INTERNATIONAL STANDARD

ISO 4156-1

Second edition
2021-02

Straight cylindrical involute splines — Metric module, side fit —

Part 1: Generalities

Cannelures cylindriques droites à flancs en développante — Module métrique, à centrage sur flancs —

Partie 1: Généralités

Reference number
ISO 4156-1:2021(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 14, *Shafts of machinery and accessories*.

This second edition cancels and replaces the first edition (ISO 4156-1:2005), which has been technically revised.

The main changes compared to the previous edition includes:

— ISO 268-1 has been removed from Clause 2;

— ISO 4156-2 and ISO 4156-3 have been moved from Clause 2 to Bibliography;

— the definitions of base diameter, major diameter, minor diameter, depth of engagement, theoretical clearance, out-of-roundness, and auxiliary dimension have been removed;

— symbols of length and arc length between two points, according to ISO 80000-3, have been adopted and used in calculation examples in Annex A;

— in Figure 8, clearance between external spline and mating part has been corrected;

— in Figure 10, measurement of space width, effective and tooth thickness, effective have been corrected;

— in Figure 11, the figure title has been changed;

— in Figure 15, the indication of form tooth height and minor tooth height has been corrected;

— in Table 11, the tolerance on D_{ii} for diameter > 80 to 120 in column H 11 has been corrected;

— the previous Tables 14 to 17 have been corrected and moved to Annex A;

— in A.4, the calculation of $M_{Re\ min}$ has been completed.

A list of all parts in the ISO 4156 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

ISO 4156 (all parts) provides the data and indications necessary for the design, manufacture and inspection of straight (non-helical) side-fitting cylindrical involute splines.

Straight cylindrical involute splines manufactured in accordance with ISO 4156 (all parts) are used for clearance, sliding and interference connections of shafts and hubs. They contain all the necessary characteristics for the assembly, transmission of torque, and economic production.

The nominal pressure angles are 30°, 37,5° and 45°. For electronic data processing purposes, the form of expression 37,5° has been adopted instead of 37°30′. ISO 4156 (all parts) establishes a specification based on the following modules:

— for pressure angles of 30° and 37,5° the module increments are:

0,5; 0,75; 1; 1,25; 1,5; 1,75; 2; 2,5; 3; 4; 5; 6; 8; 10;

— for pressure angle of 45° the module increments are:

0,25; 0,5; 0,75; 1; 1,25; 1,5; 1,75; 2; 2,5.

Straight cylindrical involute splines — Metric module, side fit —

Part 1: Generalities

1 Scope

This document provides the data and indications necessary for the design and manufacture of straight (non-helical) side-fitting cylindrical involute splines.

Limiting dimensions, tolerances, manufacturing deviations and their effects on the fit between connecting coaxial spline elements are defined in the formulae and given in the tables. Unless otherwise specified, linear dimensions are expressed in millimetres and angular dimensions in degrees.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1101, *Geometrical product specifications (GPS) — Geometrical tolerancing — Tolerances of form, orientation, location and run-out*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1
spline joint
connecting, coaxial elements that transmit torque through the simultaneous engagement of equally spaced teeth situated around the periphery of a cylindrical external member with similar spaced mating spaces situated around the inner surface of the related cylindrical internal member

3.2
involute spline
member of *spline joint* (3.1) having teeth or spaces that have involute flank profiles

3.3
internal spline
spline formed on the inner surface of a cylinder

3.4
external spline
spline formed on the outer surface of a cylinder

3.5
fillet
concave surface of the tooth or space connecting the involute flank and the root circle

Note 1 to entry: For generated splines this curved surface, as generated, varies and cannot be properly specified by a radius of any given value. For splines formed directly by a tool (e.g. broached, net formed) the fillet may be specified as a true radius.

3.6
fillet root
spline having a tooth or space profile in which the opposing involute flanks are connected to the root circle (D_{ei} or D_{ie} diameter) by a single *fillet* (3.5)

3.7
flat root
spline having a tooth or space profile in which each of the opposing involute flanks are connected to the root circle (D_{ei} or D_{ie} diameter) by a *fillet* (3.5)

3.8
module
m
ratio of the *circular pitch* (3.12), expressed in millimetres, to the number π

3.9
pitch circle
reference circle to which all spline dimensions are related, and the circle on which the specified *pressure angle* (3.13) has its nominal value

3.10
pitch diameter
D
diameter of the *pitch circle* (3.9), in millimetres, equal to the number of teeth multiplied by the *module* (3.8)

3.11
pitch point
intersection of the spline tooth profile with the *pitch circle* (3.9)

3.12
circular pitch
p
length of arc of the *pitch circle* (3.9) between two consecutive *pitch points* (3.11) of left- (or right-) hand flanks, which has a value of the number π multiplied by the *module* (3.8)

3.13
pressure angle
α
acute angle between a radial line passing through any point on a tooth flank and the tangent plane to the flank at that point

3.14
standard pressure angle
α_D
pressure angle (3.13) at the specified *pitch point* (3.11)

3.15
base circle
circle from which *involute spline* (3.2) tooth profiles are generated

3.16
base pitch
p_b
arc length of the *base circle* (3.15) between two consecutive corresponding flanks

3.17
form diameter
D_F
diameter used to define the depth of involute profile control

Note 1 to entry: In the case of an *external spline* (3.4) it is located near and above the minor diameter, and on an *internal spline* (3.3) near and below the major diameter.

3.18
basic circular space width
E
for 30°, 37,5° and 45° *pressure angle* (3.13) splines, half the *circular pitch* (3.12), measured at the *pitch diameter* (3.10)

3.19
basic circular tooth thickness
S
for 30°, 37,5° and 45° *pressure angle* (3.13) splines, half the *circular pitch* (3.12), measured at the *pitch diameter* (3.10)

3.20
actual space width
practically measured circular space width, on the *pitch circle* (3.9), of any single space width within the limit values E_{max} and E_{min}

3.21
effective space width, circular
E_v
space width where an imaginary perfect *external spline* (3.4) would fit without clearance or interference, given by the size of the tooth thickness of this external spline, considering engagement of the entire axial length of the splined assembly

Note 1 to entry: The minimum effective space width ($E_{v\ min}$, always equal to E) of the *internal spline* (3.3) is always basic, as shown in Table 3.

3.22
actual tooth thickness
practically measured circular tooth thickness, on the *pitch circle* (3.9), of any single tooth within the limit values S_{max} and S_{min}

3.23
effective tooth thickness, circular
S_v
tooth thickness where an imaginary perfect *internal spline* (3.3) would fit without clearance or interference, given by the size of the space width of this internal spline, considering engagement of the entire axial length of the splined assembly

3.24
effective clearance
c_v
⟨looseness or interference⟩ *effective space width, circular* (3.21) of the *internal spline* (3.3) minus the *effective tooth thickness, circular* (3.23) of the *external spline* (3.4)

Note 1 to entry: For looseness, c_v is positive; for interference, c_v is negative.

3.25
form clearance
c_F
radial clearance between the form diameter of the *internal spline* (3.3) and the major diameter of the *external spline* (3.4), or between the minor diameter of the internal spline and the form diameter of the external spline

Note 1 to entry: It allows eccentricity of their respective *pitch circles* (3.9).

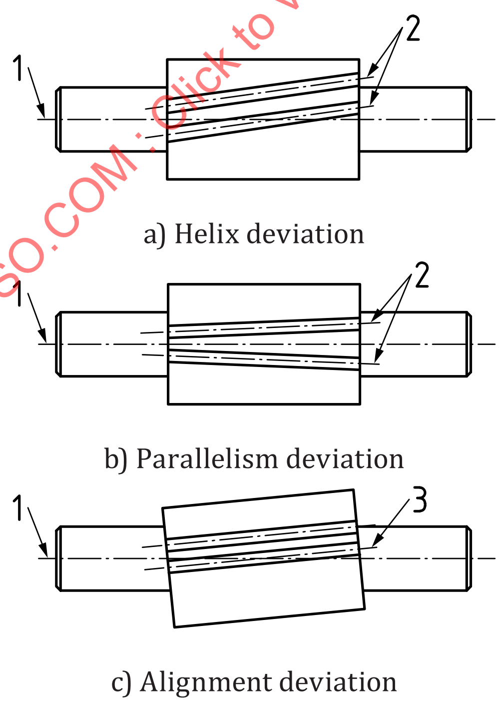
3.26
total pitch deviation
F_p
absolute value of the difference between the greatest positive and negative deviations from the theoretical spacing, measured at the *pitch circle* (3.9)

Note 1 to entry: See ISO 1328-1.

3.27
total profile deviation
F_α
absolute value of the difference between the greatest positive and negative deviations from the theoretical tooth profile, measured normal to the flanks

3.28
total helix deviation
F_β
absolute value of the difference between the two extreme deviations from the theoretical direction parallel to the reference axis

Note 1 to entry: This includes *parallelism deviation* (3.29) and *alignment deviation* (3.30), see Figure 1.



Key
1 reference axis
2 centreline of teeth
3 effective spline axis

Figure 1 — Helix deviations

3.29
parallelism deviation
deviation of parallelism of a single spline tooth to any other single spline tooth

Note 1 to entry: See Figure 1 b).

3.30
alignment deviation
deviation of the effective spline axis with respect to the reference axis

Note 1 to entry: See Figure 1 c).

3.31
effective deviation
accumulated effect of the spline deviations on the fit with the mating part

3.32
deviation allowance
λ
permissible deviation between minimum actual and minimum *effective space width, circular* (3.21) or maximum effective and maximum *actual tooth thickness* (3.22)

3.33
machining tolerance
T
permissible deviation between maximum actual and minimum *actual space width* (3.20) or tooth thickness

3.34
effective clearance tolerance
T_v
permissible deviation between maximum effective and minimum *effective space width, circular* (3.21) or tooth thickness

3.35
total tolerance
$T + \lambda$
⟨general⟩ *machining tolerance* (3.33) plus the *deviation allowance* (3.32)

3.36
total tolerance
⟨internal spline⟩ difference between the minimum *effective space width, circular* (3.21) and the maximum *actual space width* (3.20)

3.37
total tolerance
⟨external spline⟩ difference between the maximum *effective tooth thickness, circular* (3.23) and the minimum *actual tooth thickness* (3.22)

3.38
basic dimension
numerical value to describe the theoretically exact size, shape or location of a feature

Note 1 to entry: It is the basis from which permissible deviations are established by tolerances.

4 Symbols, subscripts and abbreviated terms

4.1 General symbols

The general symbols used to designate the various spline terms and dimensions are given below.

b	Spline length	mm
c_F	Form clearance	mm
c_v	Effective clearance (looseness or interference)	µm
$c_{v\,max}$	Maximum effective clearance	µm
$c_{v\,min}$	Minimum effective clearance	µm
d_{ce}	Ball or pin contact diameter, external spline	mm
d_{ci}	Ball or pin contact diameter, internal spline	mm
D	Pitch diameter	mm
D_{Fe}	Form diameter, external spline	mm
$D_{Fe\,max}$	Maximum form diameter, external spline	mm
D_{Fi}	Form diameter, internal spline	mm
$D_{Fi\,min}$	Minimum form diameter, internal spline	mm
D_{Re}	Diameter of measuring ball or pin for external spline	mm
D_{Ri}	Diameter of measuring ball or pin for internal spline	mm
D_b	Base diameter	mm
D_{ee}	Major diameter, external spline	mm
$D_{ee\,max}$	Maximum major diameter, external spline	mm
$D_{ee\,min}$	Minimum major diameter, external spline	mm
D_{ei}	Major diameter, internal spline	mm
$D_{ei\,max}$	Maximum major diameter, internal spline	mm
$D_{ei\,min}$	Minimum major diameter, internal spline	mm
D_{ie}	Minor diameter, external spline	mm
$D_{ie\,max}$	Maximum minor diameter, external spline	mm
$D_{ie\,min}$	Minimum minor diameter, external spline	mm
D_{ii}	Minor diameter, internal spline	mm
$D_{ii\,max}$	Maximum minor diameter, internal spline	mm
$D_{ii\,min}$	Minimum minor diameter, internal spline	mm
E	Basic circular space width	mm
E_{max}	Maximum actual space width	mm
E_{min}	Minimum actual space width	mm
E_v	Effective space width, circular	mm

$E_{v\,max}$	Maximum effective space width	mm
$E_{v\,min}$	Minimum effective space width	mm
es_v	Fundamental deviation, external	µm
F_p	Total pitch deviation	µm
F_α	Total profile deviation	µm
F_β	Total helix deviation	µm
h_s	Form tooth height	mm
i	Tolerance unit	µm
inv α	Involute α $(=\tan\alpha-\pi\cdot\alpha/180°)$	—
K_e	Approximation factor for external spline	—
K_i	Approximation factor for internal spline	—
k	Number of measured teeth	—
L	Length of the arc	mm
l_{BA}	Length between two points (e.g. point B and point A)	mm
M_{Re}	Measurement over two balls or pins, external splines	mm
M_{Ri}	Measurement between two balls or pins, internal	mm
m	Module	mm
p	Circular pitch	mm
p_b	Base pitch	mm
S	Basic circular tooth thickness	mm
S_{max}	Maximum actual tooth thickness	mm
S_{min}	Minimum actual tooth thickness	mm
S_v	Effective tooth thickness, circular	mm
$S_{v\,max}$	Maximum effective tooth thickness	mm
$S_{v\,min}$	Minimum effective tooth thickness	mm
s_{DE}	Arc length between two points (e.g. point D and point E)	mm
T	Machining tolerance	µm
T_v	Effective clearance tolerance	µm
W	Measurement over k teeth, external spline	mm
z	Number of teeth (for external and internal splines, z has a positive sign)	—
α	Pressure angle	°

α_{ce}	Pressure angle at ball or pin diameter, external spline	°
α_{ci}	Pressure angle at ball or pin diameter, internal spline	°
α_D	Standard pressure angle at pitch diameter	°
α_e	Pressure angle at ball or pin centre, external spline	°
α_{Fe}	Pressure angle at form diameter, external spline	°
α_{Fi}	Pressure angle at form diameter, internal spline	°
α_i	Pressure angle at ball or pin centre, internal spline	°
λ	Deviation allowance	µm
ρ_{Fe}	Fillet radius of the basic rack, external spline	mm
ρ_{Fi}	Fillet radius of the basic rack, internal spline	mm
φ_f	Tolerance factor	—
k; js; h; f; e; d	Fundamental deviation of the external spline	µm

4.2 Subscripts

The following subscripts are used as part of the above general symbols to designate relative conditions or locations:

b	at the base
c	at contact point
D	standard
d	tolerance based on pitch diameter (D)
E	tolerance based on basic circular space width (E) or basic circular tooth thickness (S)
e	major or external (in the last case in the last position)
F	pertaining to form diameter
f	factor
i	minor or internal (in the last case in the last position)
R	pertaining to gauge
v	effective

NOTE In electronic data processing (EDP), it is not always possible to present symbols in their theoretically correct form because of limitations of connected printing equipment. For this reason, some alternative symbols for EDP usage are given in Table 1 (for example, the symbol for D_b for base diameter can be printed as DB).

4.3 Formulae for dimensions and tolerances for all fit classes

The formulae for dimensions and tolerances for all fit classes are given in Table 1.

Table 1 — Formulae for dimensions and tolerances for all fit classes

Term	Symbol	Formula	EDP representation
Pitch diameter	D	$m \cdot z$	D
Base diameter	D_b	$m \cdot z \cdot \cos\alpha_D$	DB
Circular pitch	p	$m \cdot \pi$	P
Base pitch	p_b	$m \cdot \pi \cdot \cos\alpha_D$	PB
Fundamental deviation, external	es_v	Resulting from fundamental deviation k, js, h, f, e and d	ESV
Minimum major diameter, internal:			
30°, flat root	$D_{ei\,min}$	$m \cdot (z+1,5)$	DEIMIN
30°, fillet root	$D_{ei\,min}$	$m \cdot (z+1,8)$	DEIMIN
37,5°, fillet root	$D_{ei\,min}$	$m \cdot (z+1,4)$	DEIMIN
45°, fillet root	$D_{ei\,min}$	$m \cdot (z+1,2)$	DEIMIN
Maximum major diameter, internal	$D_{ei\,max}$	$D_{ei\,min}+(T+\lambda)/\tan\alpha_D$ [a]	DEIMAX
Minimum form diameter, internal:			
30° flat root and fillet root	$D_{Fi\,min}$	$m \cdot (z+1)+2 \cdot c_F$	DFIMIN
37,5° fillet root	$D_{Fi\,min}$	$m \cdot (z+0,9)+2 \cdot c_F$	DFIMIN
45° fillet root	$D_{Fi\,min}$	$m \cdot (z+0,8)+2 \cdot c_F$	DFIMIN
Minimum minor diameter, internal	$D_{ii\,min}$	$D_{Fe\,max}+2 \cdot c_F$ [b]	DIIMIN
Maximum minor diameter, internal:			
$m \leq 0,75$	$D_{ii\,max}$	$D_{ii\,min}$ + IT 10	DIIMAX
$0,75 < m < 2$	$D_{ii\,max}$	$D_{ii\,min}$ + IT 11	DIIMAX
$m \geq 2$	$D_{ii\,max}$	$D_{ii\,min}$ + IT 12	DIIMAX
Basic circular space width	E	$0,5 \cdot \pi \cdot m$	E
Minimum effective space width	$E_{v\,min}$	$0,5 \cdot \pi \cdot m$	EVMIN
Maximum actual space width:			
class 4 to class 7	E_{max}	$E_{v\,min}+(T+\lambda)$ [c]	EMAX
Minimum actual space width	E_{min}	$E_{v\,min}+\lambda$	EMIN
Maximum effective space width	$E_{v\,max}$	$E_{v\,min}+T_v$	EVMAX
Maximum major diameter, external:			
30°, flat root and fillet root	$D_{ee\,max}$	$m \cdot (z+1)+es_v/\tan\alpha_D$ [d]	DEEMAX

[a] $(T+\lambda)$ for class 7 – see 9.1.

[b] For all classes of fit, always take the $D_{Fe\,max}$ value corresponding to the H/h fit.

[c] See Clauses 8, 9 and ISO 4156-2.

[d] Take $es_v = 0$ for fundamental deviation js and k.

[e] For h_s, see Figure 15 and Table 2.

[f] See 9.1.

[g] See ISO 4156-3 concerning the choice of balls or pins.

Table 1 *(continued)*

Term	Symbol	Formula	EDP representation
37,5°, fillet root	$D_{ee\,max}$	$m\cdot(z+0,9)+es_v\,/\tan\alpha_D$ [d]	DEEMAX
45°, fillet root	$D_{ee\,max}$	$m\cdot(z+0,8)+es_v\,/\tan\alpha_D$ [d]	DEEMAX
Minimum major diameter, external:			
$m \leq 0,75$	$D_{ee\,min}$	$D_{ee\,max} - \text{IT } 10$	DEEMIN
$0,75 < m < 2$	$D_{ee\,min}$	$D_{ee\,max} - \text{IT } 11$	DEEMIN
$m \geq 2$	$D_{ee\,min}$	$D_{ee\,max} - \text{IT } 12$	DEEMIN
Maximum form diameter, external[e]	$D_{Fe\,max}$	$2\cdot\sqrt{(0,5D_b)^2+\left(0,5D\cdot\sin\alpha_D-\dfrac{h_s-\dfrac{0,5\cdot es_v}{\tan\alpha_D}}{\sin\alpha_D}\right)^2}$	DFEMAX
Maximum minor diameter, external:			
30°, flat root	$D_{ie\,max}$	$m\cdot(z-1,5)+es_v\,/\tan\alpha_D$	DIEMAX
30°, fillet root	$D_{ie\,max}$	$m\cdot(z-1,8)+es_v\,/\tan\alpha_D$	DIEMAX
37,5°, fillet root	$D_{ie\,max}$	$m\cdot(z-1,4)+es_v\,/\tan\alpha_D$	DIEMAX
45°, fillet root	$D_{ie\,max}$	$m\cdot(z-1,2)+es_v\,/\tan\alpha_D$	DIEMAX
Minimum minor diameter, external	$D_{ie\,min}$	$D_{ie\,max}-(T+\lambda)/\tan\alpha_D$ [a]	DIEMIN
Basic circular tooth thickness	S	$0,5\cdot\pi\cdot m$	S
Maximum effective tooth thickness	$S_{v\,max}$	$S+es_v$	SVMAX
Minimum actual tooth thickness:			
class 4 to class 7	S_{min}	$S_{v\,max}-(T+\lambda)$ [c]	SMIN
Maximum actual tooth thickness	S_{max}	$S_{v\,max}-\lambda$	SMAX
Minimum effective tooth thickness	$S_{v\,min}$	$S_{v\,max}-T_v$	SVMIN
Total tolerance, space width or tooth thickness	$(T+\lambda)$	[f]	TLAM
Maximum effective clearance	$c_{v\,max}$	$E_{v\,max}-S_{v\,min}$	CVMAX
Minimum effective clearance	$c_{v\,min}$	$E_{v\,min}-S_{v\,max}$	CVMIN
Form clearance	c_F	[e]	CF
Form tooth height	h_s	[e]	HS
Ball or pin diameter, internal spline	D_{Ri}	[g]	DRI
Ball or pin diameter, external spline	D_{Re}	[g]	DRE

[a] $(T+\lambda)$ for class 7 – see 9.1.

[b] For all classes of fit, always take the $D_{Fe\,max}$ value corresponding to the H/h fit.

[c] See Clauses 8, 9 and ISO 4156-2.

[d] Take es_v = 0 for fundamental deviation js and k.

[e] For h_s, see Figure 15 and Table 2.

[f] See 9.1.

[g] See ISO 4156-3 concerning the choice of balls or pins.

Table 1 *(continued)*

Term	Symbol	Formula	EDP representation
Measurement between balls or pins	M_{Ri}	[g]	MRI
Measurement over balls or pins	M_{Re}	[g]	MRE
Change factor, internal	K_{i}	[g]	KI
Change factor, external	K_{e}	[g]	KE

[a] $(T+\lambda)$ for class 7 – see 9.1.

[b] For all classes of fit, always take the $D_{\mathrm{Fe\,max}}$ value corresponding to the H/h fit.

[c] See Clauses 8, 9 and ISO 4156-2.

[d] Take $es_{\mathrm{v}} = 0$ for fundamental deviation js and k.

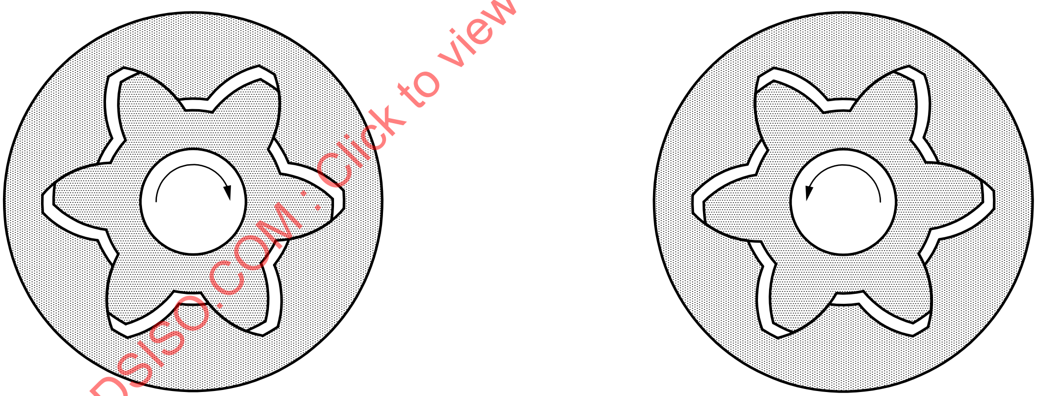
[e] For h_{s}, see Figure 15 and Table 2.

[f] See 9.1.

[g] See ISO 4156-3 concerning the choice of balls or pins.

5 Concept of side fit splines

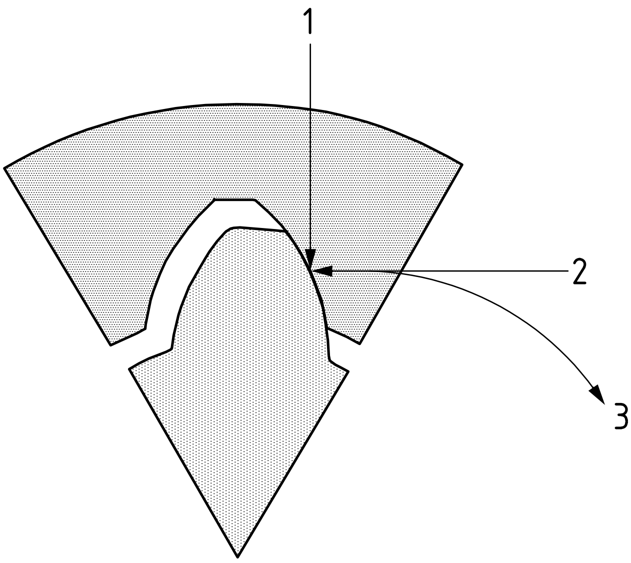
This document defines side fit involute splines with pressure angles of 30°, 37,5° and 45°. The transmission of torque is achieved by contact of the tooth flanks only. This is possible in the clockwise or anticlockwise direction of rotation (see Figure 2). The opposite tooth flanks, major and minor diameters shall have clearance.



a) Clockwise rotation **b) Anticlockwise rotation**

Figure 2 — Side fit tooth flank contact

The nature of the involute profile divides the torque into two directions resulting in a centring force (see Figure 3). This centring force enables side fit involute splines to be centralized by the tooth flanks.



Key

1 centring force
2 rotation force
3 torque

Figure 3 — Centring force

The sizes of space width and tooth thickness (see Figure 4) are defined as the length of the arc at the theoretical pitch circle diameter.

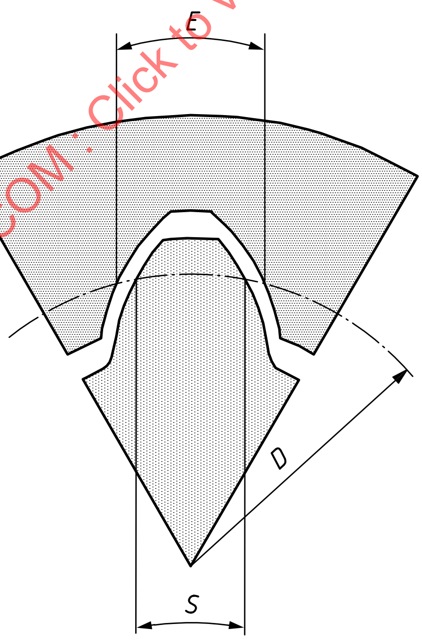


Figure 4 — Space width and tooth thickness

The major and minor diameters (see Figure 5) always have clearance and do not contact each other.

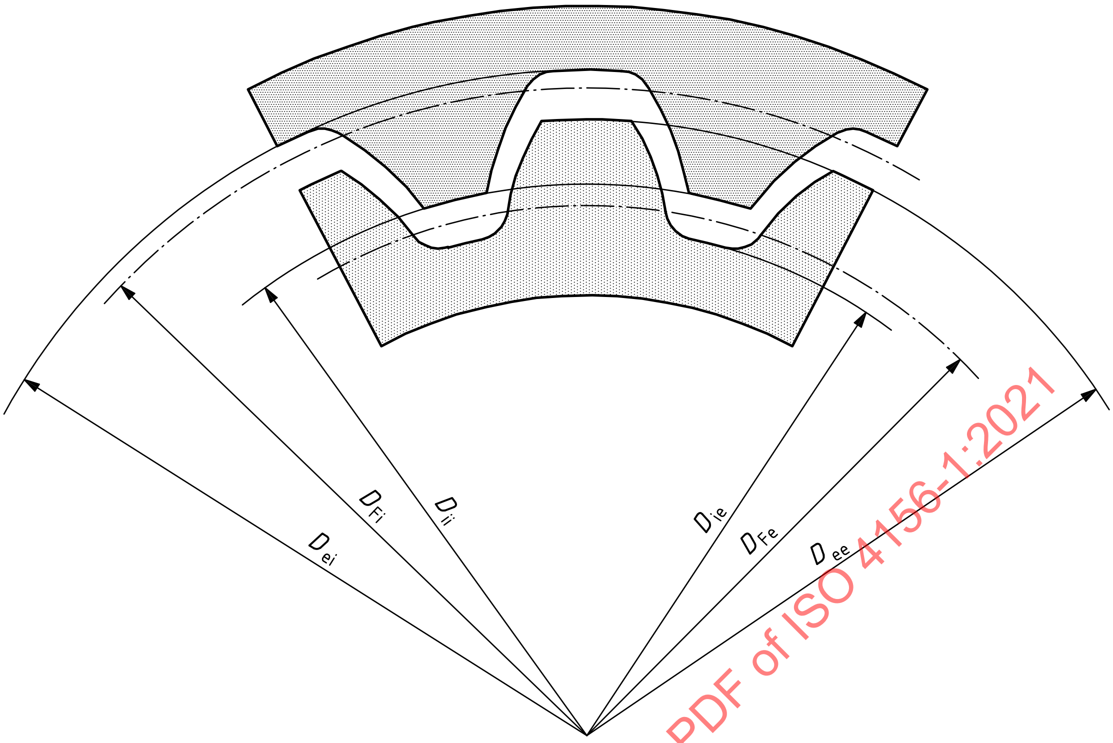


Figure 5 — Diameters

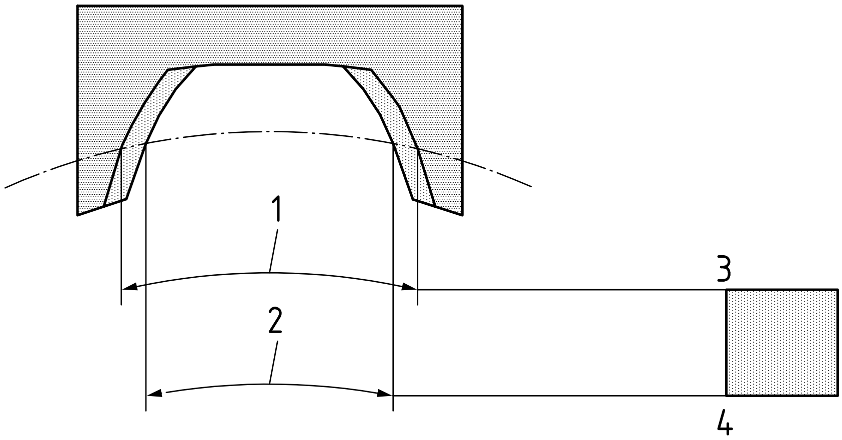
6 Effective fit concept

To be able to machine the spaces of internal splines and the teeth of external splines, a machining tolerance commonly referred to as the actual machining tolerance is necessary. Four classes of machining tolerance (classes 4, 5, 6 and 7) are provided for the different needs of industrial use. The machining tolerance, *T* (see Figure 6), is applied to the space width of internal splines and to the tooth thickness of external splines.

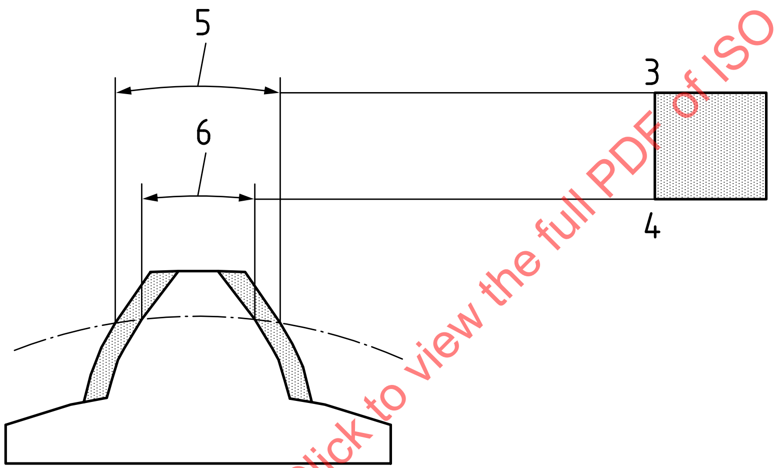
The upper machining tolerance limit is referred to as maximum actual and the lower one is referred to as minimum actual.

Similar to cylindrical fits between hubs and shafts, form deviations of the geometry (see Figure 7) affect the maximum material condition and hence the fit. The form deviation is the deviation compared to the perfect cylinder. The form deviations of splines are much more complex and occur on each flank of every space or tooth. These form deviations have an accumulative effect which is referred to as effective deviation.

The form deviations consist of three types: profile deviation, pitch deviation and helix deviation. The positive material elements of these deviations result in a reduction of effective space width of an internal spline, or an increase in the effective tooth thickness of an external spline, and hence a reduction in the effective clearance. This effect can only be detected by the use of an imaginary perfect mating spline that fits without looseness or interference.



a) Internal spline

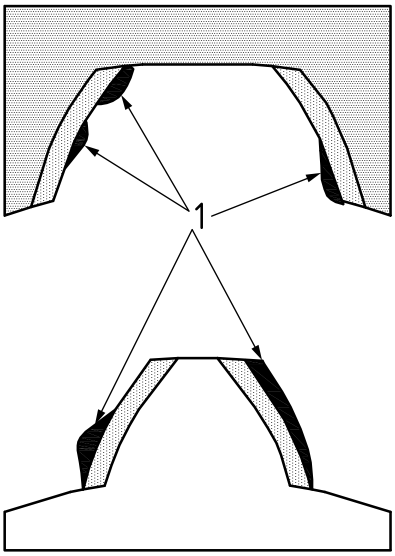


b) External spline

Key

1 largest space width
2 smallest space width
3 maximum actual tolerance
4 minimum actual tolerance
5 largest tooth thickness
6 smallest tooth thickness

Figure 6 — Machining tolerance, *T*

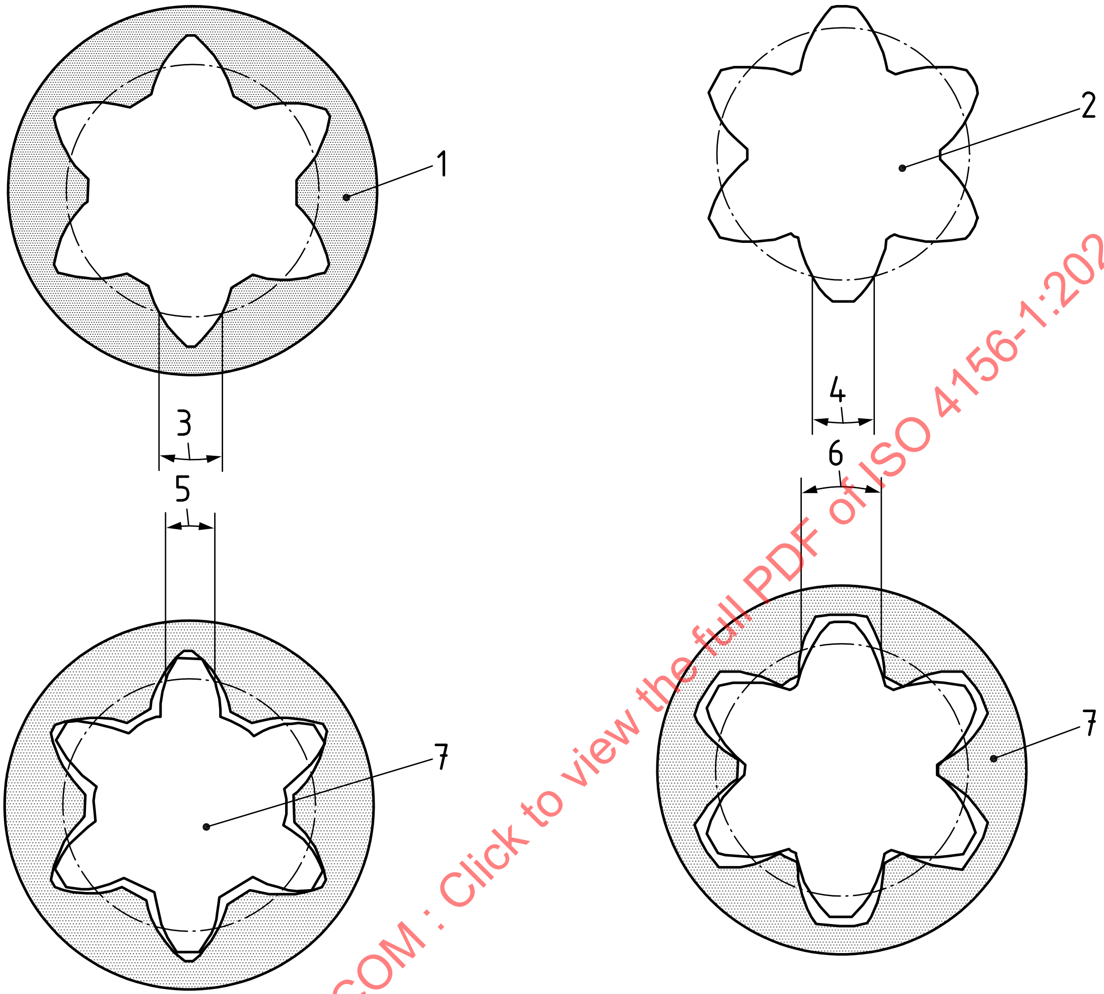


Key

1 form deviation

Figure 7 — Form deviations

The positive material elements of profile deviation (see Figure 8) result in a smaller space or a larger tooth thickness which has an effect on the fit with a mating part.

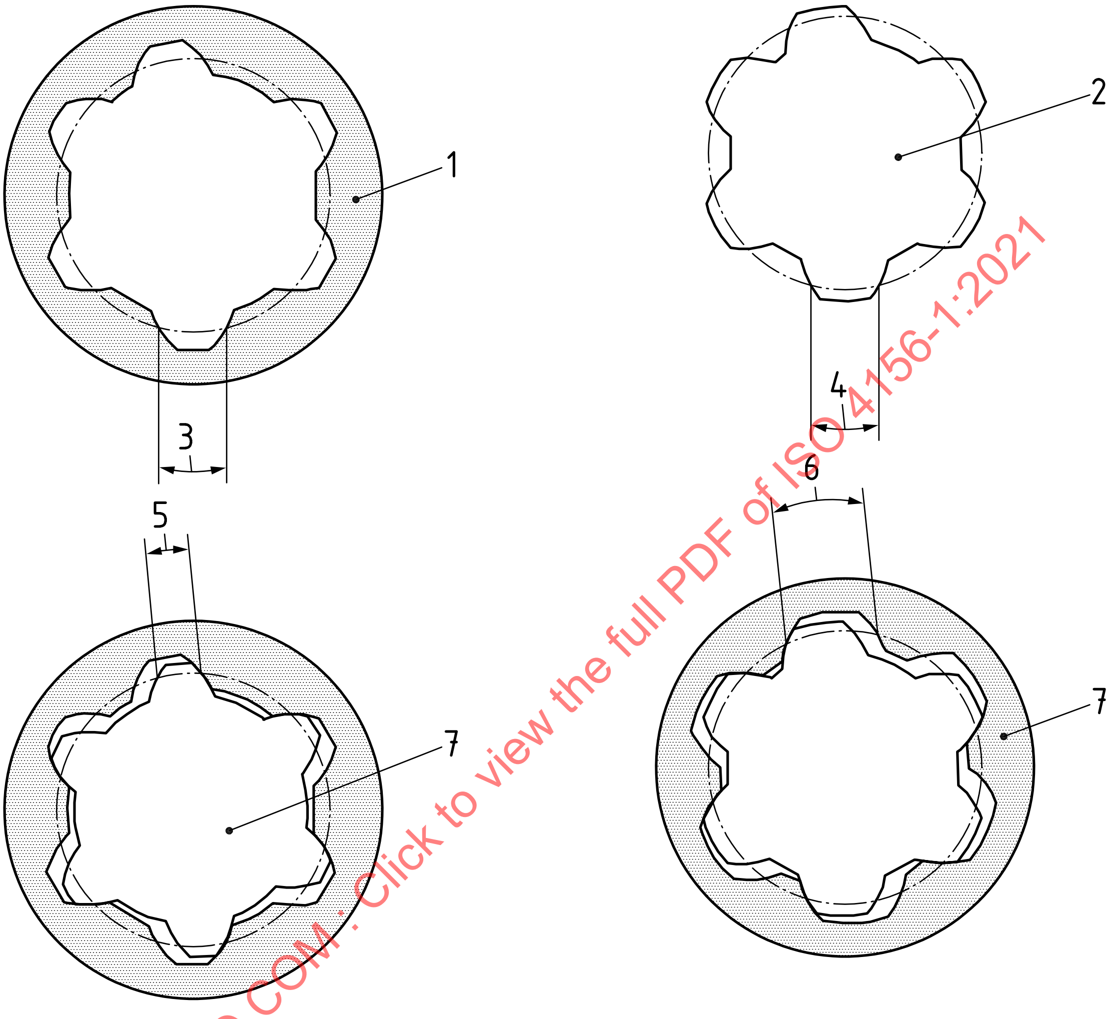


Key

1 internal spline
2 external spline
3 space width, actual
4 tooth thickness, actual
5 space width, effective
6 tooth thickness, effective
7 mating part

Figure 8 — Profile deviation

The positive material elements of pitch deviation (see Figure 9) also result in a smaller space width or a larger tooth thickness which again affects the fit with a mating part.

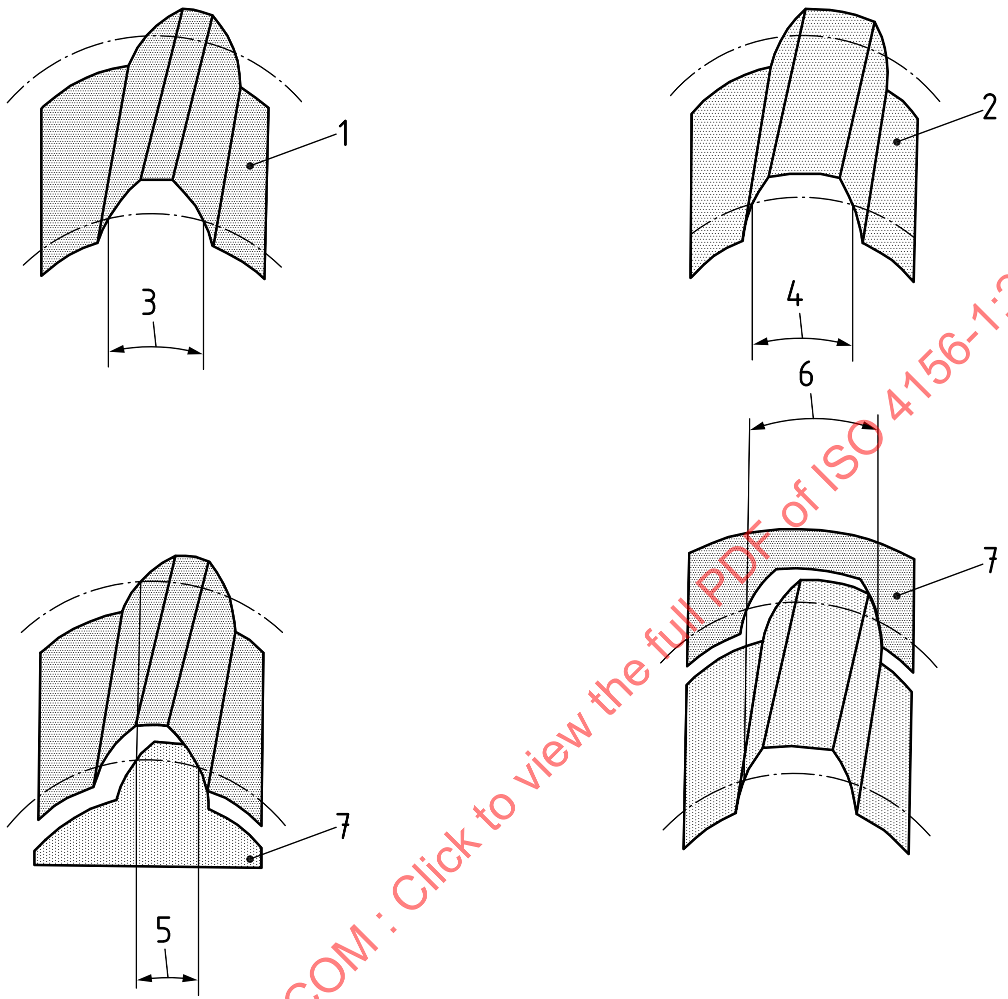


Key

1 internal spline
2 external spline
3 space width, actual
4 tooth thickness, actual
5 space width, effective
6 tooth thickness, effective
7 mating part

Figure 9 — Pitch deviation

The positive material elements of helix deviation (see Figure 10) also result in a smaller space width or a larger tooth thickness which again affects the fit with a mating part.

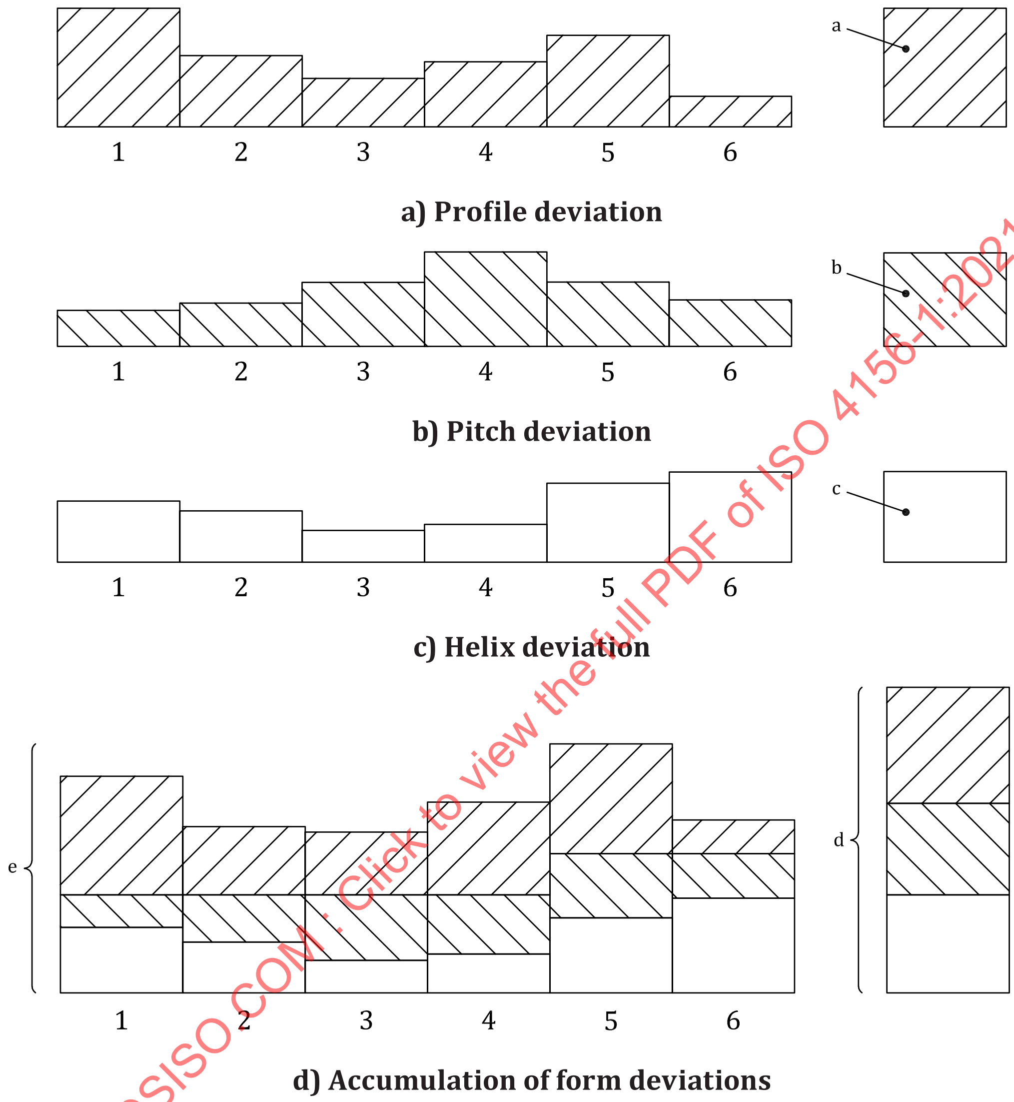


Key

1 internal spline
2 external spline
3 space width, actual
4 tooth thickness, actual
5 space width, effective
6 tooth thickness, effective
7 mating part

Figure 10 — Helix deviation

The accumulated form deviations (see Figure 11) of each flank result in an effective size of space width or tooth thickness.



Key

1-6 tooth numbers

a Maximum at tooth 1.

b Maximum at tooth 4.

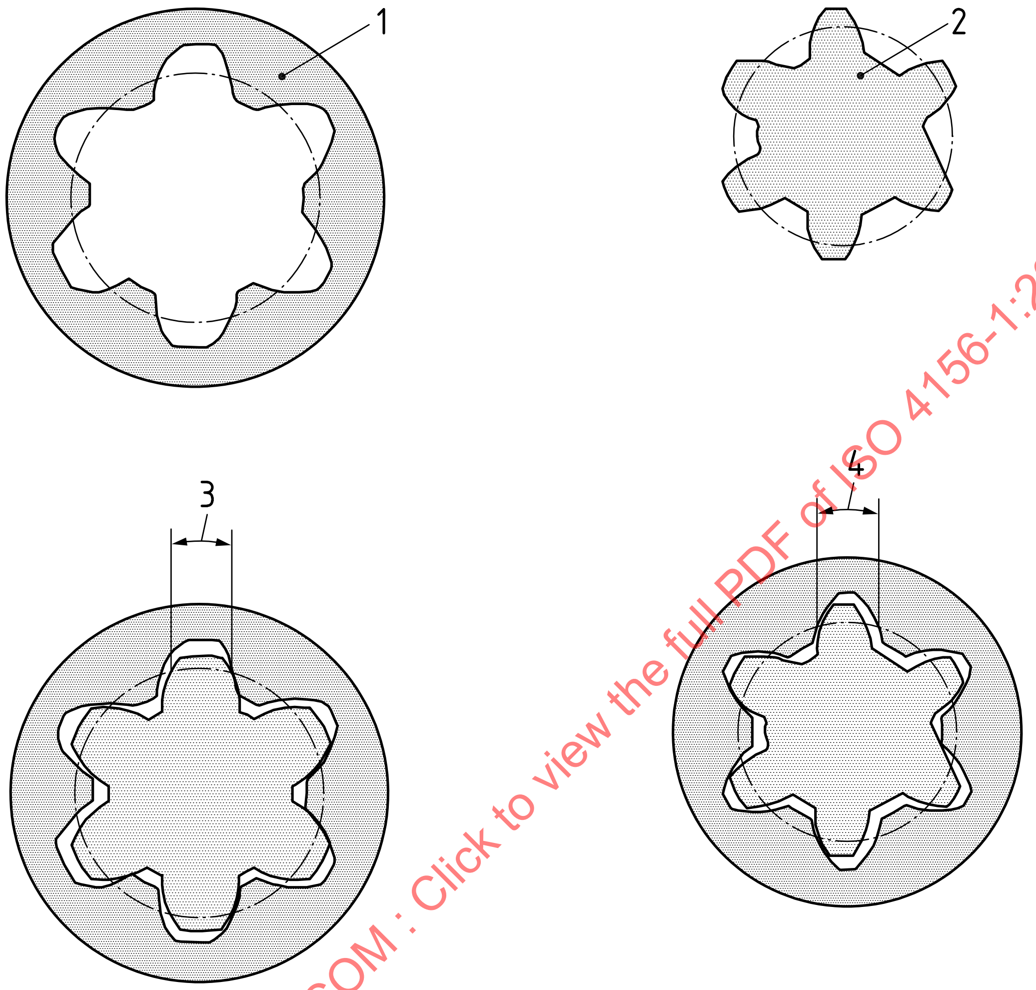
c Maximum at tooth 6.

d Theoretical maximum.

e Effective deviation.

Figure 11 — Influence of individual form deviations

The true effective size of a spline part with accumulated form deviations can only be found using an imaginary perfect mating spline that fits without looseness or interference (see Figure 12).

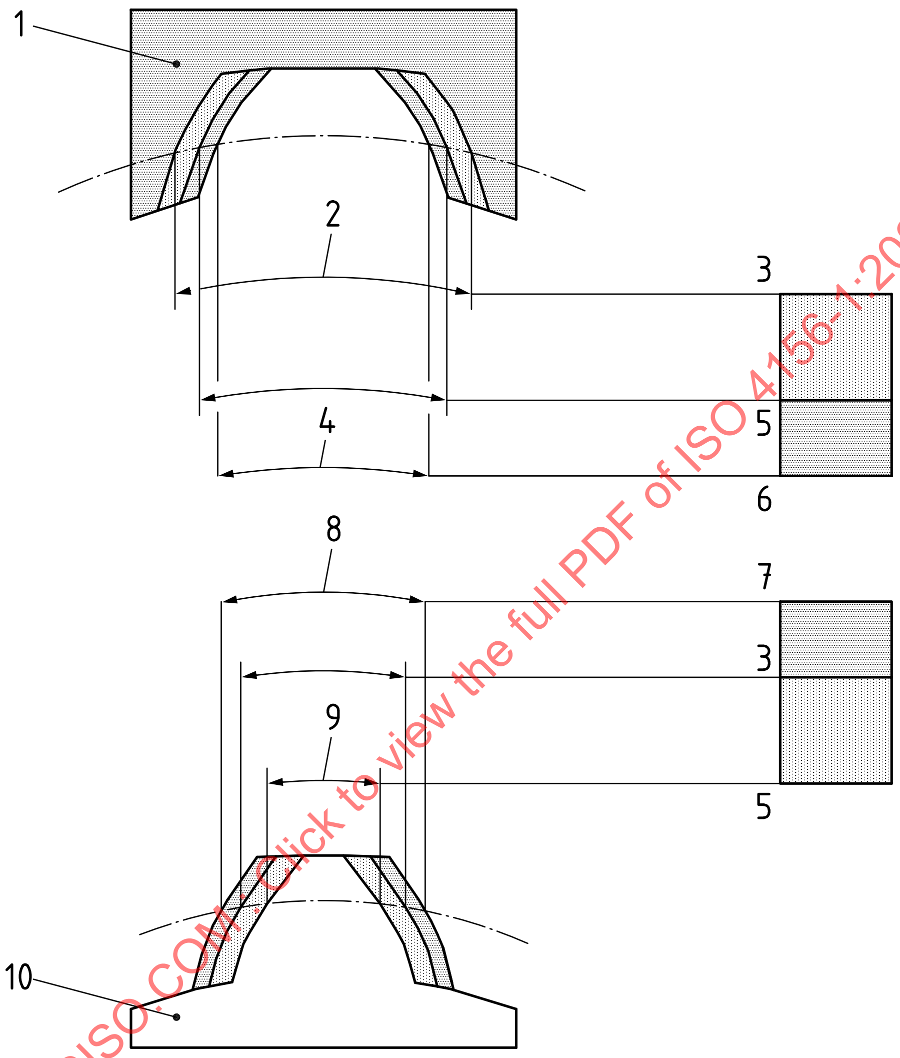


Key

1 internal spline with form deviations
2 external spline with form deviations
3 internal effective space width
4 external effective tooth thickness

Figure 12 — True effective space width and tooth thickness

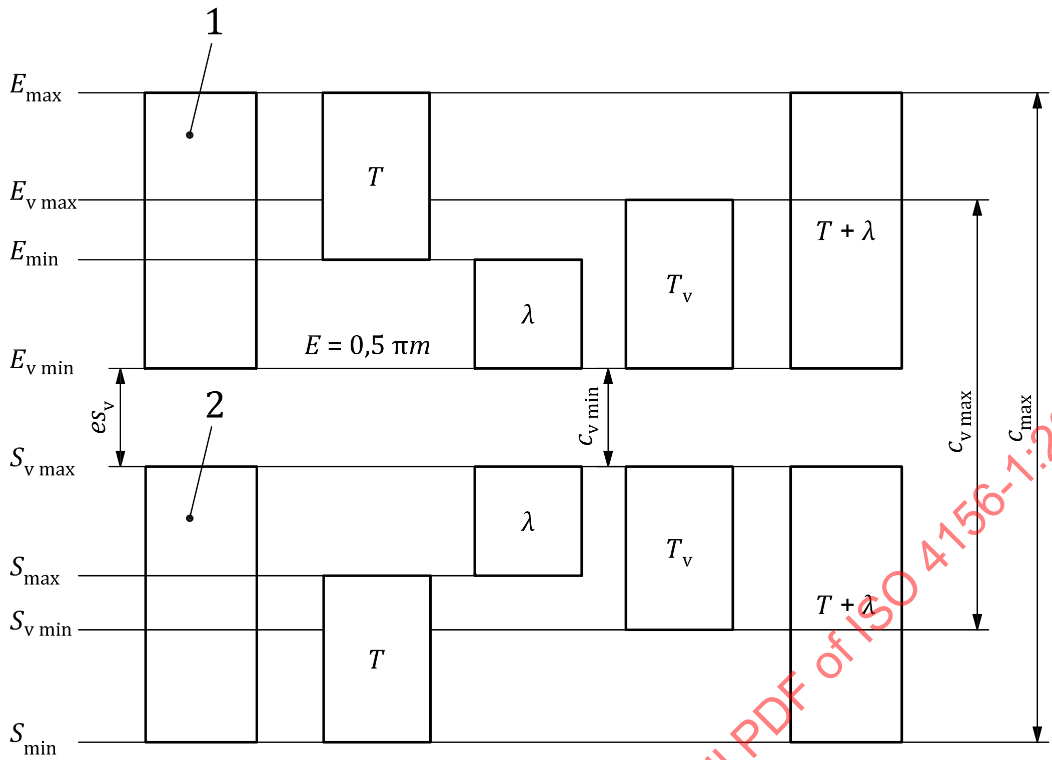
In addition to the machining tolerance and because of the form deviations, spline parts have an effective tolerance (see Figure 13). For internal parts, this creates a minimum effective tolerance limit of space width, and for external parts, a maximum effective tolerance limit of tooth thickness. See also Figure 14.



Key

1 internal spline
2 largest space width
3 maximum actual tolerance
4 smallest space width
5 minimum actual tolerance
6 minimum effective tolerance of space width
7 maximum effective tolerance of tooth thickness
8 largest tooth thickness
9 smallest tooth thickness
10 external spline

Figure 13 — Actual and effective tolerances



Key

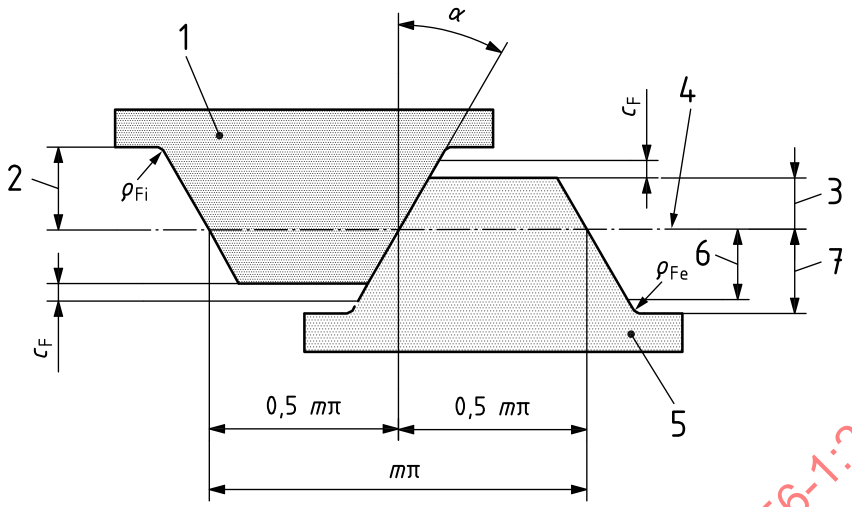
1 space width, internal

2 tooth thickness, external

Figure 14 — Graphical display of space width and tooth thickness theoretical tolerance zones

7 Basic rack profiles for spline

The basic rack is a section of the tooth surface on an involute spline of infinitely large diameter on a plane at right angles to the tooth surfaces, the profile of which is used as the basis for defining the standard tooth dimensions of a system of involute splines. The reference line is a straight line crossing the profile of the basic rack, with reference to which the tooth dimensions are specified. The profile of the basic rack for standard pressure angle splines is represented in Figure 15 and Table 2.



Key

1 internal spline
2 major space height
3 major tooth height
4 pitch line
5 external spline
6 form tooth height
7 minor tooth height

NOTE For internal splines (hub), the form diameter, obtained by generating from the basic rack, is always greater than the form diameter shown in the tables of dimensions in ISO 4156-2, which correspond in all fit cases to the major maximum diameter of the shaft increased to diametrical form clearance ($2c_F$). For external splines (shafts), c_F is obtained by generation from the basic rack ($D_{Fe\,max}$) and for H/h fit (see footnote [b] to Table 1).

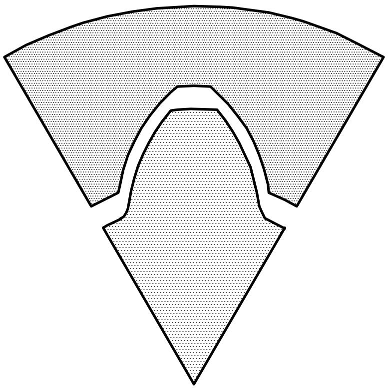
Figure 15 — Basic rack profile

Table 2 — Dimensions of basic rack

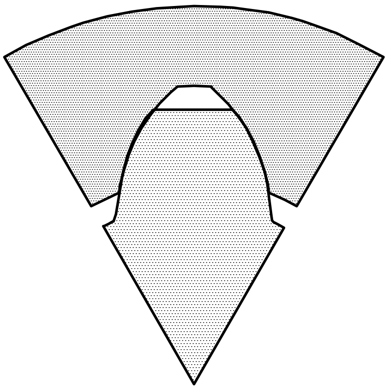
Parameter	Pressure angle			
	30°		37,5°	45°
	Flat root	Fillet root		
Major space height	0,75 *m*	0,9 *m*	0,7 *m*	0,6 *m*
Major tooth height	0,5 *m*	0,5 *m*	0,45 *m*	0,4 *m*
Form tooth height, h_s	0,6 *m*	0,6 *m*	0,55 *m*	0,5 *m*
Minor tooth height	0,75 *m*	0,9 *m*	0,7 *m*	0,6 *m*
Root radius, ρ_{Fi}	0,2 *m*	0,4 *m*	0,3 *m*	0,25 *m*
Root radius, ρ_{Fe}	0,2 *m*	0,4 *m*	0,3 *m*	0,25 *m*
Form clearance, c_F	0,1 *m*	0,1 *m*	0,1 *m*	0,1 *m*

8 Spline fit classes

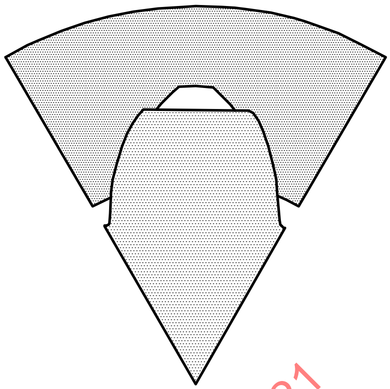
To achieve different amounts of clearance or interference between the space width and the tooth thickness, this document has a number of fit classes (see Figure 16). These result in different amounts of clearance or interference.



a) Loose fit



b) Zero fit

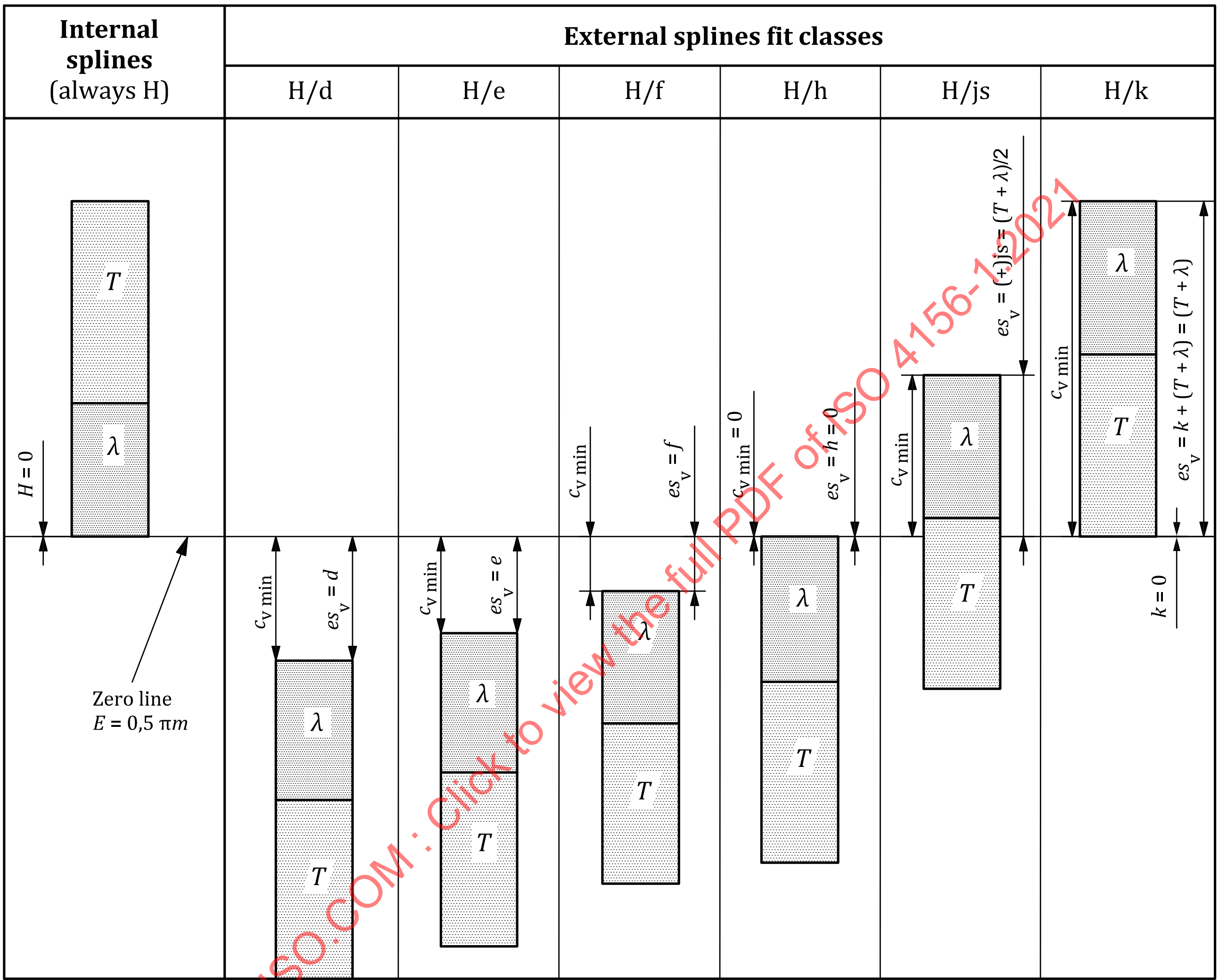


c) Press fit

Figure 16 — Types of fit

This document provides standard fundamental deviation k, js, h, f, e and d for application to the basic circular tooth thickness (S) at the pitch diameter of the external spline, in order to establish the spline fit classes without looseness or having maximum effective interferences or minimum effective clearances (see Table 6), and thus standardizing on composite GO gauges. Table 3 represents in graphical form the fundamental deviations and spline class tolerance zones for the six spline fit classes.

Table 3 — Graphical representation of fundamental deviations for spline fit classes



The required maximum effective interference or minimum effective looseness (see Table 4) shall be obtained by adjusting from the zero line the maximum effective tooth thickness by the fundamental deviation es_v (see Tables 3 and 5), whilst maintaining machining tolerance T and deviation allowance λ. The spline dimensions in all spline tables of ISO 4156-2 apply to class H/h.

Table 4 — Effective interference and effective looseness of spline fit classes

Spline fit class		Minimum effective looseness
H/d	es_v = fundamental deviation d	$c_{v\,min} = -es_v$
H/e	es_v = fundamental deviation e	$c_{v\,min} = -es_v$
H/f	es_v = fundamental deviation f	$c_{v\,min} = -es_v$
H/h	es_v = fundamental deviation h = 0	$c_{v\,min} = -es_v = 0$
		Maximum effective interference
H/js	es_v = fundamental deviation js	$c_{v\,min} = -es_v = -(T+\lambda)/2$
H/k	$es_v = (T+\lambda)$	$c_{v\,min} = -es_v = -(T+\lambda)$

Table 5 — Fundamental deviation es_v

Pitch diameter D mm	Fundamental deviation es_v µm at pitch diameter D						
	Relative to basic circular tooth thickness S for externals						Relative to basic circular space width E for internals
	For						
	d	e	f	h	js	k	H
≤ 3	−20	−14	−6	0	a	b	0
> 3 to 6	−30	−20	−10	0			0
> 6 to 10	−40	−25	−13	0			0
> 10 to 18	−50	−32	−16	0			0
> 18 to 30	−65	−40	−20	0			0
> 30 to 50	−80	−50	−25	0			0
> 50 to 80	−100	−60	−30	0			0
> 80 to 120	−120	−72	−36	0			0
> 120 to 180	−145	−85	−43	0			0
> 180 to 250	−170	−100	−50	0			0
> 250 to 315	−190	−110	−56	0			0
> 315 to 400	−210	−125	−62	0			0
> 400 to 500	−230	−135	−68	0			0
> 500 to 630	−260	−145	−76	0			0
> 630 to 800	−290	−160	−80	0			0
> 800 to 1 000	−320	−170	−86	0			0

[a] $+(T + \lambda)/2$ relative to tolerance class considered; for $T + \lambda$, see 9.1.

[b] $+(T + \lambda)$ relative to tolerance class considered; for $T + \lambda$, see 9.1.

9 Space width and tooth thickness tolerances

9.1 Total tolerance $T + \lambda$

This document includes four classes of total tolerance $(T + \lambda)$ on space width and tooth thickness. The tolerance classes are indicated in Table 6 incorporating tolerance units (i).

Table 6 — Total space width and tooth thickness tolerance (T+λ)

Spline tolerance class	**Total tolerance** $(T + \lambda)$ µm
4	$T+\lambda=(10\cdot i_d+40\cdot i_E)$
5	$T+\lambda=(16\cdot i_d+64\cdot i_E)$
6	$T+\lambda=(25\cdot i_d+100\cdot i_E)$
7	$T+\lambda=(40\cdot i_d+160\cdot i_E)$

where

$$i_d=0{,}45\cdot\sqrt[3]{D}+0{,}001\cdot D \text{ for } D\leq 500 \text{ mm} \quad (1)$$

$$i_d = 0{,}004 \cdot D + 2{,}1 \text{ for } D > 500 \text{ mm} \quad (2)$$

$$i_E = 0{,}45 \cdot \sqrt[3]{E\,(\text{or } S)} + 0{,}001 \cdot E\,(\text{or } S) \quad (3)$$

9.2 Deviation allowance, λ

The deviation allowance, being the accumulation of the total pitch deviation, total profile deviation and total helix deviation, has an effect on the effective fit of an involute spline. The effect of these individual spline deviations on the fit is less than their total, because areas of more than minimum clearance can have form, helix, or pitch deviations without changing the fit. It is also unlikely that these deviations would occur in their maximum amounts simultaneously on the same spline. For this reason, total pitch deviation, profile deviation and total helix deviation are added together statistically and 60 % of this total is taken to determine the effect that these deviations have on the spline fit. On this basis, the deviation allowance is calculated as follows:

$$\lambda = 0{,}6\sqrt{F_p^2 + F_\alpha^2 + F_\beta^2} \quad (4)$$

NOTE Adding the values of the single deviations when they are individually rounded to a micrometer, this can result in a difference of one micrometer to those expressed in Tables A.1 to A.4.

In 9.3 to 9.5, the values of F_p, F_α and F_β are referenced to the datum of the effective spline axis. See ISO 4156-3.

9.3 Total pitch deviation, F_p

The total pitch deviation is the cumulative pitch deviation between the two greatest opposite pitch deviations over circumference. The formulae given in Table 7 are used to calculate the total pitch deviation F_p expressed in micrometres.

Table 7 — Total pitch deviation

Spline tolerance class	Total pitch deviation F_p µm
4	$F_p = 2{,}5 \cdot \sqrt{L} + 6{,}3$
5	$F_p = 3{,}55 \cdot \sqrt{L} + 9$
6	$F_p = 5 \cdot \sqrt{L} + 12{,}5$
7	$F_p = 7{,}1 \cdot \sqrt{L} + 18$

L is the length of the arc, in millimetres:

$$L = m \cdot z \cdot \pi / 2 \quad (5)$$

9.4 Total profile deviation, F_α

The total profile deviation is the absolute value of the difference between the greatest positive and negative deviations from the theoretical tooth profile, measured normal to the flanks. A positive deviation is in the direction of the space, and a negative deviation is in the direction of the tooth, as shown in Figure 17. The formulae given in Table 8 are used to calculate the total profile deviation F_α expressed in micrometres.

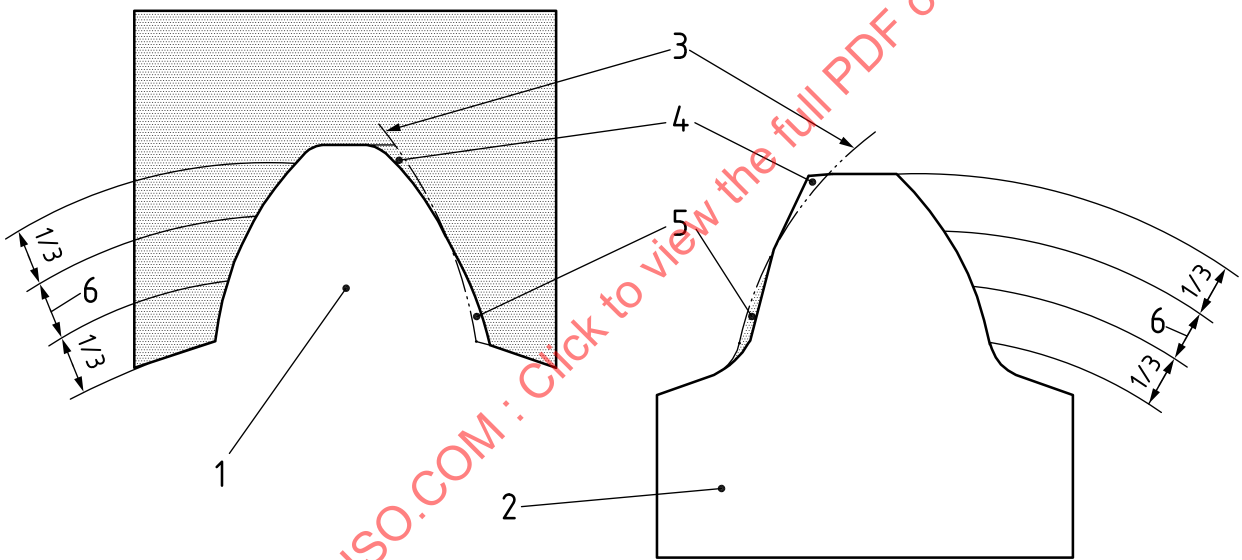
Table 8 — Total profile deviation

Spline tolerance class	Total profile deviation F_α µm
4	$F_\alpha = 1{,}6 \cdot \varphi_f + 10$
5	$F_\alpha = 2{,}5 \cdot \varphi_f + 16$
6	$F_\alpha = 4 \cdot \varphi_f + 25$
7	$F_\alpha = 6{,}3 \cdot \varphi_f + 40$

φ_f is the tolerance factor:

$$\varphi_f = m + 0{,}012\ 5 \cdot m \cdot z \tag{6}$$

The permissible positive deviation on external splines and the negative deviation on internal splines from the design profile within the central one-third of the flank depth to the form diameter shall not exceed one-third of the calculated values. See Figure 17.



Key

1 space (internal)
2 tooth (external)
3 reference profile
4 positive profile deviation
5 negative profile deviation
6 centre third

Figure 17 — Profile deviations

9.5 Total helix deviation, F_β

The total helix deviation is the absolute value of the difference between the two extreme deviations from the theoretical direction, measured normal to the flank for the full length of spline. The formulae given in Table 9 are used to calculate the total helix deviation F_β, expressed in micrometres.

Table 9 — Total helix deviation

Spline tolerance class	Total helix deviation F_β µm
4	$F_\beta = 0{,}8 \cdot \sqrt{b} + 4$
5	$F_\beta = \sqrt{b} + 5$
6	$F_\beta = 1{,}25 \cdot \sqrt{b} + 6{,}3$
7	$F_\beta = 2 \cdot \sqrt{b} + 10$
b is the spline length, in millimetres.	

9.6 Machining tolerance, T

The machining tolerance (T) is derived from the difference between the total tolerance ($T + \lambda$) and the deviation allowance λ, i.e. ($T + \lambda$) − λ. The division of total class tolerance between machining tolerance and deviation allowance outlined in Clause 6 is recommended for general guidance. Design requirements or specific processes utilized for spline manufacture may require a different division.

$$T = (T + \lambda) - \lambda \tag{7}$$

9.7 Effective clearance tolerance, T_v

If T_v is necessary, it is recommended that it be made equal to T.

9.8 Use of effective and actual dimensions for space width and tooth thickness

9.8.1 Minimum material

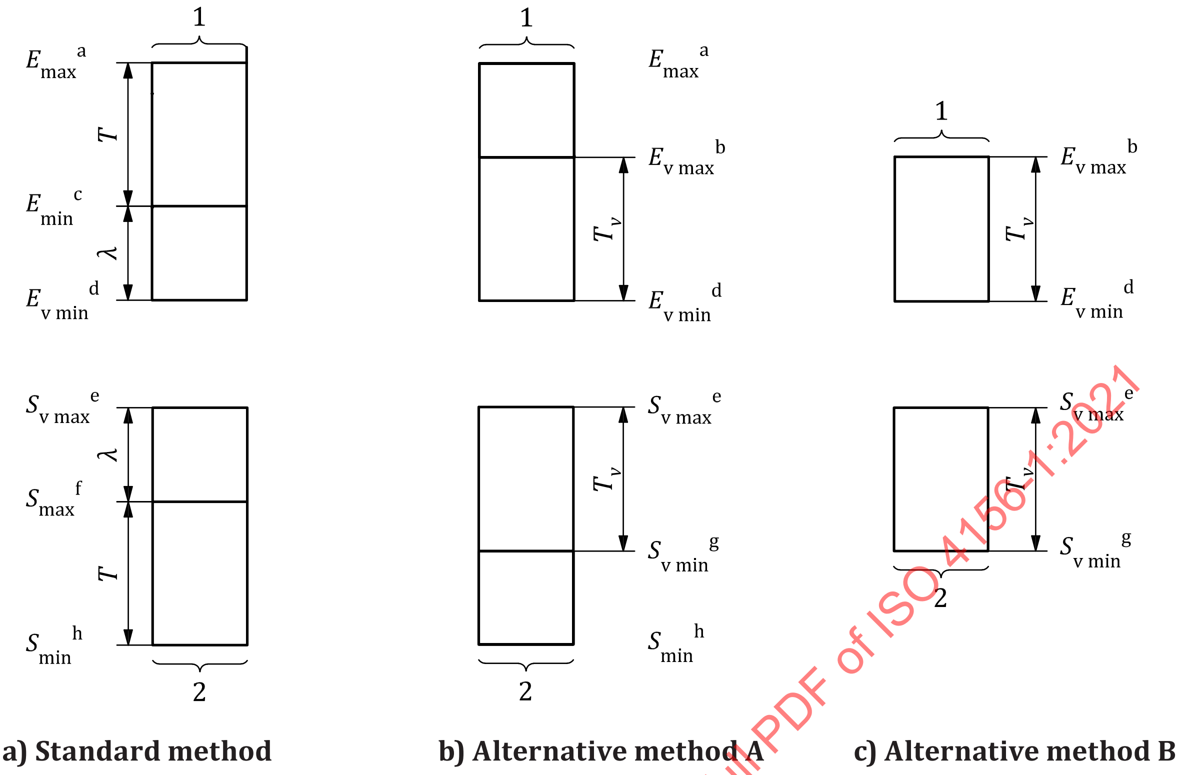
It is limited by maximum actual space width E_{max} and minimum actual tooth thickness S_{min} (see Figure 18 and Table 10).

9.8.2 Maximum material (minimum effective clearance)

It is limited by minimum effective space width $E_{v\,min}$ and maximum effective tooth thickness $S_{v\,max}$ (see Figure 18 and Table 10).

9.8.3 Maximum effective clearance

It is limited by maximum effective space width $E_{v\,max}$ and minimum effective tooth thickness $S_{v\,min}$ (see Figure 18 and Table 10). If it is necessary to control the effective tolerance band, then additional inspection is required.



Key

1 internal space width

2 external tooth thickness

[a] E_{max} is the maximum actual tolerance limit.

[b] $E_{v\ max}$ is the maximum effective tolerance limit.

[c] E_{min} is the minimum actual auxiliary.

[d] $E_{v\ min}$ is the minimum effective tolerance limit.

[e] $S_{v\ max}$ is the maximum effective tolerance limit.

[f] S_{max} is the maximum actual auxiliary.

[g] $S_{v\ min}$ is the minimum effective tolerance limit.

[h] S_{min} is the minimum actual tolerance limit.

Figure 18 — Graphical display of space width and tooth thickness tolerance zones according to inspection methods

Table 10 — Relationship between parameters and control method

	Minimum material	Minimum effective clearance	Maximum effective clearance
Parameters	S_{min}/E_{max}	$S_{v\ max}/E_{v\ min}$	$S_{v\ min}/E_{v\ max}$
Standard method	X	X	—
Method A	X	X	X
Method B	—	X	X

10 Minor and major diameters

10.1 Tolerances

See Table 11.

Table 11 — Tolerances for minor diameter internal spline, D_{ii} and for major diameter external spline, D_{ee}

Diameter $D_{ii\,min}$ or $D_{ee\,max}$ mm	Tolerances on D_{ii} µm for modules m			Tolerances on D_{ee} µm for modules m		
	$m \leq 0,75$	$0,75 < m < 2$	$m \geq 2$	$m \leq 0,75$	$0,75 < m < 2$	$m \geq 2$
	H10	H11	H12	h10	h11	h12
≤ 3	+40 0			0 −40		
> 3 to 6	+48 0	+75 0		0 −48	0 −75	
> 6 to 10	+58 0	+90 0		0 −58	0 −90	
> 10 to 18	+70 0	+110 0	+180 0	0 −70	0 −110	0 −180
> 18 to 30	+84 0	+130 0	+210 0	0 −84	0 −130	0 −210
> 30 to 50	+100 0	+160 0	+250 0	0 −100	0 −160	0 −250
> 50 to 80	+120 0	+190 0	+300 0	0 −120	0 −190	0 −300
> 80 to 120		+220 0	+350 0		0 −220	0 −350
> 120 to 180		+250 0	+400 0		0 −250	0 −400
> 180 to 250			+460 0			0 −460
> 250 to 315			+520 0			0 −520
> 315 to 400			+570 0			0 −570
> 400 to 500			+630 0			0 −630
> 500 to 630			+700 0			0 −700
> 630 to 800			+800 0			0 −800
> 800 to 1 000			+900 0			0 −900

10.2 Adjustment to minor diameters (D_{ie}), form diameters (D_{Fe}) and major diameters (D_{ee}) of external splines

When fundamental deviation d, e and f are applied to external splines, it is necessary to adjust the major, form and minor diameters. When applying js and k classes, the minor and form diameters only are adjusted, see formulae and notes in Table 1.

11 Manufacturing and design considerations

11.1 Radii

External splines may be produced by generating with a pinion-type shaper cutter or a hob, or by cutting with a no-generating motion using a tool formed to the contour of a tooth space. External splines are also made by cold forming, and in these cases are usually of the fillet root design. Internal splines are usually produced by broaching, by form cutting, or by generating with a shaper cutter. Even when full-tip radius tools are used, each of these cutting methods produces a fillet contour with individual characteristics. Generated spline fillets are curves related to the prolate epicycloid for external splines and the prolate hypocycloid for internal splines. These fillets have a minimum radius of curvature at the point where the fillet is tangent to the external spline minor diameter circle or the internal spline major diameter circle, and a rapidly increasing radius of curvature up to the point where the fillet becomes tangent to the involute profile.

The values in Table 12 may be used as minimum radii of curvature for stress calculations and specified as the minimum fillet radius. These values are based on the fillet radius shown on the basic rack profiles. For cutting of internal and external splines by generating, with the same pinion-type shaper cutter, the tool design shall be made according to the dimensions of the internal splines to be obtained.

Table 12 — Minimum root radius of internal or external splines

Minimum root radius mm for			
$\alpha_D = 30°$ flat root	$\alpha_D = 30°$ fillet root	$\alpha_D = 37{,}5°$	$\alpha_D = 45°$
0,2 m	0,4 m	0,3 m	0,25 m

11.2 Profile shifts

In the manufacture of external splines with a standard fundamental deviation, standard cutters conforming to the basic rack can be used by shifting the basic rack reference line radially from its position touching the spline pitch circle. The spline tooth profiles remain as involutes; merely different parts of the same involutes, lying further inward or outward with respect to the pitch circle, are used. The amount of this radial displacement is equivalent to $0{,}5\, es_v / \tan \alpha_D$.

The effect of the various magnitudes of fundamental deviation (es_v) on the tooth form have an effect on the circular tip thickness, at the major diameter of the external spline, which varies when the fundamental deviation (es_v) varies. The following checks should be made when applying fundamental deviations:

a) Check for tip thickness at major maximum diameter of external spline

A tip thickness of less than 0,25 m should be avoided. Tip thickness is computed as follows, considering $D_{ee\,max}$:

$$S_{ee} = D_{ee\,max}\left[\left(S_{min} / D\right) + \operatorname{inv}\alpha_D - \operatorname{inv}\alpha_{Dee}\right] \tag{8}$$

where

S_{ee} is the tip thickness;

α_{Dee} is the pressure angle at the maximum major diameter,

$$\cos\alpha_{Dee} = \frac{D_b}{D_{ee\,max}} \tag{9}$$

$$\mathrm{inv}\alpha_D = \tan\alpha_D - \alpha_D \cdot \pi/180 \tag{10}$$

$$\mathrm{inv}\alpha_{Dee} = \tan\alpha_{Dee} - \alpha_{Dee} \cdot \pi/180 \tag{11}$$

b) Undercut fillet at the root circle (interference)

When a fit with looseness is selected, generating tools may produce undercut fillets on the external spline. Calculation using the appropriate tool data should be made to establish the presence or absence of undercutting and, if it exists, further calculations or a layout are then required to determine whether the degree of undercutting is acceptable.

Low tooth numbers and pressure angles introduce the risk of undercutting, which occurs with basic rack generation when

$$\left(h_s + 0{,}5 \cdot \Delta D_{ie}\right) > 0{,}5 \cdot D \cdot \sin^2 \alpha_D \tag{12}$$

11.3 Eccentricity and misalignment

11.3.1 Eccentricity

The design assembly can make it necessary to apply a tolerance to the position of the effective spline axis and the functional axis of the part. To facilitate this, symbols from ISO 1101 shall be used. This document does not recommend any values, but they shall instead be defined according to the assembly requirements.

11.3.2 Misalignment

If the assembly requires looseness or interference and the axis of the mating splines are misaligned, it is necessary to increase the minimum effective clearance. This shall be accomplished by reducing the effective and actual tooth thickness of the external spline by an amount which is sufficient to allow the misaligned splines to assemble.

11.3.3 Major and minor diameters

The eccentricity of major and minor diameters does not cause interference with the form diameters of the mating spline even when there is maximum clearance. However, if misalignment exists within the assembly, it is necessary to reduce the major diameter of the external spline or increase the minor diameter of the internal spline.

12 Spline data

12.1 Basic dimensions

Spline data is used for engineering and manufacturing purposes.

12.2 Combination of types

Flat root side fit splines may be used with fillet root splines where the larger radius is desired for control of stress concentrations or manufacturing purposes. These combinations of spline roots may also be permitted as design options.

The major diameter of a flat root internal spline and, if needed, the diameter at the tangency point of radius should be placed between the form diameter and the maximum major diameter of the fillet root internal spline.

12.3 Designation

The mating parts of straight cylindrical involute splines (metric modules, side fit) shall be designated in the following order:

Internal spline	INT	
External spline	EXT	
Mating splines	INT/EXT	
Number of teeth	z (preceded by the number)	
Module	m (preceded by the value)	
Pressure angle	30° flat root	30 P
Pressure angle	30° fillet root	30 R
Pressure angle	37,5°	37,5
Pressure angle	45°	45
Tolerance classes	4, 5, 6, 7	
Fit class, internal spline	H	
Fit class, external spline	k; js; h; f; e; d	
ISO 4156		

EXAMPLE Mating splines, 24 teeth; module 2,5; pressure angle 30° fillet root; tolerance class 5; fit

Mating: INT/EXT 24z × 2,5m × 30R × 5H/5f ISO 4156

Internal spline: INT 24z × 2,5m × 30R × 5H ISO 4156

External spline: EXT 24z × 2,5m × 30R × 5f ISO 4156

12.4 Drawing data

It is important that uniform specifications be used to show complete information on detail drawings of splines. Much misunderstanding can be avoided by following the suggested arrangement of dimensions and data, as shown in Figure 19 and Table 13. The number of X indicates the number of decimal places normally used. With this charted type of spline specification, it is usually not necessary to give a graphical illustration of the spline teeth. Annex A provides a guide for the calculation of drawing data for all spline fit classes.

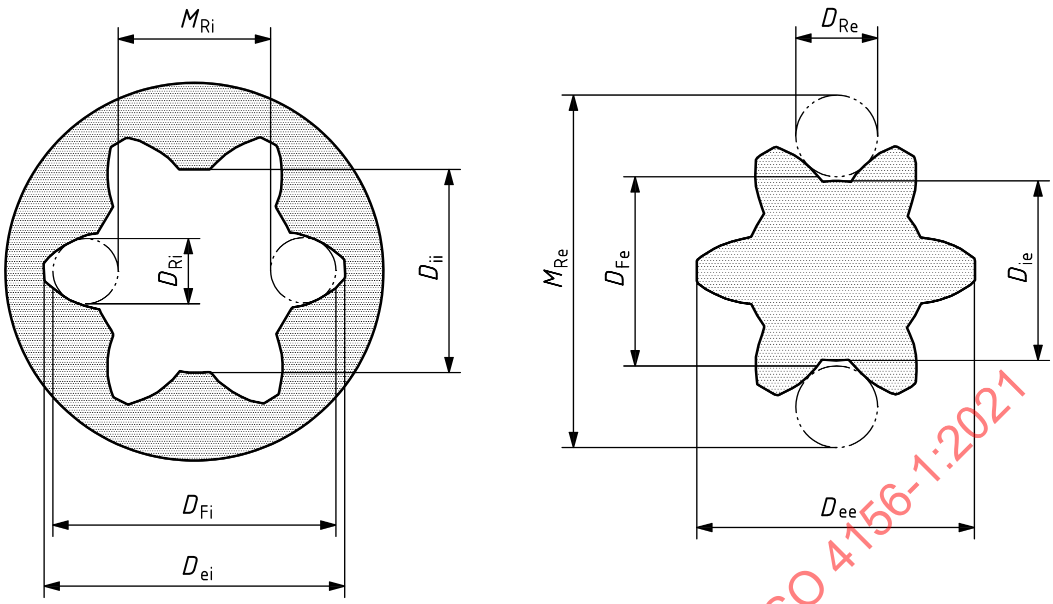


Figure 19 — Internal and external spline drawing dimensions

Table 13 — Spline terms and symbols

Internal spline ISO 4156			**External spline ISO 4156**		
Designation	see 12.3		Designation	see 12.3	
Number of teeth	z	.....	Number of teeth	z	.....
Module	m	.....	Module	m	.....
Pressure angle	α_D	XX°	Pressure angle	α_D	XX°
Pitch diameter	D	XX,XXXX [a]	Pitch diameter	D	XX,XXXX [a]
Base diameter	D_b	XX,XXXX [a]	Base diameter	D_b	XX,XXXX [a]
Major diameter	D_{ei}	XX,XX max.	Major diameter	D_{ee}	XX,XX h...
Form diameter	D_{Fi}	XX,XX min.	Form diameter	D_{Fe}	XX,XX max.
Minor diameter	D_{ii}	XX,XX H...	Minor diameter	D_{ie}	XX,XX min.
Space width [b]			Tooth thickness [b]		
max. actual	E_{max}	XX,XXX	max. effective	$S_{v\ max}$	XX,XXX
max. effective	$E_{v\ max}$	XX,XXX	max. actual	S_{max}	XX,XXX aux.
min. actual	E_{min}	XX,XXX aux.	min. effective	$S_{v\ min}$	XX,XXX
min. effective	$E_{v\ min}$	XX,XXX	min. actual	S_{min}	XX,XXX
Measurement between balls or pins	M_{Ri}	XX,XXX max. [a]	Measurement over balls or pins	M_{Re}	XX,XXX max. aux. [a]
Measurement between balls or pins	M_{Ri}	XX,XXX min. aux. [a]	Measurement over balls or pins	M_{Re}	XX,XXX min. [a]
Ball or pin diameter	D_{Ri}	X,XXX	Ball or pin diameter	D_{Re}	X,XXX
Fillet radius	See Table 12		Fillet radius	See Table 12	

[a] Calculated dimensions.

[b] See 9.8.

Annex A
(informative)

Drawing data example calculations

A.1 General

ISO 4156-2 provides drawing data for spline fit class H/h. The example calculations in A.2 to A.6 are provided to show how this data is calculated, and also to provide a guide for the calculation of drawing data for the non-tabulated spline fit classes.

A.2 INT 25z × 1,0 m × 30P × 5H - ISO 4156

NOTE 1 Unless otherwise stated, all formulae are provided in Table 1.

Number of teeth $z=25$

Module $m=1,0$

Pressure angle $\alpha_D=30°$

Pitch diameter $D=m\cdot z=1,0\cdot 25=25,000\ 0$

Base diameter $D_b=m\cdot z\cdot\cos\alpha_D=1,0\cdot 25\cdot\cos(30°)=21,650\ 635\ 09=21,650\ 6$

Minimum major diameter (flat root) not tabulated but necessary for calculating maximum major diameter

$$D_{ei\ min}=m\cdot(z+1,5)=1,0\cdot(25+1,5)=26,50$$

Maximum major diameter $D_{ei\ max}=D_{ei\ min}+(T+\lambda)/\tan\alpha_D$

From 9.1 class 7 $(T+\lambda)=40\cdot i_d+160\cdot i_E$

$$i_d=0,45\cdot\sqrt[3]{D}+0,001\cdot D=0,45\cdot\sqrt[3]{25,0}+0,001\cdot 25,0=1,340\ 8$$

$$i_E=0,45\cdot\sqrt[3]{E}+0,001\cdot E$$

$$E=0,5\cdot\pi\cdot m=0,5\cdot\pi\cdot 1,0=1,571$$

$$i_E=0,45\cdot\sqrt[3]{1,571}+0,001\cdot 1,571=0,524\ 7$$

$$T+\lambda=40\cdot 1,340\ 8+160\cdot 0,524\ 7=137,584\ \mu m=0,138\ mm$$

$$D_{ei\ max}=26,50+0,138/\tan(30°)=26,74$$

Form diameter $D_{Fi\ min}=m\cdot(z+1)+2\cdot c_F$

$$c_F=0,1\cdot 1,0=0,1$$

$$D_{Fi\ min}=1,0\cdot(25+1)+2\cdot 0,1=26,20$$

Minor diameter $D_{ii\ min}=D_{Fe\ max}+2\cdot c_F$

$$D_{\text{Fe max}} = 2 \cdot \sqrt{(0{,}5 \cdot D_b)^2 + \left(0{,}5 \cdot D \cdot \sin\alpha_D - \frac{h_s - \frac{0{,}5 \cdot es_v}{\tan\alpha_D}}{\sin\alpha_D}\right)^2}$$

From Table 2 $h_s = 0{,}6 \cdot m = 0{,}6 \cdot 1{,}0 = 0{,}6$

From Table 5 $es_v = 0$ (see Table 1, NOTE 2)

$$D_{\text{Fe max}} = 2 \cdot \sqrt{(0{,}5 \cdot 21{,}650\ 64)^2 + \left(0{,}5 \cdot 25{,}0 \cdot \sin(30°) - \frac{0{,}6 - \frac{0{,}5 \cdot 0}{\tan(30°)}}{\sin(30°)}\right)^2}$$

$D_{\text{Fe max}} = 23{,}89$

$D_{\text{ii min}} = 23{,}89 + 2 \cdot 0{,}1 = 24{,}09$

$D_{\text{ii}} = 24{,}09\ \text{H11}\ (+0{,}130/0)$

Minimum effective space width $E_{\text{v min}} = 0{,}5 \cdot \pi \cdot m = 0{,}5 \cdot \pi \cdot 1{,}0$

$E_{\text{v min}} = 1{,}571$

Maximum actual space width $E_{\text{max}} = E_{\text{v min}} + (T + \lambda)$

From 9.1 class 5 $(T + \lambda) = 16 \cdot i_d + 64 \cdot i_E$

From above formula $i_d = 1{,}3408$ and $i_E = 0{,}5247$

$T + \lambda = 16 \cdot 1{,}340\ 8 + 64 \cdot 0{,}524\ 7 = 55{,}03\ \mu\text{m} = 0{,}055\ \text{mm}$

$E_{\text{max}} = 1{,}571 + 0{,}055 = 1{,}626$

Minimum actual space width $E_{\text{min}} = E_{\text{v min}} + \lambda$

NOTE 2 λ can be calculated from total pitch deviation (F_p) total profile deviation (F_α) and total helix deviation (F_β), see 9.2; or else can be obtained from Table A.1.

Total pitch deviation $F_p = 3{,}55 \cdot \sqrt{L} + 9$ (formula from Table 7)

where

$$L = m \cdot z \cdot \frac{\pi}{2} = 1{,}0 \cdot 25 \cdot \frac{\pi}{2} = 39{,}269\ 908\ 17$$

$F_p = 3{,}55 \cdot \sqrt{39{,}269\ 908\ 17} + 9 = 31{,}25\ \mu\text{m} = 0{,}031\ \text{mm}$

Total profile deviation $F_\alpha = 2{,}5 \cdot \varphi_f + 16$ (formula from Table 8)

where $\varphi_f = m + 0{,}012\ 5 \cdot m \cdot z = 1{,}0 + 0{,}012\ 5 \cdot 1{,}0 \cdot 25 = 1{,}312\ 50$

$F_\alpha = 2{,}5 \cdot 1{,}312\ 50 + 16 = 19{,}28\ \mu\text{m} = 0{,}019\ \text{mm}$

Total helix deviation $F_\beta = \sqrt{b} + 5$ (formula from Table 9)

where b is the length of spline (assumed to be one half of the pitch diameter)

$$b = \frac{D}{2} = \frac{25}{2} = 12{,}50$$

$$F_\beta = \sqrt{12{,}50} + 5 = 8{,}54\ \mu\text{m} = 0{,}009\ \text{mm}$$

Deviation allowance

$$\lambda = 0{,}6 \cdot \sqrt{F_p^2 + F_\alpha^2 + F_\beta^2}$$

$$\lambda = 0{,}6 \cdot \sqrt{0{,}031^2 + 0{,}019^2 + 0{,}009^2}$$

$$\lambda = 0{,}022$$

$$E_{\min} = 1{,}571 + 0{,}022 = 1{,}593$$

Maximum effective space width

$$E_{\text{v max}} = E_{\max} - \lambda$$

$$E_{\text{v max}} = 1{,}626 - 0{,}022 = 1{,}604$$

Measuring ball or pin diameter D_{Ri} (formulae taken from ISO 4156–3)

$$s_{\text{DE i}} = E \cdot \cos\alpha_D + D_b \cdot \text{inv}\alpha_D$$

where

$$\text{inv}\alpha_D = \tan\alpha_D - \left(\alpha_D \cdot \frac{2 \cdot \pi}{360°}\right) = \tan(30°) - \left(30° \cdot \frac{2 \cdot \pi}{360°}\right) = 0{,}053\ 75$$

$$s_{\text{DE i}} = 1{,}571 \cdot \cos(30°) + 21{,}650\ 64 \cdot 0{,}053\ 75 = 2{,}524\ 25$$

$$l_{\text{BA}} = \frac{D_b \cdot \tan\alpha_D}{2} = \frac{21{,}650\ 64 \cdot \tan(30°)}{2} = 6{,}250\ 00$$

$$l_{\text{BO i}} = \frac{D_b \cdot \tan\left(\alpha_D + \text{inv}\alpha_D - \frac{s_{\text{DE i}}}{D_b}\right)}{2}$$

$$l_{\text{BO i}} = \frac{21{,}650\ 64 \cdot \tan\left[\left(30° \cdot \frac{2 \cdot \pi}{360°} + 0{,}053\ 75 - \frac{2{,}524\ 25}{21{,}650\ 64}\right) \cdot \frac{360°}{2 \cdot \pi}\right]}{2} = 5{,}373\ 62$$

Calculated

$$D_{\text{Ri}} = 2 \cdot (l_{\text{BA}} - l_{\text{BO i}}) = 2 \cdot (6{,}250 - 5{,}373\ 62) = 1{,}752\ 76$$

from ISO 3 R40 no. series

$$D_{\text{Ri}} = 1{,}800$$

Maximum measurement between balls or pins $M_{\text{Ri max}}$ (formulae taken from ISO 4156-3)

from above formula

$$D_{\text{Ri}} = 1{,}800$$

$$\text{inv}\alpha_i = \frac{E}{D} + \left(\text{inv}\alpha_D - \frac{D_{\text{Ri}}}{D_b}\right)$$

$$E = E_{\max} = 1{,}626$$

$$\text{inv}\alpha_i = \frac{1{,}626}{25{,}000} + \left(0{,}053\ 75 - \frac{1{,}800}{21{,}650\ 64}\right) = 0{,}035\ 651\ 58$$

$$\alpha_i = 26{,}403\ 30°$$

for odd numbers of teeth

$$M_{\text{Ri max}} = \frac{D_b \cdot \cos\frac{90°}{z}}{\cos\alpha_i} - D_{\text{Ri}}$$

$$M_{\text{Ri max}} = \frac{21{,}650\ 64 \cdot \cos\left(\frac{90°}{25}\right)}{\cos(26{,}403\ 30°)} - 1{,}800 = 22{,}324$$

Minimum measurement between balls or pins $M_{\text{Ri min}}$

$$E=E_{\min}=1{,}593$$

$$\text{inv}\alpha_{\text{i}}=\frac{1{,}593}{25{,}000}+\left(0{,}053\ 75-\frac{1{,}800}{21{,}650\ 64}\right)=0{,}034\ 331\ 58$$

$$\alpha_{\text{i}}=26{,}092\ 26°$$

$$M_{\text{Ri min}}=\frac{21{,}650\ 64\cdot\cos\left(\frac{90°}{25}\right)}{\cos(26{,}092\ 26°)}-1{,}800=22{,}260$$

Fillet radius $\rho_{\text{Fi}}=0{,}2\ m$ (formula from Table 2)

$$\rho_{\text{Fi}}=0{,}2\cdot 1{,}0=0{,}2$$

A.3 INT 25z × 1,0 m × 30R × 7H - ISO 4156

In this example, a length of spline of 25,0 mm has been used to calculate total helix deviation and hence the deviation allowance.

NOTE 1 Unless otherwise stated all formulae are provided in Table 1.

Number of teeth $z=25$

Module $m=1{,}0$

Pressure angle $\alpha_{\text{D}}=30°$

Pitch diameter $D=m\cdot z=1{,}0\cdot 25=25{,}000\ 0$

Base diameter $D_{\text{b}}=m\cdot z\cdot\cos\alpha_{\text{D}}=1{,}0\cdot 25\cdot\cos(30°)=21{,}650\ 635\ 09=21{,}650\ 6$

Minimum major diameter (fillet root) not tabulated but necessary for calculating maximum major diameter

$$D_{\text{ei min}}=m\cdot(z+1{,}8)=1{,}0\cdot(25+1{,}8)=26{,}80$$

Maximum major diameter $D_{\text{ei max}}=D_{\text{ei min}}+(T+\lambda)/\tan\alpha_{\text{D}}$

From 9.1 class 7 $(T+\lambda)=40\cdot i_{\text{d}}+160\cdot i_{\text{E}}$

$$i_{\text{d}}=0{,}45\cdot\sqrt[3]{D}+0{,}001\cdot D=0{,}45\cdot\sqrt[3]{25{,}0}+0{,}001\cdot 25{,}0=1{,}340\ 8$$

$$i_{\text{E}}=0{,}45\cdot\sqrt[3]{E}+0{,}001\cdot E$$

$$E=0{,}5\cdot\pi\cdot m=0{,}5\cdot\pi\cdot 1{,}0=1{,}571$$

$$i_{\text{E}}=0{,}45\cdot\sqrt[3]{1{,}571}+0{,}001\cdot 1{,}571=0{,}524\ 7$$

$$T+\lambda=40\cdot 1{,}340\ 8+160\cdot 0{,}524\ 7=137{,}584\ \mu\text{m}=0{,}138\ \text{mm}$$

$$D_{\text{ei max}}=26{,}80+0{,}138/\tan(30°)=27{,}04$$

Form diameter $D_{\text{Fi min}}=m\cdot(z+1)+2\cdot c_{\text{F}}$

$$c_{\text{F}}=0{,}1\cdot 1{,}0=0{,}1$$

$$D_{\text{Fi min}}=1{,}0\cdot(25+1)+2\cdot 0{,}1=26{,}20$$

Minor diameter $D_{ii\,min} = D_{Fe\,max} + 2 \cdot c_F$

$$D_{Fe\,max} = 2 \cdot \sqrt{(0{,}5 \cdot D_b)^2 + \left(0{,}5 \cdot D \cdot \sin\alpha_D - \frac{h_s - \frac{0{,}5 \cdot es_v}{\tan\alpha_D}}{\sin\alpha_D}\right)^2}$$

From Table 2 $h_s = 0{,}6 \cdot m = 0{,}6 \cdot 1{,}0 = 0{,}6$

From Table 5 $es_v = 0$ (see Table 1, NOTE 2)

$$D_{Fe\,max} = 2 \cdot \sqrt{(0{,}5 \cdot 21{,}650\ 64)^2 + \left(0{,}5 \cdot 25{,}0 \cdot \sin(30°) - \frac{0{,}6 - \frac{0{,}5 \cdot 0}{\tan(30°)}}{\sin(30°)}\right)^2}$$

$D_{Fe\,max} = 23{,}89$

$D_{ii\,min} = 23{,}89 + 2 \cdot 0{,}1 = 24{,}09$

$D_{ii} = 24{,}09\ \text{H11}\ (+0{,}130/0)$

Minimum effective space width $E_{v\,min} = 0{,}5 \cdot \pi \cdot m = 0{,}5 \cdot \pi \cdot 1{,}0$

$E_{v\,min} = 1{,}571$

Maximum actual space width $E_{max} = E_{v\,min} + (T + \lambda)$

From above formula class 7 $(T + \lambda) = 0{,}138$

$E_{max} = 1{,}571 + 0{,}138 = 1{,}709$

Minimum actual space width $E_{min} = E_{v\,min} + \lambda$

NOTE 2 λ can be calculated from total pitch deviation (F_p) total profile deviation (F_α) and total helix deviation (F_β), see 9.2; or λ can be obtained from Table A.1.

Total pitch deviation $F_p = 7{,}1 \cdot \sqrt{L} + 18$ (formula from Table 7)

where $L = m \cdot z \cdot \dfrac{\pi}{2} = 1{,}0 \cdot 25 \cdot \dfrac{\pi}{2} = 39{,}269\ 908\ 17$

$F_p = 7{,}1 \cdot \sqrt{39{,}269\ 908\ 17} + 18 = 62{,}49\ \mu m = 0{,}062\ mm$

Total profile deviation $F_\alpha = 6{,}3 \cdot \varphi_f + 40$ (formula from Table 8)

where $\varphi_f = m + 0{,}012\ 5 \cdot m \cdot z = 1{,}0 + 0{,}012\ 5 \cdot 1{,}0 \cdot 25 = 1{,}312\ 50$

$F_\alpha = 6{,}3 \cdot 1{,}312\ 50 + 40 = 48{,}27\ \mu m = 0{,}048\ mm$

Total helix deviation $F_\beta = 2 \cdot \sqrt{b} + 10$ (formula from Table 9)

where b is the length of spline (assumed to be 25,0 mm)

$F_\beta = 2 \cdot \sqrt{25{,}0} + 10 = 20{,}00\ \mu m = 0{,}020\ mm$

Deviation allowance $\lambda = 0{,}6 \cdot \sqrt{F_p^2 + F_\alpha^2 + F_\beta^2}$

$$\lambda = 0{,}6 \cdot \sqrt{0{,}062^2 + 0{,}048^2 + 0{,}020^2}$$

$$\lambda = 0{,}049$$

$$E_{\min} = 1{,}571 + 0{,}049 = 1{,}620$$

Maximum effective space width $E_{\mathrm{v\,max}} = E_{\max} - \lambda$

$$E_{\mathrm{v\,max}} = 1{,}709 - 0{,}049 = 1{,}660$$

Measuring ball or pin diameter D_{Ri} (formulae taken from ISO 4156–3)

$$s_{\mathrm{DE\,i}} = E \cdot \cos\alpha_{\mathrm{D}} + D_{\mathrm{b}} \cdot \mathrm{inv}\alpha_{\mathrm{D}}$$

where

$$\mathrm{inv}\alpha_{\mathrm{D}} = \tan\alpha_{\mathrm{D}} - \left(\alpha_{\mathrm{D}} \cdot \frac{2 \cdot \pi}{360^\circ}\right) = \tan(30^\circ) - \left(30^\circ \cdot \frac{2 \cdot \pi}{360^\circ}\right) = 0{,}053\ 75$$

$$s_{\mathrm{DE\,i}} = 1{,}571 \cdot \cos(30^\circ) + 21{,}650\ 64 \cdot 0{,}053\ 75 = 2{,}524\ 25$$

$$l_{\mathrm{BA}} = \frac{D_{\mathrm{b}} \cdot \tan\alpha_{\mathrm{D}}}{2} = \frac{21{,}650\ 64 \cdot \tan(30^\circ)}{2} = 6{,}250\ 00$$

$$l_{\mathrm{BO\,i}} = \frac{D_{\mathrm{b}} \cdot \tan\left(\alpha_{\mathrm{D}} + \mathrm{inv}\alpha_{\mathrm{D}} - \frac{s_{\mathrm{DE\,i}}}{D_{\mathrm{b}}}\right)}{2}$$

$$l_{\mathrm{BO\,i}} = \frac{21{,}650\ 64 \cdot \tan\left[\left(30^\circ \cdot \frac{2 \cdot \pi}{360^\circ} + 0{,}053\ 75 - \frac{2{,}524\ 25}{21{,}650\ 64}\right) \cdot \frac{360^\circ}{2 \cdot \pi}\right]}{2} = 5{,}373\ 62$$

Calculated $D_{\mathrm{Ri}} = 2 \cdot (l_{\mathrm{BA}} - l_{\mathrm{BO\,i}}) = 2 \cdot (6{,}250 - 5{,}373\ 62) = 1{,}752\ 76$

from ISO 3 R40 no. series $D_{\mathrm{Ri}} = 1{,}800$

Maximum measurement between balls or pins $M_{\mathrm{Ri\,max}}$ (formulae taken from ISO 4156-3)

from above formula $D_{\mathrm{Ri}} = 1{,}800$

$$\mathrm{inv}\alpha_{\mathrm{i}} = \frac{E}{D} + \left(\mathrm{inv}\alpha_{\mathrm{D}} - \frac{D_{\mathrm{Ri}}}{D_{\mathrm{b}}}\right)$$

$$E = E_{\max} = 1{,}709$$

$$\mathrm{inv}\alpha_{\mathrm{i}} = \frac{1{,}709}{25{,}000} + \left(0{,}053\ 75 - \frac{1{,}800}{21{,}650\ 64}\right) = 0{,}038\ 971\ 58$$

$$\alpha_{\mathrm{i}} = 27{,}150\ 21^\circ$$

for odd numbers of teeth

$$M_{\mathrm{Ri\,max}} = \frac{D_{\mathrm{b}} \cdot \cos\frac{90^\circ}{z}}{\cos\alpha_{\mathrm{i}}} - D_{\mathrm{Ri}}$$

$$M_{\mathrm{Ri\,max}} = \frac{21{,}650\ 64 \cdot \cos\left(\frac{90^\circ}{25}\right)}{\cos(27{,}150\ 21^\circ)} - 1{,}800 = 22{,}484$$

Minimum measurement between balls or pins $M_{\mathrm{Ri\,min}}$

$$E = E_{\min} = 1{,}620$$

$$\mathrm{inv}\alpha_\mathrm{i} = \frac{1{,}620}{25{,}000} + \left(0{,}053\ 75 - \frac{1{,}800}{21{,}650\ 64}\right) = 0{,}035\ 411\ 58$$

$$\alpha_\mathrm{i} = 26{,}347\ 37°$$

$$M_\mathrm{Ri\ min} = \frac{21{,}650\ 64 \cdot \cos\left(\frac{90°}{25}\right)}{\cos(26{,}347\ 37°)} - 1{,}800 = 22{,}313$$

Fillet radius $\rho_\mathrm{Fi} = 0{,}4 \cdot m$ (formula from Table 2)

$$\rho_\mathrm{Fi} = 0{,}4 \cdot 1{,}0 = 0{,}4$$

A.4 EXT 25z × 1,0 m × 30P × 4h - ISO 4156

NOTE 1 Unless otherwise stated all formulae are provided in Table 1.

Number of teeth $z = 25$

Module $m = 1{,}0$

Pressure angle $\alpha_\mathrm{D} = 30°$

Pitch diameter $D = m \cdot z = 1{,}0 \cdot 25 = 25{,}000\ 0$

Base diameter $D_\mathrm{b} = m \cdot z \cdot \cos\alpha_\mathrm{D} = 1{,}0 \cdot 25 \cdot \cos(30°) = 21{,}650\ 635\ 09 = 21{,}650\ 6$

Major diameter

$$D_\mathrm{ee\ max} = m \cdot (z+1) + \frac{es_\mathrm{v}}{\tan\alpha_\mathrm{D}}$$

From Table 5 $es_\mathrm{v} = 0$

$$D_\mathrm{ee\ max} = 1{,}0 \cdot (25+1) + \frac{0}{\tan(30°)} = 26{,}00$$

$$D_\mathrm{ee} = 26{,}00\ \mathrm{h}11(0/-0{,}130)$$

Form diameter

$$D_\mathrm{Fe\ max} = 2 \cdot \sqrt{(0{,}5 \cdot D_\mathrm{b})^2 + \left(0{,}5 \cdot D \cdot \sin\alpha_\mathrm{D} - \frac{h_\mathrm{s} - \frac{0{,}5 \cdot es_\mathrm{v}}{\tan\alpha_\mathrm{D}}}{\sin\alpha_\mathrm{D}}\right)^2}$$

From Table 2 $h_\mathrm{s} = 0{,}6 \cdot m = 0{,}6 \cdot 1{,}0 = 0{,}6$

From Table 5 $es_\mathrm{v} = 0$

$$D_\mathrm{Fe\ max} = 2 \cdot \sqrt{(0{,}5 \cdot 21{,}650\ 64)^2 + \left(0{,}5 \cdot 25{,}0 \cdot \sin(30°) - \frac{0{,}6 - \frac{0{,}5 \cdot 0}{\tan(30°)}}{\sin(30°)}\right)^2}$$

$$D_\mathrm{Fe\ max} = 23{,}89$$

Maximum minor diameter (flat root) — not tabulated but necessary for calculating minimum minor diameter.

$$D_{\text{ie max}} = m \cdot (z - 1,5) + \frac{es_{\text{v}}}{\tan \alpha_{\text{D}}} = 1,0 \cdot (25 - 1,5) + \frac{0}{\tan(30°)} = 23,50$$

Minimum minor diameter $D_{\text{ie min}} = D_{\text{ie max}} - \dfrac{(T+\lambda)}{\tan \alpha_{\text{D}}}$

From 9.1 class 7 $(T+\lambda) = 40 \cdot i_{\text{d}} + 160 i_{\text{E}}$

$$i_{\text{d}} = 0,45 \cdot \sqrt[3]{D} + 0,001 \cdot D = 0,45 \cdot \sqrt[3]{25,0} + 0,001 \cdot 25,0 = 1,340\ 8$$

$$i_{\text{E}} = 0,45 \cdot \sqrt[3]{E} + 0,001 \cdot E$$

$$E = 0,5 \cdot \pi \cdot m = 0,5 \cdot \pi \cdot 1,0 = 1,571$$

$$i_{\text{E}} = 0,45 \cdot \sqrt[3]{1,571} + 0,001 \cdot 1,571 = 0,524\ 7$$

$$T + \lambda = 40 \cdot 1,340\ 8 + 160 \cdot 0,524\ 7 = 137,584\ \mu\text{m} = 0,138\ \text{mm}$$

$$D_{\text{ie min}} = 23,50 - \frac{0,138}{\tan(30°)} = 23,26$$

Maximum effective tooth thickness $S_{\text{v max}} = S + es_{\text{v}}$

$$S = 0,5 \cdot \pi \cdot m = 0,5 \cdot \pi \cdot 1,0 = 1,571$$

$$S_{\text{v max}} = 1,571 + 0 = 1,571$$

Minimum actual tooth thickness $S_{\text{min}} = S_{\text{v max}} - (T - \lambda)$

From 9.1 class 4 $(T+\lambda) = 10 \cdot i_{\text{d}} + 40 \cdot i_{\text{E}}$

from above formula $i_{\text{d}} = 1,340\ 8$ and $i_{\text{E}} = 0,524\ 7$

$$T + \lambda = 10 \cdot 1,340\ 8 + 40 \cdot 0,524\ 7 = 34,396\ \mu\text{m} = 0,034\ \text{mm}$$

$$S_{\text{min}} = 1,571 - 0,034 = 1,537$$

Maximum actual tooth thickness $S_{\text{max}} = S_{\text{v max}} - \lambda$

NOTE 2 λ can be calculated from total pitch deviation (F_{p}) total profile deviation (F_{α}) and total helix deviation (F_{β}), see 9.2; or λ can be obtained from Table A.1.

Total pitch deviation $F_{\text{p}} = 2,5 \cdot \sqrt{L} + 6,3$ (formula from Table 7)

where $L = m \cdot z \cdot \dfrac{\pi}{2} = 1,0 \cdot 25 \cdot \dfrac{\pi}{2} = 39,269\ 908\ 17$

$$F_{\text{p}} = 2,5 \cdot \sqrt{39,269\ 908\ 17} + 6,3 = 21,97\ \mu\text{m} = 0,022\ \text{mm}$$

total profile deviation $F_{\alpha} = 1,6 \cdot \varphi_{\text{f}} + 10$ (formula from Table 8)

where $\varphi_{\text{f}} = m + 0,012\ 5 \cdot m \cdot z = 1,0 + 0,012\ 5 \cdot 1,0 \cdot 25 = 1,312\ 50$

Total helix deviation $F_{\beta} = 0,8 \cdot \sqrt{b} + 4$ (formula from Table 9)

where b is the length of spline (assume to be one half of the pitch diameter)

$$b = \frac{D}{2} = \frac{25}{2} = 12,50$$

$$F_{\beta}=0{,}8\cdot\sqrt{12{,}50}+4=6{,}83\ \mu\text{m}=0{,}007\ \text{mm}$$

Deviation allowance $\lambda=0{,}6\cdot\sqrt{F_{\text{p}}^{2}+F_{\alpha}^{2}+F_{\beta}^{2}}$

$$\lambda=0{,}6\cdot\sqrt{0{,}022^{2}+0{,}012^{2}+0{,}007^{2}}$$

$$\lambda=0{,}016$$

$$S_{\max}=1{,}571-0{,}016=1{,}555$$

Minimum effective tooth thickness $S_{\text{v min}}=S_{\min}+\lambda$

$$S_{\text{v min}}=1{,}537+0{,}016=1{,}553$$

Measuring ball or pin diameter D_{Re} (formulae taken from ISO 4156–3)

$$s_{\text{DE e}}=p_{\text{b}}-\left(S\cdot\cos\alpha_{\text{D}}+D_{\text{b}}\cdot\text{inv}\alpha_{\text{D}}\right)$$

where $p_{\text{b}}=m\cdot\pi\cdot\cos\alpha_{\text{D}}=1{,}0\cdot\pi\cdot\cos(30°)=2{,}720\ 70$

and

$$\text{inv}\alpha_{\text{D}}=\tan\alpha_{\text{D}}-\left(\alpha_{\text{D}}\cdot\frac{2\cdot\pi}{360°}\right)=\tan(30°)-\left(30°\cdot\frac{2\cdot\pi}{360°}\right)=0{,}053\ 75$$

$$s_{\text{DE e}}=2{,}720\ 70-(1{,}571\cdot\cos(30°)+21{,}650\ 64\cdot0{,}053\ 75)=0{,}196\ 45$$

$$l_{\text{BA}}=\frac{D_{\text{b}}\cdot\tan\alpha_{\text{D}}}{2}=\frac{21{,}650\cdot64\cdot\tan(30°)}{2}=6{,}250\ 00$$

$$l_{\text{BO e}}=\frac{D_{\text{b}}\cdot\tan\left(\alpha_{\text{D}}+\text{inv}\alpha_{\text{D}}-\frac{s_{\text{DE e}}}{D_{\text{b}}}\right)}{2}$$

$$l_{\text{BO e}}=\frac{21{,}650\ 64\cdot\tan\left[\left(30°\cdot\frac{2\cdot\pi}{360°}+0{,}053\ 75-\frac{0{,}196\ 45}{21{,}650\ 64}\right)\cdot\frac{360°}{2\cdot\pi}\right]}{2}=7{,}192\ 20$$

Calculated $D_{\text{Re}}=2\cdot(l_{\text{BO e}}-l_{\text{BA}})=2\cdot(7{,}192\ 20-6{,}250\ 00)=1{,}884\ 40$

from ISO 3 R40 no. series $D_{\text{Re}}=1{,}900$

Maximum measurement over balls or pins $M_{\text{Re max}}$ (formulae taken from ISO 4156-3)

from above formula $D_{\text{Re}}=1{,}900$

$$\text{inv}\alpha_{\text{e}}=\frac{S}{D}+\left(\text{inv}\alpha_{\text{D}}+\frac{D_{\text{Re}}}{D_{\text{b}}}-\frac{\pi}{z}\right)$$

$$S=S_{\max}=1{,}555$$

$$\text{inv}\alpha_{\text{e}}=\frac{1{,}555}{25{,}000}+\left(0{,}053\ 75+\frac{1{,}900}{21{,}650\ 64}-\frac{\pi}{25}\right)=0{,}078\ 043\ 51$$

$$\alpha_{\text{e}}=33{,}609\ 83°$$

for odd numbers of teeth

$$M_{\text{Re max}}=\frac{D_{\text{b}}\cdot\cos\frac{90°}{z}}{\cos\alpha_{\text{e}}}+D_{\text{Re}}$$

$$M_{\text{Re max}} = \frac{21{,}650\ 64 \cdot \cos\left(\frac{90^\circ}{25}\right)}{\cos(33{,}609\ 83^\circ)} + 1{,}900 = 27{,}845$$

Minimum measurement between balls or pins $M_{\text{Re min}}$

$$S = S_{\min} = 1{,}537$$

$$\text{inv}\alpha_e = \frac{1{,}537}{25{,}000} + \left(0{,}053\ 75 + \frac{1{,}900}{21{,}650\ 64} - \frac{\pi}{25}\right) = 0{,}077\ 323\ 51$$

$$\alpha_e = 33{,}516\ 11^\circ$$

$$M_{\text{Re min}} = \frac{21{,}650\ 64 \cdot \cos\left(\frac{90^\circ}{25}\right)}{\cos(33{,}516\ 11^\circ)} + 1{,}900 = 27{,}817$$

Fillet radius $\rho_{\text{Fe}} = 0{,}2 \cdot \text{m}$ (formula from Table 2)

$$\rho_{\text{Fe}} = 0{,}2 \cdot 1{,}0 = 0{,}2$$

A.5 EXT 25z × 1,0 m × 30R × 6e - ISO 4156

NOTE 1 Unless otherwise stated all formulae are provided in Table 1.

Number of teeth $z = 25$

Module $m = 1{,}0$

Pressure angle $\alpha_D = 30^\circ$

Pitch diameter $D = m \cdot z = 1{,}0 \cdot 25 = 25{,}000\ 0$

Base diameter $D_b = m \cdot z \cdot \cos\alpha_D = 1{,}0 \cdot 25 \cdot \cos(30^\circ) = 21{,}650\ 635\ 09 = 21{,}650\ 6$

Major diameter $D_{\text{ee max}} = m \cdot (z+1) + \frac{es_v}{\tan\alpha_D}$

From Clause 10, adjustment is made to the major, form and minor diameters for fundamental deviation e.

So, from Table 5 $es_v = -0{,}040$

$$D_{\text{ee max}} = 1{,}0 \cdot (25+1) + \left(\frac{-0{,}040}{\tan(30^\circ)}\right) = 25{,}930\ 7$$

$$D_{\text{ee}} = 25{,}93\ \text{h}11(0/-0{,}130)$$

Form diameter

$$D_{\text{Fe max}} = 2 \cdot \sqrt{(0{,}5 \cdot D_b)^2 + \left(0{,}5 \cdot D \cdot \sin\alpha_D - \frac{h_s - \frac{0{,}5 \cdot es_v}{\tan\alpha_D}}{\sin\alpha_D}\right)^2}$$

From Table 2 $h_s = 0{,}6 \cdot m = 0{,}6 \cdot 1{,}0 = 0{,}6$

From Table 5 $es_v = -0{,}040$

$$D_{\text{Fe max}} = 2 \cdot \sqrt{(0{,}5 \cdot 21{,}650\ 64)^2 + \left(0{,}5 \cdot 25{,}0 \cdot \sin(30°) - \frac{0{,}6 - \frac{0{,}5 \cdot (-0{,}040)}{\tan(30°)}}{\sin(30°)} \right)^2}$$

$$D_{\text{Fe max}} = 23{,}83$$

Maximum minor diameter (fillet root) — not tabulated but necessary for calculating minimum minor diameter.

$$D_{\text{ie max}} = m \cdot (z - 1{,}8) + \frac{es_{\text{v}}}{\tan \alpha_{\text{D}}} = 1{,}0 \cdot (25 - 1{,}8) + (-0{,}069) = 23{,}13$$

Minimum minor diameter $D_{\text{ie min}} = D_{\text{ie max}} - \dfrac{(T+\lambda)}{\tan \alpha_{\text{D}}}$

From 9.1 class 7 $(T+\lambda) = 40 \cdot i_{\text{d}} + 160 \cdot i_{\text{E}}$

$$i_{\text{d}} = 0{,}45 \cdot \sqrt[3]{D} + 0{,}001 \cdot D = 0{,}45 \cdot \sqrt[3]{25{,}0} + 0{,}001 \cdot 25{,}0 = 1{,}340\ 8$$

$$i_{\text{E}} = 0{,}45 \cdot \sqrt[3]{E} + 0{,}001 \cdot E$$

$$E = 0{,}5 \cdot \pi \cdot m = 0{,}5 \cdot \pi \cdot 1{,}0 = 1{,}571$$

$$i_{\text{E}} = 0{,}45 \cdot \sqrt[3]{1{,}571} + 0{,}001 \cdot 1{,}571 = 0{,}524\ 7$$

$$T + \lambda = 40 \cdot 1{,}340\ 8 + 160 \cdot 0{,}524\ 7 = 137{,}584\ \mu\text{m} = 0{,}138\ \text{mm}$$

$$D_{\text{ie min}} = 23{,}13 - \frac{0{,}138}{\tan(30°)} = 22{,}89$$

Maximum effective tooth thickness $S_{\text{v max}} = S + es_{\text{v}}$

$$S = 0{,}5 \cdot \pi \cdot m = 0{,}5 \cdot \pi \cdot 1{,}0 = 1{,}571$$

$$S_{\text{v max}} = 1{,}571 + (-0{,}040) = 1{,}531$$

Minimum actual tooth thickness $S_{\text{min}} = S_{\text{v max}} - (T+\lambda)$

From 9.1 class 6 $(T+\lambda) = 25 \cdot i_{\text{d}} + 100 i_{\text{E}}$

from above formula $i_{\text{d}} = 1{,}340\ 8$ and $i_{\text{E}} = 0{,}524\ 7$

$$T + \lambda = 25 \cdot 1{,}340\ 8 + 100 \cdot 0{,}524\ 7 = 85{,}990\ \mu\text{m} = 0{,}086\ \text{mm}$$

$$S_{\text{min}} = 1{,}531 - 0{,}086 = 1{,}445$$

Maximum actual tooth thickness $S_{\text{max}} = S_{\text{v max}} - \lambda$

NOTE 2 λ can be calculated from total pitch deviation (F_{p}) total profile deviation (F_{α}) and total helix deviation (F_{β}), see 9.2; or λ can be obtained from Table A.1.

Total pitch deviation $F_{\text{p}} = 5 \cdot \sqrt{L} + 12{,}5$ (formula from Table 7)

where

$$L = m \cdot z \cdot \frac{\pi}{2} = 1{,}0 \cdot 25 \cdot \frac{\pi}{2} = 39{,}269\ 908\ 17$$

$$F_{\text{p}} = 5 \cdot \sqrt{39{,}269\ 908\ 17} + 12{,}5 = 43{,}83\ \mu\text{m} = 0{,}044\ \text{mm}$$

Total profile deviation $F_{\alpha}=4\cdot\varphi_{f}+25$ (formula from Table 8)

where $\varphi_{f}=m+0{,}012\ 5\cdot m\cdot z=1{,}0+0{,}012\ 5\cdot 1{,}0\cdot 25=1{,}312\ 50$

$F_{\alpha}=4\cdot 1{,}312\ 50+25=30{,}25\ \mu m=0{,}030\ mm$

Total helix deviation $F_{\beta}=1{,}25\cdot\sqrt{b}+6{,}3$ (formula from Table 9)

where b is the length of spline (assumed to be one half of the pitch diameter)

$$b=\frac{D}{2}=\frac{25}{2}=12{,}50$$

$$F_{\beta}=1{,}25\cdot\sqrt{12{,}50}+6{,}3=10{,}72\ \mu m=0{,}011\ mm$$

Deviation allowance $\lambda=0{,}6\cdot\sqrt{F_{p}^{2}+F_{\alpha}^{2}+F_{\beta}^{2}}$

$$\lambda=0{,}6\cdot\sqrt{0{,}044^{2}+0{,}030^{2}+0{,}011^{2}}$$

$$\lambda=0{,}033$$

$$S_{max}=1{,}531-0{,}033=1{,}498$$

Minimum effective tooth thickness $S_{v\ min}=S_{min}+\lambda$

$$S_{v\ min}=1{,}445+0{,}033=1{,}478$$

Measuring ball or pin diameter D_{Re} (formulae taken from ISO 4156-3)

$$s_{DE\ e}=p_{b}-(S\cdot\cos\alpha_{D}+D_{b}\cdot\mathrm{inv}\alpha_{D})$$

where $p_{b}=m\cdot\pi\cdot\cos\alpha_{D}=1{,}0\cdot\pi\cdot\cos(30°)=2{,}720\ 70$

and $$\mathrm{inv}\alpha_{D}=\tan\alpha_{D}-\left(\alpha_{D}\cdot\frac{2\cdot\pi}{360°}\right)=\tan(30°)-\left(30°\cdot\frac{2\cdot\pi}{360°}\right)=0{,}053\ 75$$

$$s_{DE\ e}=2{,}720\ 70-(1{,}571\cdot\cos(30°)+21{,}650\ 64\cdot 0{,}053\ 75)=0{,}196\ 45$$

$$l_{BA}=\frac{D_{b}\cdot\tan\alpha_{D}}{2}=\frac{21{,}650\ 64\cdot\tan(30°)}{2}=6{,}250\ 00$$

$$l_{BO\ e}=\frac{D_{b}\cdot\tan\left(\alpha_{D}+\mathrm{inv}\alpha_{D}-\frac{s_{DE\ e}}{D_{b}}\right)}{2}$$

$$l_{BO\ e}=\frac{21{,}650\ 64\cdot\tan\left[\left(30°\cdot\frac{2\cdot\pi}{360°}+0{,}053\ 75+\frac{0{,}196\ 45}{21{,}650\ 64}\right)\cdot\frac{360°}{2\cdot\pi}\right]}{2}=7{,}192\ 20$$

Calculated $D_{Re}=2\cdot(l_{BO\ e}-l_{BA})=2\cdot(7{,}192\ 20-6{,}250\ 00)=1{,}884\ 40$

from ISO 3 R40 no. series $D_{Re}=1{,}900$

Maximum measurement over balls or pins $M_{Re\ max}$ (formulae taken from ISO 4156-3)

from above formula $D_{Re}=1{,}900$

$$\mathrm{inv}\alpha_{e}=\frac{S}{D}+\left(\mathrm{inv}\alpha_{D}+\frac{D_{Re}}{D_{b}}-\frac{\pi}{z}\right)$$

$$S = S_{max} = 1{,}498$$

$$\text{inv}\alpha_e = \frac{1{,}498}{25{,}000} + \left(0{,}053\ 75 + \frac{1{,}900}{21{,}650\ 64} - \frac{\pi}{25}\right) = 0{,}075\ 763\ 51$$

$$\alpha_e = 33{,}310\ 74°$$

for odd numbers of teeth

$$M_{Re\ max} = \frac{D_b \cdot \cos\frac{90°}{z}}{\cos\alpha_e} + D_{Re}$$

$$M_{Re\ max} = \frac{21{,}650\ 64 \cdot \cos\left(\frac{90°}{25}\right)}{\cos(33{,}310\ 74°)} + 1{,}900 = 27{,}756$$

Minimum measurement over balls or pins $M_{Re\ min}$

$$S = S_{min} = 1{,}445$$

$$\text{inv}\alpha_e = \frac{1{,}445}{25{,}000} + \left(0{,}053\ 75 + \frac{1{,}900}{21{,}650\ 64} - \frac{\pi}{25}\right) = 0{,}073\ 643\ 51$$

$$\alpha_e = 33{,}026\ 41°$$

$$M_{Re\ min} = \frac{21{,}650\ 64 \cdot \cos\left(\frac{90°}{25}\right)}{\cos(33{,}026\ 41°)} + 1{,}900 = 27{,}672$$

Fillet radius $\rho_{Fe} = 0{,}4 \cdot m$ (formula from Table 2)

$$\rho_{Fe} = 0{,}4 \cdot 1{,}0 = 0{,}4$$

A.6 EXT 25z × 1,0 m × 30P × 5js - ISO 4156

NOTE 1 Unless otherwise stated all formulae are provided in Table 1.

Number of teeth $z = 25$

Module $m = 1{,}0$

Pressure angle $\alpha_D = 30°$

Pitch diameter $D = m \cdot z = 1{,}0 \cdot 25 = 25{,}000\ 0$

Base diameter $D_b = m \cdot z \cdot \cos\alpha_D = 1{,}0 \cdot 25 \cdot \cos(30°) = 21{,}650\ 635\ 09 = 21{,}650\ 6$

Major diameter

$$D_{ee\ max} = m \cdot (z+1) + \frac{es_v}{\tan\alpha_D}$$

From Clause 10, no adjustment is applied to the major diameter, but adjustment is applied to the form and minor diameters for fundamental deviation js.

So,

$$\frac{es_v}{\tan\alpha_D} = 0$$

$$D_{ee\ max} = 1{,}0 \cdot (25+1) + 0 = 26{,}00$$

$$D_{ee} = 26{,}00\ \text{h11}(0/-0{,}130)$$

Form diameter

$$D_{\text{Fe max}} = 2 \cdot \sqrt{(0{,}5 \cdot D_b)^2 + \left(0{,}5 \cdot D \cdot \sin\alpha_D - \frac{h_s - \dfrac{0{,}5 \cdot es_v}{\tan\alpha_D}}{\sin\alpha_D} \right)^2}$$

From Table 2 $h_s = 0{,}6 \cdot m = 0{,}6 \cdot 1{,}0 = 0{,}6$

From Table 5, Footnote [a] $es_v = \dfrac{T+\lambda}{2}$

From 9.1 class 5 $(T+\lambda) = 16 \cdot i_d + 64 \cdot i_E$

$$i_d = 0{,}45 \cdot \sqrt[3]{D} + 0{,}001 \cdot D = 0{,}45 \cdot \sqrt[3]{25{,}0} + 0{,}001 \cdot 25{,}0 = 1{,}340\ 8$$

$$i_E = 0{,}45 \cdot \sqrt[3]{E} + 0{,}001 \cdot E$$

$$E = 0{,}5 \cdot \pi \cdot m = 0{,}5 \cdot \pi \cdot 1{,}0 = 1{,}571$$

$$i_E = 0{,}45 \cdot \sqrt[3]{1{,}571} + 0{,}001 \cdot 1{,}571 = 0{,}524\ 7$$

$$T+\lambda = 16 \cdot 1{,}340\ 8 + 64 \cdot 0{,}524\ 7 = 55{,}03\ \mu\text{m} = 0{,}055\ \text{mm}$$

$$es_v = \frac{0{,}055}{2} = 0{,}028$$

$$D_{\text{Fe max}} = 2 \cdot \sqrt{(0{,}5 \cdot 21{,}650\ 64)^2 + \left(0{,}5 \cdot 25{,}0 \cdot \sin(30^\circ) - \frac{0{,}6 - \dfrac{0{,}5 \cdot 0{,}028}{\tan(30^\circ)}}{\sin(30^\circ)} \right)^2} = 23{,}93$$

Maximum minor diameter (flat root) — not tabulated but necessary for calculating minimum minor diameter.

$$D_{\text{ie max}} = m \cdot (z - 1{,}5) + \frac{es_v}{\tan\alpha_D} = 1{,}0 \cdot (25 - 1{,}5) + \frac{0{,}028}{\tan(30^\circ)} = 23{,}55$$

Min. minor diameter $D_{\text{ie min}} = D_{\text{ie max}} - \dfrac{(T+\lambda)}{\tan\alpha_D}$

From 9.1 class 7 $(T+\lambda) = 40 \cdot i_d + 160 i_E$

from above formula $i_d = 1{,}340\ 8$ and $i_E = 0{,}524\ 7$

$$T+\lambda = 40 \cdot 1{,}340\ 8 + 160 \cdot 0{,}524\ 7 = 137{,}584\ \mu\text{m} = 0{,}138\ \text{mm}$$

$$D_{\text{ie min}} = 23{,}55 - \frac{0{,}138}{\tan(30^\circ)} = 23{,}31$$

Maximum effective tooth thickness $S_{\text{v max}} = S + es_v$

$$S = 0{,}5 \cdot \pi \cdot m = 0{,}5 \cdot \pi \cdot 1{,}0 = 1{,}571$$

$$S_{\text{v max}} = 1{,}571 + 0{,}028 = 1{,}599$$

Minimum actual tooth thickness $S_{\min} = S_{\text{v max}} - (T+\lambda)$

from above formula class 5 $(T+\lambda) = 0{,}055$

$$S_{min}=1{,}599-0{,}055=1{,}544$$

Maximum actual tooth thickness $S_{max}=S_{v\,max}-\lambda$

NOTE 2 λ can be calculated from total pitch deviation (F_p) total profile deviation (F_α) and total helix deviation (F_β), see 9.2, or λ can be obtained from Table A.1.

Total pitch deviation $F_p=3{,}55\cdot\sqrt{L}+9$ (formula from Table 7)

where

$$L=m\cdot z\cdot\frac{\pi}{2}=1{,}0\cdot 25\cdot\frac{\pi}{2}=39{,}269\ 908\ 17$$

$$F_p=3{,}55\cdot\sqrt{39{,}269\ 908\ 17}+9=31{,}25\ \mu m=0{,}031\ mm$$

Total profile deviation $F_\alpha=2{,}5\cdot\varphi_f+16$ (formula from Table 8)

where

$$\varphi_f=m+0{,}012\ 5\cdot m\cdot z=1{,}0+0{,}012\ 5\cdot 1{,}0\cdot 25=1{,}312\ 50$$

$$F_\alpha=2{,}5\cdot 1{,}312\ 50+16=19{,}28\ \mu m=0{,}019\ mm$$

Total helix deviation $F_\beta=\sqrt{b}+5$ (formula from Table 9)

where b is the length of spline (assumed to be one half of the pitch diameter)

$$b=\frac{D}{2}=\frac{25}{2}=12{,}50$$

$$F_\beta=\sqrt{12{,}50}+5=8{,}54\ \mu m=0{,}009\ mm$$

Deviation allowance

$$\lambda=0{,}6\cdot\sqrt{F_p^2+F_\alpha^2+F_\beta^2}$$

$$\lambda=0{,}6\cdot\sqrt{0{,}031^2+0{,}019^2+0{,}009^2}$$

$$\lambda=0{,}022$$

$$S_{max}=1{,}599-0{,}022=1{,}577$$

Minimum effective tooth thickness $S_{v\,min}=S_{min}+\lambda$

$$S_{v\,min}=1{,}544+0{,}022=1{,}566$$

Measuring ball or pin diameter D_{re} (formulae taken from ISO 4156–3)

$$s_{DE\,e}=p_b-(S\cdot\cos\alpha_D+D_b\cdot\mathrm{inv}\alpha_D)$$

where

$$p_b=m\cdot\pi\cdot\cos\alpha_D=1{,}0\cdot\pi\cdot\cos(30°)=2{,}720\ 70$$

and

$$\mathrm{inv}\alpha_D=\tan\alpha_D-\left(\alpha_D\cdot\frac{2\cdot\pi}{360°}\right)=\tan(30°)-\left(30°\cdot\frac{2\cdot\pi}{360°}\right)=0{,}053\ 75$$

$$s_{DE\,e}=2{,}720\ 70-(1{,}571\cdot\cos(30°)+21{,}650\ 64\cdot 0{,}053\ 75)=0{,}196\ 45$$

$$l_{BA}=\frac{D_b\cdot\tan\alpha_D}{2}=\frac{21{,}650\ 64\cdot\tan(30°)}{2}=6{,}250\ 00$$

$$l_{BO\,e}=\frac{D_b\cdot\tan\left(\alpha_D+\mathrm{inv}\alpha_D-\frac{DE_e}{D_b}\right)}{2}$$

