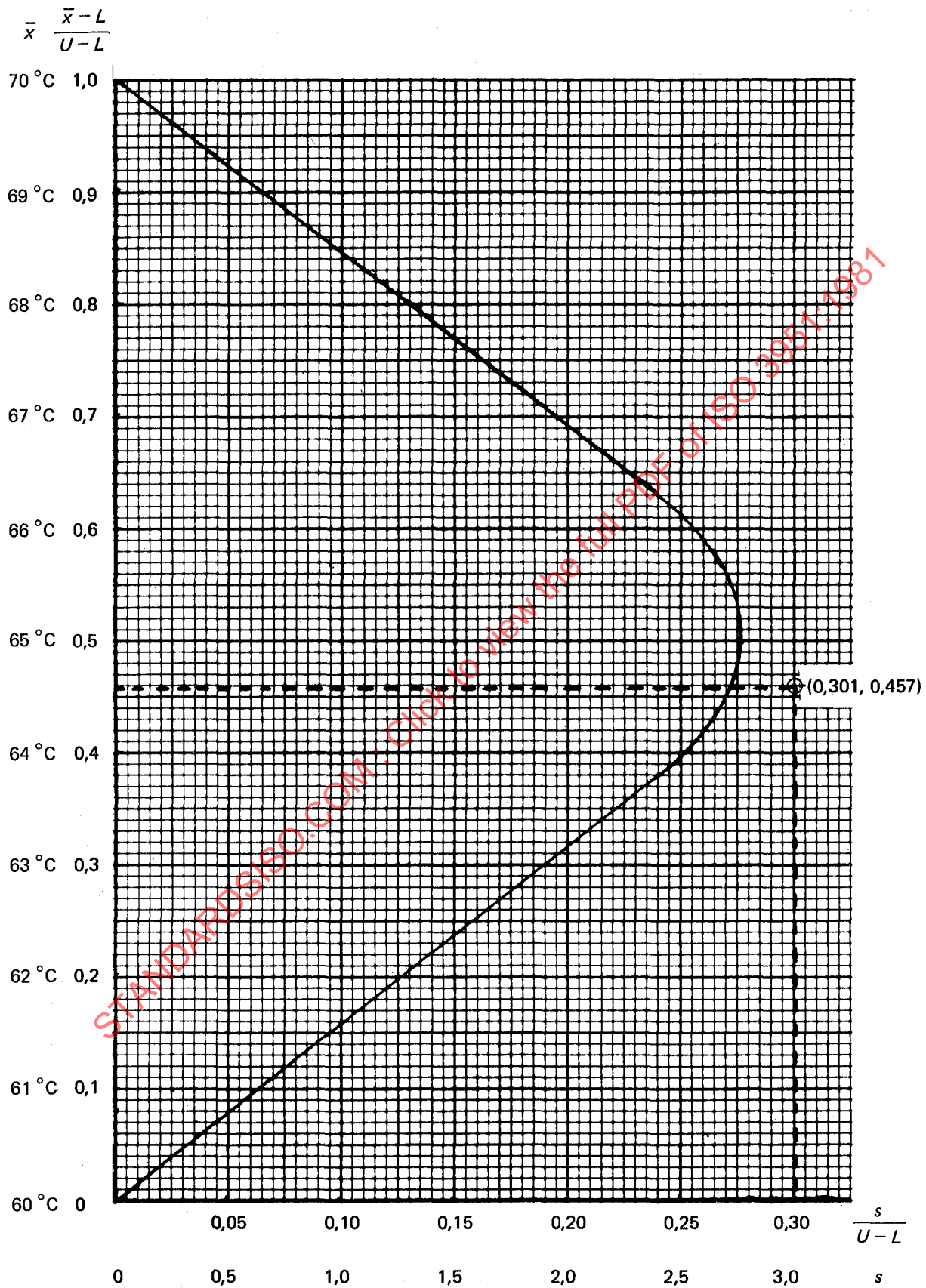


Figure 3 — Acceptance curve for F code-letter, AQL = 1,5 %, MSD = 2,76 : "s" method

15 Standard procedure for "σ" method

15.1 Selecting a plan

This method is only to be used when there is valid evidence that the standard deviation of the process can be considered constant and taken to be "σ".

From table I-A obtain the code letter; then, for normal inspection, from table III-A with code letter and the specified AQL, obtain the sample size n and acceptability constant k .

Take a random sample of this size, measure the characteristic under inspection x for all items of the sample and calculate the mean $\bar{x}$.

15.2 Acceptability criteria for single or separate specification limits

It is possible to obtain an acceptability criterion by following the procedure given for the "s" method, by substituting σ , the given value of the standard deviation of the process, for the s derived for the individual samples and comparing the calculated value of Q with the value of the acceptability constant k obtained from table III-A.

However, if $Q_U = (U - \bar{x})/\sigma > k_U$, the acceptability criterion for an upper specification is written as $\bar{x} \leq U - k_U \sigma$.

Since U , k_U and σ are all known in advance, the value of $\bar{x}_U = U - k_U \sigma$ should be determined before inspection begins. The acceptability criterion then becomes, for an upper specification limit,

accept if $\bar{x} \leq \bar{x}_U = U - k_U \sigma$

reject if $\bar{x} > \bar{x}_U = U - k_U \sigma$

similarly, for a lower specification limit,

accept if $\bar{x} \geq \bar{x}_L = L + k_L \sigma$

reject if $\bar{x} < \bar{x}_L = L + k_L \sigma$

When both separate limits are given,

accept if both $\bar{x} \leq \bar{x}_U$ and $\bar{x} \geq \bar{x}_L$

reject if either $\bar{x} > \bar{x}_U$ or $\bar{x} < \bar{x}_L$

Example

The specified minimum yield point for certain steel castings is 400 N/mm². A lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. σ is known to be 21 N/mm². From table I-A it is seen that the code letter is I and then, from table III-A, that for AQL of 1,5 % the sample size n is 10 and the acceptability constant k is 1,70. Suppose the yield points of the sample specimens are

431; 417; 469; 407; 452; 427; 421; 476; 400; 445;

and compliance with the acceptability criterion is to be determined.

Information needed	Value obtained
Acceptability constant : k	1,70
Known σ	21
Product : $k\sigma$	35,70
Specification limit : L	400
Sum : $L + k\sigma = \bar{x}_L$	435,70
Sum of measurements : $\sum x$	4 345
Sample size : n	10
Sample mean : $\bar{x}$	434,5
Acceptability criterion : Is $\bar{x} \geq \bar{x}_L$?	No
The lot does not meet the acceptability criterion and is not to be accepted.	

15.3 Acceptability criterion for a combined double specification limit

The following procedure is recommended :

a) Using the lot size and given inspection level, find the code letter from table I-A and the sample size from table I-B using the code letter and AQL.

b) From the chart indexed with the code letter in the SIGMA (σ) series, select the acceptance curve for the AQL given for this plan.

NOTE — In order to provide a reasonable scale for the most frequently used values, the AQLs 6,5 % and 10 % have had to be omitted in certain places (for example, diagram σ -D). However, the procedure for their construction can be found in annex B.

c) Calculate the value $\sigma/(U - L)$ and draw a vertical line through this point.

d) If this line intercepts the acceptance curve (see note 1), measure the values of $(\bar{x} - L)/(U - L)$ at the intersection points. These give the highest and lowest "standardized" acceptance values of the mean. (See figure 4.) The highest and lowest values, $\bar{x}_U$ and $\bar{x}_L$, are then deduced as shown in the example.

NOTES

1 If the line does not intercept the acceptance curve, the process can never be acceptable and sampling inspection is pointless.

2 If the line intercepts the straight line portions of the acceptance curve, this is equivalent to two separate specification limits and the acceptance criteria are those given in 15.2.

e) In accordance with 15.3 a), take a sample of the appropriate size and obtain the mean of the characteristic under inspection.

f) The acceptability criterion is : accept if the mean lies between the highest and lowest acceptable values of the mean (i.e. if $\bar{x}_L < \bar{x} < \bar{x}_U$), otherwise reject.

Example

The specification for electrical resistance of a certain electrical component is $520 \pm 50 \Omega$. Production is at a rate of 3 500 items per inspection lot. Inspection level II, normal in-

spection, with a single AQL 1 %, is to be used for the two specification limits (470 and 570). σ is known to be 21,0. From table I-A the code letter is L; from table I-B it is seen that a sample size of 25 is required. Suppose the values of the sample resistance are as follows :

515; 491; 479; 507; 543; 521; 536; 483; 509; 514; 507; 484;
526; 552; 499; 530; 512; 492; 521; 467; 489; 513; 535; 501;
529.

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Figure 4 — Acceptance curve for code letter L, AQL = 1,0 % : "σ" method

Information needed	Value obtained
Sample size : n	25
Upper specification limit : U	570
Lower specification limit : L	470
Known σ :	21,0
"Standardized" $\sigma : \frac{\sigma}{U-L} = \frac{21,0}{100}$	0,21

"Standardized" limiting values from acceptance curve (see figure 4)

Maximum	Minimum
$\frac{\bar{x}_U - L}{U - L} = 0,576$	$\frac{\bar{x}_L - L}{U - L} = 0,424$
$\bar{x}_U = 57,6 + 470$	$\bar{x}_L = 42,4 + 470$
Limits of $\bar{x}$	
$\bar{x}_U = 527,6$	$\bar{x}_L = 512,4$
Sum of measurements :	12 755
Mean of measurement :	510,2

Since $\bar{x}$ at 510,2 lies outside the acceptance limits for $\bar{x}$ of 527,6 and 512,4 the lot is not acceptable.

NOTES

1 All the calculations other than the last two lines should be completed before sampling begins.

2 If σ had been known to be 25, the vertical line through its "standardized" value [i.e. $\sigma/(U-L) = 0,25$] does not intercept the acceptance curve for an AQL = 1 % and sampling inspection would not have taken place.

3 If σ had been 15, the acceptance limits for $\bar{x}$ could be obtained either from the chart or by calculation, since the intercept is on the straight line portion of the curve.

From table III-A

$$k = 1,97$$

$$k\sigma = 29,6$$

$$\bar{x}_U = U - k\sigma = 540,4$$

$$\bar{x}_L = L + k\sigma = 499,6$$

and in that case the lot, whose mean was 510,2 would be acceptable.

16 Procedure during continuing inspection

As a variable sampling inspection plan can only operate efficiently if

- the characteristic being inspected is normally distributed,
- records are kept,

c) the switching rules are obeyed,

it is necessary to ensure that these requirements are being met.

17 Normality and outliers

17.1 Normality

Normality should have been checked by the responsible authority before sampling begins. In case of doubt, a statistician should advise whether the distribution appears suitable for sampling by variables or whether use should be made of the graphical tests for normality given in ISO 2854, section two.

17.2 Outliers

An outlier (or an outlying observation) is one that appears to deviate markedly from other observations in the sample in which it occurs. A single outlier, even when it lies within specification limits, may produce an increase in variability, change the mean and consequently lead to rejection of the lot. When an outlying observation is suspected, a test for outliers should precede, and be made independently of, a decision on acceptability of a lot. (See, for example, ISO 5725.) When outliers are detected, the disposition of the lot should be a matter for negotiation between the vendor and vendee.

18 Records

18.1 Control charts

One of the advantages of inspection by variables is that trends in the quality level of the product can be detected and warning given before an unacceptable standard is reached, but this is only possible if adequate records are kept.

Whatever the method used, s , σ , (or R), records should be kept of the values of $\bar{x}$ and s (or R), preferably in the form of control chart.

This procedure should be applied especially with the " σ " method in order to verify that the values of s obtained from the samples fall within the limits of the prescribed value of σ .

When a combined double specification limit has been laid down, the value of the MSD, given in table IV (or MAR given in table RIV), should be plotted on the s (or R) control chart, as an indication of an unacceptable value.

NOTE — Control charts are used to detect trends. The ultimate decision as to the acceptability of an individual lot is governed by the procedure given in clauses 14 and 15.

18.2 Rejected lots

Particular care shall be taken to record all rejected lots and to see that switching rules are implemented. Any lot not accepted by the sampling plan shall not be resubmitted either in whole or in part without the permission of the responsible authority.

19 Operation of switching rules

The standard switching rules are as follows

19.1 Normal inspection is used at the start of inspection (unless otherwise designated) and shall continue to be used during the course of inspection until tightened inspection becomes necessary or reduced inspection is allowed.

19.2 Tightened inspection shall be instituted when two lots on original normal inspection are rejected within any five or less successive lots.

Tightened inspection is achieved by increasing the values of the acceptability constant. The values are tabulated in table II-B for the "s" method (table RII-B for the "R" method) and table III-B for the "σ" method. In the "s" method (and also in the "R" method), there is no change in the size of the sample, unless the sample size is so small that the tables indicate, with a downward arrow, that an increase of sampling is necessary.

19.3 Tightened inspection shall be relaxed when five successive lots on original inspection have been accepted on tightened inspection; then reinstate normal inspection.

19.4 Reduced inspection may be instituted after ten successive lots have been accepted under normal inspection, provided that

- a) these lots would have been acceptable if the AQL had been one step tighter;

NOTE — If a value of k for this tighter AQL is not given in table II-A ("s" method), a sufficiently accurate acceptance curve may be extrapolated by eye in comparison with the other curves of the appropriate diagram in the "s" series. The same procedure applies to the "σ" method (and the "R" method).

- b) production is in statistical control;
- c) reduced inspection is considered desirable by the responsible authority.

Reduced inspection is conducted on a much smaller sample than normal inspection and the value of the acceptability constant is also decreased. The values of n and k for reduced inspection are given in table II-C for the "s" method, table III-C for the "σ" method (table RII-C for the "R" method).

19.5 Reduced inspection shall cease and normal inspection be reinstated if any of the following occur on original inspection:

- a) a lot is rejected;
- b) production becomes irregular or delayed;

- c) other conditions warrant that normal inspection shall be instituted.

20 Discontinuing inspection

If tightened inspection has had to be maintained continuously for over ten successive lots, sampling inspection is to be discontinued pending action by the producer to improve the quality of the submitted product.

21 Acceptance curves for tightened and reduced inspection

- a) Separate acceptance curves have not been produced for tightened or reduced inspection, so when combined double specification limits have to be met under tightened or reduced inspection, it is necessary to find the curve among those given for use in normal inspection which has the required sample size and acceptability constant.

- b) When switching to tightened inspection, it will always be the curve just inside that being used in normal inspection (i.e. that with the next lower preferred AQL). When switching to reduced inspection, it is necessary to find the values of the sample size and acceptability constant from table II-C or table III-C for the "s" and "σ" methods respectively (table RII-C for the "R" method) and then refer back to tables II-A or III-A (or RII-A) to find the normal inspection plan giving these values (i.e. code letter and AQL).

- c) This code letter and AQL, then determine the acceptance curve to be used.

Example

If in normal inspection the code letter is I and the AQL is 0,25, when using the "s" method, from table II-A, $n = 25$ and $k = 2,26$. On switching to reduced inspection, from table II-C, $n = 10$ and $k = 1,98$. Referring back to table II-A, $n = 10$ and $k = 1,98$ is found to correspond to a code letter F and AQL of 0,40. The acceptance curve given for these values in diagram s-F should, accordingly, be used.

22 Switching to "σ" method

If it appears that the value of s (or $\bar{R}$) is in control, the root mean square value of s (or $\bar{R}/c$) may be presumed to be σ , the "known" standard deviation of the process, and the "σ" method may be adopted. In order to verify that the variability remains under control, the value of s should still be calculated and plotted on a control chart.

NOTE — This switch should not be undertaken without the permission of the responsible authority.

Section four : Tables and diagrams

Tables

I-A	Code letters and inspection levels
I-B	Code letters and sample sizes for normal inspection
II-A	Single sampling plans for normal inspection (master table) : "s" method
II-B	Single sampling plans for tightened inspection (master table) : "s" method
II-C	Single sampling plans for reduced inspection (master table) : "s" method
III-A	Single sampling plans for normal inspection (master table) : " σ " method
III-B	Single sampling plans for tightened inspection (master table) : " σ " method

III-C

Single sampling plans for reduced inspection (master table) : " σ " method

IV

Values of f for maximum standard deviation (MSD) : "s" method

V (V-B to V-P) Tables and operating characteristic curves for sampling plans (code letter B to P)

Diagrams

A

Sample sizes of standard sampling plans for specified qualities at 95 % and 10 % probabilities of acceptance

s-D to s-P

Acceptance curves for combined double specification limits : "s" method

 σ -C to σ -P

Acceptance curves for combined double specification limits : " σ " method

Table I-A — Code letters and inspection levels

Lot or batch size	Special inspection levels		General inspection levels		
	S-3	S-4	I	II	II
2 to 8					C
9 to 15					D
16 to 25					E
26 to 50			C	D	F
51 to 90		B	D	E	G
91 to 150		C	E	F	H
151 to 280	B	D	F	G	I
281 to 500	C	E	G	H/I*	J
501 to 1 200	D	F	H	J	K
1 201 to 3 200	E	G	I	K	L
3 201 to 10 000	F	H	J	L	M
10 001 to 35 000	G	I	K	M	N
35 001 to 150 000	H	J	L	N	P
150 001 to 500 000	I	K	M	P	
500 001 and over	J	L	N		

* Use H for lot size 281 to 400 and I for lot size 401 to 500.

NOTES

1 The code letters and inspection levels in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.

2 Symbol



There is no suitable sampling plan in this area; use the first sampling plan below or above the arrow. This refers to both the sample size and acceptability constant k .

Table I-B — Code letters and sample sizes for normal inspection

Method		"s"	"σ"											Equivalent attributes sample	
Acceptable quality level		All AQL	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	Code letter	Sample size
Code letter	B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	B	3
	C	4												C	5
	D	5												D	8
	E	7	↓	↓	2	2	3	3	3	4	4	5	5	E	13
	F	10			3	3	4	4	4	5	5	6	7	F	20
	G	15			4	5	5	6	6	7	8	9	11	G	32
	H	20	4	5	6	6	7	7	8	9	10	12	14	H	50
	I	25	5	6	7	8	8	9	10	11	13	15	17	—	—
	J	35	6	9	9	10	11	12	14	15	18	20	24	J	80
	K	50	8	12	13	14	16	17	19	22	25	29	33	K	125
	L	75	11	17	19	21	23	25	28	32	36	42	49	L	200
	M	100	16	23	25	27	30	33	36	42	48	55	64	M	315
	N	150	22	34	37	40	44	49	54	61	70	82	95	N	500
	P	200	31	45	49	54	59	65	71	81	93	109	127	P	800

NOTES

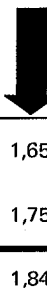
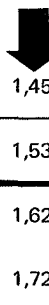
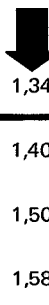
- 1 All AQL values are in percent defective.
- 2 The code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical in MIL STD 414.
- 3 Symbol



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant k .

"s" method

Table II-A — Single sampling plans for normal inspection (master table) : "s" method

Sample size code letter	Sample size	Acceptable quality level (normal inspection)										
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
		<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>
B	3								1,12	0,958	0,765	0,566
C	4								1,17	1,01	0,814	0,617
D	5					1,65	1,53	1,40	1,24	1,07	0,874	0,675
E	7					1,75	1,62	1,50	1,33	1,15	0,955	0,755
F	10		2,24	2,11	1,98	1,84	1,72	1,58	1,41	1,23	1,03	0,828
G	15	2,42	2,32	2,20	2,06	1,91	1,79	1,65	1,47	1,30	1,09	0,886
H	20	2,47	2,36	2,24	2,11	1,96	1,82	1,69	1,51	1,33	1,12	0,917
I	25	2,50	2,40	2,26	2,14	1,98	1,85	1,72	1,53	1,35	1,14	0,936
J	35	2,54	2,45	2,31	2,18	2,03	1,89	1,76	1,57	1,39	1,18	0,969
K	50	2,60	2,50	2,35	2,22	2,08	1,93	1,80	1,61	1,42	1,21	1,00
L	75	2,66	2,55	2,41	2,27	2,12	1,98	1,84	1,65	1,46	1,24	1,03
M	100	2,69	2,58	2,43	2,29	2,14	2,00	1,86	1,67	1,48	1,26	1,05
N	150	2,73	2,61	2,47	2,33	2,18	2,03	1,89	1,70	1,51	1,29	1,07
P	200	2,73	2,62	2,47	2,33	2,18	2,04	1,89	1,70	1,51	1,29	1,07

NOTES

- 1 All AQL values are in percent defective.
- 2 The code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.
- 3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.



The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"s" method

Table II-B — Single sampling plans for tightened inspection (master table) : "s" method

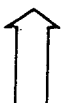
Sample size code letter	Sample size	Acceptable quality levels (tightened inspection)										
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
		<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>
B	3	↓	↓	↓	↓	↓	↓	↓	↓	1,12	0,958	0,765
C	4	↓	↓	↓	↓	↓	↓	1,45	1,34	1,17	1,01	0,814
D	5	↓	↓	↓	↓	↓	1,65	1,53	1,40	1,24	1,07	0,874
E	7	↓	↓	↓	2,00	1,88	1,75	1,62	1,50	1,33	1,15	0,955
F	10	↓	↓	2,24	2,11	1,98	1,84	1,72	1,58	1,41	1,23	1,03
G	15	2,53	2,42	2,32	2,20	2,06	1,91	1,79	1,65	1,47	1,30	1,09
H	20	2,58	2,47	2,36	2,24	2,11	1,96	1,82	1,69	1,51	1,33	1,12
I	25	2,61	2,50	2,40	2,26	2,14	1,98	1,85	1,72	1,53	1,35	1,14
J	35	2,65	2,54	2,45	2,31	2,18	2,03	1,89	1,76	1,57	1,39	1,18
K	50	2,71	2,60	2,50	2,35	2,22	2,08	1,93	1,80	1,61	1,42	1,21
L	75	2,77	2,66	2,55	2,41	2,27	2,12	1,98	1,84	1,65	1,46	1,24
M	100	2,80	2,69	2,58	2,43	2,29	2,14	2,00	1,86	1,67	1,48	1,26
N	150	2,84	2,73	2,61	2,47	2,33	2,18	2,03	1,89	1,70	1,51	1,29
P	200	2,85	2,73	2,62	2,47	2,33	2,18	2,04	1,89	1,70	1,51	1,29

NOTES

- 1 All AQL values are in percent defective.
- 2 The code letters in this International Standard correspond to those given in ISO 2859; they are not identical with those given in MIL STD 414.
- 3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



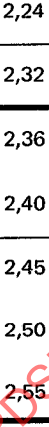
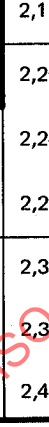
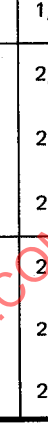
The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.



The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"s" method

Table II-C — Single sampling plans for reduced inspection (master table) : "s" method

Sample size code letter	Sample size	Acceptable quality levels (reduced inspection)										
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
		<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>
B	3							1,12	0,958	0,765	0,566	0,341
C	3							1,12	0,958	0,765	0,566	0,341
D	3							1,12	0,958	0,765	0,566	0,341
E	3							1,12	0,958	0,765	0,566	0,341
F	4							1,45	1,34	1,17	1,01	0,814
G	5				1,65	1,53	1,40	1,24	1,07	0,874	0,675	0,455
H	7				1,75	1,62	1,50	1,33	1,15	0,955	0,755	0,536
I	10				1,84	1,72	1,58	1,41	1,23	1,03	0,828	0,611
J	15				2,32	2,20	2,06	1,91	1,79	1,65	1,47	1,30
K	20				2,36	2,24	2,11	1,96	1,82	1,69	1,51	1,33
L	25	2,40	2,26	2,14	1,98	1,85	1,72	1,53	1,35	1,14	0,936	0,712
M	35	2,45	2,31	2,18	2,03	1,89	1,76	1,57	1,39	1,18	0,969	0,745
N	50	2,50	2,35	2,22	2,08	1,93	1,80	1,61	1,42	1,21	1,00	0,774
P	75	2,55	2,41	2,27	2,12	1,98	1,84	1,65	1,46	1,24	1,03	0,804

NOTES

- 1 All AQL values are in percent defective.
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- 3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.



The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"σ" method

Table III-A — Single sampling plans for normal inspection (master table) : "σ" method

Code letter	Acceptable quality levels (normal inspection)											
	0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00	
	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	
B												
C												
D												
E												
F												
G	4 2,39	4 2,30	4 2,14	5 2,05	5 1,88	6 1,78	6 1,62	7 1,45	8 1,28	9 1,07	11 0,877	
H	5 2,46	5 2,34	6 2,23	6 2,08	7 1,95	7 1,80	8 1,68	9 1,49	10 1,31	12 1,11	14 0,906	
I	6 2,49	6 2,37	7 2,25	8 2,13	8 1,96	9 1,83	10 1,70	11 1,51	13 1,34	15 1,13	17 0,924	
J	8 2,54	9 2,45	9 2,29	10 2,16	11 2,01	12 1,88	14 1,75	15 1,56	18 1,38	20 1,17	24 0,964	
K	11 2,59	12 2,49	13 2,35	14 2,21	16 2,07	17 1,93	19 1,79	22 1,61	25 1,42	29 1,21	33 0,995	
L	16 2,65	17 2,54	19 2,41	21 2,27	23 2,12	25 1,97	28 1,84	32 1,65	36 1,46	42 1,24	49 1,03	
M	22 2,69	23 2,57	25 2,43	27 2,29	30 2,14	33 2,00	36 1,86	42 1,67	48 1,48	55 1,26	64 1,05	
N	31 2,72	34 2,62	37 2,47	40 2,33	44 2,17	49 2,03	54 1,89	61 1,69	70 1,51	82 1,29	95 1,07	
P	42 2,73	45 2,62	49 2,48	54 2,34	59 2,18	65 2,04	71 1,89	81 1,70	93 1,51	109 1,29	127 1,07	

NOTES

- 1 All AQL values are in percent defective.
- 2 The code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.
- 3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



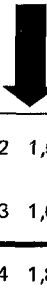
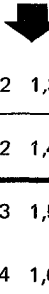
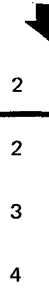
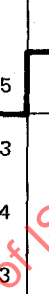
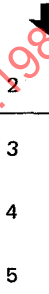
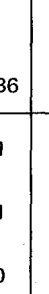
The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.



The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"σ" method

Table III-B — Single sampling plans for tightened inspection (master table) : "σ" method

Code letter	Acceptable quality levels (tightened inspection)											
	0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00	
	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	<i>n</i> <i>k</i>	
B												
C												
D												
E												
F												
G	3 2,49	4 2,39	4 2,30	4 2,14	5 2,05	5 1,88	6 1,78	6 1,62	7 1,45	8 1,28	9 1,07	
H	4 2,55	5 2,46	5 2,34	6 2,23	6 2,08	7 1,95	7 1,80	8 1,68	9 1,49	10 1,31	12 1,11	
I	6 2,59	6 2,49	6 2,37	7 2,25	8 2,13	8 1,96	9 1,83	10 1,70	11 1,51	13 1,34	15 1,13	
J	7 2,63	8 2,54	9 2,45	9 2,29	10 2,16	11 2,01	12 1,86	14 1,75	15 1,56	18 1,38	20 1,17	
K	11 2,72	11 2,59	12 2,49	13 2,35	14 2,21	16 2,07	17 1,93	19 1,79	22 1,61	25 1,42	29 1,21	
L	15 2,77	16 2,65	17 2,54	19 2,41	21 2,27	23 2,12	25 1,97	28 1,84	32 1,65	36 1,46	42 1,24	
M	20 2,80	22 2,69	23 2,57	25 2,43	27 2,29	30 2,14	33 2,00	36 1,86	42 1,67	48 1,48	55 1,26	
N	30 2,84	31 2,72	34 2,62	37 2,47	40 2,33	44 2,17	49 2,03	54 1,89	61 1,69	70 1,51	82 1,29	
P	40 2,85	42 2,73	45 2,62	49 2,48	54 2,34	59 2,18	65 2,04	71 1,89	81 1,70	93 1,51	109 1,29	

NOTES

- 1 All AQL values are in percent defective.
- 2 The code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.
- 3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.



The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"σ" method

Table III-C — Single sampling plans for reduced inspection (master table) : "σ" method

Code letter	Acceptable quality levels (reduced inspection)											
	0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00	
	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k
B												
C												
D												
E												
F												
G				2 1,58	2 1,42	2 1,33	3 1,17	3 1,01	3 0,825	4 0,641	4 0,429	
H		2 1,94	2 1,81	3 1,69	3 1,56	3 1,44	4 1,28	4 1,11	5 0,919	5 0,728	6 0,515	
I	3 2,19	3 2,07	3 1,91	4 1,80	4 1,69	4 1,53	5 1,39	5 1,20	6 0,991	7 0,797	8 0,584	
J	4 2,30	4 2,14	5 2,05	5 1,88	6 1,78	6 1,62	7 1,45	8 1,28	9 1,07	11 0,877	12 0,649	
K	5 2,34	6 2,23	6 2,08	7 1,95	7 1,80	8 1,68	9 1,49	10 1,31	12 1,11	14 0,906	16 0,685	
L	6 2,37	7 2,25	8 2,13	8 1,96	9 1,83	10 1,70	11 1,51	13 1,34	15 1,13	17 0,924	20 0,706	
M	9 2,45	9 2,29	10 2,16	11 2,01	12 1,88	14 1,75	15 1,56	18 1,38	20 1,17	24 0,964	27 0,737	
N	12 2,49	13 2,35	14 2,21	16 2,07	17 1,93	19 1,79	22 1,61	25 1,42	29 1,21	33 0,995	38 0,770	
P	17 2,54	19 2,41	21 2,27	23 2,12	25 1,97	28 1,84	32 1,65	36 1,46	42 1,24	49 1,03	56 0,803	

NOTES

- 1 All AQL values are in percent defective.
- 2 The code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.
- 3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant k .



The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.



The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

Table IV — Values of f for maximum standard deviation (MSD) : "s" method

Sample size	Acceptable quality levels (normal inspection)												
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00	
3									0,436	0,453	0,475	0,502	0,538
4							0,339	0,353	0,374	0,399	0,432	0,472	0,528
5						0,294	0,308	0,323	0,346	0,372	0,408	0,452	0,511
7				0,242	0,253	0,266	0,280	0,295	0,318	0,345	0,381	0,425	0,485
10			0,214	0,224	0,235	0,248	0,261	0,276	0,298	0,324	0,359	0,403	0,460
15	0,188	0,195	0,202	0,211	0,222	0,235	0,248	0,262	0,284	0,309	0,344	0,386	0,442
20	0,183	0,190	0,197	0,206	0,216	0,229	0,242	0,255	0,277	0,302	0,336	0,377	0,432
25	0,180	0,187	0,193	0,203	0,212	0,225	0,238	0,251	0,273	0,297	0,331	0,372	0,426
35	0,176	0,183	0,189	0,198	0,208	0,220	0,232	0,245	0,266	0,291	0,323	0,364	0,416
50	0,172	0,178	0,184	0,194	0,203	0,214	0,227	0,241	0,261	0,284	0,317	0,356	0,408
75	0,168	0,174	0,181	0,189	0,199	0,211	0,223	0,235	0,255	0,279	0,310	0,348	0,399
100	0,166	0,172	0,179	0,187	0,197	0,208	0,220	0,233	0,253	0,276	0,307	0,345	0,395
150	0,163	0,170	0,175	0,185	0,193	0,206	0,216	0,230	0,249	0,271	0,302	0,341	0,388
200	0,163	0,168	0,175	0,183	0,193	0,203	0,215	0,228	0,248	0,269	0,302	0,338	0,386
	0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00		
	Acceptable quality levels (tightened inspection)												
			0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
			Acceptable quality levels (reduced inspection)										

NOTE — The MSD is obtained by multiplying the factor f by the difference between the upper specification limit U and lower specification limit L , i.e. $MSD = f(U - L)$.

The MSD serves as a guide for the magnitude of the estimate of lot standard deviation when using plans for the double specification limit case, based on the estimate of lot standard deviation of unknown variability. The estimate of lot standard deviation, if it is less than the MSD, helps to ensure, but does not guarantee, lot acceptability.

B

Table V-B — Code letter B

Chart V-B — Operating characteristic curves for single sampling plans

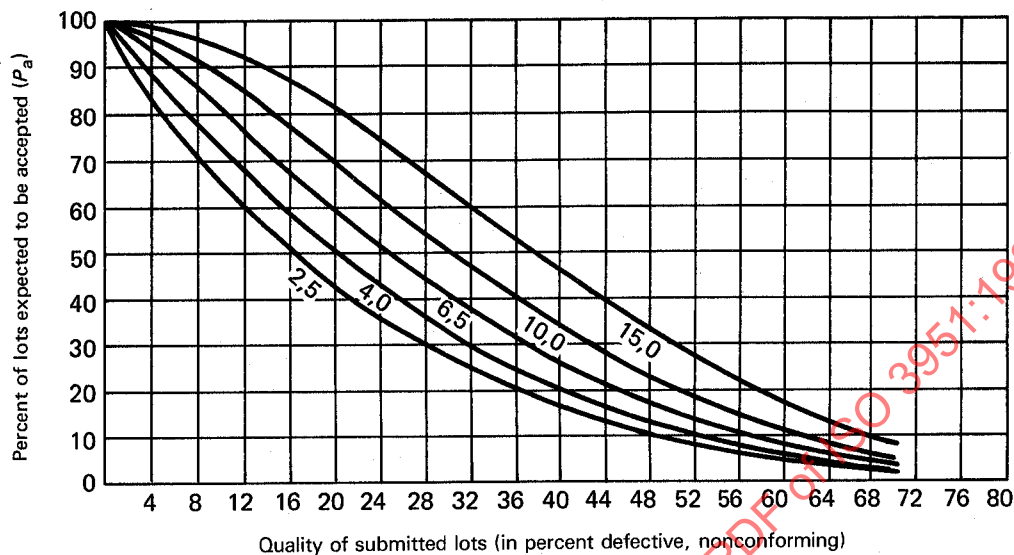


Table V-B-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0								0,18	0,44	1,07	2,28	99,0
95,0								1,04	1,89	3,52	6,02	95,0
90,0								2,26	3,65	6,03	9,39	90,0
75,0								6,66	9,18	12,95	17,71	75,0
50,0								16,68	20,30	25,22	30,97	50,0
25,0								32,40	36,35	41,45	47,14	25,0
10,0								49,34	52,83	57,24	62,08	10,0
5,0								59,45	62,44	66,20	70,30	5,0
1,0								75,99	77,93	80,34	82,98	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptable quality level (tightened inspection)											

Table V-C — Code letter C

C

Chart V-C — Operating characteristic curves for single sampling plans

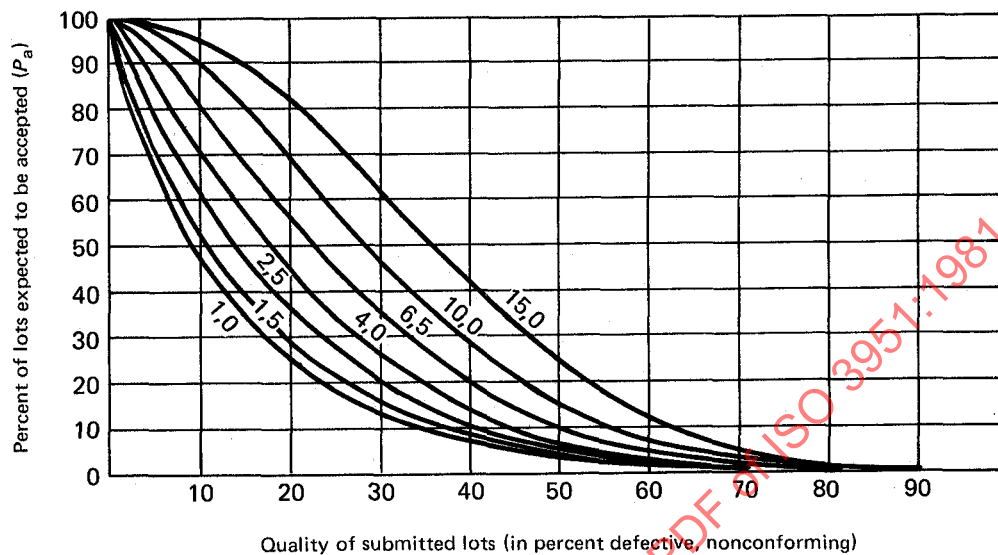


Table V-C-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0							0,13	0,32	0,69	1,55	3,05	99,0
95,0						0,44	0,69	1,32	2,29	4,13	6,85	95,0
90,0						1,02	1,48	2,53	3,98	6,51	10,00	90,0
75,0						3,36	4,37	6,37	8,81	12,61	17,35	75,0
50,0						9,52	11,28	14,44	17,93	22,89	28,61	50,0
25,0						20,81	23,11	27,00	31,02	36,43	42,37	25,0
10,0						34,88	37,26	41,15	45,05	50,13	55,55	10,0
5,0						44,29	46,53	50,14	53,72	58,33	68,20	5,0
1,0						61,76	63,48	66,23	68,95	72,37	75,98	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
Acceptable quality level (tightened inspection)												

D

Table V-D — Code letter D

Chart V-D — Operating characteristic curves for single sampling plans

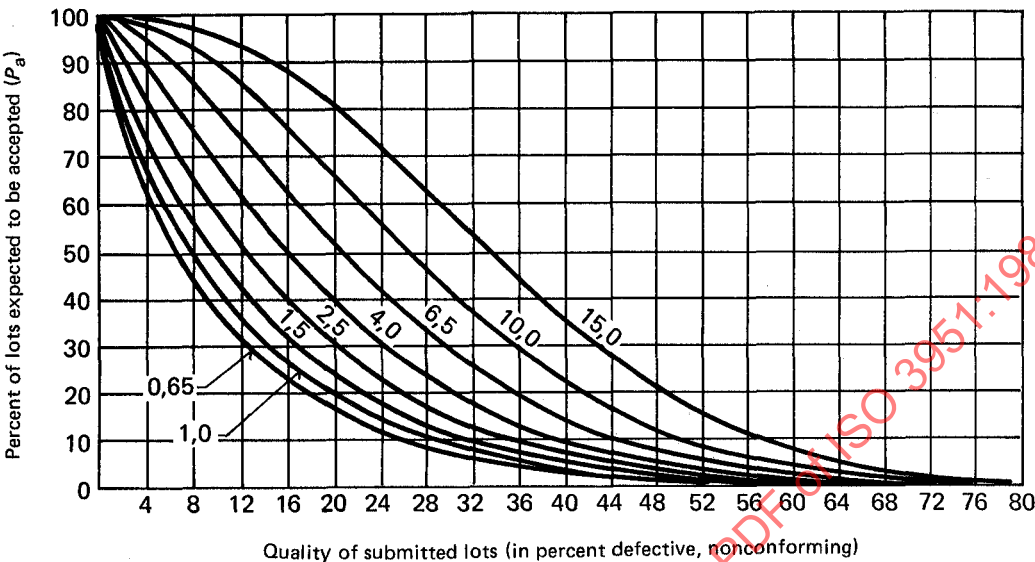


Table V-D-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a	
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5		10,0
99,0						0,04	0,09	0,18	0,40	0,84	1,79	3,44	99,0
95,0						0,28	0,46	0,77	1,38	2,43	4,30	7,11	95,0
90,0						0,64	0,98	1,51	2,48	3,99	6,49	10,00	90,0
75,0						2,15	2,93	4,02	5,78	8,23	11,89	16,55	75,0
50,0						6,34	7,82	9,71	12,47	15,97	20,75	26,40	50,0
25,0						14,64	16,79	19,36	22,88	27,06	32,43	38,46	25,0
10,0						25,94	28,40	31,24	34,98	39,25	44,55	50,32	10,0
5,0						34,06	36,52	39,33	42,97	47,06	52,06	57,42	5,0
1,0						50,47	52,63	55,04	58,11	61,51	65,57	69,89	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
Acceptable quality level (tightened inspection)													

Table V-E — Code letter E

E

Chart V-E — Operating characteristic curves for single sampling plans

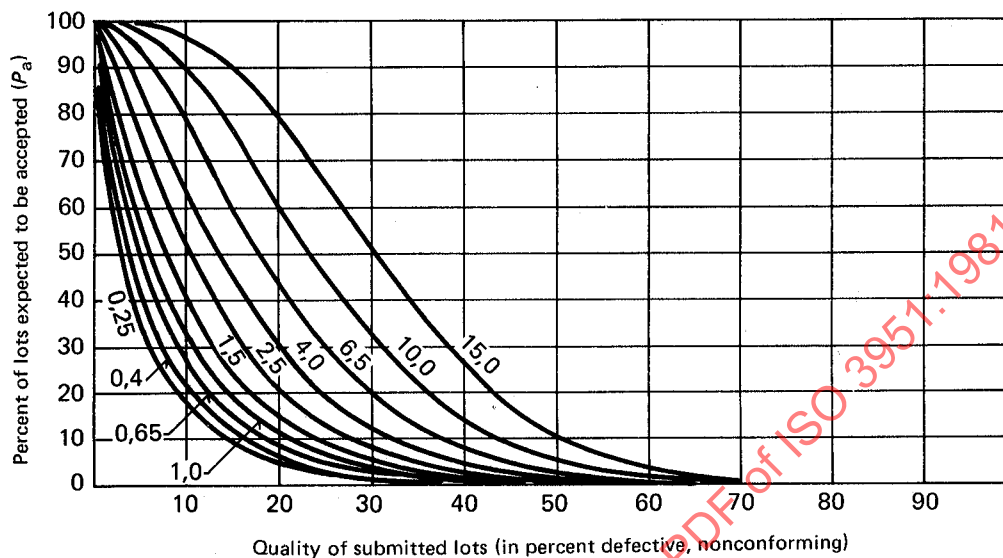


Table V-E-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											p_a
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0			0,02	0,03	0,07	0,14	0,25	0,53	1,09	2,19	4,04	99,0
95,0			0,11	0,18	0,32	0,53	0,83	1,50	2,65	4,57	7,46	95,0
90,0			0,25	0,40	0,64	1,01	1,48	2,47	4,04	6,50	9,99	90,0
75,0			0,90	1,27	1,83	2,58	3,47	5,15	7,56	11,00	15,49	75,0
50,0			2,89	3,72	4,83	6,18	7,69	10,28	13,66	18,11	23,53	50,0
25,0			7,38	8,80	10,57	12,60	14,71	18,11	22,27	27,41	33,35	25,0
10,0			14,42	16,33	18,60	21,09	23,58	27,43	31,93	37,28	43,25	10,0
5,0			20,09	22,20	24,65	27,29	29,88	33,82	38,33	43,60	49,38	5,0
1,0			33,10	35,32	37,83	40,45	42,95	46,72	50,89	55,64	60,73	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
Acceptable quality level (tightened inspection)												

F

Table V-F — Code letter F

Chart V-F — Operating characteristic curves for single sampling plans

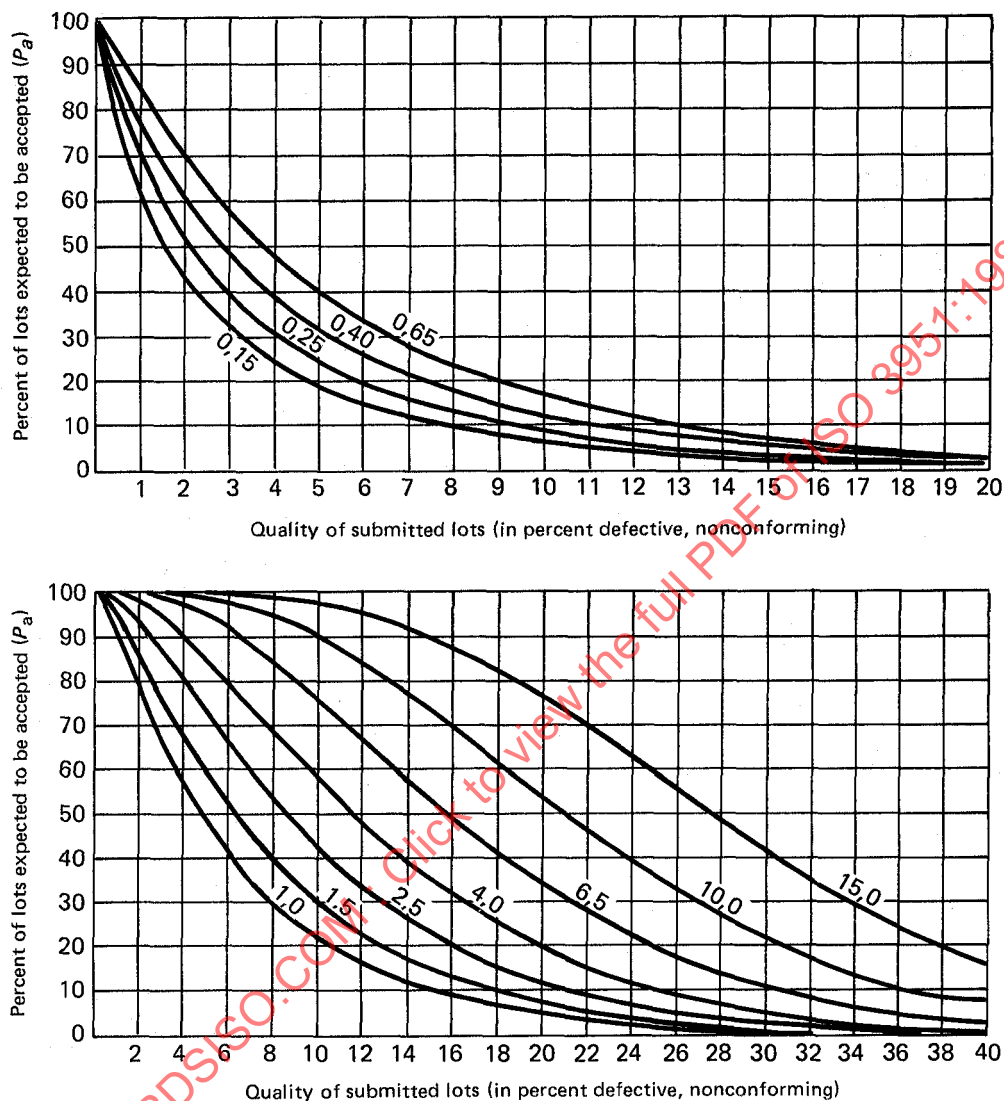


Table V-F-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0			0,01	0,03	0,05	0,11	0,19	0,35	0,69	1,35	2,62	4,69	99,0
95,0			0,07	0,12	0,21	0,36	0,57	0,94	1,65	2,83	4,84	7,81	95,0
90,0			0,15	0,25	0,40	0,66	0,98	1,52	2,50	4,04	6,52	10,01	90,0
75,0			0,49	0,74	1,08	1,61	2,21	3,15	4,70	6,93	10,25	14,60	75,0
50,0			1,53	2,08	2,79	3,77	4,82	6,33	8,62	11,69	15,91	21,09	50,0
25,0			3,93	4,95	6,16	7,72	9,29	11,41	14,45	18,25	23,20	28,96	25,0
10,0			7,95	9,44	11,15	13,23	15,23	17,84	21,40	25,66	30,99	36,98	10,0
5,0			11,40	13,17	15,13	17,47	19,68	22,49	26,27	30,68	36,09	42,06	5,0
1,0			20,10	22,24	24,53	27,19	29,58	32,59	36,50	40,92	46,18	51,82	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table V-G — Code letter G

G

Table V-G — Operating characteristic curves for single sampling plans

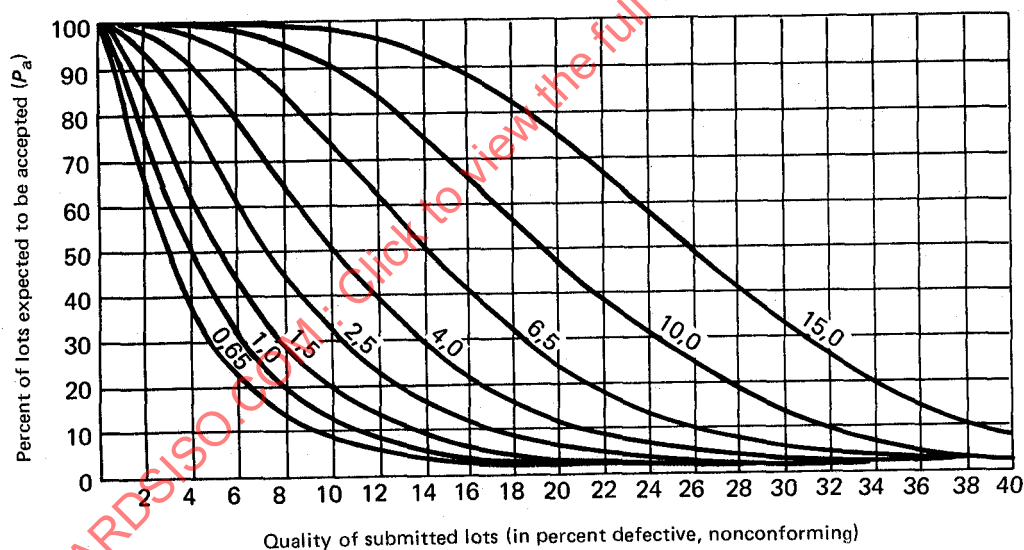
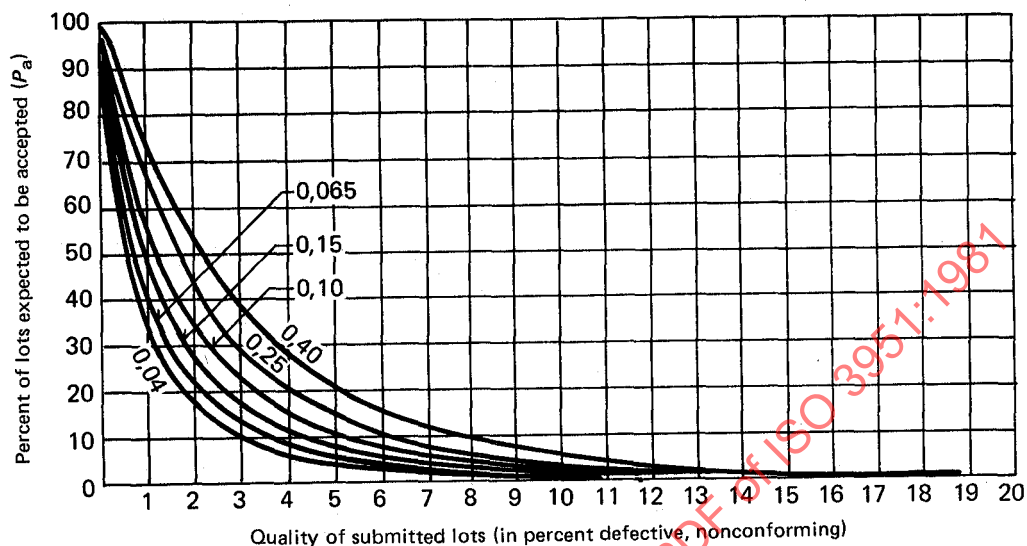


Table V-G-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,01	0,01	0,02	0,04	0,08	0,17	0,28	0,49	0,96	1,71	3,23	5,58	99,0
95,0	0,03	0,06	0,09	0,15	0,25	0,45	0,68	1,09	1,91	3,09	5,30	8,41	95,0
90,0	0,07	0,11	0,17	0,26	0,43	0,72	1,06	1,61	2,67	4,14	6,76	10,30	90,0
75,0	0,22	0,32	0,45	0,65	0,98	1,50	2,07	2,94	4,49	6,50	9,83	14,09	75,0
50,0	0,67	0,90	1,17	1,57	2,20	3,09	3,99	5,32	7,51	10,15	14,27	19,25	50,0
25,0	1,73	2,18	2,67	3,38	4,41	5,77	7,09	8,92	11,77	15,02	19,84	25,38	25,0
10,0	3,58	4,31	5,07	6,13	7,58	9,41	11,12	13,38	16,77	20,48	25,76	31,63	10,0
5,0	5,27	6,19	7,13	8,40	10,11	12,22	14,13	16,63	20,28	24,20	29,67	35,63	5,0
1,0	9,91	11,18	12,45	14,11	16,24	18,76	21,00	23,83	27,82	31,97	37,57	43,50	1,0
Acceptable quality level (tightened inspection)													
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		

H

Table V-H — Code letter H

Chart V-H — Operating characteristic curves for single sampling plans

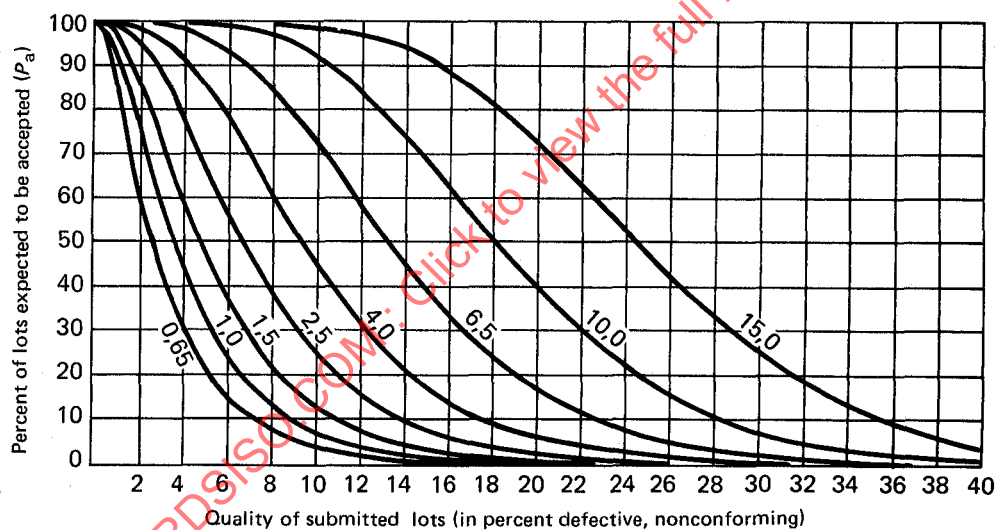
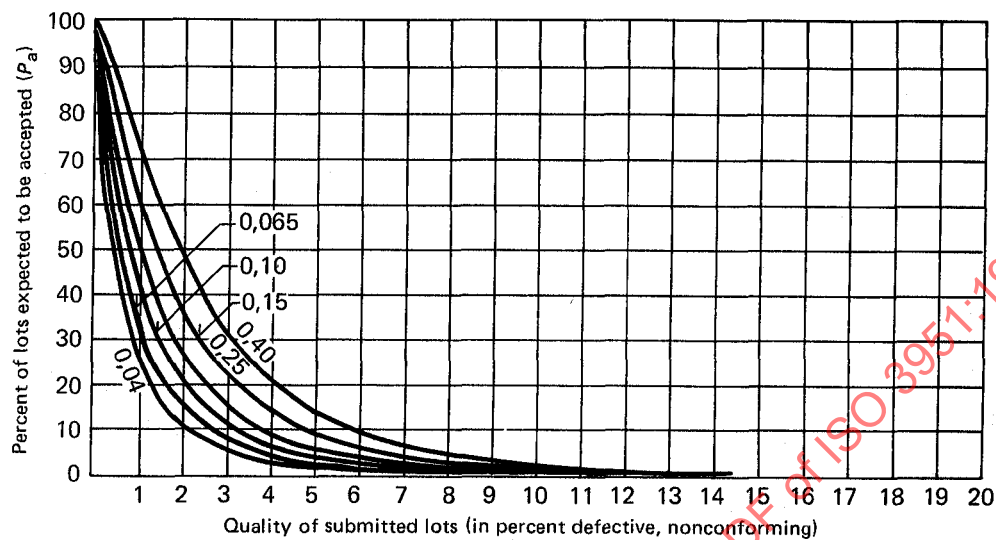


Table V-H-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,02	0,04	0,06	0,11	0,21	0,37	0,61	1,15	2,04	3,73	6,25	99,0
95,0	0,04	0,07	0,11	0,17	0,29	0,49	0,79	1,21	2,07	3,39	5,69	95,0
90,0	0,08	0,12	0,19	0,29	0,45	0,75	1,15	1,69	2,77	4,35	7,01	90,0
75,0	0,21	0,31	0,44	0,63	0,93	1,42	2,06	2,85	4,35	6,43	9,71	75,0
50,0	0,56	0,76	1,01	1,38	1,90	2,69	3,66	4,81	6,85	9,51	13,49	50,0
25,0	1,31	1,68	2,13	2,74	3,56	4,75	6,12	7,67	10,29	13,52	18,13	25,0
10,0	2,58	3,16	3,85	4,73	5,88	7,46	9,23	11,14	14,25	17,94	23,01	10,0
5,0	3,71	4,44	5,29	6,36	7,71	9,54	11,53	13,65	17,03	20,95	26,24	5,0
1,0	6,81	7,85	9,22	10,44	12,17	14,42	16,79	19,24	23,02	27,26	32,79	1,0
Acceptable quality level (tightened inspection)												
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	

Table V-I — Code letter I

Chart V-I — Operating characteristic curves for single sampling plans

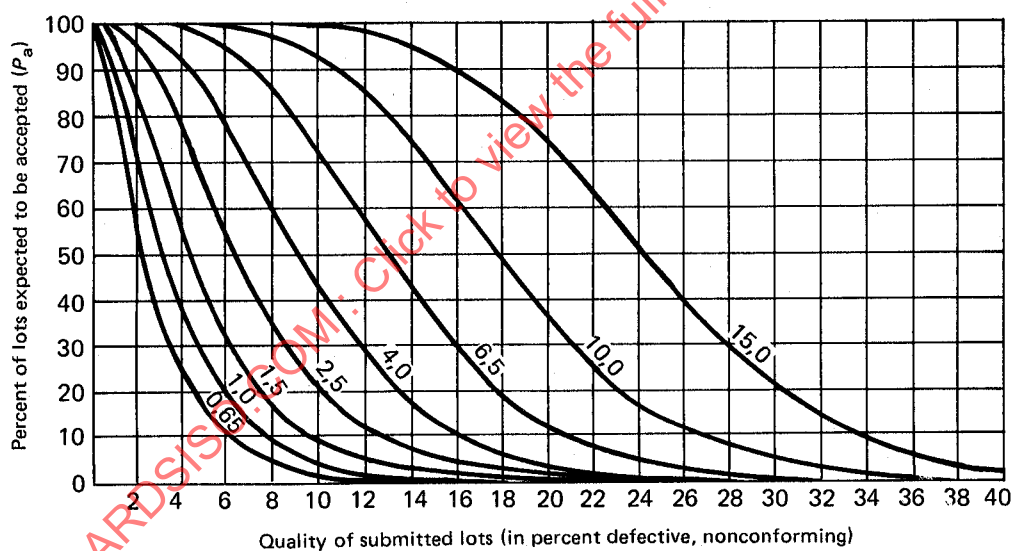
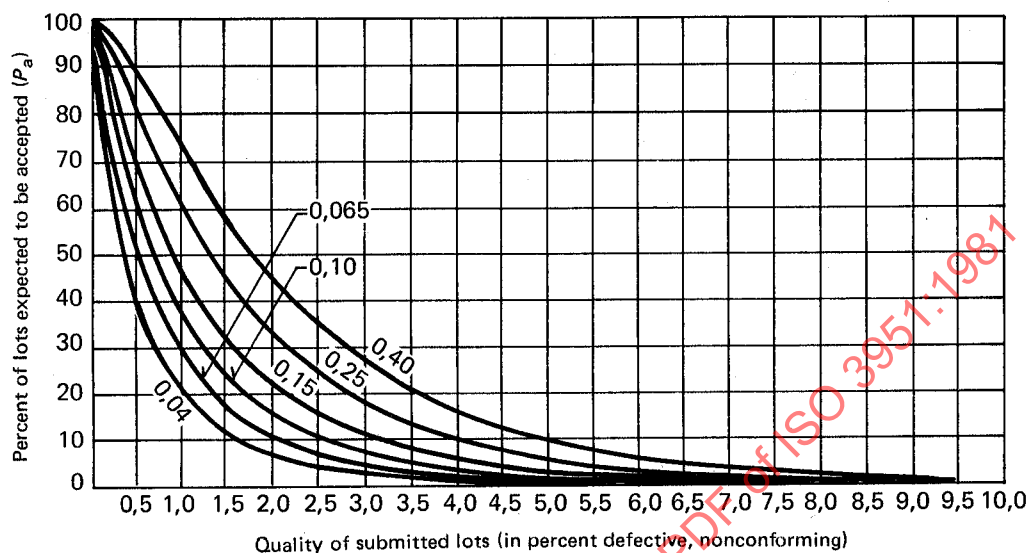


Table V-I-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,02	0,03	0,04	0,08	0,14	0,26	0,44	0,70	1,33	2,31	4,12	6,79	99,0
95,0	0,05	0,08	0,12	0,20	0,32	0,56	0,85	1,28	2,23	3,61	5,98	9,27	95,0
90,0	0,09	0,13	0,19	0,32	0,48	0,80	1,19	1,73	2,89	4,51	7,21	10,83	90,0
75,0	0,21	0,30	0,41	0,64	0,91	1,42	2,00	2,78	4,34	6,39	9,65	13,82	75,0
50,0	0,50	0,68	0,89	1,28	1,73	2,53	3,39	4,47	6,54	9,12	13,00	17,74	50,0
25,0	1,09	1,42	1,77	2,41	3,09	4,25	5,43	6,87	9,47	12,57	17,03	22,27	25,0
10,0	2,05	2,55	3,08	3,99	4,93	6,46	7,97	9,73	12,81	16,34	21,24	26,82	10,0
5,0	2,89	3,52	4,17	5,26	6,37	8,14	9,83	11,78	15,14	18,89	24,01	29,75	5,0
1,0	5,17	6,06	6,97	8,43	9,85	12,04	14,08	16,36	20,14	24,24	29,66	35,56	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table V-J — Code letter J

Chart V-J — Operating characteristic curves for single sampling plans

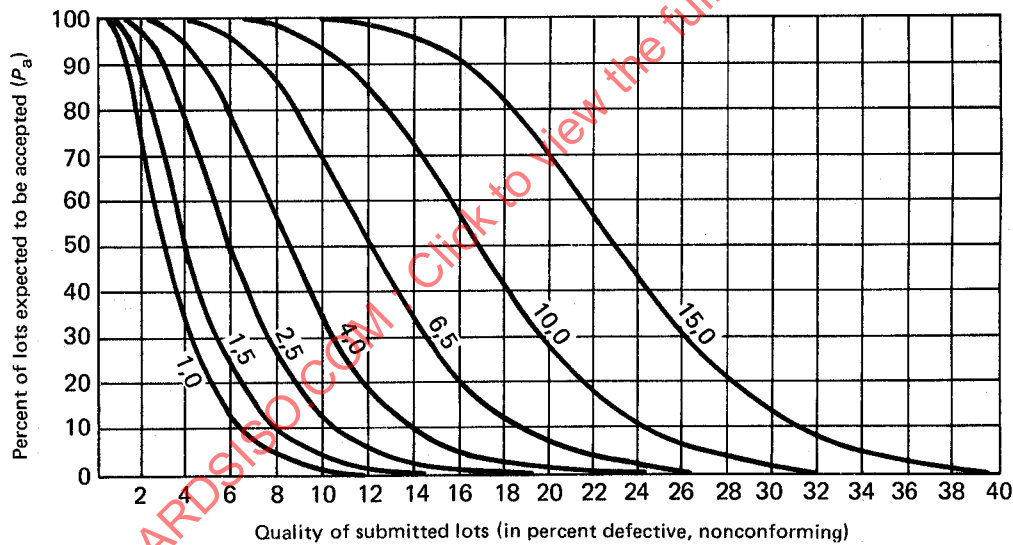
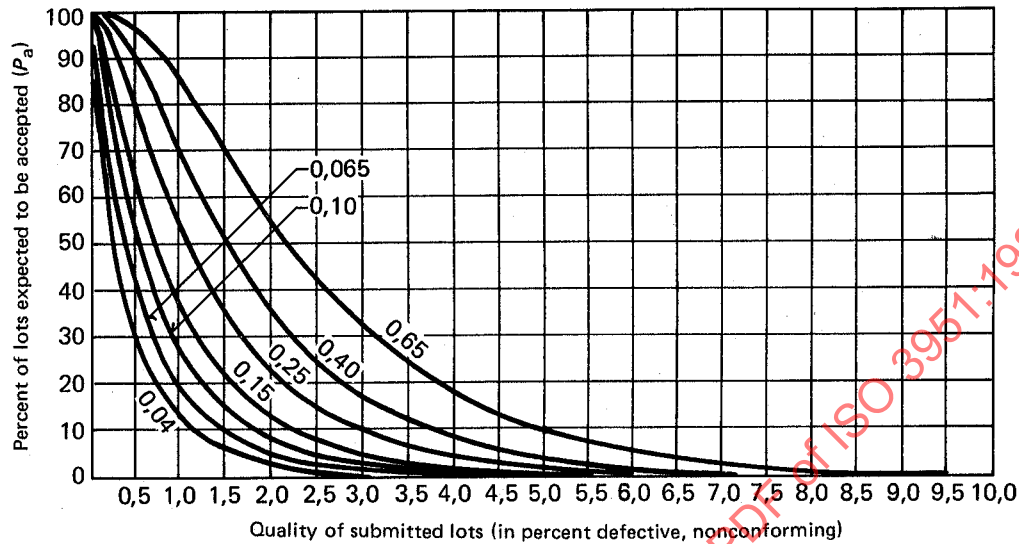


Table V-J-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
	0,02	0,04	0,06	0,11	0,18	0,32	0,54	0,84	1,54	2,61	4,54	7,44	
99,0	0,02	0,04	0,06	0,11	0,18	0,32	0,54	0,84	1,54	2,61	4,54	7,44	99,0
95,0	0,06	0,09	0,13	0,23	0,36	0,60	0,94	1,40	2,38	3,80	6,21	9,65	95,0
90,0	0,10	0,15	0,20	0,33	0,51	0,82	1,25	1,80	2,97	4,59	7,28	11,01	90,0
75,0	0,20	0,29	0,39	0,60	0,88	1,34	1,94	2,69	4,19	6,18	9,34	13,56	75,0
50,0	0,43	0,59	0,76	1,10	1,54	2,21	3,05	4,05	5,98	8,41	12,10	16,82	50,0
25,0	0,86	1,12	1,39	1,92	2,55	3,50	4,62	5,91	8,29	11,16	15,35	20,53	25,0
10,0	1,50	1,90	2,29	3,02	3,87	5,10	6,50	8,07	10,85	14,11	18,71	24,23	10,0
5,0	2,05	2,54	3,01	3,89	4,89	6,29	7,87	9,60	12,62	16,09	20,90	26,60	5,0
1,0	3,50	4,12	4,84	6,02	7,30	9,04	10,95	12,98	16,42	20,24	25,39	31,32	1,0
Acceptable quality level (tightened inspection)													
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		

Table V-K — Code letter K

K

Chart V-K — Operating characteristic curves for single sampling plans

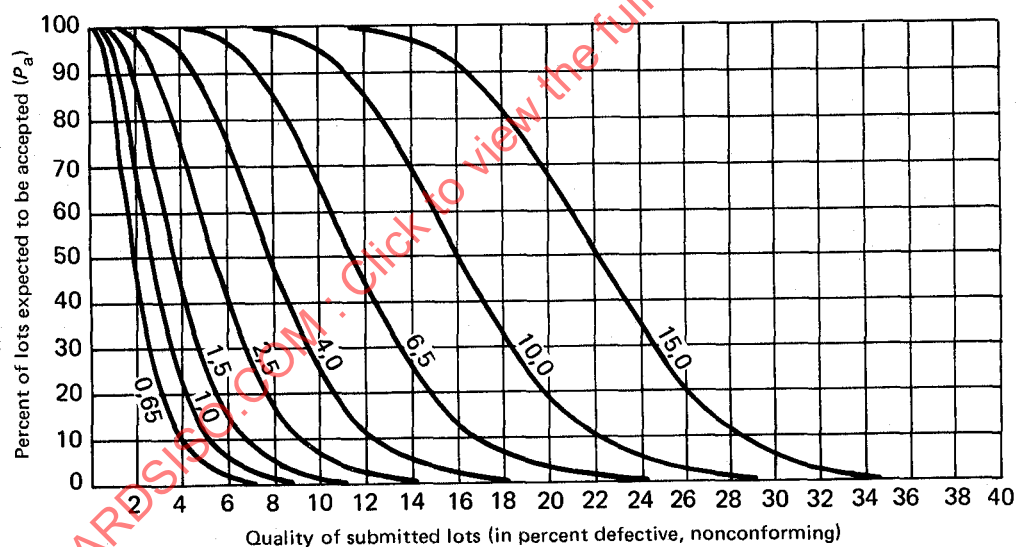
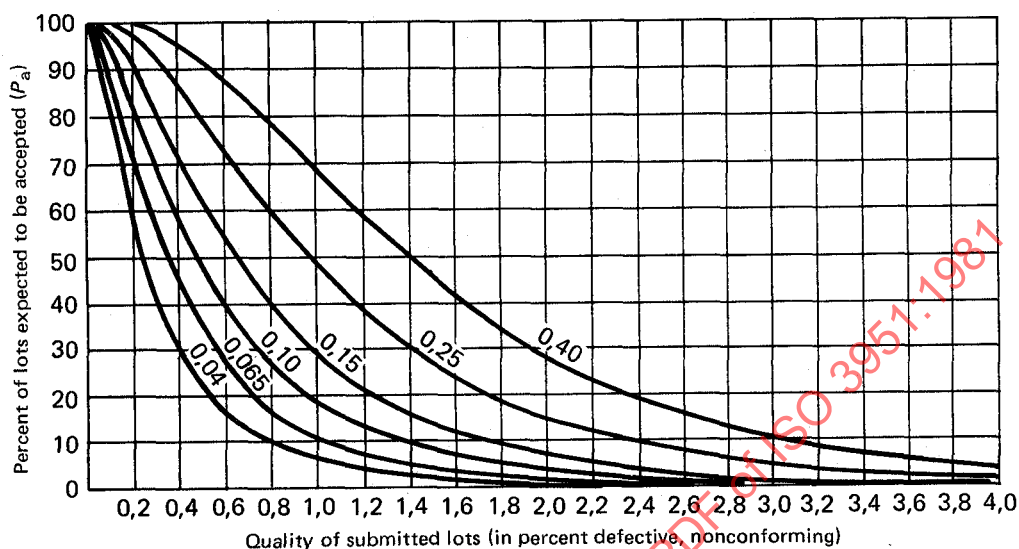


Table V-K-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (nominal inspection)												P_a
	0,03	0,05	0,08	0,14	0,23	0,38	0,64	0,98	1,74	2,96	5,03	8,05	
99,0	0,03	0,05	0,08	0,14	0,23	0,38	0,64	0,98	1,74	2,96	5,03	8,05	99,0
95,0	0,07	0,10	0,15	0,26	0,40	0,64	1,02	1,49	2,51	4,04	6,52	10,00	95,0
90,0	0,10	0,15	0,21	0,35	0,54	0,83	1,29	1,84	3,01	4,73	7,44	11,16	90,0
75,0	0,19	0,27	0,37	0,58	0,85	1,26	1,87	2,59	4,04	6,08	9,19	13,31	75,0
50,0	0,35	0,49	0,65	0,98	1,37	1,94	2,76	3,68	5,48	7,90	11,45	16,00	50,0
25,0	0,64	0,86	1,10	1,58	2,12	2,89	3,95	5,11	7,27	10,09	14,06	19,01	25,0
10,0	1,06	1,36	1,70	2,35	3,07	4,03	5,33	6,72	9,23	12,39	16,72	21,98	10,0
5,0	1,40	1,77	2,18	2,94	3,77	4,87	6,32	7,84	10,55	13,92	18,45	23,88	5,0
1,0	2,29	2,81	3,36	4,36	5,42	6,78	8,52	10,30	13,39	17,10	21,97	27,65	1,0
Acceptable quality level (tightened inspection)													
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		

Table V-L — Code letter L

Chart V-L — Operating characteristic curves for single sampling plans

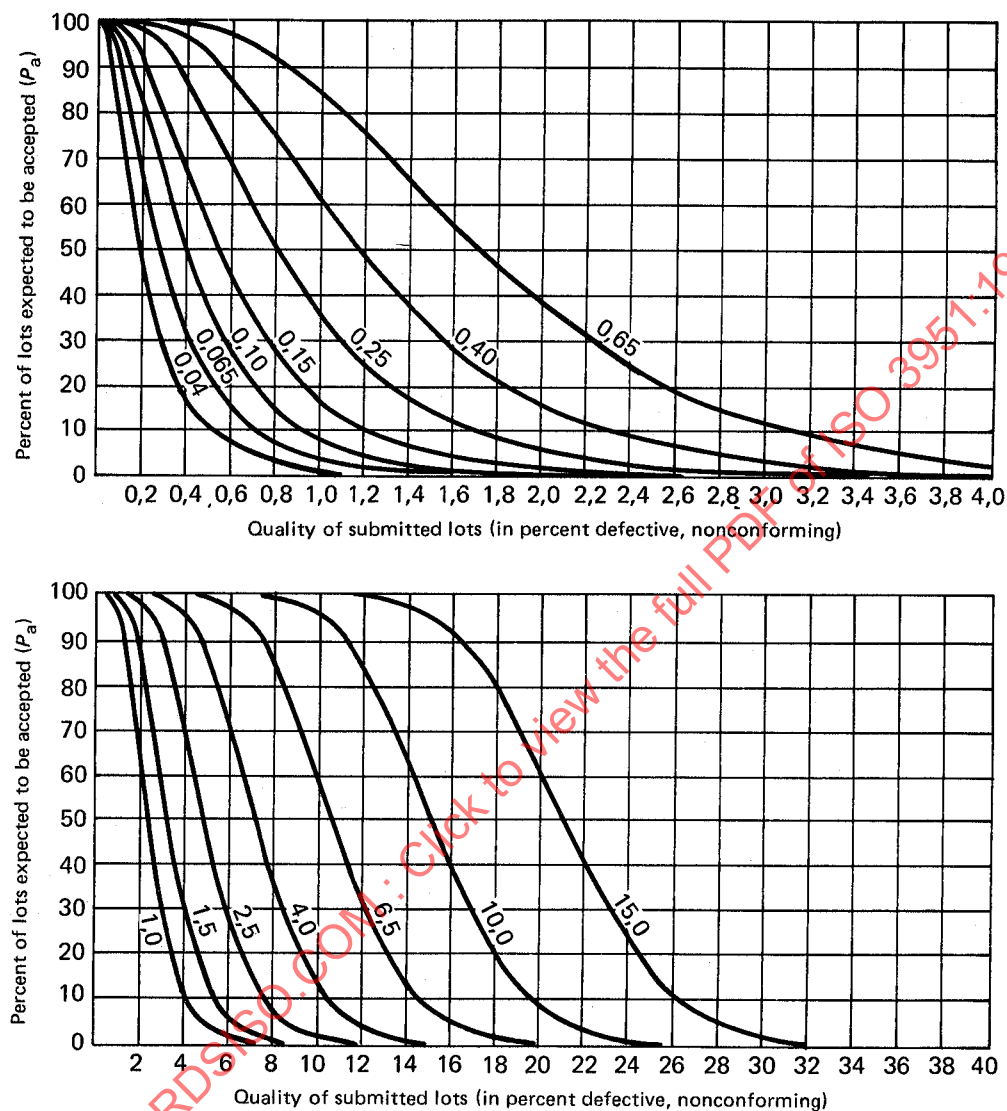


Table V-L-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
	0,04	0,06	0,09	0,16	0,27	0,46	0,73	1,12	1,95	3,24	5,52	8,67	
99,0	0,04	0,06	0,09	0,16	0,27	0,46	0,73	1,12	1,95	3,24	5,52	8,67	99,0
95,0	0,07	0,11	0,17	0,27	0,43	0,70	1,06	1,58	2,62	4,18	6,81	10,34	95,0
90,0	0,10	0,15	0,22	0,35	0,55	0,86	1,29	1,88	3,05	4,76	7,59	11,31	90,0
75,0	0,17	0,24	0,35	0,53	0,80	1,21	1,76	2,49	3,89	5,86	9,02	13,08	75,0
50,0	0,29	0,40	0,56	0,82	1,19	1,74	2,43	3,34	5,02	7,29	10,84	15,24	50,0
25,0	0,48	0,65	0,87	1,23	1,73	2,44	3,30	4,41	6,38	8,97	12,89	17,62	25,0
10,0	0,74	0,97	1,27	1,74	2,37	3,24	4,28	5,58	7,82	10,70	14,94	19,95	10,0
5,0	0,95	1,23	1,57	2,12	2,84	3,82	4,97	6,38	8,79	11,84	16,26	21,42	5,0
1,0	1,47	1,84	2,30	3,01	3,91	5,10	6,47	8,11	10,84	14,19	18,93	24,34	1,0
Acceptable quality level (tightened inspection)													
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		

Table V-M — Code letter M

M

Chart V-M — Operating characteristic curves for single sampling plans

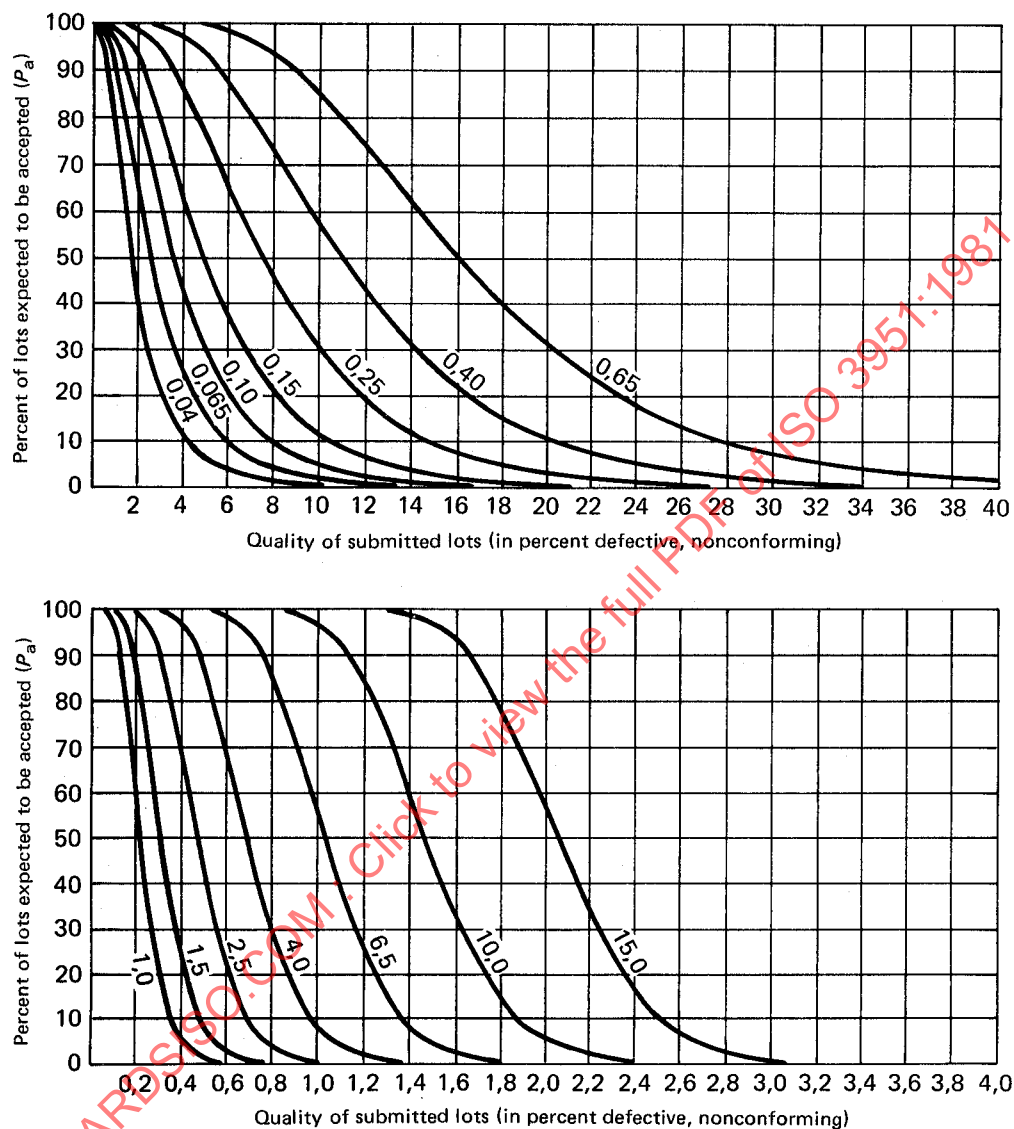


Table V-M-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,05	0,07	0,11	0,19	0,31	0,52	0,81	1,24	2,12	3,46	5,81	9,03	99,0
95,0	0,08	0,12	0,18	0,29	0,47	0,74	1,12	1,66	2,73	4,31	6,97	10,51	95,0
90,0	0,10	0,15	0,22	0,37	0,57	0,89	1,33	1,93	3,11	4,82	7,65	11,36	90,0
75,0	0,16	0,23	0,33	0,53	0,79	1,20	1,74	2,46	3,84	5,78	8,90	12,90	75,0
50,0	0,26	0,37	0,51	0,77	1,12	1,64	2,31	3,18	4,80	7,00	10,45	14,75	50,0
25,0	0,41	0,56	0,75	1,11	1,56	2,22	3,02	4,07	5,93	8,41	12,18	16,78	25,0
10,0	0,61	0,80	1,05	1,50	2,06	2,86	3,81	5,01	7,11	9,84	13,89	18,73	10,0
5,0	0,75	0,99	1,28	1,79	2,43	3,31	4,35	5,65	7,89	10,77	14,99	19,97	5,0
1,0	1,12	1,43	1,81	2,46	3,24	4,30	5,52	7,02	9,53	12,68	17,19	22,41	1,0
Acceptable quality level (tightened inspection)													
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		

N

Table V-N — Code letter N

Chart V-N — Operating characteristic curves for single sampling plans

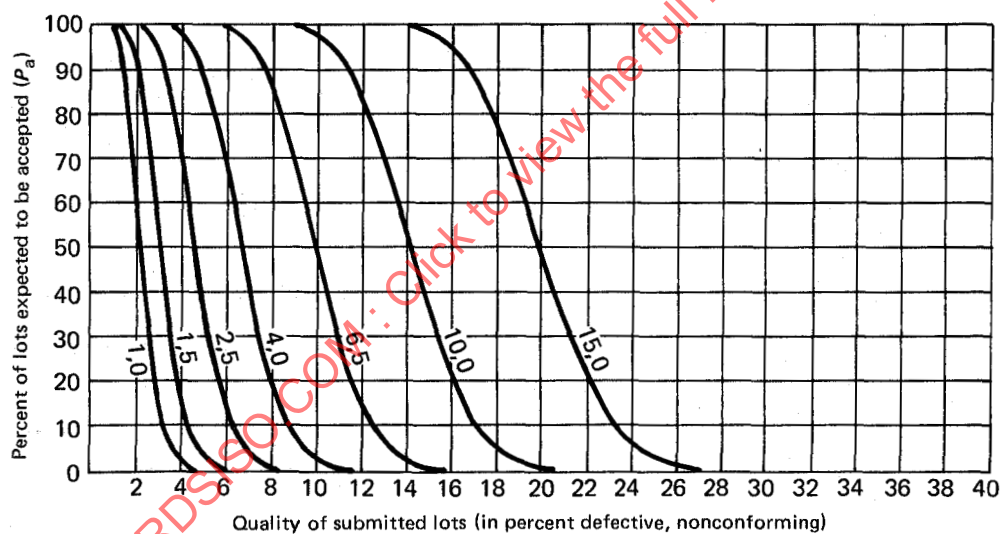
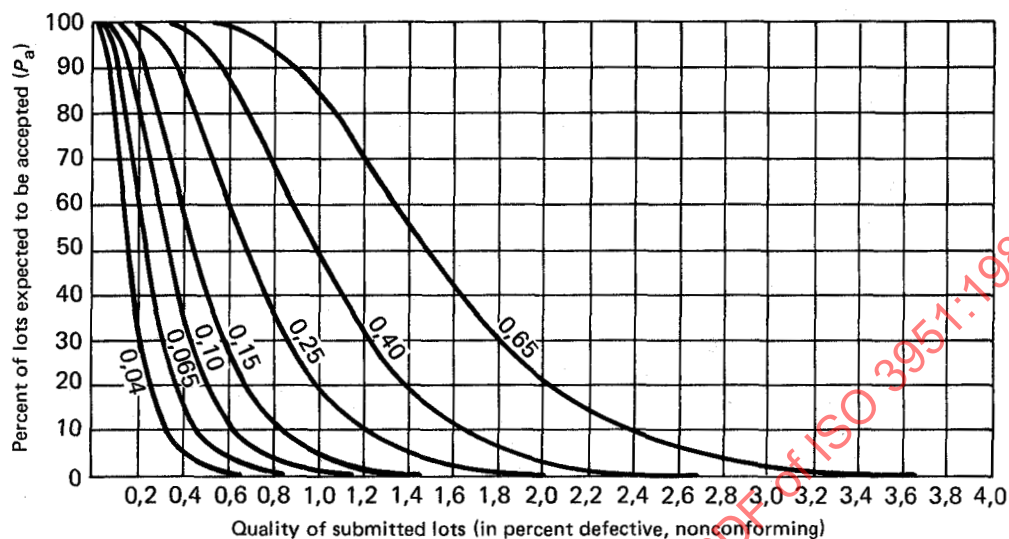


Table V-N-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
	0,05	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,05	0,08	0,13	0,22	0,35	0,57	0,91	1,36	2,29	3,69	6,10	9,55	99,0
95,0	0,08	0,13	0,19	0,31	0,48	0,77	1,18	1,73	2,82	4,41	7,07	10,80	95,0
90,0	0,11	0,16	0,24	0,37	0,57	0,89	1,35	1,96	3,13	4,84	7,64	11,51	90,0
75,0	0,15	0,22	0,33	0,50	0,75	1,14	1,69	2,39	3,73	5,62	8,66	12,77	75,0
50,0	0,23	0,32	0,46	0,69	1,00	1,48	2,14	2,96	4,49	6,59	9,90	14,28	50,0
25,0	0,34	0,46	0,64	0,93	1,32	1,90	2,68	3,64	5,36	7,69	11,26	15,89	25,0
10,0	0,46	0,62	0,85	1,21	1,68	2,36	3,26	4,34	6,26	8,78	12,58	17,44	10,0
5,0	0,56	0,74	1,00	1,40	1,93	2,68	3,65	4,81	6,84	9,48	13,43	18,40	5,0
1,0	0,79	1,03	1,35	1,84	2,48	3,36	4,48	5,79	8,04	10,91	15,11	20,31	1,0
Acceptable quality level (tightened inspection)													
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		

Table V-P — Code letter P

P

Chart V-P — Operating characteristic curves for single sampling plans

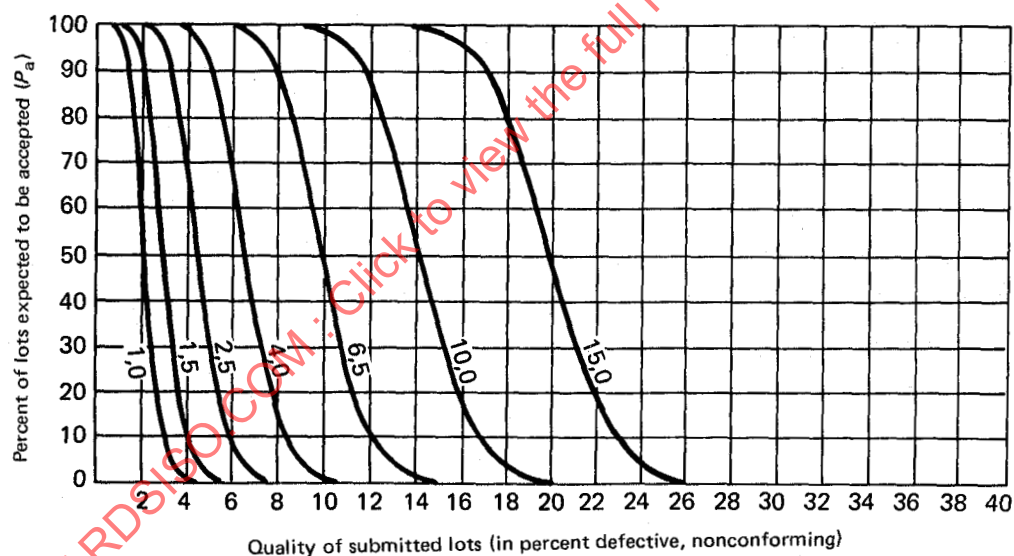
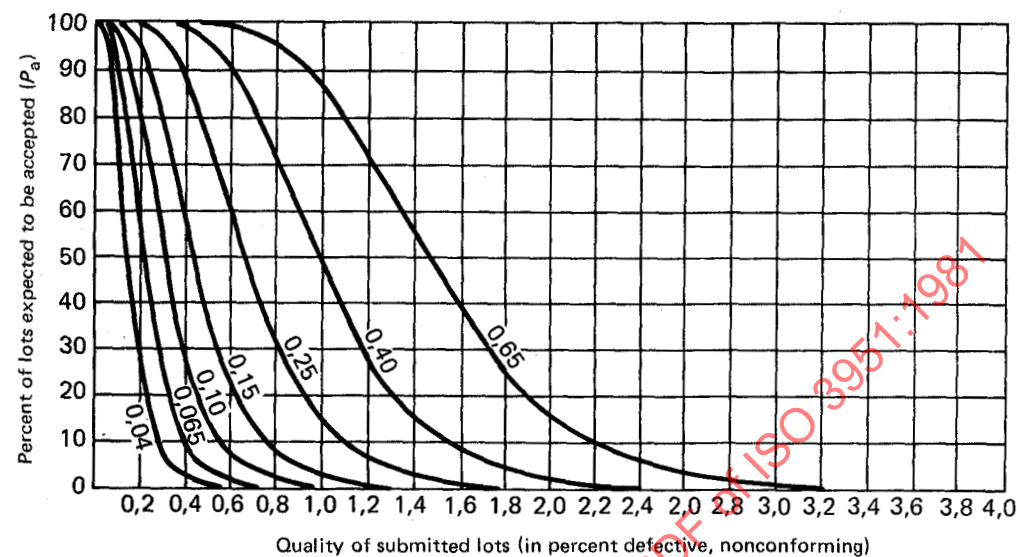


Table V-P-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,064	0,101	0,150	0,254	0,405	0,65	0,99	1,52	2,52	4,00	6,52	10,10	99,0
95,0	0,094	0,143	0,210	0,344	0,534	0,84	1,25	1,86	3,00	4,66	7,40	11,22	95,0
90,0	0,114	0,173	0,249	0,402	0,617	0,95	1,40	2,07	3,29	5,04	7,91	11,85	90,0
75,0	0,157	0,233	0,330	0,519	0,779	1,18	1,70	2,46	3,82	5,73	8,80	12,96	75,0
50,0	0,222	0,321	0,445	0,683	1,000	1,48	2,08	2,96	4,48	6,58	9,88	14,27	50,0
25,0	0,309	0,437	0,594	0,889	1,272	1,83	2,54	3,53	5,23	7,52	11,05	15,66	25,0
10,0	0,411	0,571	0,763	1,116	1,567	2,22	3,02	4,12	5,98	8,45	12,19	16,98	10,0
5,0	0,485	0,666	0,882	1,275	1,770	2,47	3,33	4,51	5,47	9,04	12,90	17,80	5,0
1,0	0,657	0,884	1,150	1,621	2,206	3,02	4,00	5,32	7,46	10,23	14,31	19,41	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Diagram A — Sample sizes of standard sampling plans for specified qualities at 95 % and 10 % probabilities of acceptance

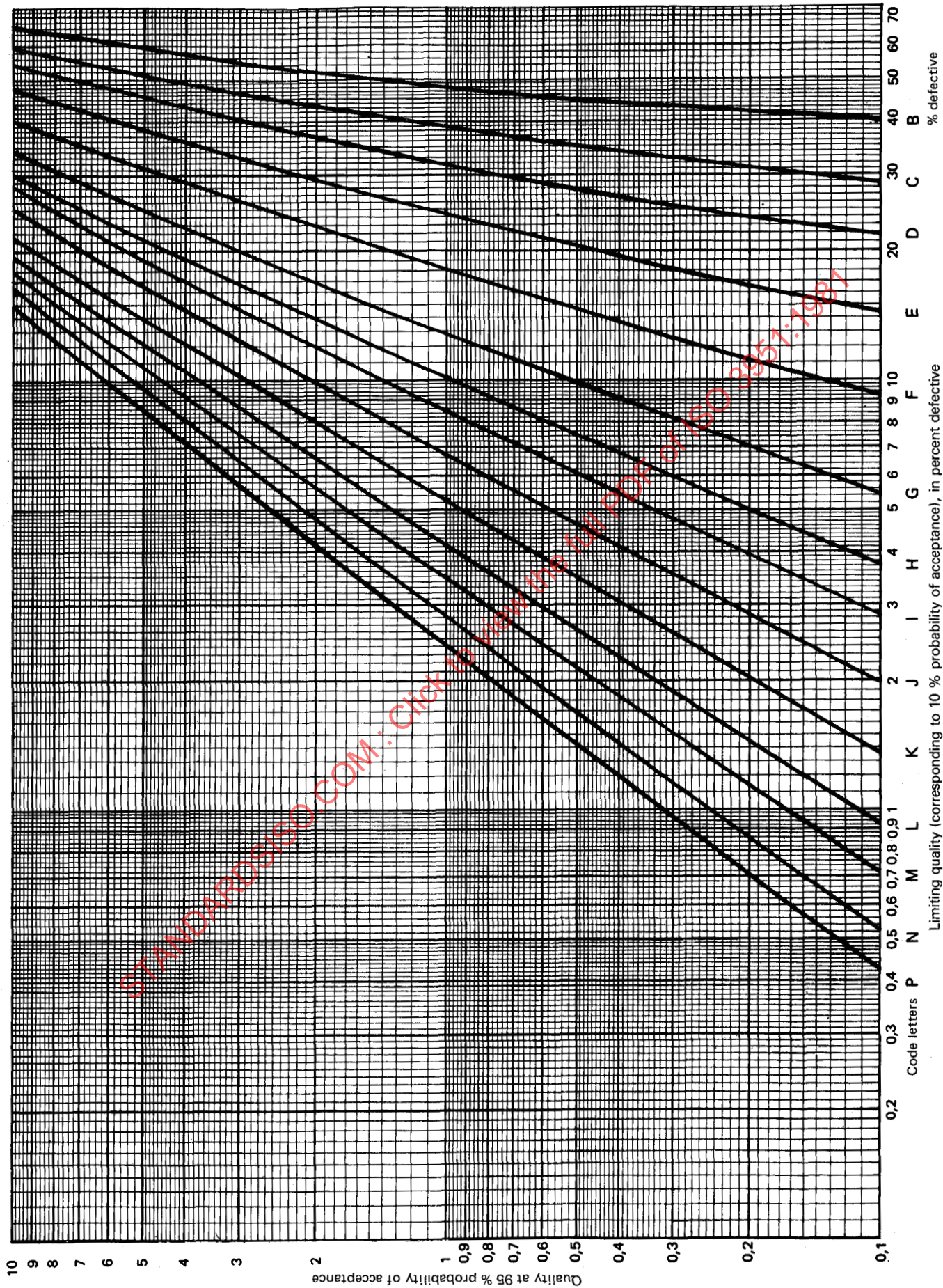
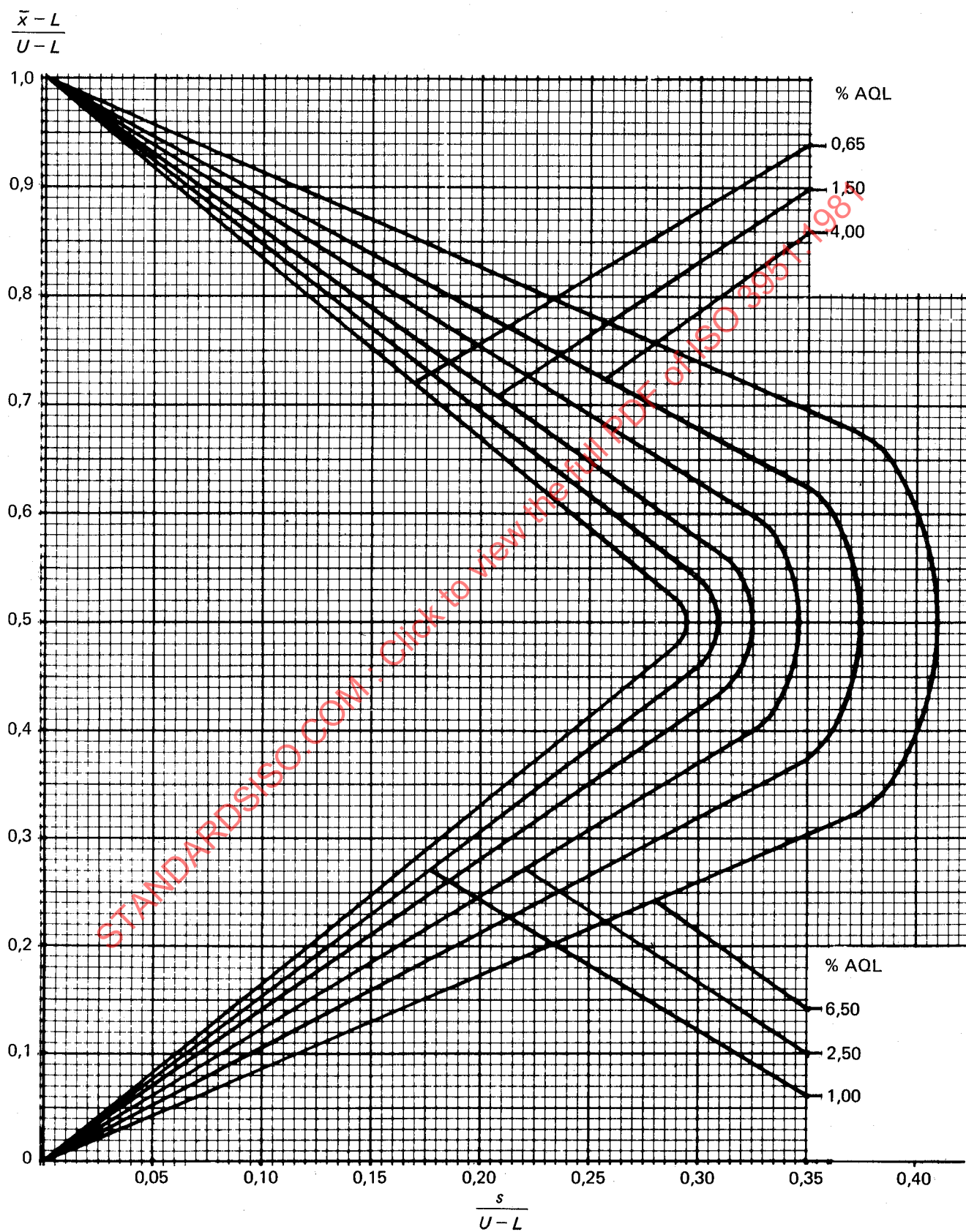


Diagram s-D — Acceptance curves for combined double specification limit :
 "s" method — Code letter D (sample size 5)

D

E

Diagram s-E — Acceptance curves for combined double specification limits :
"s" method — Code letter E (sample size 7)

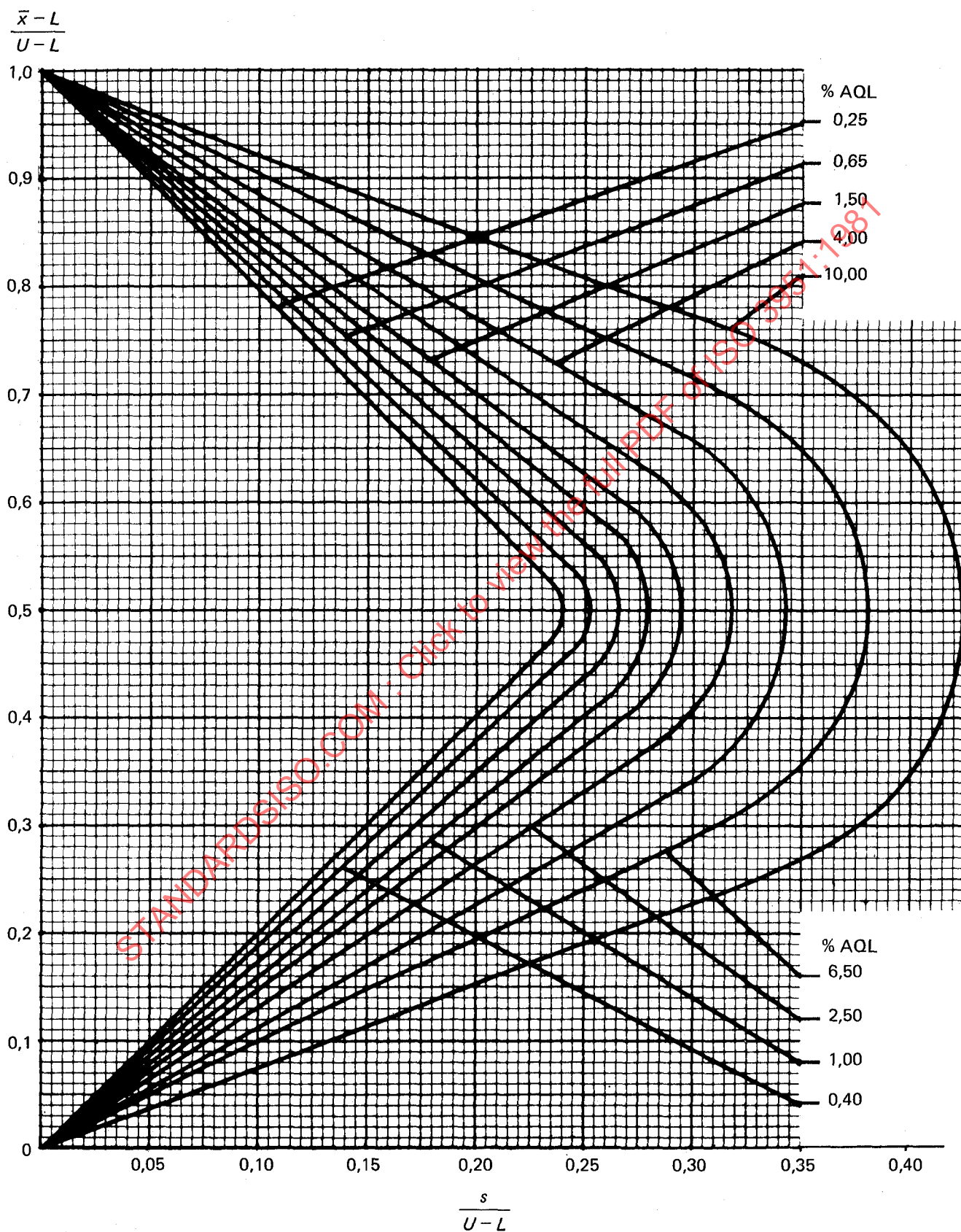
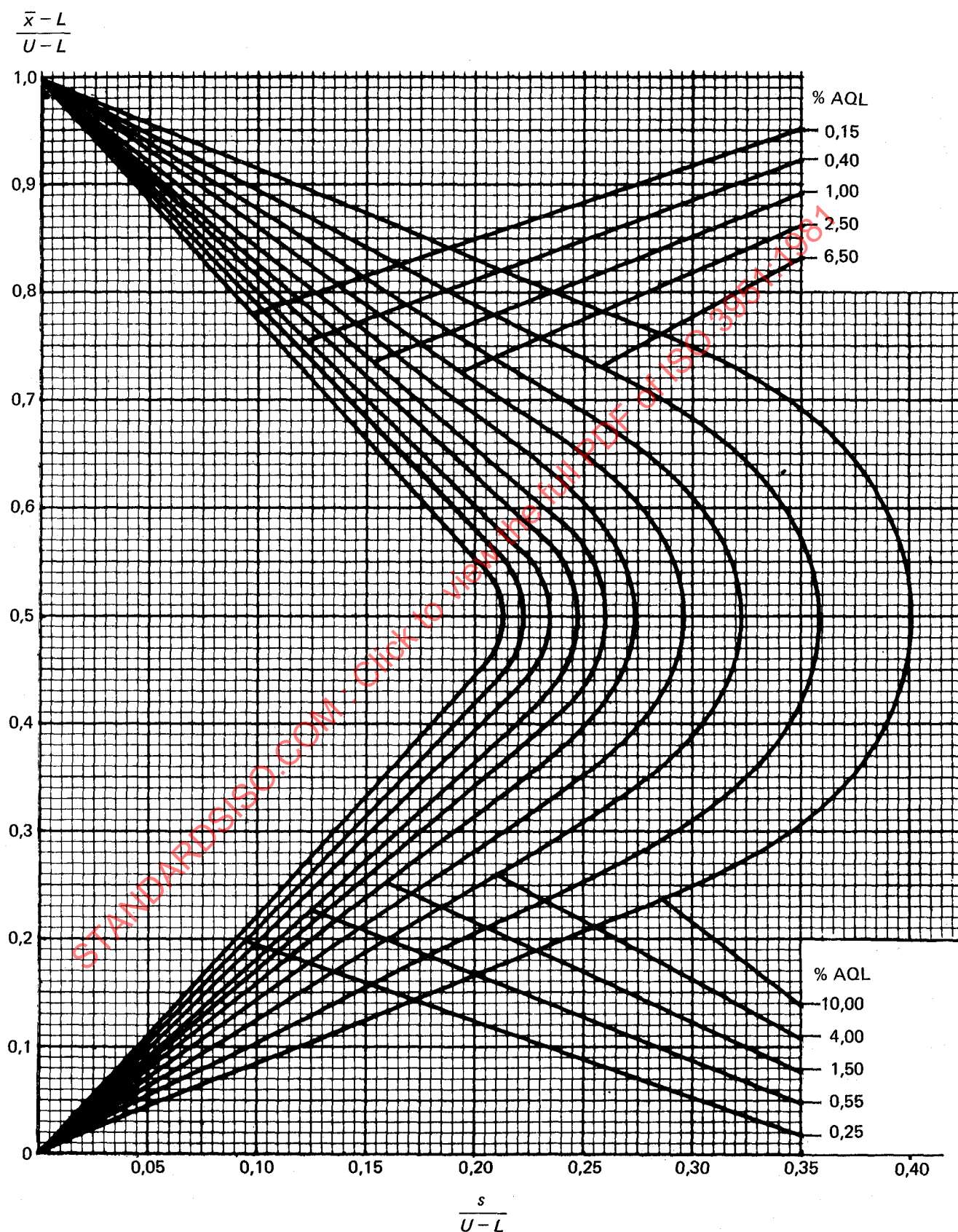


Diagram s-F — Acceptance curves for combined double specification limits :
 "s" method — Code letter F (sample size 10)

F

G

Diagram s-G — Acceptance curves for combined double specification limits :
"s" method — Code letter G (sample size 15)

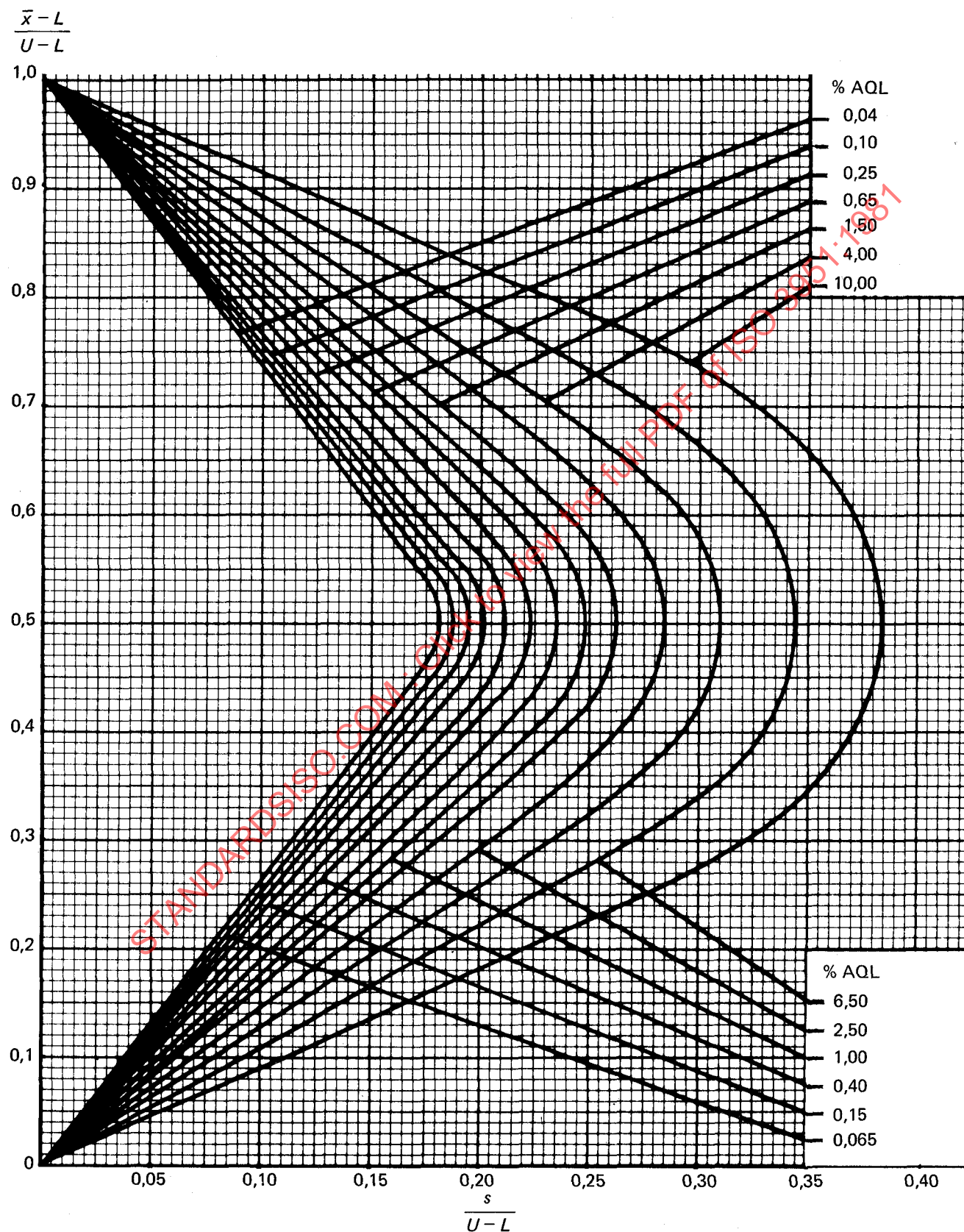


Diagram s-H — Acceptance curves for combined double specification limits :
 "s" method — Code letter H (sample size 20)

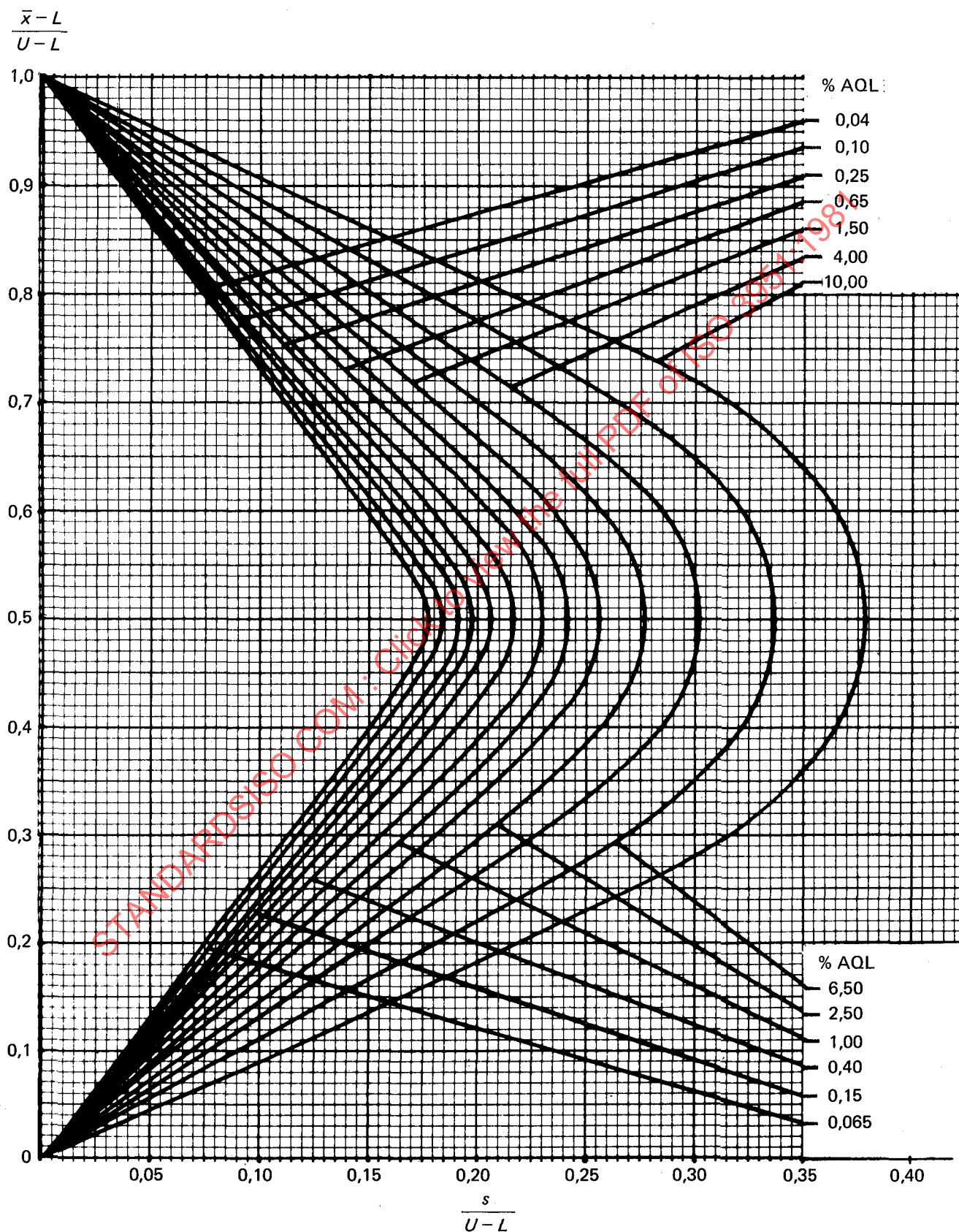
H

Diagram s-l — Acceptance curves for combined double specification limits :
"s" method — Code letter I (sample size 25)

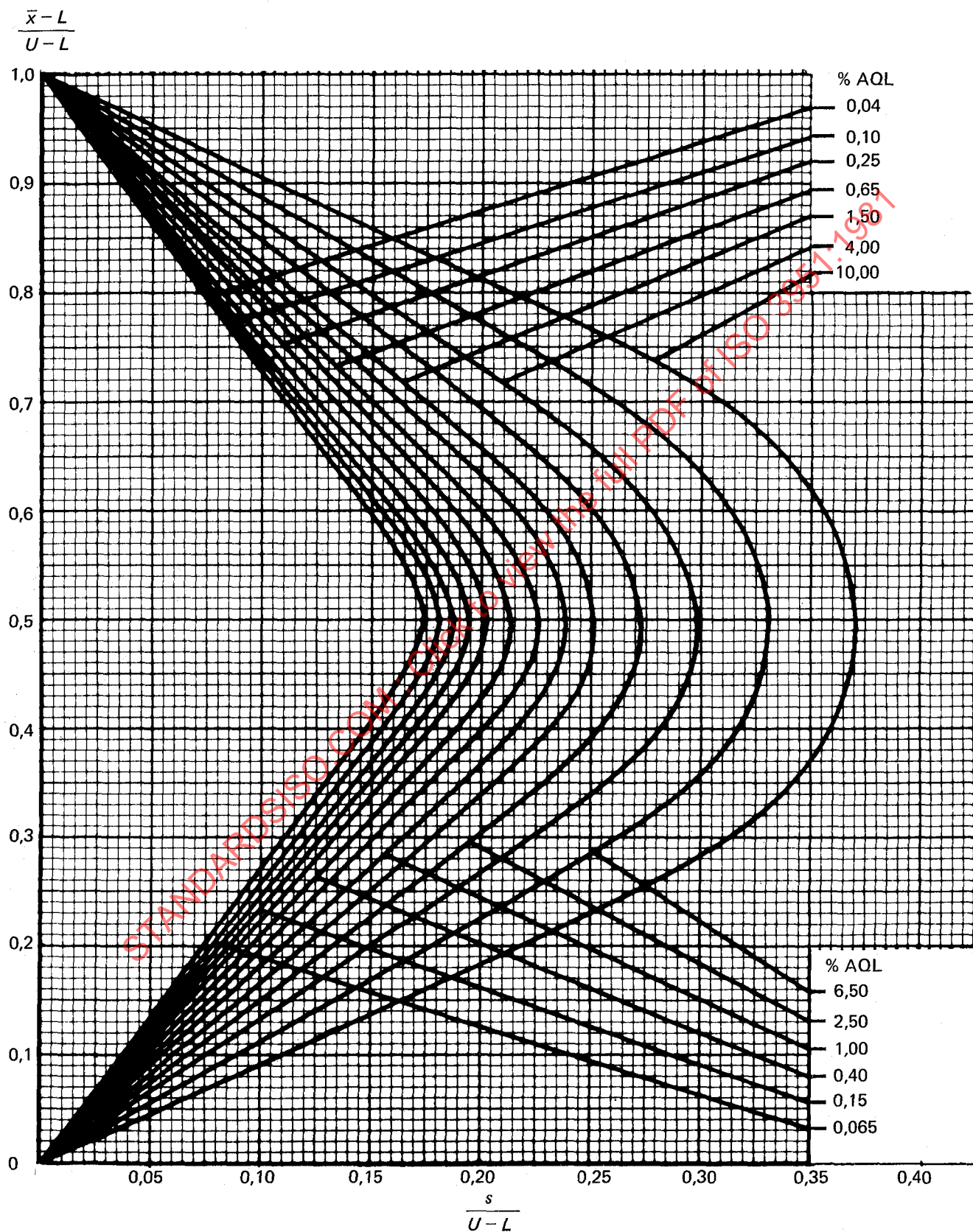
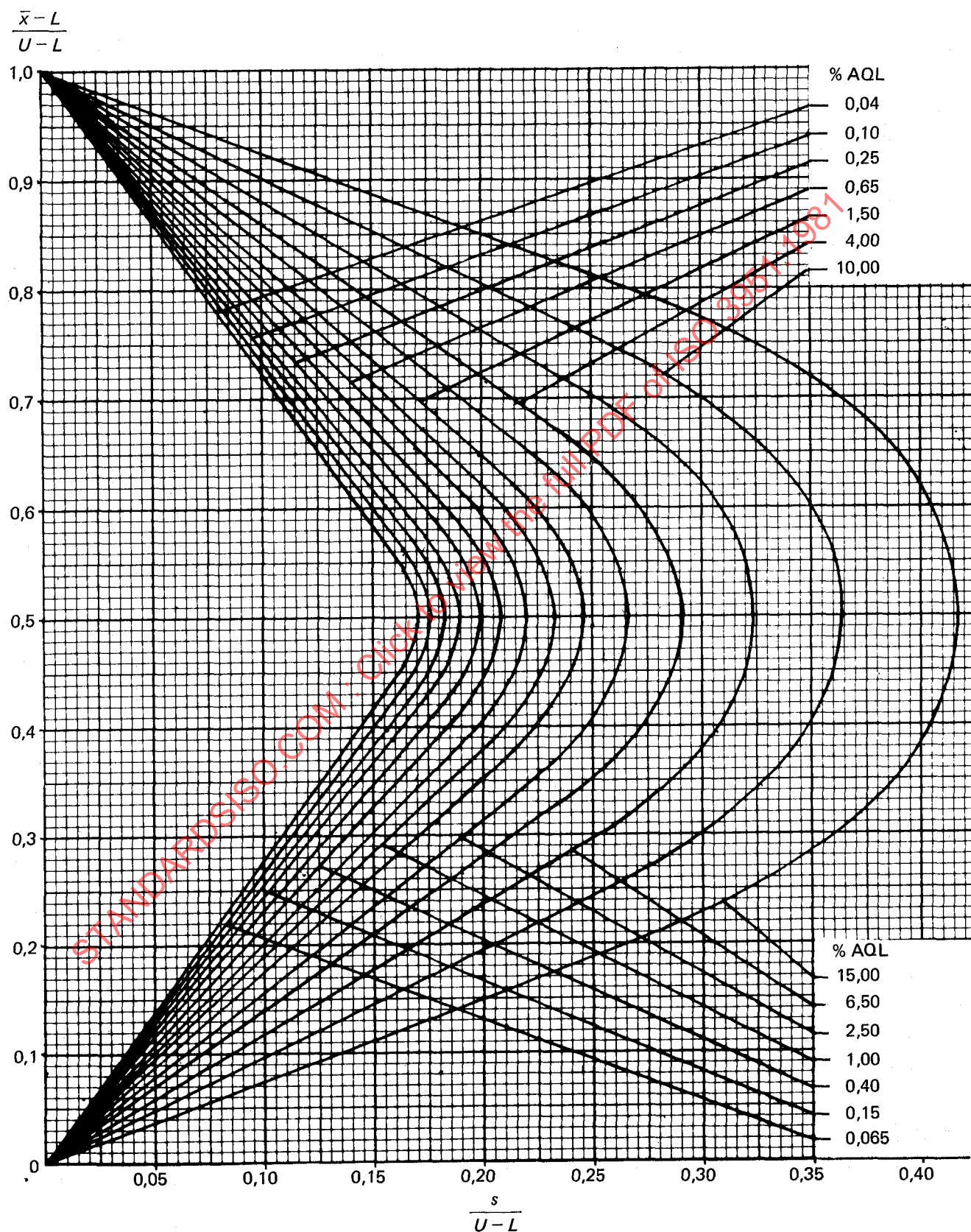


Diagram s-J — Acceptance curves for combined double specification limits :
 "s" method — Code letter J (sample size 35)



K

Diagram s-K — Acceptance curves for combined double specification limits :
"s" method — Code letter K (sample size 50)

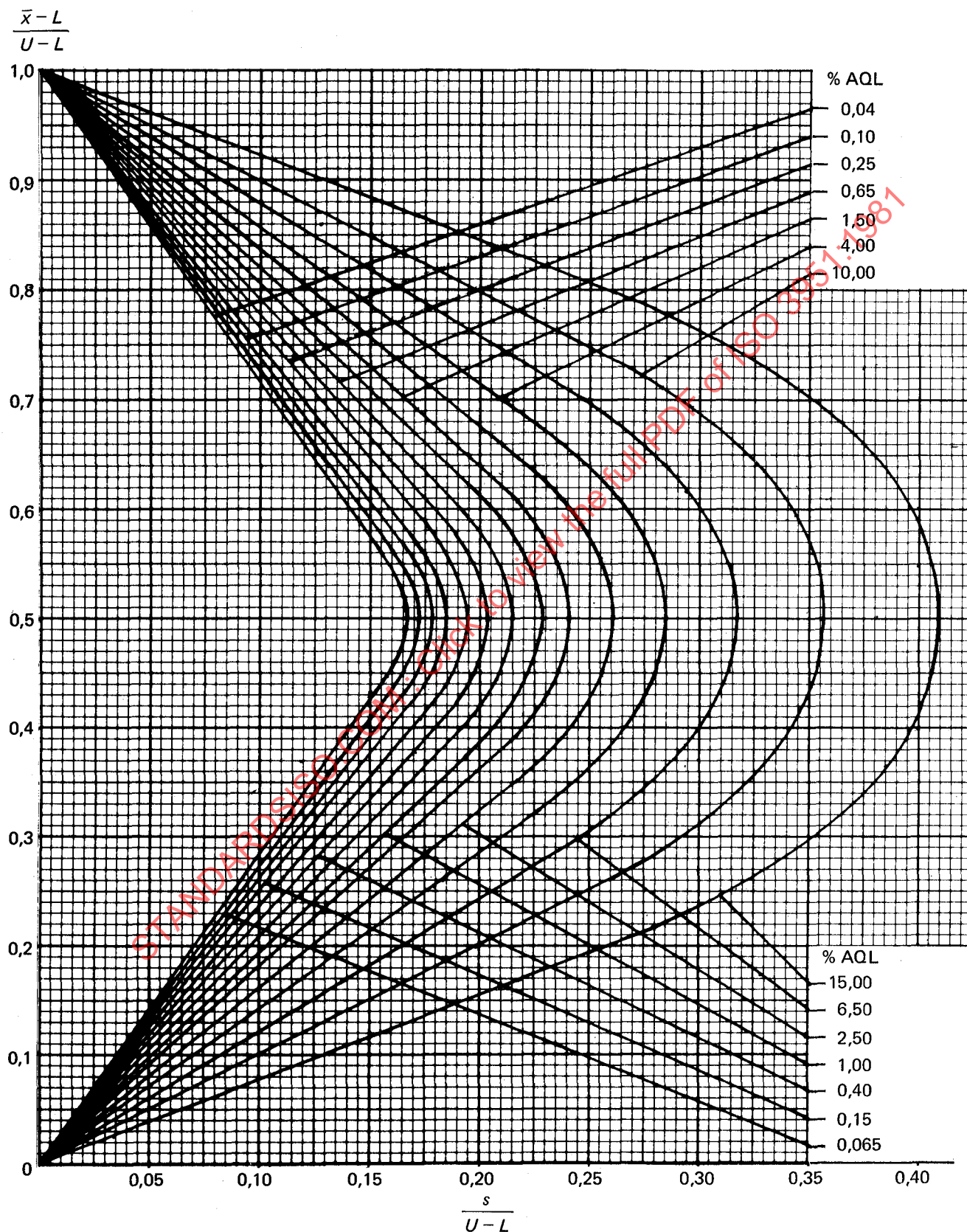
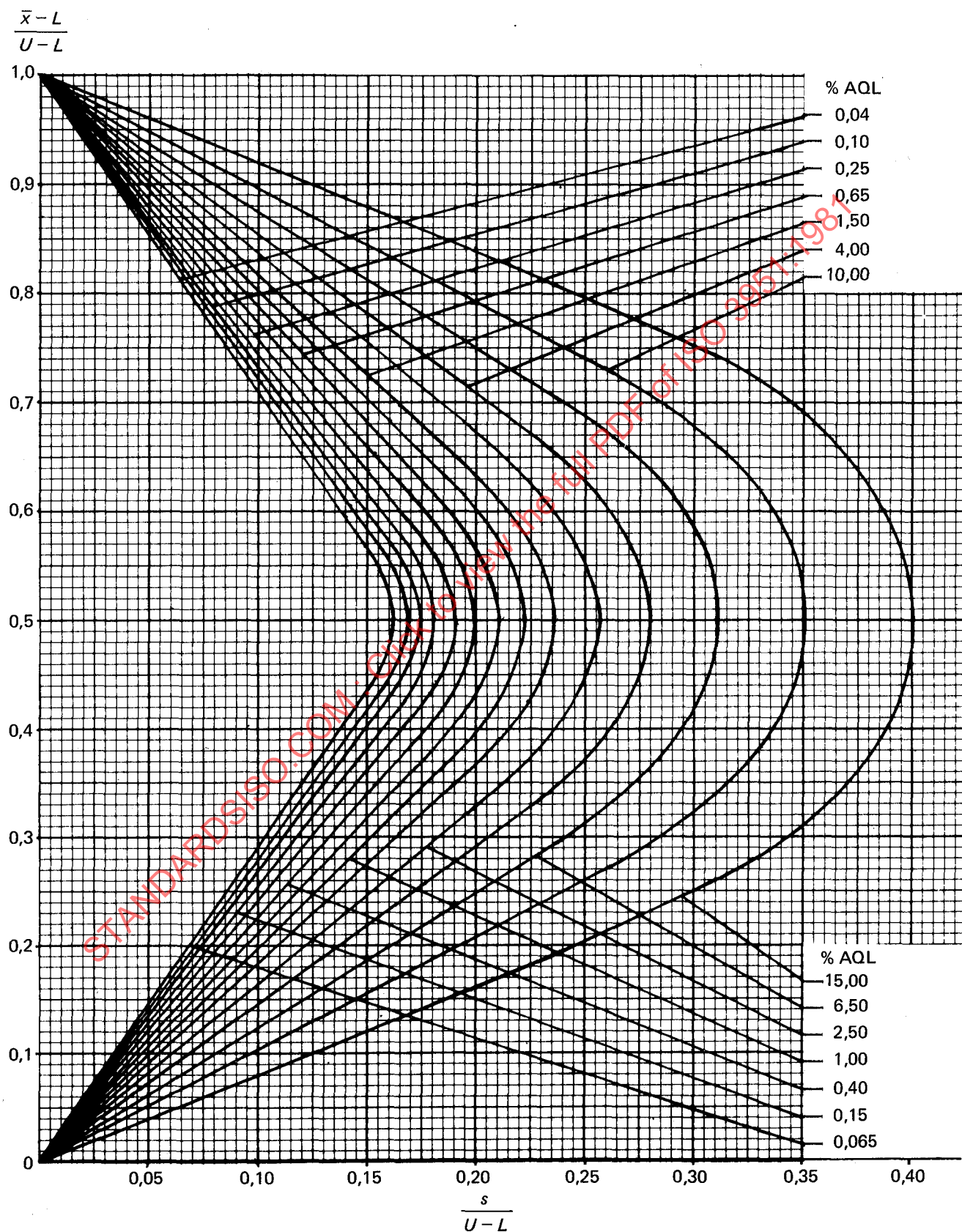


Diagram s-L — Acceptance curves for combined double specification limits :
 "s" method — Code letter L (sample size 75)

L



M

Diagram s-M — Acceptance curves for combined double specification limits :
"s" method — Code letter M (sample size 100)

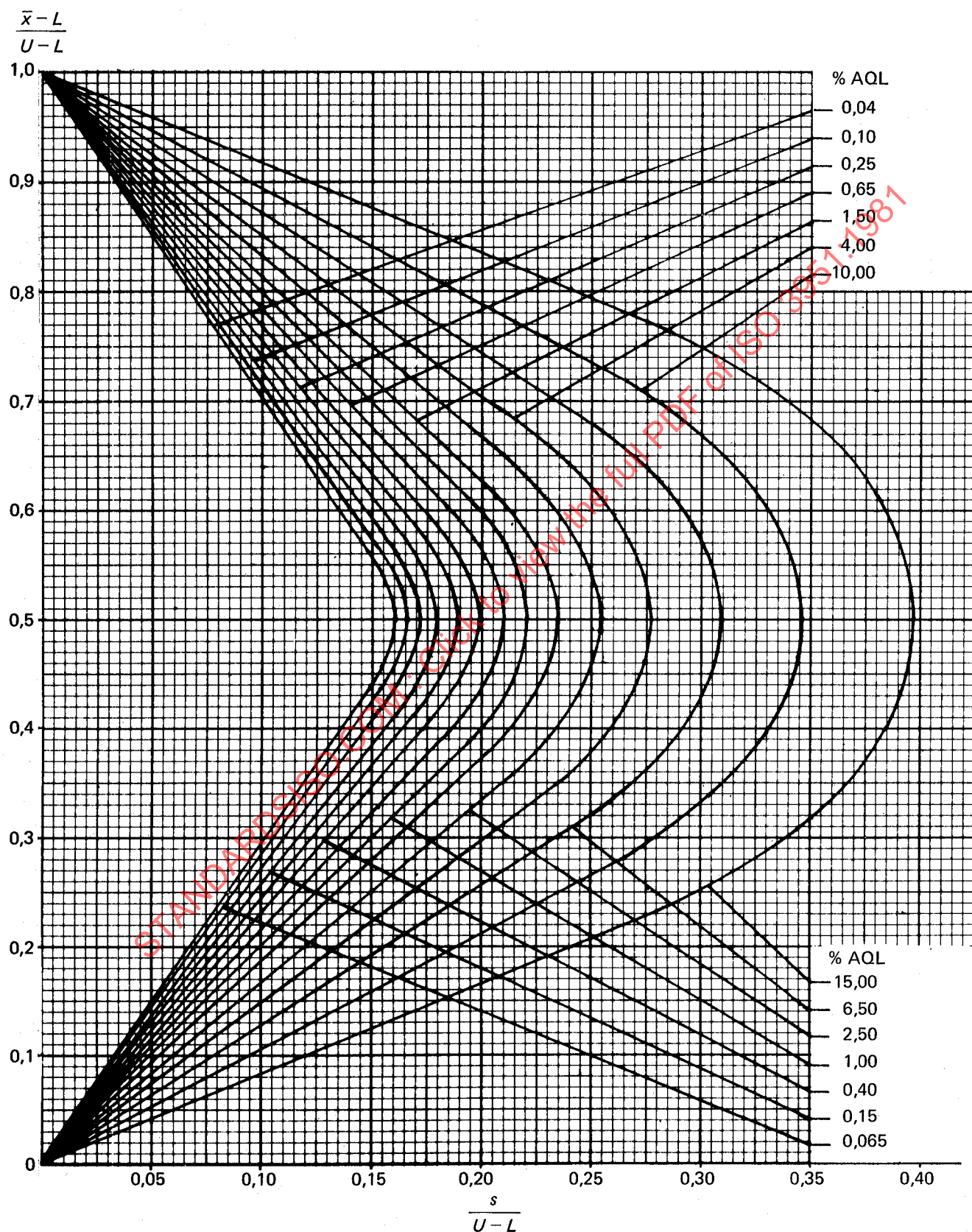
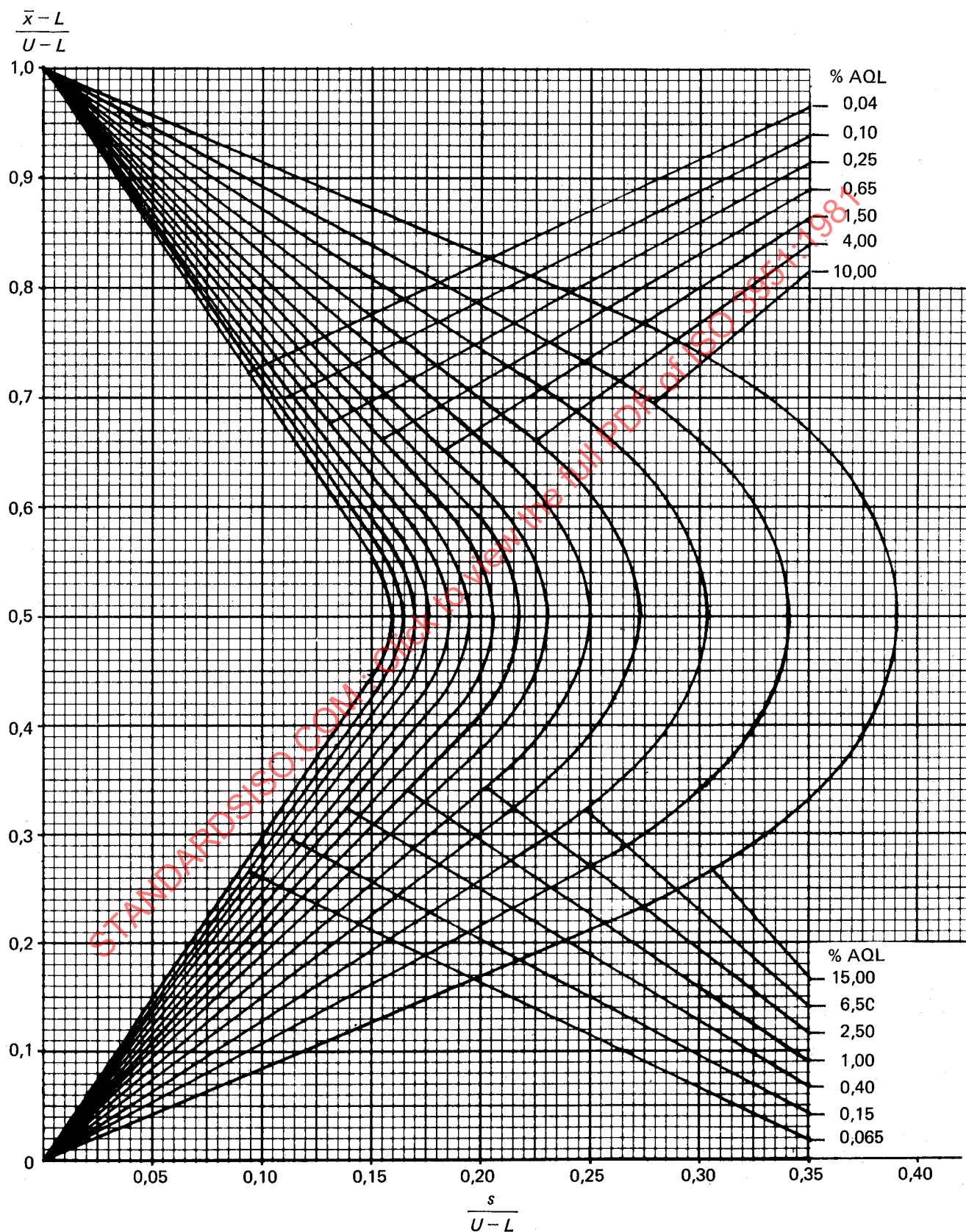


Diagram s-N — Acceptance curves for combined double specification limits :
 "s" method — Code letter N (sample size 150)

N



P

Diagram s-P — Acceptance curves for combined double specification limits :
"s" method — Code letter P (sample size 200)

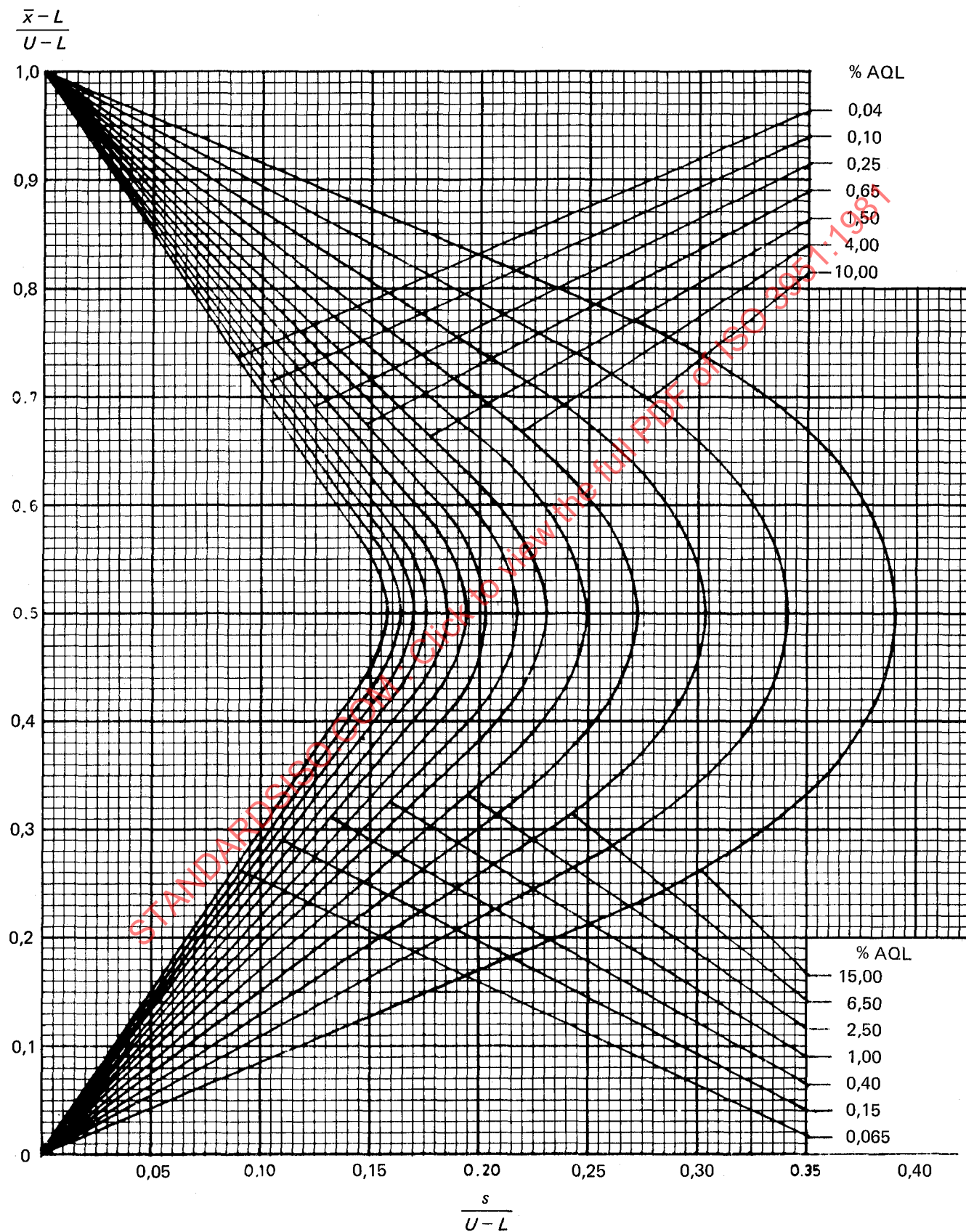
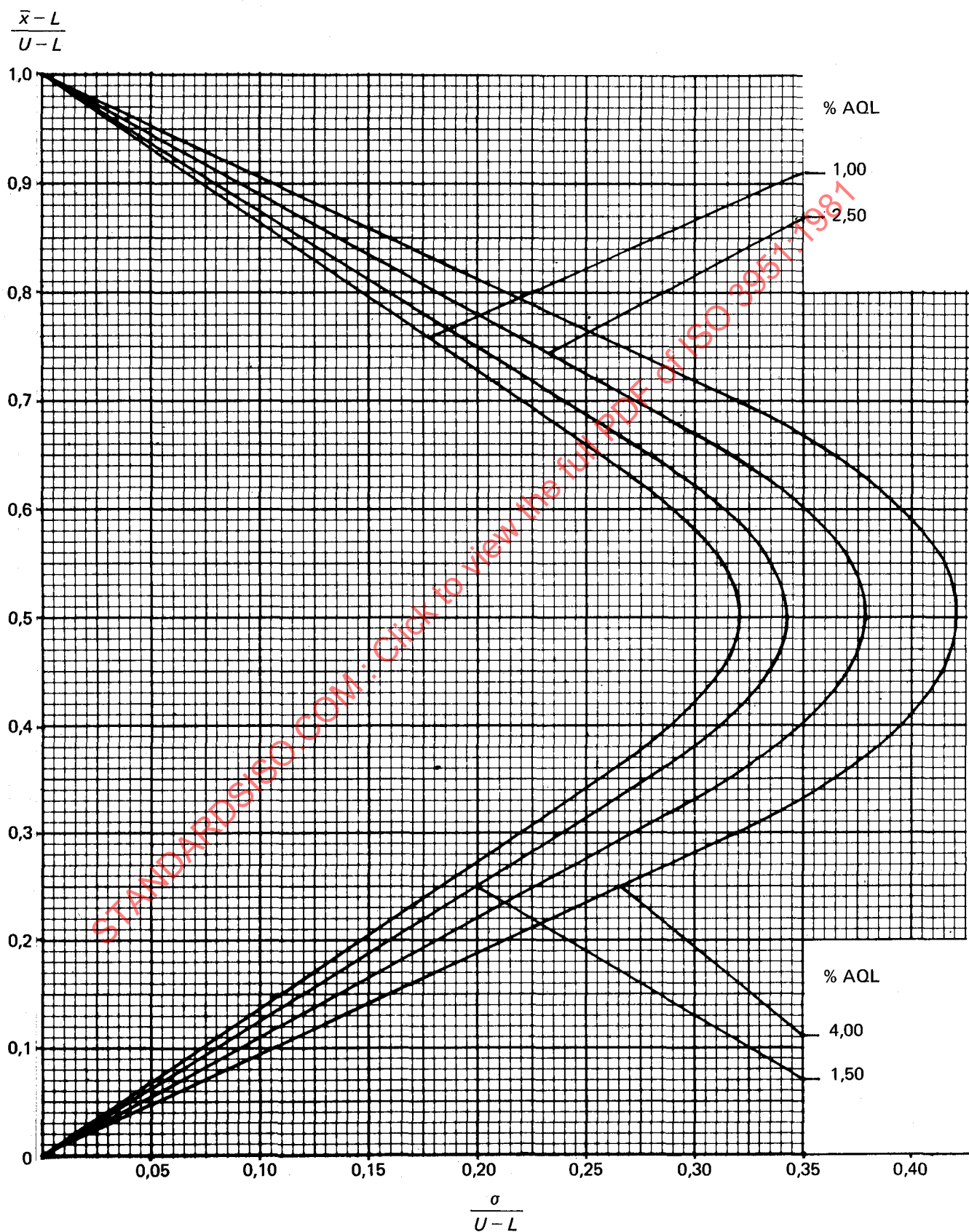


Diagram σ -C — Acceptance curves for combined double specification limits :
 "σ" method — Code letter C

C



D

Diagram σ -D — Acceptance curves for combined double specification limits :
"σ" method — Code letter D

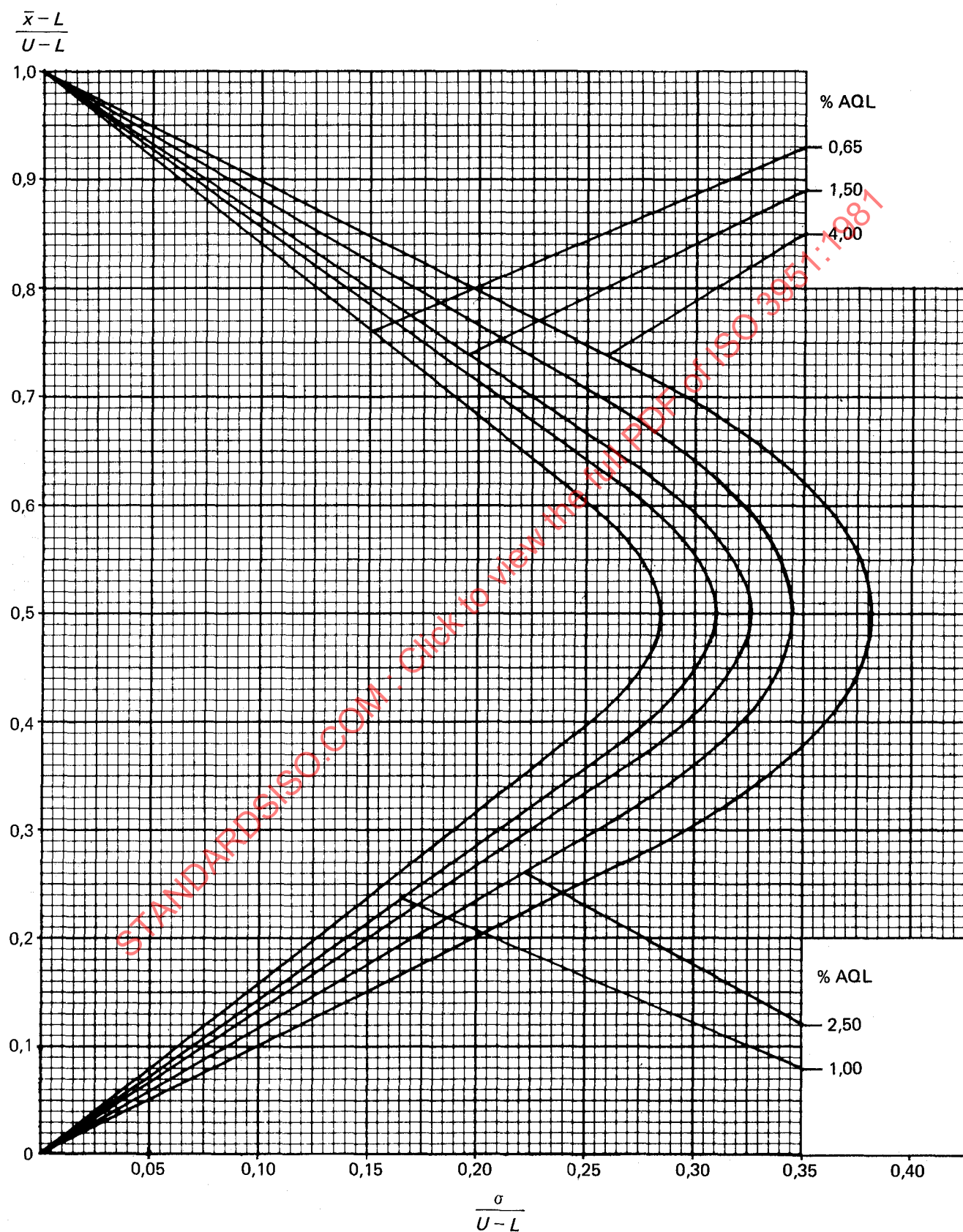
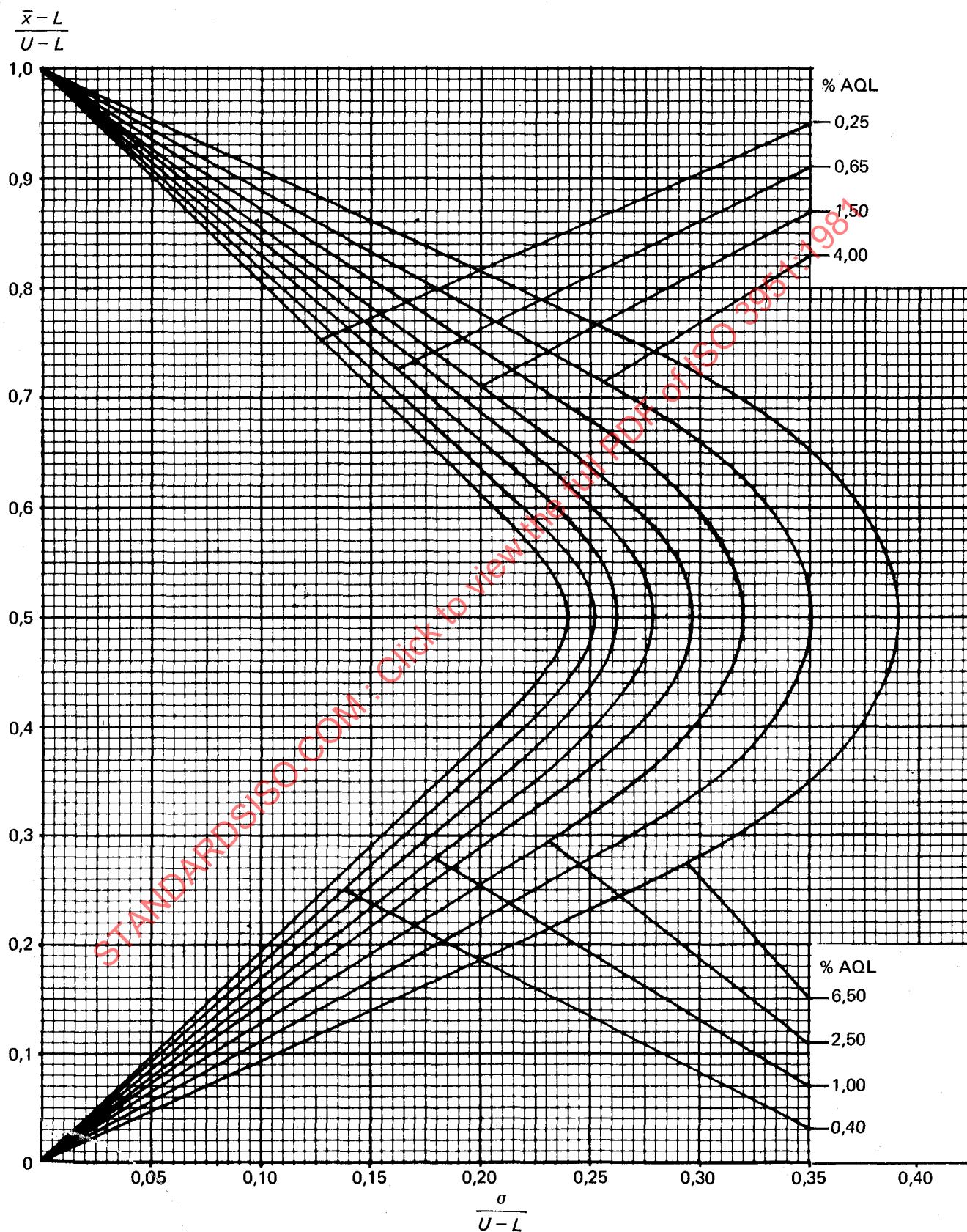


Diagram σ -E — Acceptance curves for combined double specification limits :
 "σ" method — Code letter E

E



F

Diagram σ -F — Acceptance curves for combined double specification limits :
" σ " method — Code letter F

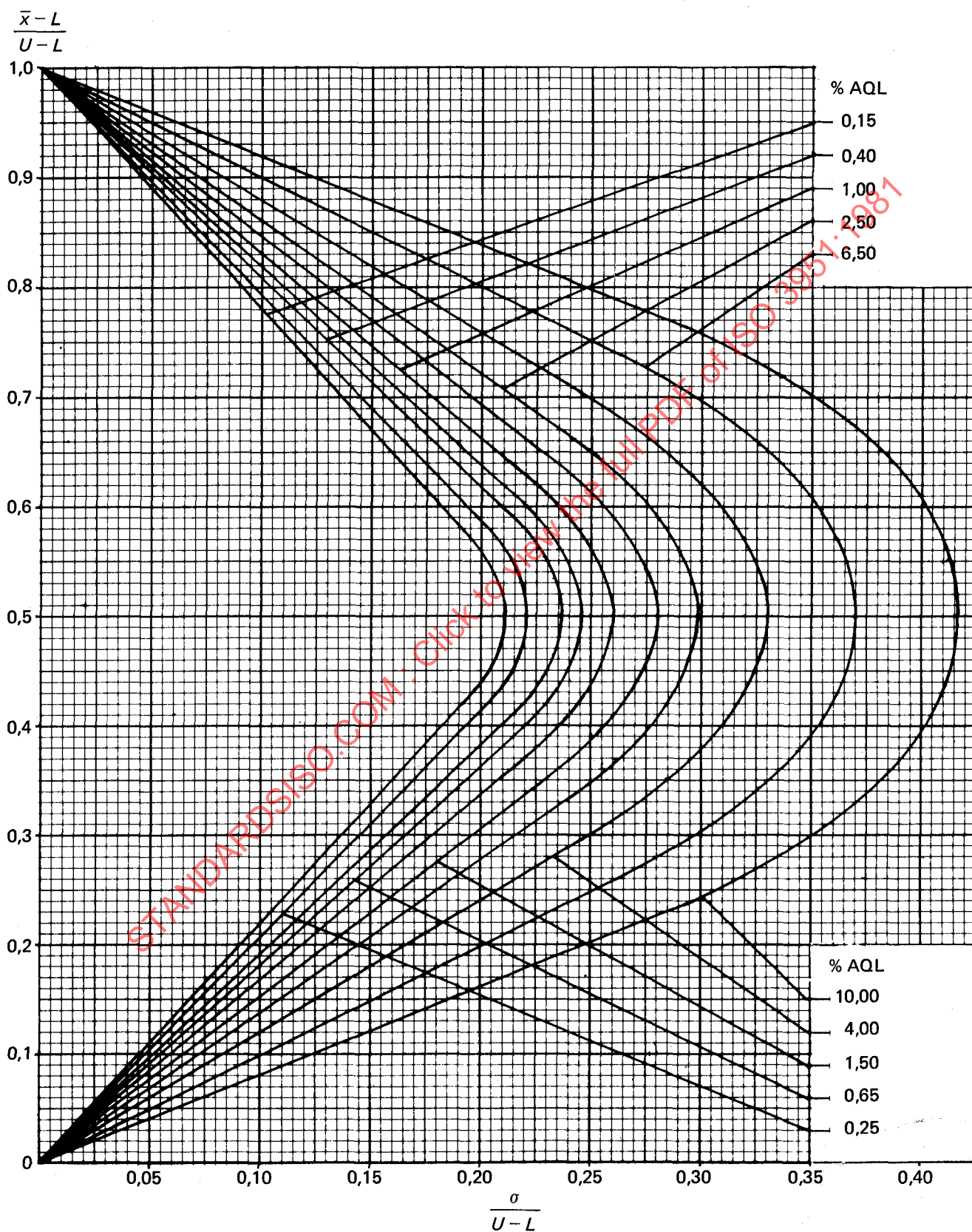
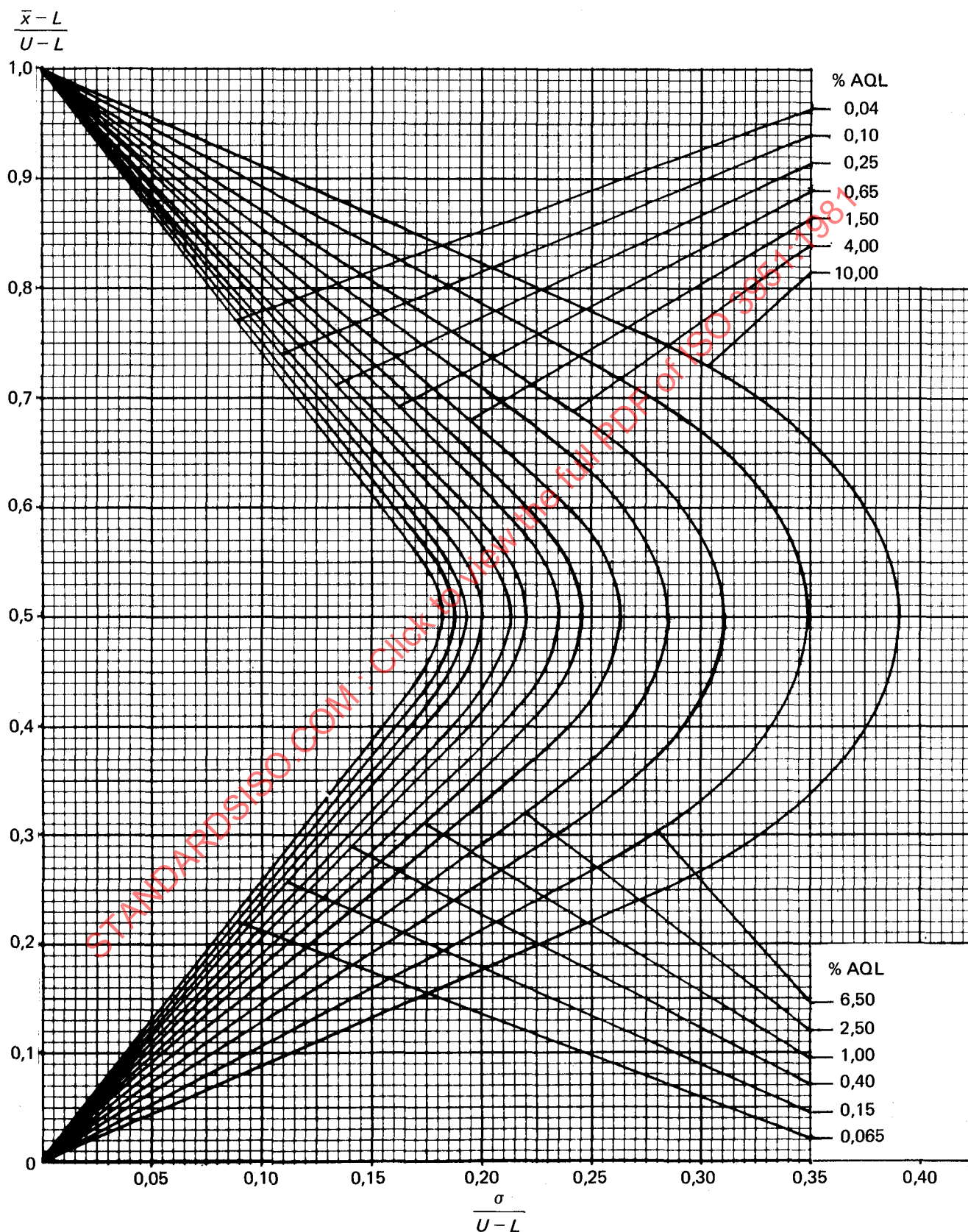


Diagram σ -G — Acceptance curves for combined double specification limits :
 " σ " method — Code letter G

G



H

Diagram σ -H — Acceptance curves for combined double specification limits :
"σ" method — Code letter H

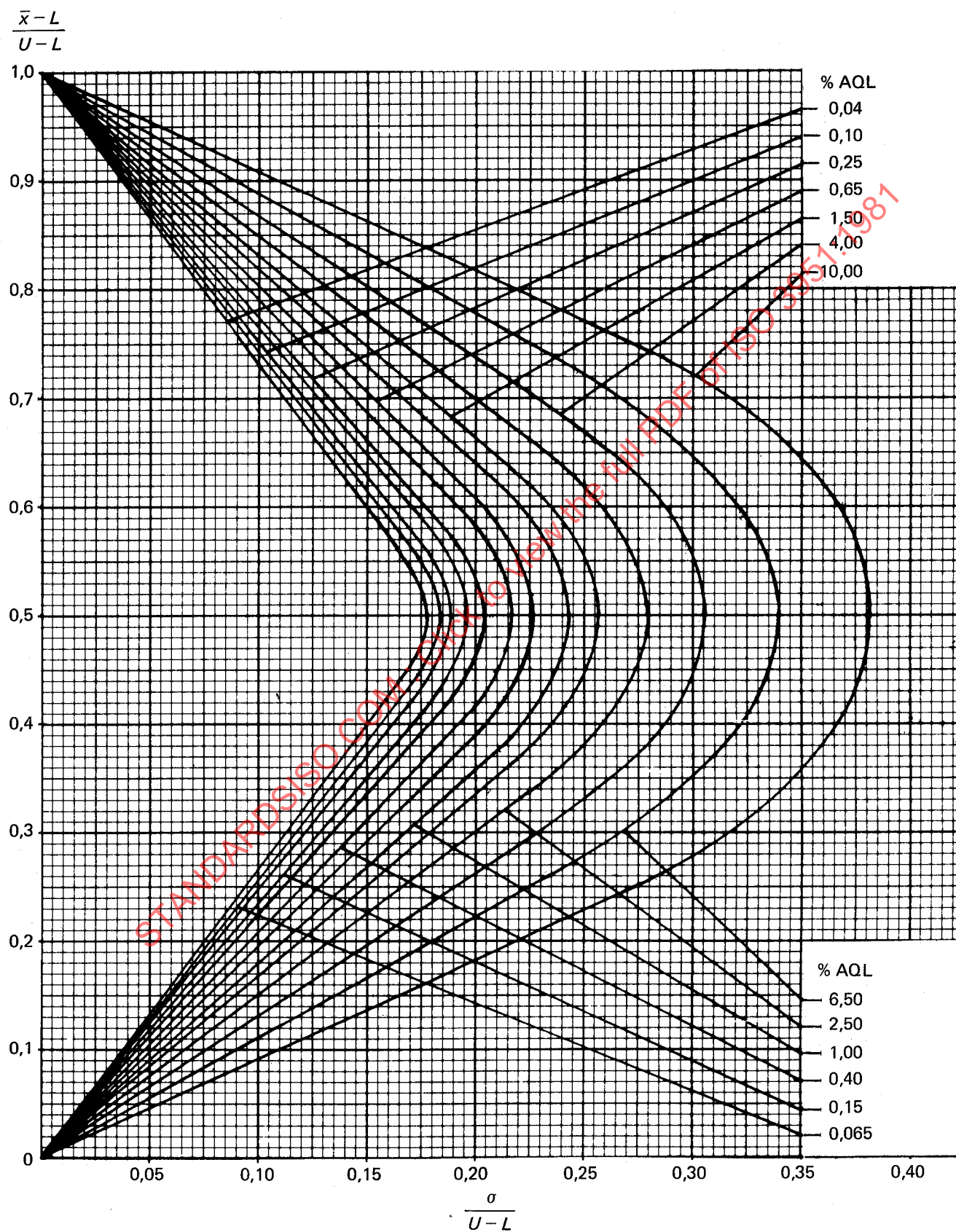
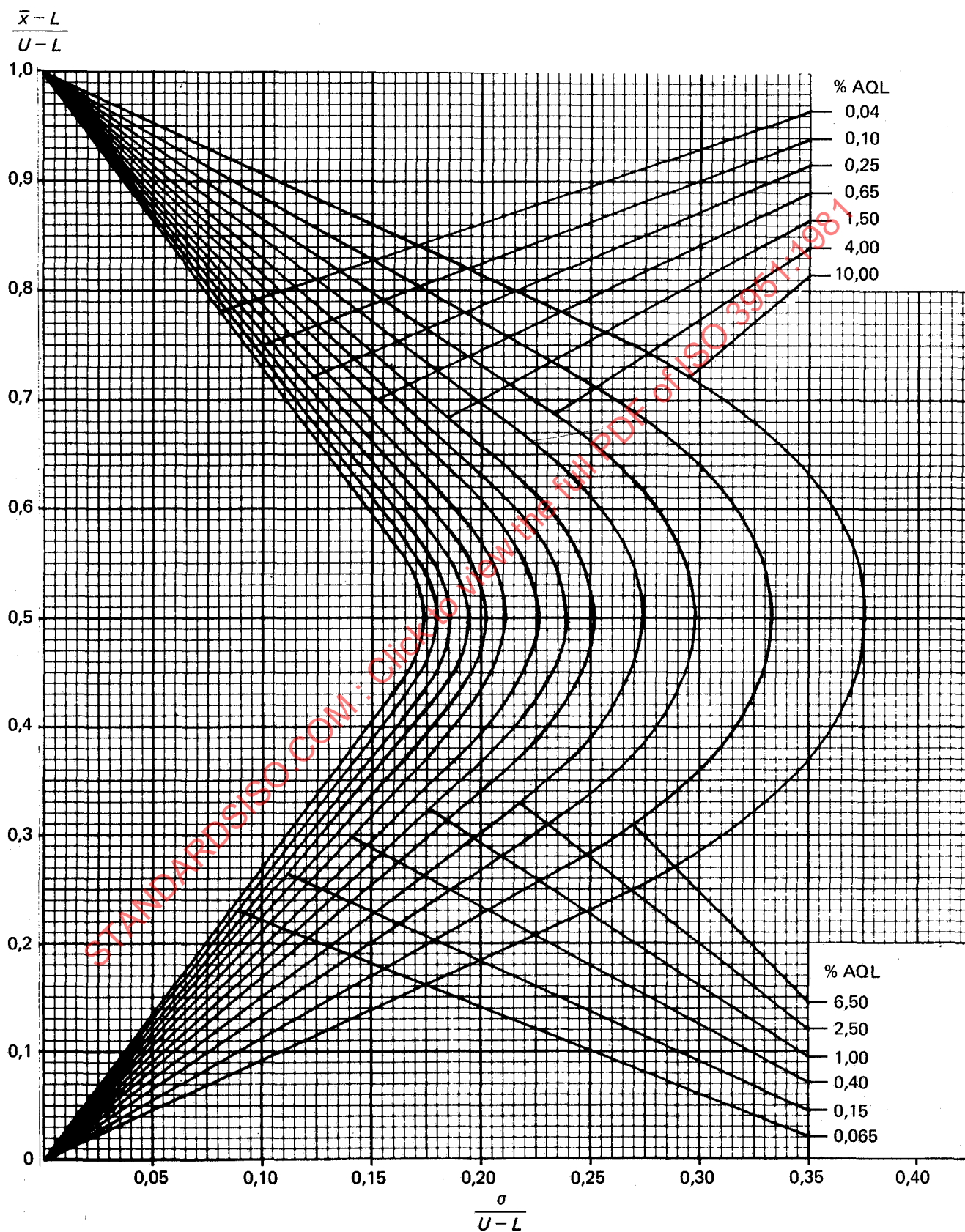


Diagram σ -I — Acceptance curves for combined double specification limits :
 "σ" method — Code letter I



J

Diagram σ -J — Acceptance curves for combined double specification limits :
"σ" method — Code letter J

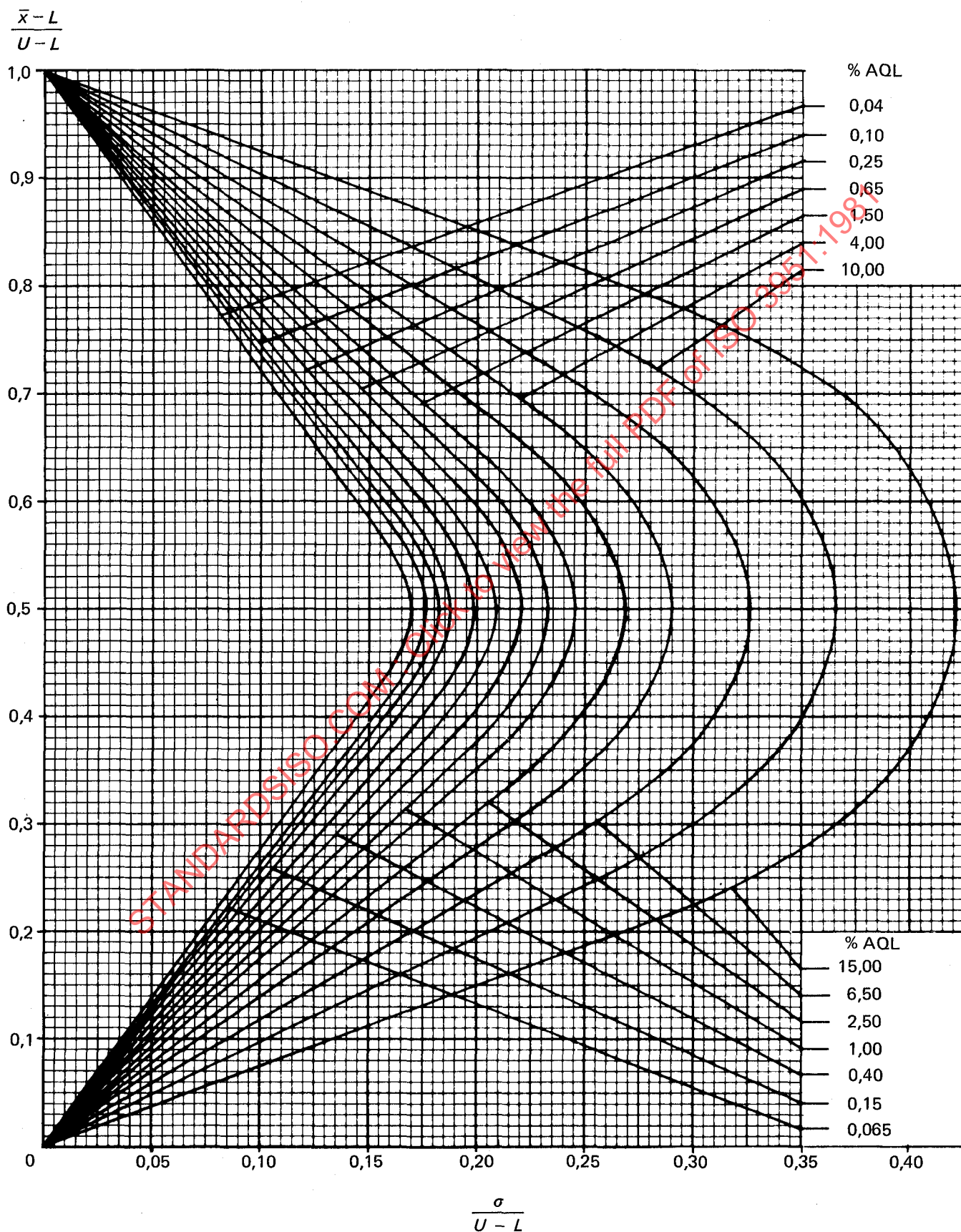
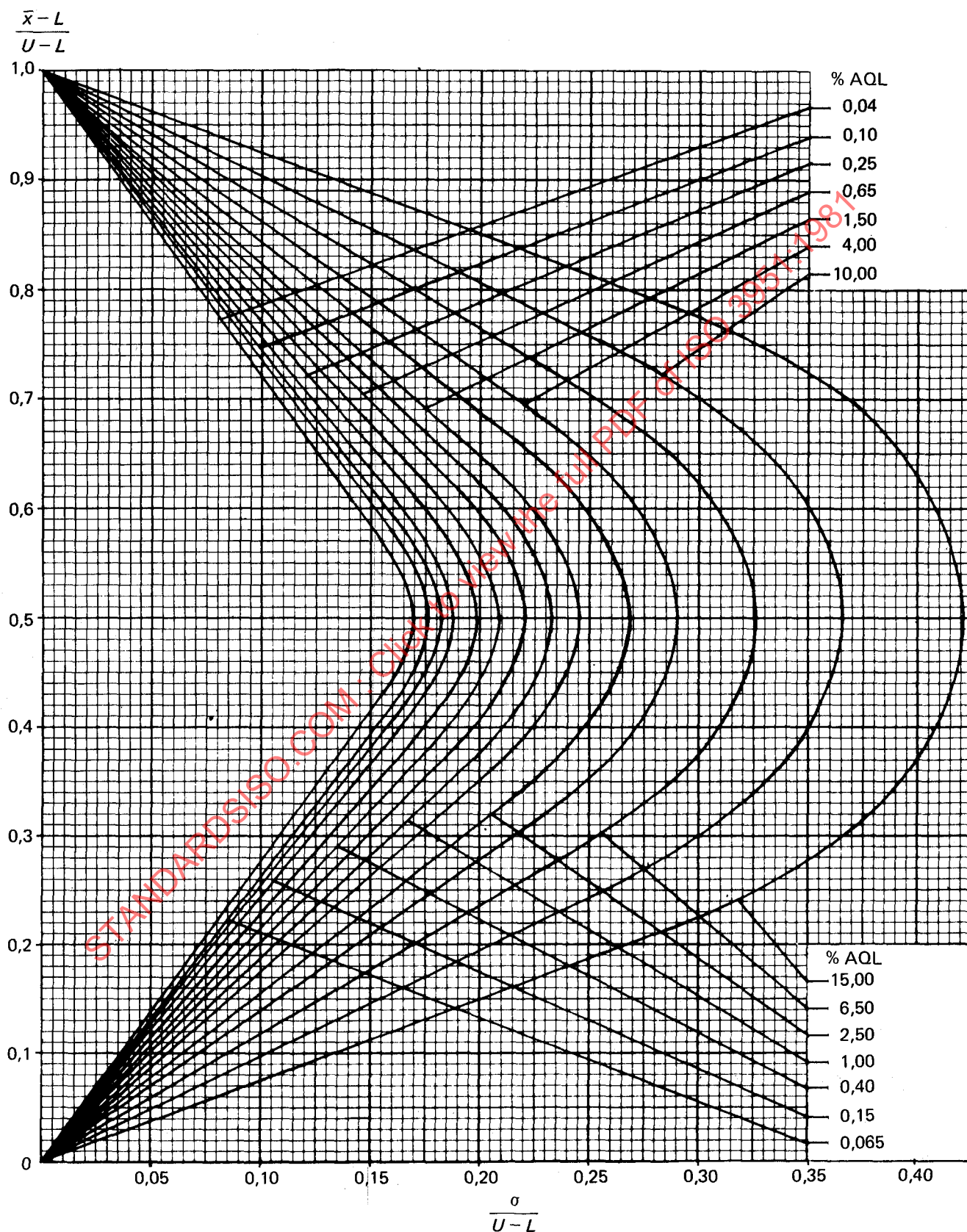


Diagram σ -K — Acceptance curves for combined double specification limits :
 "σ" method — Code letter K

K

L

Diagram σ -L — Acceptance curves for combined double specification limits :
" σ " method — Code letter L

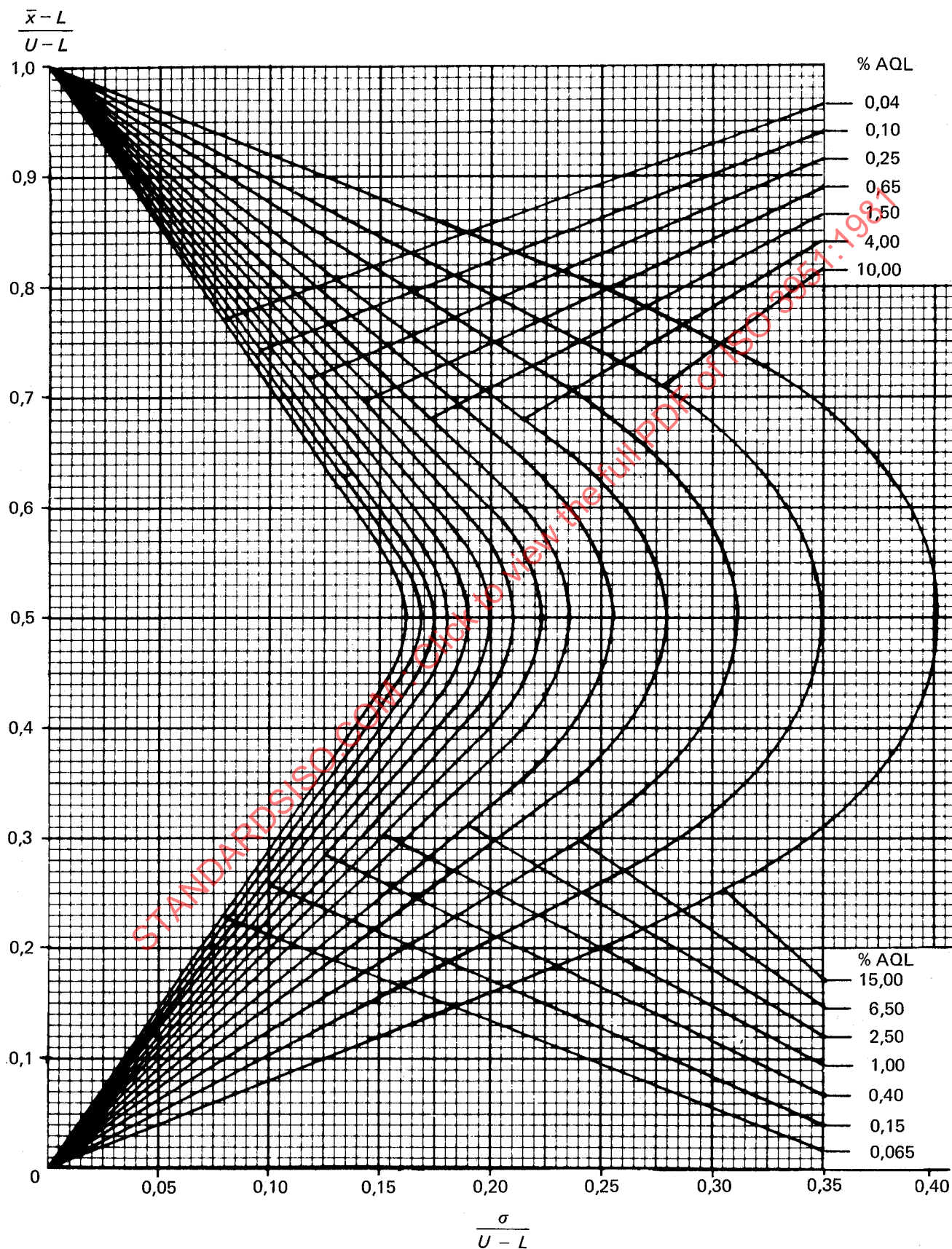
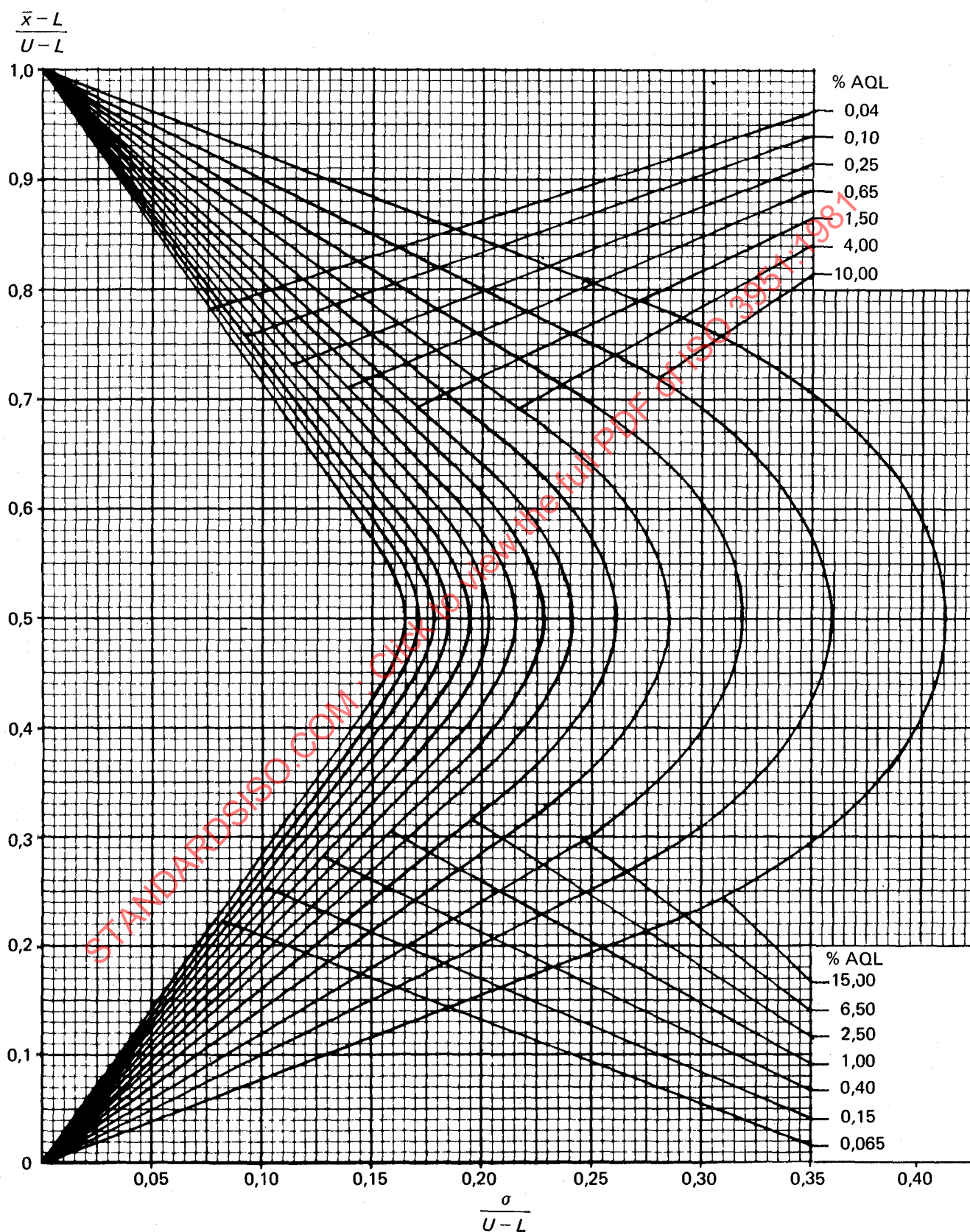


Diagram σ -M — Acceptance curves for combined double specification limits :
 "σ" method — Code letter M

M



N

Diagram σ -N — Acceptance curves for combined double specification limits :
"σ" method — Code letter N

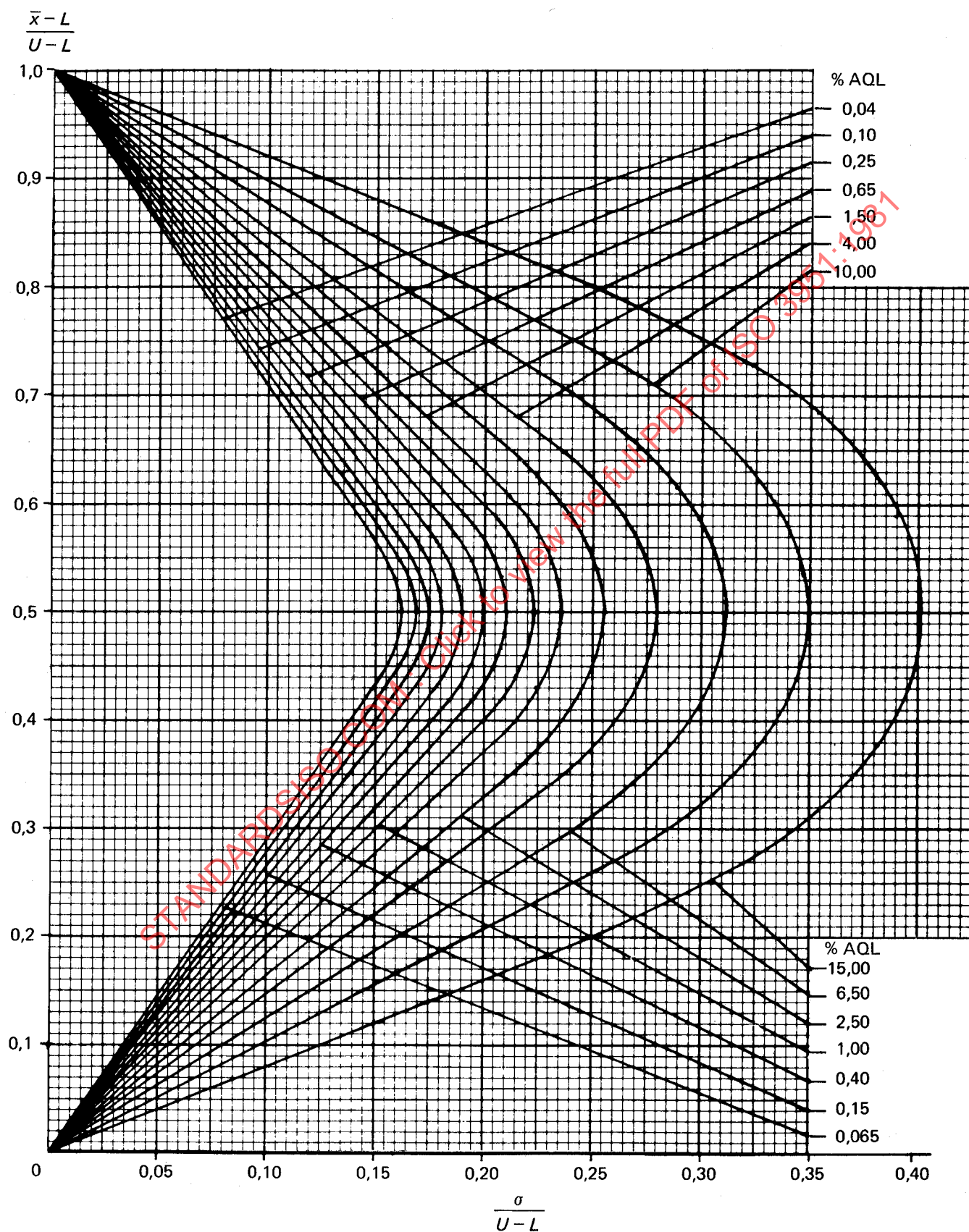
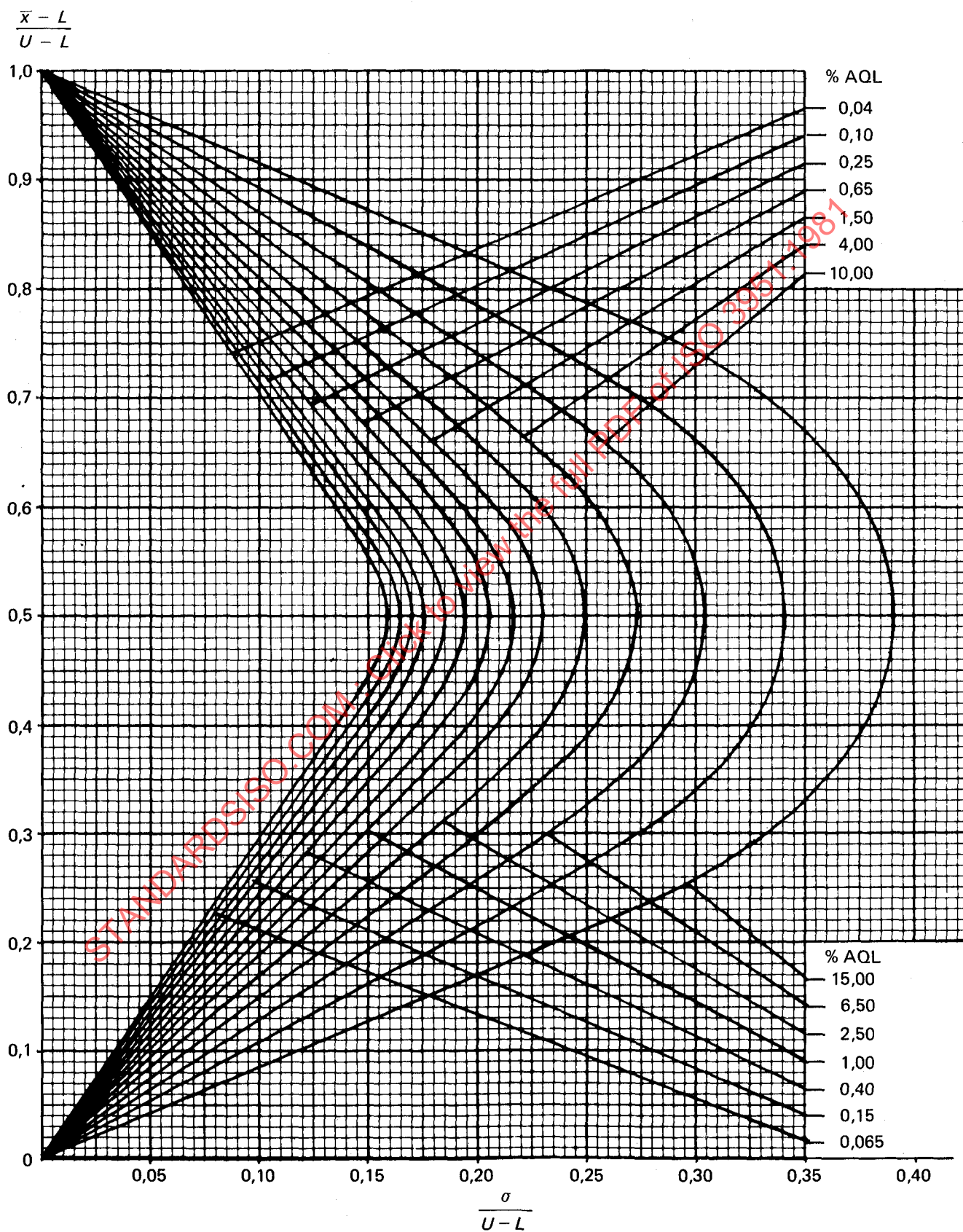


Diagram σ -P — Acceptance curves for combined double specification limits :
 "σ" method — Code letter P

P



Annex A

Procedure for obtaining s

A.1 The estimate from a sample of the standard deviation of a lot is given in the symbol s and its value may be obtained from the mathematical formula

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

where x_i is the measured characteristic of the i th item in a sample of n items and $\bar{x}$ is the mean value of the x_i , i.e.

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

A.2 The above formula for s is not recommended for the purpose of computation, since the calculations can be simplified by subtracting an arbitrary whole number a from the x_i and using the alternative equivalent formula for s :

$$s^2 = \left\{ \sum_{i=1}^n (x_i - a)^2 - n(a - \bar{x})^2 \right\} / (n-1)$$

For example, consider the measurements given in the example to 14.4. Take $a = 65$ and set out the calculations in tabular forms as follows :

x_i	a	$(x_i - a)$	$(x_i - a)^2$
63,5	65	-1,5	2,25
62	65	-3,0	9,00
65,2	65	0,2	0,04
61,7	65	-3,3	10,89
69	65	4,0	16,00
67,1	65	2,1	4,41
60	65	-5,0	25,00
66,4	65	1,4	1,96
62,8	65	-2,2	4,84
68	65	3,0	9,00

$$\begin{aligned} \sum x_i &= 645,7 \\ n &= 10 \\ \bar{x} &= 64,57 \end{aligned}$$

$$\begin{aligned} \sum (x_i - a)^2 &= 83,39 \\ n(a - \bar{x})^2 &= 1,85 \end{aligned}$$

$$\begin{aligned} \text{Difference} &= 81,54 = (n-1)s^2 \\ (n-1) &= 9 \\ s^2 &= 9,06 \\ s &= 3,01 \end{aligned}$$

$$\begin{aligned} a &= 65 \\ (a - \bar{x}) &= 0,43 \\ (a - \bar{x})^2 &= 0,185 \\ n(a - \bar{x})^2 &= 1,85 \end{aligned}$$

A.3 When a desk calculator is available, the value of a in clause A.2 may be taken as zero and the formula becomes

$$s^2 = \left\{ \sum_{i=1}^n (x_i)^2 - n(\bar{x})^2 \right\} / (n - 1)$$

With a modern machine, it is possible to obtain the cumulative

sum of both x_i and $(x_i)^2$ simultaneously and it is unnecessary to write down the individual values of x_i and $(x_i)^2$. But to illustrate the process, these values are given in the following example. Note how large the figures in the fourth column can become and when using an electronic calculator ensure that they are not truncated. The figures are taken from the example given sub-clause C.9.2.5 annex C, but in this case s will be calculated.

x_i	Cumulative sum of x_i	x_i^2	Cumulative sum of x_i^2
515	515	265 225	265 225
491	1 006	241 081	506 306
479	1 485	229 441	735 747
507	1 992	257 049	992 796
543	2 535	294 849	1 287 645
521	3 056	271 441	1 559 086
536	3 592	287 296	1 846 382
483	4 075	233 289	2 079 671
509	4 584	259 081	2 338 752
514	5 098	264 196	2 602 948
507	5 605	257 049	2 859 997
484	6 089	234 256	3 094 253
526	6 615	276 676	3 370 929
552	7 167	304 704	3 675 633
499	7 666	249 001	3 924 634
530	8 196	280 900	4 205 534
512	8 708	262 144	4 467 678
492	9 200	242 064	4 709 742
521	9 721	271 441	4 981 183
467	10 188	218 089	5 199 272
489	10 677	239 121	5 438 393
513	11 190	263 169	5 701 562
535	11 725	286 225	5 987 787
501	12 226	251 001	6 238 788
529	12 755	279 841	6 518 629

$$\begin{aligned}
 \text{so } \sum_{i=1}^n x_i &= 12\,755 \\
 n &= 25 \\
 \bar{x} &= \sum_{i=1}^n x_i / n = 510,2 \\
 (\bar{x})^2 &= 260\,304,04 \\
 n(\bar{x})^2 &= 6\,507\,601 \\
 \sum_{i=1}^n x_i^2 &= 6\,518\,629 \\
 n(\bar{x})^2 &= 6\,507\,601 \\
 \sum_{i=1}^n x_i^2 - n(\bar{x})^2 &= 11\,028 = (n - 1)s^2 \\
 n - 1 &= 24 \\
 s^2 &= 459,5 \\
 s &= 21,43
 \end{aligned}$$

Thus the mean $\bar{x} = 510,2$ and the estimate of the lot standard deviation $s = 21,4$.

A.4 If the variation between readings is small the arithmetical labour can be reduced by only considering that part of the measurement which changes from item; e.g. if the values of x_i had been 27,515; 27,491; 27,479; etc., they could be considered as if there were $y_i = 1\,000 (x_i - 27)$ i.e. y_i is 515; 491; 479; etc. These are then the figures in the previous example; thus $\bar{y} = 27,510$ and $s_y = 0,021$.

A.5 Other forms of the equation for s can be derived. Depending on the mechanism of the calculator, either of the following may be useful :

$$s = \sqrt{\frac{\sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2 / n}{n - 1}}$$

or

$$s = \sqrt{\frac{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}{n(n - 1)}}$$

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Annex B

Statistical theory

B.1 The normal distribution

B.1.1 The theory governing the calculation of risks in inspection by variables depends on the properties of the normal distribution and, therefore, this International Standard is only accurately applicable when there is reason to believe that the frequency distribution of the measured characteristic is normal or nearly normal.

B.1.2 A normal distribution can be defined completely in terms of the mean μ and standard deviation σ of the population; when these two parameters are known, it is possible to calculate the probability with which any measured value x will fall between two given values and, in particular, the probability of any measured value occurring outside given upper or lower limits. (See figure 5.)

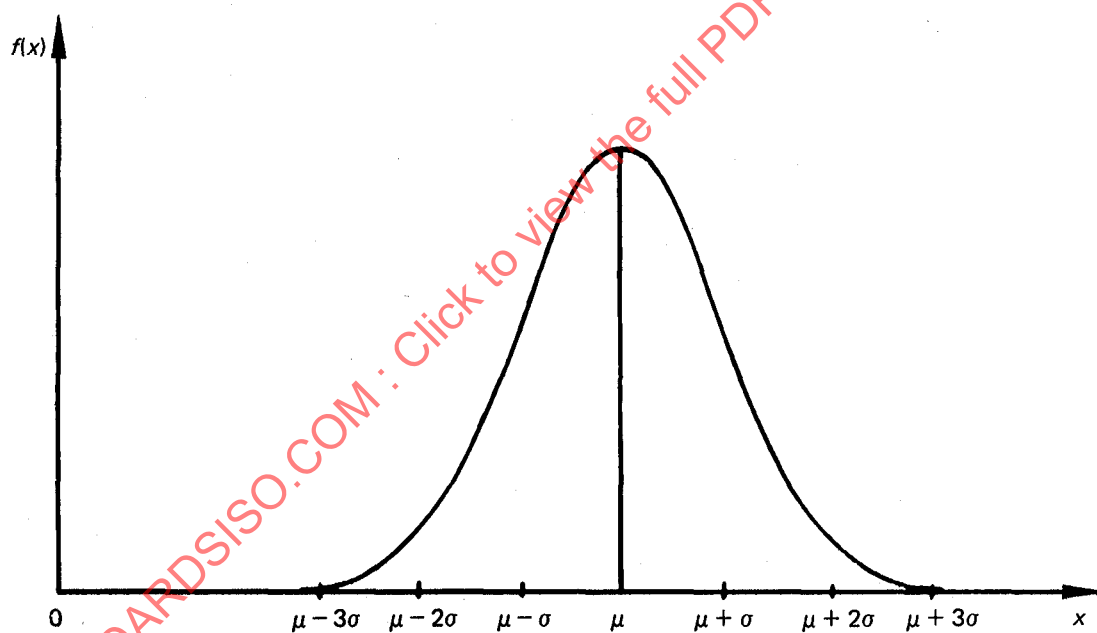


Figure 5 — Normal distribution

B.1.3 In order to facilitate the tabulation of these probabilities, the measured value x is transformed into a normal standardized variate z such that

$$z = \frac{x - \mu}{\sigma}$$

This device scales all normal distribution curves to a common shape (see figure 6), simplifies the equation of the curve and, as the total area under the curve is equal to 1, permits the calculation of a single table which, for a given value of z , gives the value of P , the cumulative probability function (the unshaded area under the curve).

B.2 The fraction defective

B.2.1 If any item is considered to be a defective when its measurement lies outside the specification limits, the fraction

defective will be represented by the area (or areas) under the distribution curve beyond the specification limit line (or lines if both upper and lower limits have been given).

NOTE — The fraction defective is, in practice, often expressed as a percentage defective; accordingly, in sections one to four it is given in this form.

B.2.2 When the measurements x are normally distributed and their mean μ and standard deviation σ are known, the fraction defective p relative to given specification limits L and/or U can be obtained from tables of the normal distribution, as $p_U = 1 - P_U$ and $p_L = P_L$.

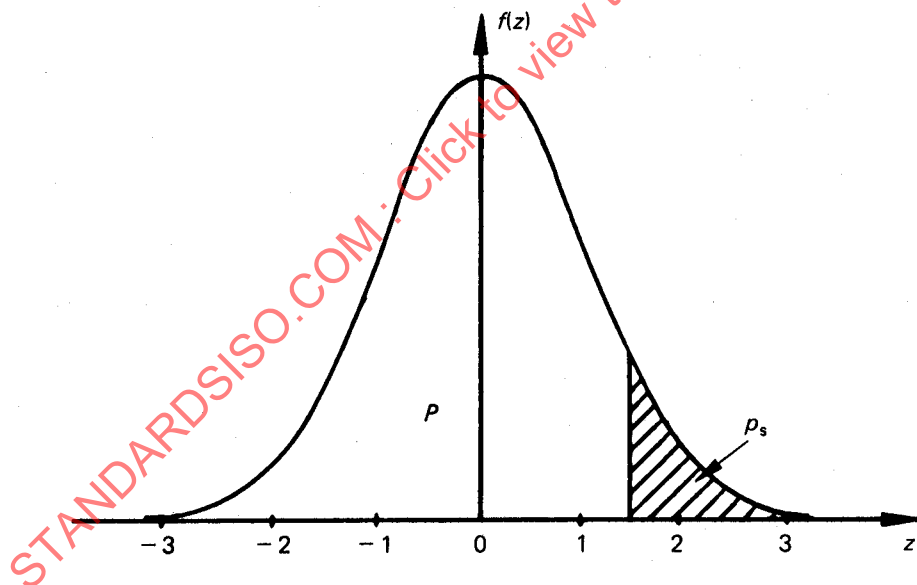


Figure 6 — Standardized normal distribution

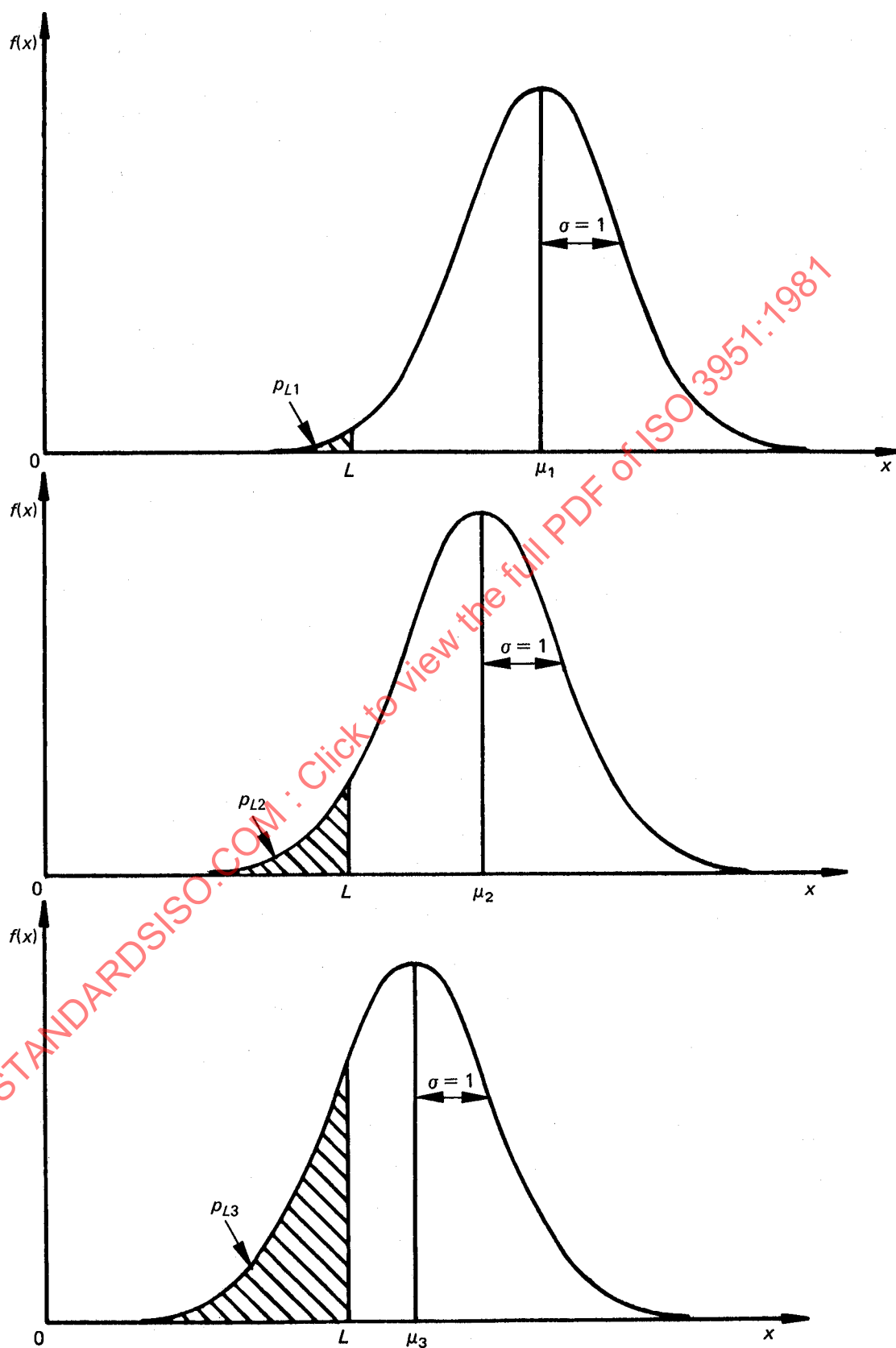


Figure 7 — Fraction defective for constant $\sigma = 1$ and different values of the mean (μ_1, μ_2, μ_3) in relation to a fixed lower specification limit

B.3 The quality parameter

B.3.1 When the normal distribution curve has been "standardized" by measuring the deviation from the mean in units of the standard deviation (i.e. to a scale $\sigma = 1$), the fraction defective will be uniquely related to the standardized deviate.

$$-z_L = \frac{(\mu - L)}{\sigma}$$

B.3.2 Accordingly, a quality parameter has been devised in terms of this difference and the standard deviation which, in relation to one specification limit, may be used in acceptance inspection instead of the fraction defective. When U , the upper specification limit, is given, this is defined as

$$q_U = \frac{U - \mu}{\sigma} = z_U$$

and when L , the lower specification limit, is given, as

$$q_L = \frac{\mu - L}{\sigma} = -z_L$$

(See figure 7.)

B.4 The AQL in terms of variables, single specification limit

The AQL is defined as the maximum percent defective that is acceptable as a process average for purpose of acceptance sampling.

When considering a single specification limit or both limits separately, the AQL can be related readily to the quality parameter q . Since there is one-to-one correspondence between the fraction beyond the given specification limit and the quality parameter, we can define a K which would be described as the smallest value of q that would be deemed acceptable as a process characteristic for purposes of acceptance sampling, i.e. for acceptability $q \geq K$.

This would then give

- in attributes : the AQL
- in variables : K

Since for a given set of specification limits, p_L or p_U depend on μ and σ , there will be a region in the μ, σ plane where p_L (or p_U) will be less than or equal to the AQL and will thus mark off processes that will be acceptable. There will conversely be another region in which p_L (or p_U) will be greater than the AQL and will thus mark off processes that are considered unacceptable. Fortunately, these regions can be separated by a single straight line.

For a single lower specification limit, the line of AQL in the μ, σ plane is given by the relationship

$$q_L = \frac{\mu - L}{\sigma} = K$$

or

$$\mu = L + K\sigma$$

For all points above this line, the process average would be less than or equal to the AQL and hence be acceptable. For points below the line, it would not be acceptable. (See figure 8.)

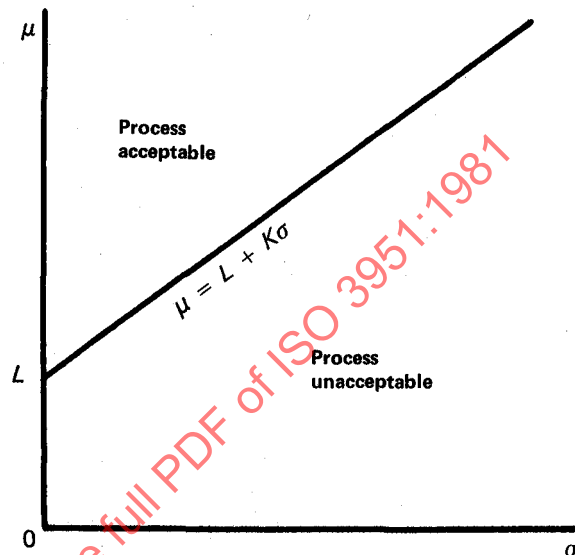


Figure 8

For a single upper specification limit, a similar relationship would be

$$q_U = \frac{U - \mu}{\sigma} = K$$

or

$$\mu = U - K\sigma$$

For points below this line, the process averages would be less than the AQL and hence acceptable; above, they would not. (See figure 9.)

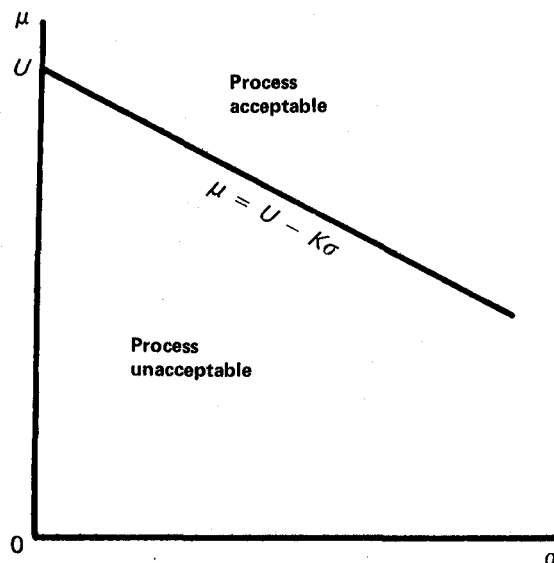


Figure 9

With two separate specification limits, the situation would be as depicted in figure 10.

B.5 The AQL and a combined double specification limit

B.5.1 Combined double specification limit

When upper and lower limits are specified and a single, overall AQL has to be met, this is known as a combined double specification limit. There is then no longer a way of obtaining a single variables figure such as K corresponding to the AQL. In-

stead, a relationship has to be found between a combination of values of the mean μ and the standard deviation σ which will just satisfy the AQL. This may be shown by a line in the μ, σ plane which may be explained as follows :

Specification of such an AQL does not specify the AQL at each limit and is not linked to p_U and p_L individually but to their sum p .

For a given value of σ , not only do the relative values of p_U and p_L depend on the value of μ but their total $p_U + p_L = p$ is also dependent on the value of μ in relation to U and L . (See figure 11.) p is a minimum when μ is midway between U and L and increases as μ approaches U or L .

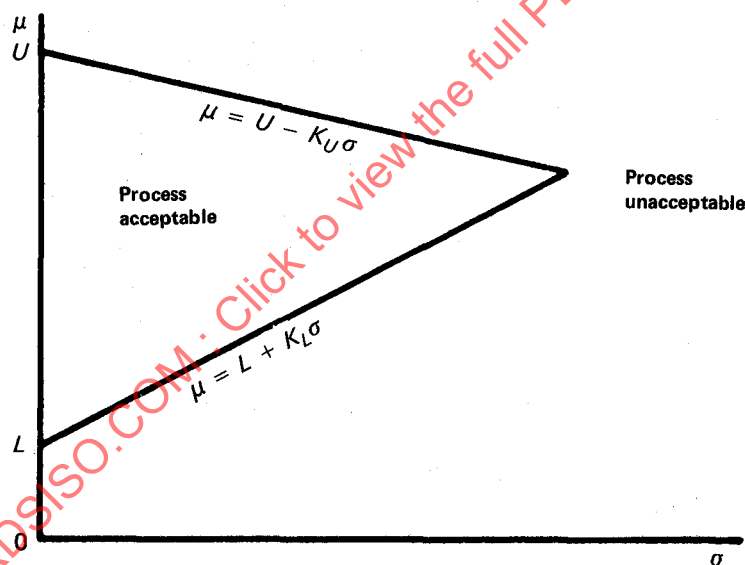


Figure 10

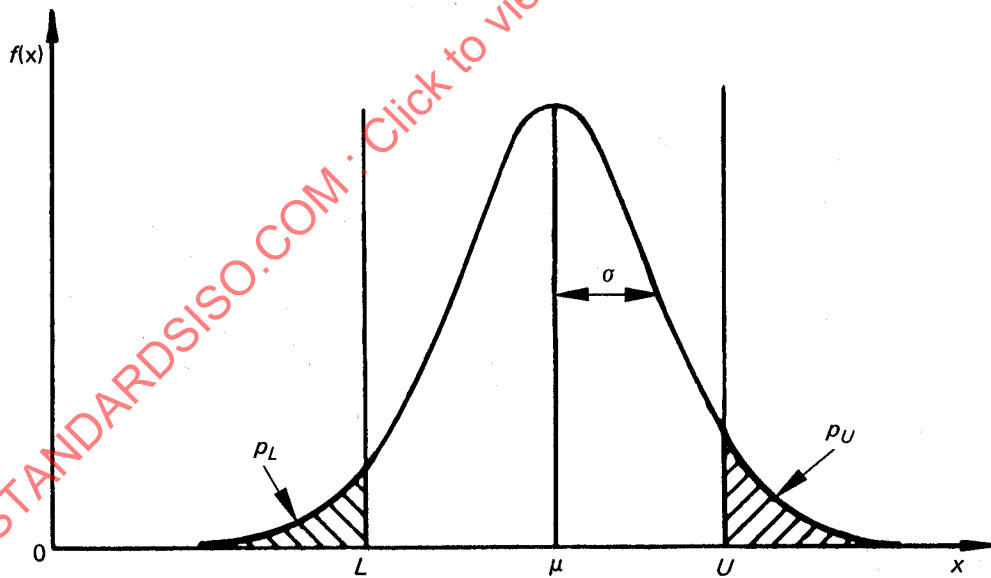
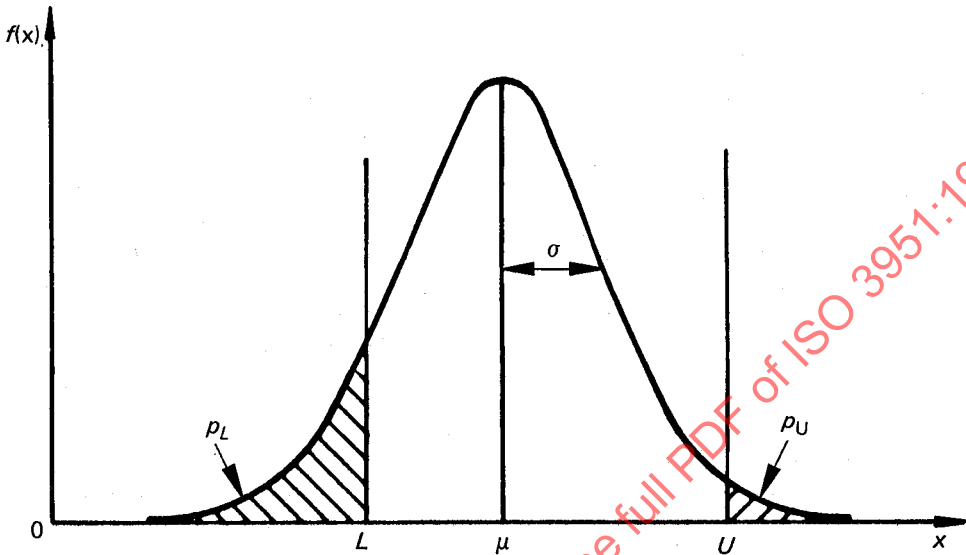


Figure 11

The extent to which the distribution curve overlaps the two limits, for a given value of μ , is governed by the value of σ . (See figure 12.)

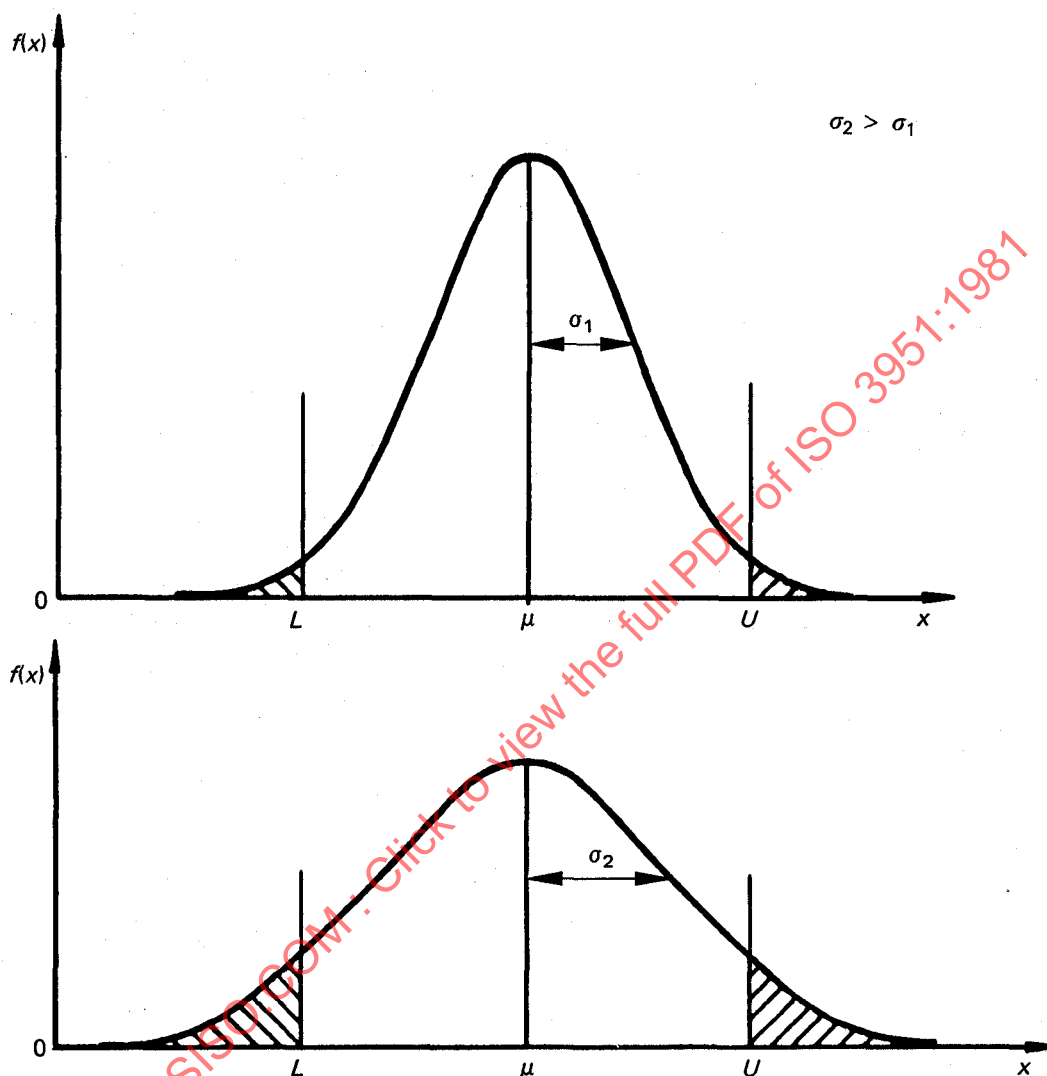


Figure 12

B.5.2 Maximum standard deviation

The largest value of σ that will just give the AQL when μ is midway between U and L may be viewed as a maximum standard deviation (MSD). If a process has a σ greater than this, its fraction defective is beyond the AQL. The converse, that a process fraction defective is less than the AQL if its standard deviation is less than this maximum, is not necessarily true.

B.5.3 Acceptance curve

When a combined double specification has been given, a series

of values of μ and σ can be calculated that give values of p_L and p_U that in total equal the AQL. These values can be plotted in the μ, σ plane and a curve drawn through them (see figure 13). This curve lies within the triangle that would be appropriate for separate upper and lower specification limits each with the given AQL (see figure 10). In the vicinity of U and L , the curve is indistinguishable from a straight line with slope $-K$ and K respectively and then becomes a smooth curve passing through the point $\sigma = \text{maximum standard deviation}$ and $\mu = (U + L)/2$. (See figure 13.)

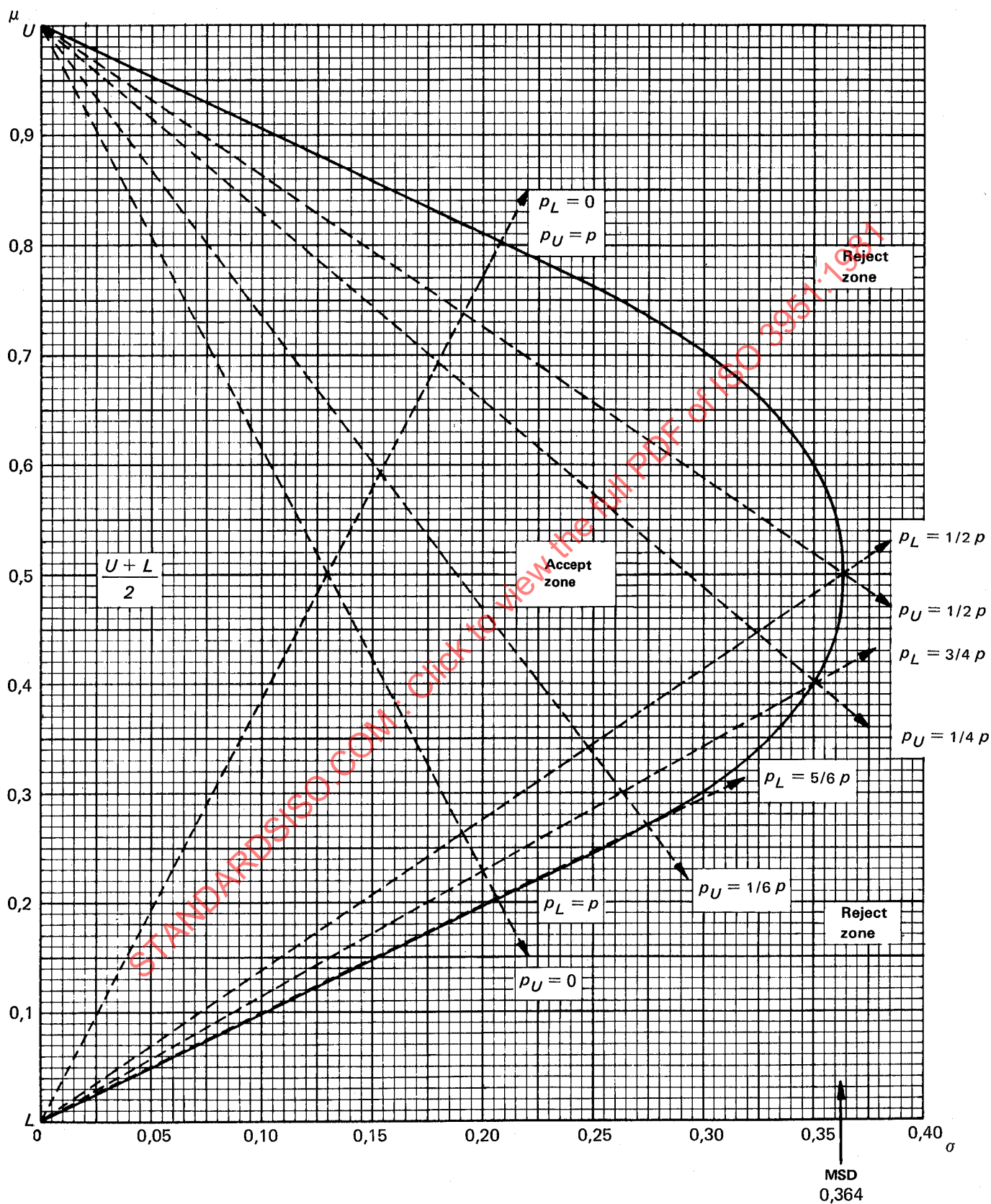


Figure 13 — Acceptance curve : "s" method — Code letter J (sample size 35) — AQL 10 %

B.6 Sampling inspection

In practice, μ is not known and has to be estimated from a sample; usually, σ has to be estimated also.

The best estimate of μ is given by $\bar{x}$, the mean of the sample. Depending on the circumstances, σ is either assumed to be known or estimated from the sample by one of two methods. These three treatments of σ lead to what are called in this International Standard the "σ" method, the "s" method (and the "R" method given in annex C), for sentencing a lot.

Using these estimates of the values of μ and σ , an estimate of the quality parameter q is obtained. This is known as the quality statistic Q .

It is possible to calculate values of K which, for a given sample size and method of assessing the process standard deviation, will ensure that, with a given probability, the lot is acceptable when the quality statistic Q is greater than or equal to this K which is then known as the acceptability constant.

B.7 The "σ" method

B.7.1 Derivation of the acceptability constant

Under certain circumstances, e.g. when the process has been running for some time and is under statistical quality control, σ is assumed to be known and constant.

The value of the means for successive lots is not assumed to be constant or known, so each has to be estimated from the mean of a random sample of items taken from the lot.

The mean of the sample $\bar{x}$ is not necessarily equal to the mean of the population μ , but $(\bar{x} - \mu)\sqrt{n}/\sigma$ is normally distributed with a zero mean and a standard deviation of 1, so it is possible to show that, for a given probability, the difference between the mean of the sample and the mean of the lot will be not more than a definite figure. For example, for a 95 % probability, this figure is $\pm 1.96 \sigma/\sqrt{n}$.

The equation for the upper quality statistic $Q_U = (U - \bar{x})/\sigma$ may be rewritten

$$Q_U \sqrt{n} = (U - \mu)\sqrt{n}/\sigma - (\bar{x} - \mu)\sqrt{n}/\sigma$$

The first term on the right-hand side is a constant and the second is the standardized normal deviate of the mean discussed above. So $Q_U \sqrt{n}$ (and similarly $Q_L \sqrt{n}$) is also normally distributed with a standard deviation of 1. Hence, it is possible to compute a value k such that, for a given sample size and AQL, the sample's quality statistic is, with a stated probability, greater than or equal to k when the lot is acceptable.

B.7.2 The acceptability criteria for a single or separate specification

For a value of the acceptability constant k computed as indicated in B.7.1 and which, for normal, tightened and reduced inspection is given in tables III-A, III-B and III-C respectively, the acceptability criterion becomes, for a single upper specification limit,

$$\text{accept if } Q_U = \frac{U - \bar{x}}{\sigma} > k$$

and, for a single lower specification limit,

$$\text{accept if } Q_L = \frac{\bar{x} - L}{\sigma} > k$$

otherwise, reject.

Where separate double specification limits are given, the acceptability criterion is

$$\text{accept if } Q_U > k_U \text{ and } Q_L > k_L$$

$$\text{reject if either } Q_U < k_U \text{ or } Q_L < k_L$$

Since σ is a known constant, it will, in practice, be more convenient to re-arrange the acceptability criteria in the form $\bar{x} \leq U - k\sigma$ for an upper specification limit and $\bar{x} \geq L + k\sigma$ for a lower limit, as the right-hand side of these criteria can be determined before inspection begins.

B.7.3 The acceptability criteria for a combined double specification

When combined double specification limits are given, in order to establish whether the lot is acceptable, the point

$$\left(\frac{\sigma}{U - L}, \frac{\bar{x} - L}{U - L} \right)$$

may be plotted on the appropriate diagram taken from section four, diagrams σ-C to σ-P, in accordance with B.5.3; however, since σ is a known constant the point will lie on a vertical line corresponding to this value of σ . It is therefore sufficient in practice to obtain $\bar{x}_U$ and $\bar{x}_L$, the acceptable upper and lower limits of $\bar{x}$. (See figure 14.)

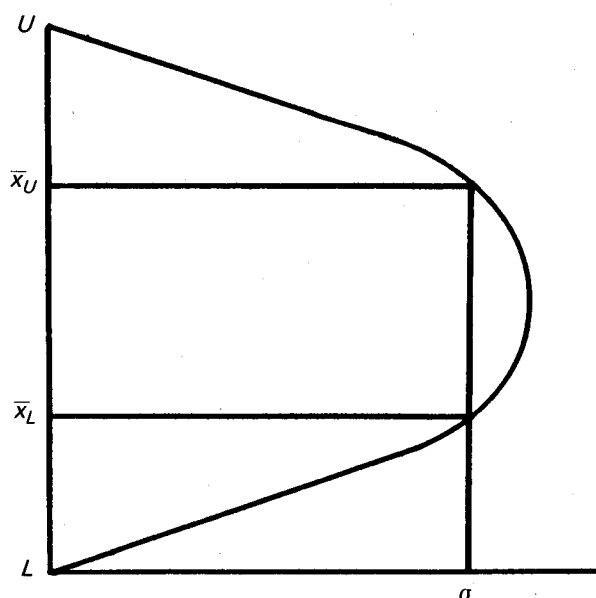


Figure 14

The acceptability criteria then becomes

$$\text{accept if } \bar{x} < \bar{x}_U \text{ and } \bar{x} > \bar{x}_L$$

otherwise, reject.

B.8 The "s" method

B.8.1 Derivation of the acceptability constant

When neither the mean nor the standard deviation of a lot can be presumed to be known, they both have to be estimated from a sample taken from that lot. In the "s" method, the mean of the sample $\bar{x}$ is taken instead of μ the mean of the lot and

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

from the sample is taken instead of σ , so

$$Q_U = \frac{U - \bar{x}}{s} \text{ and } Q_L = \frac{\bar{x} - L}{s}$$

Compared with the "σ" method, this increases the possibility of error, for when s is substituted for σ in the expression $(\bar{x} - \mu)\sqrt{n}/\sigma$ (see B.7.1), it is found that $(\bar{x} - \mu)\sqrt{n}/s$, which may be written

$$\frac{\sigma (\bar{x} - \mu)\sqrt{n}}{s \sigma}$$

is no longer normally distributed, but has a "t" distribution with $(n-1)$ degrees of freedom. For instance, if $n = 10$, there is now a 95 % probability that the difference between the mean of the sample and the mean of the lot will be not more than $\pm 2,26 s/\sqrt{10}$, compared with the $\pm 1,96 \sigma/\sqrt{n}$ given in B.7.1.

Further,

$$Q_U \sqrt{n} = \frac{(U - \bar{x})\sqrt{n}}{s}$$

which may be written as

$$Q_U \sqrt{n} = \frac{\sigma}{s} \left\{ \frac{(U - \mu)\sqrt{n}}{\sigma} - \frac{(\bar{x} - \mu)\sqrt{n}}{\sigma} \right\}$$

and

$$Q_L \sqrt{n} = \frac{(\bar{x} - L)\sqrt{n}}{s}$$

which may be written as

$$Q_L \sqrt{n} = \frac{\sigma}{s} \left\{ \frac{(\mu - L)\sqrt{n}}{\sigma} + \frac{(\bar{x} - \mu)\sqrt{n}}{\sigma} \right\}$$

are also not normally distributed, but they do follow a "non-central t" distribution and this distribution has been tabulated. The Resnikoff and Lieberman tables^[10] have sampling inspection by variables particularly in mind; the detailed theory of this application of the non-central t distribution is covered in the introduction to the tables and the arguments used fit our standard sample sizes and preferred AQL.

The acceptability constant k has been calculated, using the non-central t tables, for a standard range of sample size and AQLs, and is given in tables II-A, II-B and II-C for normal, tightened and reduced inspection respectively.

B.8.2 Acceptability criteria for a single or separate specification

The acceptability criteria are, as stated in clause B.6, similar to those given in clause B.5, but substituting the estimated values of the mean, standard deviation and quality statistic obtained from a sample and comparing the latter with the acceptability constant k calculated for this method.

The acceptability criterion for a single specification limit is

accept if $Q_U > k$ or $Q_L > k$

reject if $Q_U < k$ or $Q_L < k$

When separated double specification limits are given, the acceptability criterion is

accept if both $Q_U > k_U$ and $Q_L > k_L$

reject if either $Q_U < k_U$ and $Q_L < k_L$

The graphical method illustrated in figures 8, 9 and 10 is also applicable to both the above criteria. An example is given in 14.3 and illustrated in figure 2.

The upper boundary for the accept zone of an upper limit is given by the straight line $\bar{x} = U - ks$ and the lower boundary for the accept zone of a lower limit is given by the straight line $\bar{x} = L + ks$. When separate upper and lower specification limits are given, the accept zone is the area between these two lines and the $\bar{x}$ axis, as in figure 10.

B.8.3 Acceptability criteria for a combined double specification

When there are combined double specification limits, it was shown in B.5.3 that the acceptability of a process depended on whether the point (σ, μ) lies within the appropriate acceptance curve on the acceptance diagram. (See figure 13.)

The acceptability for a lot may be found when the values of s and $\bar{x}$, obtained from a sample, are plotted on a similar acceptance diagram, indexed in terms of the sample size and AQL.

Acceptance curves have been calculated for the standard plans given in this International Standard, making allowance for the uncertainty introduced by the need to use $\bar{x}$ as an estimate for μ and s as an estimate for σ in the "s" method.

These curves are shown in diagrams s-D to s-P in section four. They are to a common scale, i.e. instead of plotting $\bar{x}$ against s ,

$$\frac{\bar{x} - L}{U - L} \text{ is plotted against } \frac{s}{U - L}$$

when using the "s" method.

For code letters B and C (i.e. sample sizes 3 and 4), the accept zone is bounded by four straight lines, the $\bar{x}$ axis, the line $\bar{x} = U - ks$, a line parallel to the $\bar{x}$ axis through the MSD, (see table IV) and the line $\bar{x} = L + ks$, the value of k being obtained from table II-A, II-B or II-C.

The acceptability criterion is to accept if the point $[s/(U - L), (\bar{x} - L)/(U - L)]$ lies inside the accept zone and to reject if it lies outside it.

In practice, when a series of lots is being inspected, the Inspector's task is made easier if a special acceptance curve is

produced for this sampling plan in terms of co-ordinates s and $\bar{x}$. (See figure 3.)

If s is larger than the MSD (which may be found in table IV), the lot may be rejected at once. (See B.5.2.)

B.8.4 Complex specification limits

When combinations of the above specifications are given, e.g. a separate AQL ($= k_1$) for the upper limit, combined with an overall AQL, the corresponding limit lines or curves are drawn and the accept zone is the area common to both requirements. (See figure 15.)

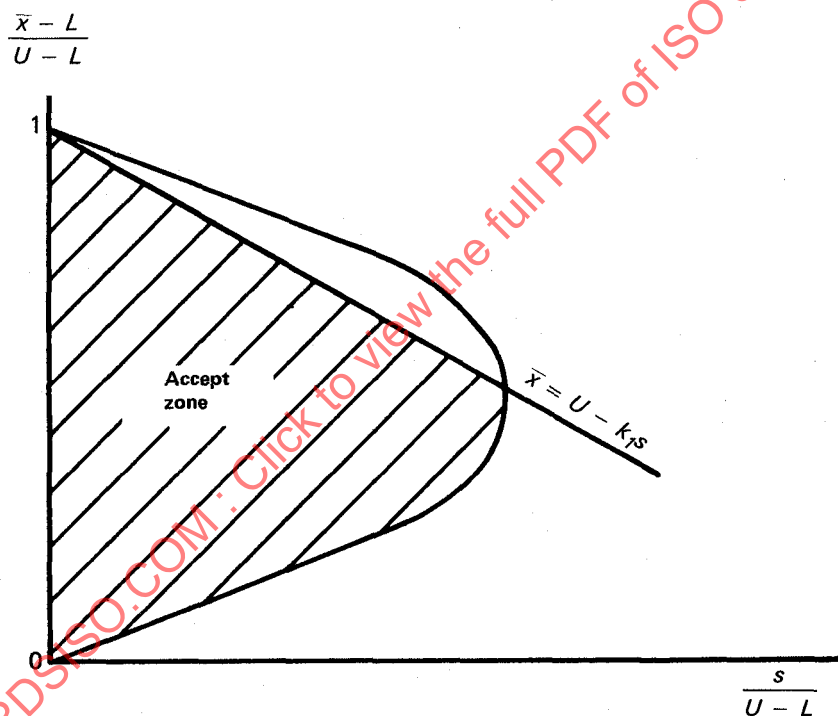


Figure 15

Annex C

Sampling plans by "R" method

C.1 Object

This method provides an alternative to the "s" method given in clause 14.

C.2 Definitions

The additional terms for use with this method are as follows :

C.2.1 "R" method : A method of assessing the acceptability of a lot by using an estimate of standard deviation of the lot based on the average range of the measurements of the items in sub-groups of a sample.

C.2.2 sub-group : Five units taken in the order in which they were measured.

C.2.3 range (R) : Restricted sense, when using the "R" method : the range of a sub-group.

NOTE — The range, in its ordinary meaning, is the difference between the greatest and the smallest observed values.

C.2.4 average range ($\bar{R}$) : The mean of the ranges of the sub-groups. For samples of less than ten items, the range of the whole sample is taken as $\bar{R}$.

C.2.5 maximum average range (MAR) : Under given conditions, the largest acceptable average range.

C.3 Symbols

Additional symbols for use with this method are as follows :

- c Scale factor given in table R1 for assessing s from $\bar{R}$.
- F A factor, given in table RIV, which relates the maximum average range to the difference between U and L .
- k The acceptability constant.
- R Range of a sub-group.
- $\bar{R}$ Average range, based on several sub-groups.

C.4 Theory

The "R" method provides an alternative way of estimating the standard deviation of a lot.

The theoretical basis of this method is that for small samples the range R and s , where both are calculated from the same normal sample, are highly correlated, which enables R to be

used as an estimator for s . This relationship is used when dealing with samples of less than ten items. However, as the sample size increases, R becomes a less precise estimator and may no longer be used, but if the sample is split randomly into small sub-groups of equal size, the average of their ranges $\bar{R}$ may be used to estimate the lot standard deviation.

In the standard plans for the "R" method the sample sizes are, with the exception of three plans having sample sizes 3, 4, and 7, all multiples of 5. A sample is split into sub-groups of five items and the range R of each sub-group is obtained and hence the average range $\bar{R}$. When the sample consists of less than 10 items, it is not split into sub-groups and the range is treated as though it were $\bar{R}$.

A scale factor c has been calculated in terms of the sub-group sizes and the number of sub-groups, such that s may be replaced by $\bar{R}/c$. This scale factor c is given for the various sample sizes, on the basis of the sub-group sizes used in this International Standard, in table R1. This table is based on table 30 of the Biometrika Tables, Volume 1.^[11]

However, the use of the scale factor c given in table R1 is not usually required as, in order to reduce the numerical work to be carried out, a special set of tables has been provided for the "R" method. Then the upper and lower quality statistics are directly defined as

$$Q_U = \frac{U - \bar{x}}{\bar{R}}$$

and

$$Q_L = \frac{\bar{x} - L}{\bar{R}}$$

Table R1 provides also a comparison for a code letter of the sample sizes for the "R" method, the "s" method and the attributes method given in ISO 2859. The sample sizes are somewhat larger in the "R" method than in the "s" method, but they are both smaller than for the attributes method.

C.5 The acceptability criteria

For each plan an acceptability constant k is calculated such that the acceptability criteria are, for a single upper specification limit,

$$\text{accept if } Q_U = \frac{U - \bar{x}}{\bar{R}} > k$$

and, for a single lower specification limit,

$$\text{accept if } Q_L = \frac{\bar{x} - L}{\bar{R}} > k$$

otherwise, reject.

When separate upper and lower specification limits are given, the acceptability criterion is

accept if both $Q_U \geq k_U$ and $Q_L \geq k_L$

reject if either $Q_U < k_U$ or $Q_L < k_L$

The graphical method described in 14.3 is applicable to this method putting $\bar{R}$ in the place of s .

When a combined double specification limit is given, the point

$$\left(\frac{\bar{R}}{U - L}, \frac{\bar{x} - L}{U - L} \right)$$

is plotted on the appropriate diagram R -D to R -P in order to establish whether the lot is acceptable. Note that in the "R" method the equivalent of the maximum standard deviation (MSD) is the maximum average range (MAR). Its value can be obtained by entering table RIV with the sample size and AQL. The point $[\bar{R} = \text{MAR}, \bar{x} = 1/2 (U + L)]$ gives the apex of the acceptance curve. If the value of $\bar{R}$ is greater than the MAR, the lot is to be rejected.

C.6 The Operating Characteristic curve

The Operating Characteristic (OC) curves calculated for the "s" method are applicable to this method with little loss of accuracy. They are identified by code letters and AQL values.

C.7 Control charts

The technique of maintaining records in the form of control charts recommended in 18.1 applies equally to the "R" method.

The MAR should be plotted on the R chart when a combined double specification limit has been specified.

If it appears that $\bar{R}$ is generally in control, the root mean square values of $\bar{R}/c$ may be presumed to be σ , the factor c being given in table RI. The "σ" method may be then adopted when permission to switch to this method has been received from the responsible authority concerned.

C.8 Choice of method

If a modern calculator is available, the use of the "s" method is normally recommended, but, if the calculation of s poses an impediment to this method, the "R" method may be used as an alternative to the "s" method.

The "R" method is simple to calculate, but requires a somewhat larger sample size.

C.9 Operation of an "R" method sampling plan

C.9.1 Selecting a plan

Check at first that the instructions given in clause 13 have been applied.

The standard procedure for an "R" method plan is as follows :

a) With the inspection level given (normally this will be II) and the lot size, obtain the code letter from table I-A.

b) Using the sample size code letter and the AQL specified, enter table RII-A and obtain the sample size n and acceptability constant k .

C.9.2 Operating a plan

C.9.2.1 Take the individual items of the sample in the random order in which they were selected and measure them for the characteristic being inspected. Record the measurements in this order.

C.9.2.2 Find the sum Σx of all these measurements and, dividing by n , the number of items in the sample, obtain the sample mean

$$\bar{x} = \frac{\Sigma x}{n}$$

C.9.2.3 Obtain the value of $\bar{R}$.

a) Where there are ten or more items, split the record of the measurements, as they were taken, into sub-groups of five. (Since the larger sample sizes for standard plans are in multiples of five, this will always be possible.) Obtain the range for each sub-group by subtracting the smallest from the largest measurement and then calculate the average range $\bar{R}$.

b) Samples of less than ten items are not subdivided, the difference between the largest and smallest measurement gives the range and this is then treated as though it were the average range $\bar{R}$.

C.9.2.4 Apply the acceptability criterion.

If a single specification limit or separate double specification limits are given, calculate the quality statistic

$$Q_U = (U - \bar{x})/\bar{R}$$

and/or

$$Q_L = (\bar{x} - L)/\bar{R}$$

as appropriate;

then compare the quality statistic (Q_U and/or Q_L) with the acceptability constant (k_U and/or k_L) obtained from table RII-A (for normal inspection). If the quality statistic is greater than or equal to this acceptability constant, accept the lot; if less, reject. Thus, if an upper specification limit U is given,

accept if $Q_U \geq k$

reject if $Q_U < k$

Or, if a lower specification limit L only is given,

accept if $Q_L \geq k$

reject if $Q_L < k$

When both U and L are given : (k values are different when AQL are different for the upper and the lower limits),

accept if **both** $Q_L \geq k_L$ and $Q_U \geq k_U$

reject if **either** $Q_L < k_L$ or $Q_U < k_U$

Example

The lower specification limit for electrical resistance of a certain electrical component is 580 Ω . A lot of 100 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 1 % is to be used. From tables I-A the code letter is F; from RII-A it is seen that a sample of size 10 is required and that the acceptability constant k is 0,073. Suppose that values of the sample resistances are obtained in the following order :

First sub-group 610; 629; 593; 617 ($R = 629 - 593 = 36$).

Second sub-group 623; 589; 608; 591; 611
($R = 623 - 589 = 34$).

Compliance with the acceptability criterion is to be determined.

Information needed

Value obtained

Sample size : n

10

Sample mean $\bar{x} : \Sigma x/n$

608,6

Average range $\bar{R} : \Sigma R/\text{No. of sub-}$

$$\text{groups} = \frac{36 + 34}{2}$$

35

Specification limit (lower) : L

580

$$Q_L = (\bar{x} - L)/\bar{R}$$

0,817

Acceptability constant : k (see table RII-A)

0,703

Acceptability criterion : compare Q_L with k $0,817 > 0,703$

The lot meets the acceptability criterion, since Q_L is greater than k .

C.9.2.5 When a graphical criterion is desired, draw the line

$\bar{x} = U - k \bar{R}$ (for an upper limit), straight line through the point ($\bar{R} = 0, \bar{x} = U$) with a slope $-k$,

and/or

$\bar{x} = L + k \bar{R}$ (for a lower limit), straight line through the point ($\bar{R} = 0, \bar{x} = L$) with a slope k ,

as appropriate, on graph paper with $\bar{x}$ as the vertical axis and $\bar{R}$ as the horizontal axis. So the graph can be prepared before beginning the inspection of a series of lots (following the procedure given in 14.3 for the "s" method). Then use the values of $\bar{R}$ and $\bar{x}$ from the measurements obtained from each sample and plot the point ($\bar{R}, \bar{x}$) on the graph. If this point lies in the accept zone, accept the lot, if outside, reject it.

Example

Using the figures given in the example above, mark the point $L = 580$ on the $\bar{x}$ (vertical) axis, draw a line through this point with a slope k , [as $k = 0,703$, this means the line will pass through points ($\bar{R} = 10, \bar{x} = 587$), ($\bar{R} = 20, \bar{x} = 594,1$), ($\bar{R} = 40, \bar{x} = 608,1$), etc.]. Select a suitable point and draw a straight line through it and ($\bar{R} = 0, \bar{x} = 580$), i.e. L . The accept zone is then the area above this line. The calculated values of $\bar{R}$ and $\bar{x}$ are 35 and 608,6. Plotting the point ($\bar{R}, \bar{x}$), it will be seen from figure 16 that it lies inside the accept zone and the lot is accepted.