
International Standard



555/3

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Liquid flow measurement in open channels — Dilution methods for measurement of steady flow — Part 3 : Constant rate injection method and integration method using radioactive tracers

Mesure de débit des liquides dans les canaux découverts — Méthodes de dilution pour le mesurage du débit en régime permanent — Partie 3 : Méthodes d'injection à débit constant et par intégration utilisant des traceurs radioactifs

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Foreword

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It has been approved by the member bodies of the following countries :

Australia	France	Romania
Chile	Germany, F.R.	Spain
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Ireland

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Liquid flow measurement in open channels — Dilution methods for measurement of steady flow —

Part 3 : Constant rate injection method and integration method using radioactive tracers

1 Scope and field of application

This International Standard deals with the measurement of steady flow in open channels by dilution methods using radioactive tracers with constant rate injection and integration techniques. These methods are applicable to the measurement of flow in channels where the flow conditions ensure efficient mixing of the injected solution throughout the flow.

The annex provides an example of the calculation of uncertainty in the flow rate.

2 References

ISO 555/1, *Liquid flow measurement in open channels — Dilution methods for measurement of steady flow — Part 1 : Constant-rate injection method.*

ISO 555/2, *Liquid flow measurement in open channels — Dilution methods for measurement of steady flow — Part 2 : Integration (sudden injection) method.*

ISO 772, *Liquid flow measurement in open channels — Vocabulary and symbols.*

ISO 5168, *Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement.*

3 Definitions

The following definitions shall be used for the purpose of this International Standard to complement those given in ISO 772.

3.1 activity : Number of disintegrations per unit time taking place in a radioactive sample. The unit of activity is the becquerel.

3.2 becquerel : Activity of a radioactive isotope the rate of disintegration of which is one disintegration per second.

The following multiples are used :

$$1 \text{ kBq} = 10^3 \text{ Bq}$$

$$1 \text{ MBq} = 10^6 \text{ Bq}$$

$$1 \text{ GBq} = 10^9 \text{ Bq}$$

3.3 carrier : Non-radioactive isotope of the radioactive tracer element usually added to radioisotope solutions when the concentration of the radioactive isotope of the element is so small that there is a danger of loss by adsorption on the walls of the recipient container.

3.4 classified worker : Person occupationally exposed to ionising radiations. The person is registered and is under medical surveillance. He is permitted to enter an area where the radiation dose rate exceeds a threshold imposed by national legislation.

3.5 electron volt (eV) : One electron volt is equivalent to the energy acquired by an electron accelerated by a potential difference of one volt.

$$1 \text{ eV} = 1,60 \times 10^{-19} \text{ joules}$$

The following multiples are employed :

$$1 \text{ keV} = 10^3 \text{ eV}$$

$$1 \text{ MeV} = 10^6 \text{ eV}$$

3.6 geometrical efficiency : Ratio of the number of particles or photons which enter the sensitive volume of a detector to the total number of particles or photons emitted by a radioactive source.

3.7 half-life : Time taken for the activity of a radioactive substance to decrease to one half of its initial value. Each radionuclide has a unique half-life.

3.8 isotopes : Nuclides which have the same number of protons in their nucleus, and hence the same atomic number, but differ in the number of neutrons, and hence in their atomic weights. Their chemical properties are identical.

3.9 rad : Unit of absorbed dose; $1 \text{ rad} = 10^{-2} \text{ J/kg}$ of absorbent material.

3.10 radioactive isotope, radioisotope : Nuclides whose nucleus is unstable and which on becoming stable emit energy in the form of alpha or beta particles, gamma or X rays.

3.11 radioisotopic (isotopic solution) : Solution containing radioactive isotopes.

3.12 Roentgen : Unit of exposure, corresponding to the production of two ions of opposite charges in 1 kg of air under normal temperature and pressure conditions, i.e. the liberation of $2,58 \times 10^{-4}$ coulombs in 1 kg of air.

3.13 scintillator : Solid or liquid which after absorption of electrically-charged particles emits visible or ultraviolet radiation detectable by a photomultiplier. Gamma rays produce, in this medium, interactions producing electrically charged particles which can be detected.

3.14 unclassified worker : Person who is not normally occupationally exposed to ionising radiation. This person is restricted to areas where the dose rate is below a threshold imposed by national legislation.

3.15 volumetric activity : Activity of the radioactive isotope per unit volume of a given material.

The unit is the gigabecquerel per cubic metre.

4 Units of measurement

The units of measurement used in this International Standard are those of the International System (S.I.).

5 Principle of the constant rate injection method

A solution of concentration c_1 of a suitably selected tracer is injected at constant rate q at a cross-section at the entry to the measuring reach of the channel, in which the discharge Q remains constant over the period of the test. In a second downstream cross-section of this reach, sufficiently far from the first for the injected solution to be uniformly diluted through the cross-section, the concentration due to the tracer, c_2 , is determined for a sufficient period of time and at a sufficient number of points, along the banks if it is not possible to take samples across the channel (see 9.4 and 10.4.1), to verify on the one hand that good mixing has been obtained and on the other hand that the concentration c_2 of added tracer, has reached a steady value. In these conditions the flow of tracer at the point of injection is equal to that crossing the sampling section :

$$q c_1 = (Q + q) c_2$$

hence

$$Q = q \left(\frac{c_1 - c_2}{c_2} \right) \quad \dots(1)$$

In general c_1 is much greater than c_2 and this leads to a simplification of equation (1) as follows :

$$Q = q \left(\frac{c_1}{c_2} \right) \quad \dots(2)$$

The discharge Q can therefore be determined by comparing the concentration of the injected solution with that determined in the sampling section of the channel.

NOTE — Clauses 9 and 11 on procedure and methods of analysis give all the precautions necessary (sampling, total count) to eliminate possible interferences especially due to ambient radiation, natural radioactivity c_0 of the water flowing in the channel, etc.

6 Principle of the integration method

A solution of concentration c_1 of a suitably chosen tracer is injected for a short period in a cross-section at the entry to the measuring reach of the channel, in which the discharge Q remains constant over the period of the test.

In a second cross-section of this reach, downstream by a distance equal to the mixing length (see clause 8) the concentration of added tracer, $c_2(t)$, is determined for a period of time sufficiently long to ensure that all the tracer has passed through the second cross-section.

If all the injected tracer passes through the sampling section, the following equation holds :

$$m = V c_1 = Q \int_0^\infty c_2(t) dt \quad \dots(3)$$

where

m is the mass of tracer injected;

V is the volume of solution injected;

c_1 is the concentration of tracer in the injected solution;

Q is the discharge of the stream;

$c_2(t)$ is the concentration of tracer, above background, at the sampling point during the time-interval dt .

This equation implies that the value of the integral :

$$\int_0^\infty c_2(t) dt$$

is the same at each point of the section, and the injected solution can be considered to be well mixed with the stream water if this condition is satisfied.

It can then be deduced that

$$Q = \frac{V c_1}{\int_0^\infty c_2(t) dt} \quad \dots(4)$$

NOTE — Clauses 10 and 11 give all the precautions necessary (sampling, total count) to eliminate possible interferences especially due to ambient radiation, natural radioactivity c_0 of the water flowing in the channel, etc.

7 Choice of tracer

7.1 Requirements

The tracer to be used in the injected solution must satisfy the following conditions :

- It must have a hydrodynamic behaviour identical with that of water molecules.

This property must not be altered by the temperature, by a change of chemical form, or by any possible change of phase due to an interaction with the medium. The tracer must not be present in solution in natural waters to more than a relatively small and only slightly varying amount. It must not be retained preferentially by the constituents of the channel bed (sediments, vegetation, etc.).

- b) It must be measurable with precision at a concentration below its maximum permissible concentration (MPC) specified by national regulations.

7.2 Selection criteria

7.2.1 Type

Gamma-ray emitters are preferable to β -emitters because they are more easily measured. However, it should be considered that β -emitters are easier to transport.

7.2.2 Energy

A good balance must be found between emitters which are too energetic, and therefore require extensive shielding, and those that are too weak, so that detection is difficult.

7.2.3 Cost

Depending upon the precision sought, the least expensive tracer should be chosen.

7.2.4 Half-life

The tracer with the shortest half-life should be chosen subject to the conditions of supply, storage and safety. They must not contaminate and damage the surrounding media and biosphere.

7.3 List of recommended tracers

- Bromine-82 (half-life — 36 hours, γ -emitter of 0,55 to 1,48 MeV)
- Tritium (half life — 12,26 years, β -emitter of 0,018 MeV)

The choice is subject to the consideration of the risks of contamination of ground waters that may affect their dating by the method using measurement of tritium of thermonuclear origin.

Other tracers, such as iodine-131 (half-life — 8,04 days, γ -emitter of 0,25 to 0,8 MeV) may be used in the absence of organic matter.

Other isotopes may also be used, particularly those obtainable from isotope generators, for example :

- Caesium — 137 $\rightarrow$ barium 137 (half-life 2,6 mins, γ -emitter of 0,66 MeV);
- Tin 113 $\rightarrow$ indium 113 (half-life 104 mins, γ -emitter of 0,39 MeV).

7.4 Advantages of radioactive tracers

Measurements are not affected by any physical or chemical changes of the tracer occurring in the flow, as long as such reactions do not alter the tracer's fundamental properties (see 7.1.a).

In particular, for the case of water containing suspended solids, the measurements are unaffected if settling out does not occur within the measuring reach.

Measurement of concentrations can be carried out "in situ", hence without sampling.

For γ emitting tracers of short half-life, whose carrier is innocuous, all danger of contamination rapidly disappears; there is no permanent pollution.

The cost of radioactive tracers is often less than that of other tracers, particularly for the measurement of large flows, or numerous measurements at the same site.

7.5 Special advantages of isotope generators

It is possible to obtain at the site of investigation, from a "mother" isotope of long half-life a radioactive tracer of short half-life, at low cost. When the "daughter" isotope has a sufficiently short half-life, it is possible to make repeated measurements in a closed system.

7.6 Disadvantages of radioactive tracers

- a) For each measurement it may be necessary to obtain official authorisation, which shall be given on the basis of complete technical documentation which could involve considerable expense.
- b) It is necessary that the personnel involved are especially trained, and mostly classified workers.
- c) Depending on the half-life of the tracer, there may be supply problems.
- d) Transport and injection demand special safety precautions.
- e) Complying with the safety precautions can necessitate the use of heavy containers, and these may be difficult to handle at certain sites.

7.7 Conditions for health and safety

The handling of radioactive sources, and the injection of radioactive tracers into rivers must conform with the regulations or decreed safety laws of the country concerned. This question is dealt with in the report entitled, *Guide to the Safe Handling of Radioisotopes in Hydrology*, Safety Series No. 20, International Atomic Energy Agency — Vienna 1966.

In matters of safety, the following points are to be considered :

- a) National transport regulations for the radioactive source.

b) The national safety regulations for the storage of the radioactive isotope.

c) The national safety regulations for handling of the concentrated solution of the radioactive isotopes for injection. The personnel involved must have a basic knowledge of the precautions to take, and an adequate basic training in the handling of isotopes. An instrument to check radiation rate must be available on site, in case of danger due to an accident during handling.

d) The concentration of the tracer during injection into the river must conform with the safety regulations of the country concerned and shall be maintained at the lowest possible value.

e) A check shall be made of the tracer concentration level after the measurements are taken and in case the permissible limits for concentration level are exceeded, measures shall be taken to reduce the concentration to a normal level.

In some countries, the local or national authorities must be consulted at the time of planning the investigation.

8 Choice of measuring reach

8.1 General considerations for selection of site

The measuring reach should have a length at least equal to the mixing length corresponding to the discharge being measured.

8.1.1 Conditions for mixing

The two fundamental conditions for the application of the methods are the following :

- a) all the tracer must pass through the sampling section;
- b) the tracer must be well mixed with the river water in the sampling section :

$$\int_0^{\infty} c_2(t) dt = \text{constant at all points in the section}$$

which is equivalent, in the constant rate injection method, to the more simple condition $c_2 = \text{constant}$ in the whole section for steady 'plateau' conditions.

The mixing length is the shortest distance over which the maximum relative variation of $\int_0^{\infty} c_2(t) dt$ in the section (for the method of integration) or of the concentration of tracer (for the constant rate injection method) is less than a predetermined value ε (for example 0,5 %). The mixing length therefore does not have a fixed value; it varies with the allowable variation in concentration. The smaller the allowable variation, the longer the mixing length.

To obtain the highest precision of the discharge measurement, it is necessary to have the smallest possible value of ε in the measuring section.

It is also desirable that the distance between the injection section and the sampling section should be as small as possible : this makes for economy of time and tracer.

In consequence, a reach should be chosen in which the river is as narrow and as turbulent as possible, free of dead water zones, and with numerous transverse currents, favouring lateral mixing which is the most difficult to obtain; grassy and vegetation covered zones should be avoided, as should zones where the river divides into several arms (see figure 1).

The sampling section should be chosen in a narrow zone where there are no back-currents or dead water.

8.1.2 Special cases

- a) Inflows of water into the measuring reach

Measurements can be made where there is an inflow of water (tributaries or springs) into the measuring reach provided only that good mixing is achieved at the sampling section.

The measured flow then includes the intermediate water inflow.

- b) Abstraction or leakage from the measuring reach

If there is a leakage or abstraction between the injection section and the sampling section, the result will be questionable, except where the leakage or abstraction is perfectly known and is located at a point where good mixing is already achieved; in this case the analysis of samples will lead to the value of the river discharge upstream from the leakage or abstraction, and not to the value of the discharge at the sampling section.

- c) Dead water zones

If there are any dead water zones, these may detain tracer and release it only very slowly after the passage of the principal pulse, so that the measuring time is considerably prolonged. It is necessary to continue sampling until all the tracer collected in the dead water zones has passed the downstream sampling cross-section. An error of measurement may therefore arise, since a considerable amount of tracer may pass through the downstream section at the end of the test, when concentrations are too low to be measured.

8.2 Preliminary tests

These are meant to specify the mixing length and the corresponding optimum duration of injection.

8.2.1 Determination of the length of the measuring reach

A first test can be made by using a strong dye such as fluorescein. A concentrated solution of this dye can be injected for a relatively short time, at a point upstream of the measuring reach. Study of the diffusion of the solution will make it possible to determine whether there are dead water zones or

other zones of loss of tracer, and give a first indication of the minimum distance which must separate the injection point and a suitable sampling section.

This procedure may be improved by performing a continuous injection of fluorescein, rhodamine B or WT, the transverse distribution of which is studied at different sections downstream with the aid of a portable fluorimeter. The use of dye tracers for preliminary tests shall conform with national and local regulations relating to the release of such substances into the environment.

The minimum distance of good mixing can be estimated theoretically in a very approximate way for measurements in a channel or river.

8.2.1.1 Measurements with a line injection

Where injection is made in a channel of uniform flow by means of a device distributing the injection flow rate over the whole width of the channel in proportion to the flow rate in each elementary vertical section, the minimum length of good mixing, L , is given by the formula (in SI units)

$$L = 10 \, r \, d \quad \dots(5)$$

where

d is the mean depth;

r is the ratio of the mean velocity to the friction velocity.

The value of r will be taken to be

$$r = \sqrt{\frac{8}{\lambda}} \quad \dots(6)$$

where λ is the coefficient of the universal formula giving the head loss per unit length

$$H = \frac{\lambda}{4 R_h} \times \frac{v^2}{2g} \quad \dots(7)$$

where

H is the head loss per unit length;

R_h is the hydraulic radius;

v is the mean velocity;

g is the acceleration due to gravity.

NOTE — It is possible to express r in the following manner :

$$r = \frac{K R_h^{1/6}}{\sqrt{g}} = \frac{C}{\sqrt{g}} \quad \dots(8)$$

where

K is the Strickler coefficient, which is a function of the roughness;

R_h is the hydraulic radius;

C is Chezy's coefficient.

The Chezy coefficient can be calculated by the formula

$$C = \sqrt{g} \sqrt{\frac{8}{\lambda}} \quad \dots(9)$$

This theoretical formula has recently been the subject of satisfactory experimental verification.

8.2.1.2 Measurement with a centre point injection

For a point injection on the axis of the flow, the minimum mixing distance can be estimated by formula (10) which has not yet been checked by systematic experiment :

$$L = \frac{0,13 \, \bar{b}^2 C (0,7 C + 2 \sqrt{g})}{g d} \quad \dots(10)$$

where

$\bar{b}$ is the mean width of the wetted cross-section in the measuring reach (in metres);

d is the mean depth of water in this reach (in metres).

It is emphasised that the length obtained from formula (10) can only be used as a first indication and that the length to be used must be established by practical trials. For example, as this formula is based on a single point injection in a straight reach, the length obtained may be shorter if multiple injections are performed in the injection section.

Furthermore, some tests show that this formula (10) gives too small a value for the mixing length for small streams about 5 m wide, and too great a value for rivers about 50 m wide.

8.2.2 Duration of injection

8.2.2.1 Constant rate injection method

The duration of injection shall be such that a steady concentration regime is established in the sampling section for a sufficient length of time, usually 10 to 15 min.

The duration of injection, which is generally related to the degree of turbulence, will vary directly with the length of the measuring reach and with the extent of dead water zones and inversely with the mean water velocity.

For a given discharge, observations of the moment of arrival and disappearance of tracer in each section may be plotted as the curves 1 and 2 in figure 2. On the other hand this experiment can be used to determine the minimum mixing length (see 8.1).

The selected sampling section corresponding to the mixing length consistent with the precision sought is represented by the straight line S.

If it is desired to obtain, at the selected sampling cross-section, a steady regime for a time Δt , it is necessary to add the time Δt to the time t corresponding to the disappearance of tracer in

this section (see figure 2), and to draw, through the point obtained in this way, a curve 1' parallel to the curve 1 of arrival of tracer. The intercept at zero of this curve gives the minimum duration of injection θ . The curve 2' parallel to curve 2 determines the end of the passage of tracer.

In practice, to determine t it is possible to use a preliminary test with fluorescein or rhodamine measured by a fluorimeter, or, more simply, an experienced operator can time the passage of the visible dye cloud and adjust this duration by a coefficient based on experience.

8.2.2.2 Integration method

The duration of injection should be chosen according to the precision of the sampling method used. Injection is normally performed as quickly as possible by decantation from a vessel into the channel, but extending the period over which the tracer is injected, possibly to several minutes, lowers the maximum concentration around the point of injection and decreases density effects.

9 Procedure for the constant rate injection method

9.1 Preparation of the injection solution

It is essential that the injected solution is homogeneous. This homogeneity may be obtained by vigorous mixing performed with a mechanical stirrer or a closed-circuit pump. It is recommended that the injection solution be prepared in a separate tank from the supply tank, with water filtered by an appropriate method. If however the mixing is performed in the supply reservoir, this must be of sufficient capacity to avoid the need for the addition of liquid or tracer during the injection.

In the case of an injection of long duration, all necessary precautions should be taken to avoid a variation with time of the concentration of the solution (by evaporation for example).

9.2 Injection

The concentrated solution must be injected at the selected injection section in the reach whose discharge is to be measured, at constant rate and for the time necessary to obtain a concentration plateau of sufficient duration (see 8.2.2). Various devices can be used for carrying out this injection. For all the devices it must be possible to check the constancy of the mass flow rate of tracer, especially :

- the constancy of the concentration;
- the constancy of the flow rate of the injected solution (checking of the water-tightness of circuits, elimination of suspended impurities likely to disturb the flow in the injection circuits, etc.).

The devices that are most commonly used are :

- the constant-head tank (see figure 3);
- a volumetric pump driven by a constant speed motor (see figure 4).

9.3 Measurement of the injection rate

The chosen device for measurement of the injection rate must have a precision compatible with the required overall precision of the measurement of the stream discharge.

Three cases may be considered

- a) the chosen injection device has a measuring instrument :
 - operating in accordance with the specifications of a standard (for example an orifice plate);
 - which can be calibrated under identical measuring conditions (for example a rotameter).
- b) the chosen injection device has no measuring instrument but the flow rate may be determined during the injection by the direct measurement of fundamental quantities (for example the fall in level in a vessel or volumetric pump);
- c) the injection device is accurate by construction and can therefore itself be calibrated outside the injection period. It may be necessary for high accuracy measurements to carry out verification tests before and after the measurement.

9.4 Sampling

This document deals only with determination of concentration by taking samples. It will, however, be noted that it is possible to proceed by the detection of tracer "in situ".

For the purposes of the measurement:

- Two or three samples should be taken in the measuring section before the injection, and upstream of the injection section during and after the measurement in order to check that the initial concentration c_0 in the channel remains constant during the measurement.
- When possible, at least five samples should be taken in the measuring section from at least three points (left bank, midstream and right bank) in order to check that the condition of good mixing has been achieved. At least three of these samples should be taken from the same point during the time of passage of the tracer to demonstrate that the plateau of concentration has been achieved and to permit the calculation of the discharge.
- When it is not possible to take samples across the stream, and only in this case, samples can be taken along the bank provided it can be verified that these samples show a constant dilution ratio at at least two cross-sections.
- Three to five samples of the injected solution should be taken at the outlet from the injection apparatus just before and after the injection period to check the homogeneity of the concentration c_1 and to compare with the concentration c_2 of the tracer in the samples taken in the measuring section.
- If water from the channel is not used for dilution of the concentrated solution (see 11.1), samples of the dilution water should also be taken for the determination of c_1 .

10 Procedure for the integration method

10.1 Preparation of the solution to be injected

Whichever method of sampling is used, the solution of the radioactive tracer should be made up to a suitable volume, by addition of water from the channel, in a tank from which it can be released under conditions that give an acceptable sampling time.

The apparatus must be such as to ensure good mixing before a sample is taken for the determination of c_1 .

To minimise the possibility of adsorption of the radio-tracer, it is advantageous to add a carrier to the solution; this is a quantity of non-radioactive material that is identical to that which contains the radioactive tracer.

The injection vessel can be fitted with a device so that when it is emptied it will retain a certain volume of solution from which samples can be taken for the determination of c_1 .

10.2 Determination of the volume injected

Two procedures are available :

- a) The method of weighing, which is the more precise.

The volume injected is determined using the density of the water at the temperature at which the measurements are made.

- b) The measurement of the dimensions of the vessel that is used.

These operations are carried out before the introduction of the tracer, so that any corrections necessary can be made.

10.3 Sampling

This International Standard deals only with the determination of concentration by sampling. It should be noted, however, that it is possible to proceed by detection of the tracer "in situ".

10.3.1 Samples

The following samples are required :

- Two or three samples must be extracted from the measuring reach before the injection, also from upstream of the injection area, during and after the gauging. These are to verify that the initial concentration, c_0 in the channel remains constant during the course of the investigation.
- Samples of the injection solution must be taken.
- In cases where water from the channel is not used for the dilution of the injection solution (see 11.1), samples of the water that is used for the dilution must be taken for the determination of c_1 .

- To verify that good mixing has occurred, and for the calculation of discharge, samples should be taken from at least three points at the sampling cross-section (left bank, centre and right bank). If this is not possible (see 9.4) then the centre sample may be omitted.

The volume of each sample must be chosen to accommodate the method of analysis (see clause 11).

10.3.2 Methods of sampling in the sampling cross-section

There are several possible sampling methods (see figure 5).

- 1) By sampling at discrete and known times. (It is not necessary for the intervals between successive samples to be constant, but the intervals must be accurately known.)
- 2) By sampling at regular time intervals. (For this method to have acceptable precision, a large number of samples (at least 25) should be taken during the passage of the tracer.)
- 3) By sampling continuously at a constant rate, thus obtaining an average sample. (It is important to know the duration of the sampling, which must be longer than the time taken for the passage of the tracer.)
- 4) By sampling at a constant rate and passing the sample into a cell in which a detector is immersed.

Whichever method is chosen, it is essential that the sampling point remains the same for all samples (methods 1 and 2), or during the entire time of sampling (methods 3 and 4).

It may be desirable to set up (particularly for methods 3 and 4) a radiation detector, "in situ", upstream of the sampling site, to indicate when sampling must start. The detector should be placed a sufficient distance upstream to allow the sampling to be commenced before the arrival of the tracer at the sampling location.

Methods 3 and 4 are particularly recommended as they involve less analysis; against this, methods 1 and 2 involve the use of less equipment in the field.

In all cases the time period of sampling must be at least equal to the time taken for the passage of the tracer cloud, and synchronous with it (see figure 5).

In order to verify from the analysis that good mixing had been obtained, it would be necessary to provide at least two sets of sampling apparatus at different positions at each sampling location, these at one or more locations.

11 Methods of analysis

11.1 Dilution of the injection solution

The concentration of tracer, c_1 , in the injection solution cannot be measured directly. The concentrations c_1 and c_2 can be in a ratio of 10^8 , so it is necessary to dilute the sample of the injection solution to obtain a concentration approximately equal to

that of c_2 , the concentration of samples taken from the sampling section. This is so as to be able to measure samples of comparable concentration within the linear working range of the detection equipment.

To achieve this a sample of the concentrated injection solution is diluted by a gravimetric method (a precision balance) or a volumetric method (flasks, graduated pipettes or mini-burettes) preferably with water taken from the flow upstream of the injection site, before the investigation.

Let the dilution factor F be defined by the relation :

$$c_1 = Fc'_1 \quad \dots(11)$$

where c'_1 is the required concentration, of comparable magnitude with c_2 .

11.2 Count rate

The determination of the radioactive tracer concentration c depends on its linear relationship with the count rate N :

$$c = \nu N \quad \dots(12)$$

where ν is a proportionality constant.

The comparison of different samples must be made under strictly identical geometrical conditions (detectors of identical shape and the same counting equipment).

As a result of radioactive decay, the count rate of a sample will change with time, following the exponential law :

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-\frac{0.693}{T} t}$$

where

λ is the decay constant;

T is the half-life of the radioisotope.

This decay does not however, affect the ratio of count of two samples of the same age that were taken at the same time. If the count rates are measured at different times, decay corrections must be made.

In practice, the count rates N are not obtained directly : the activity of the channel water c_0 , cosmic radiation, the natural ambient radioactivity and the electronic noise of the detection system gives a background noise, n_0 , that must be determined from suitable samples (see 9.4 and 10.4.1) :

$$N = n - n_0 \quad \dots(13)$$

where n is the uncorrected count rate given by the measuring equipment.

11.3 Practical formulae for calculation of discharge

Combining equation (2), for the constant rate injection method, with equation (11),

$$Q = qF \frac{c'_1}{c_2} \quad \dots(14)$$

If N_1 and N_2 are the count rates corresponding to c'_1 and c_2 equation (14) becomes :

$$Q = qF \frac{N_1}{N_2} \quad \dots(15)$$

which can be written as follows, taking into account the background correction of equation (13) :

$$Q = K'F \frac{n_1 - n_0}{n_2 - n_0} \quad \dots(16)$$

where $K' = q$ (the constant flow rate of the tracer added at the injection location).

In the same way, equation (4) for the method of integration may be considered in the form :

$$Q = \frac{Vc_1}{c_2\theta}$$

where θ is the time interval considered.

Using the relationships above (equations 11, 12 and 13)

$$Q = K'F \frac{n_1 - n_0}{n_2 - n_0}$$

where $K' = \frac{V}{\theta}$

This method of calculation assumes that water extracted from the flow upstream of the injection point is used to prepare the injection solution, and for the dilution of this for the determination of N_1 . If for this operation water from another source is used, as has previously been indicated, the corresponding background correction n_e is determined, and equation (16) becomes :

$$Q = KF \frac{n_1 - n_e}{n_2 - n_0} \quad \dots(17)$$

If the dilution factor F is obtained by a gravimetric method, a correction is imposed to allow for the different densities, of the stream water, of the injection solution, and of the samples.

If the dilution of an aliquot of the injection solution is made with water at a temperature T_i , the general formula

$$Q = qF \frac{n_1 - n_e}{n_2 - n_0} \frac{\rho_i(T_i)}{\rho_d(T_d)} \frac{\rho_m(T_m)}{\rho_c(T_c)} \quad \dots(18)$$

is used, in which

ρ_i is the density of the injected solution;

ρ_c is the density of the stream water;

ρ_d is the density of the water used for the dilution;

ρ_m is the density of the sample during measurement;

T_i is the temperature of the injected solution;

T_c is the temperature of the stream water;

T_d is the temperature of the water used for dilution;

T_m is the temperature of the samples during the measurement.

The introduction of this correction necessitates the determination of densities at given temperatures. For this determination, a standard method should be used; the most suitable for the required precision, conditions of the determination and the available equipment.

If different sets of counting equipment are used to determine the count rates, it is obviously necessary to take account in the calculation of the intercalibration coefficients for the sets of equipment used.

11.4 Counting equipment

The choice of counting equipment, of sample volume and of the type of shielding necessary to reduce the background, will be made with the aim of obtaining the best signal to noise ratio (N_i/n_0) compatible with the required precision.

With γ -emitting tracers, it is best to use counting equipment comprising a sodium iodide crystal scintillation detector combined with an electronic assembly that counts and records the impulses obtained from the crystal during the measurement period.

The measurement of N_1 , N_2 , n_0 will be made in conditions of strictly identical geometry, counting equipment sensitivity, and sample temperatures.

Measurements must not commence before the detector has reached thermal equilibrium, and account must be taken of the electronic dead time of the equipment.

11.5 Method of counting

The samples taken upstream of the injection point or before the start of the exercise, will be counted before, during and after the counting of the samples containing the tracer; this is to determine the background correction n_0 and as a check that the counting equipment has not been contaminated.

To check that the sensitivity of the counting equipment remains constant, periodic counts of a radioactive source (for example ^{60}Co) should be made throughout the counting period as well as counts of the samples and diluted solutions.

It is recommended that the dilution of the injected solution be repeated at least three times, but preferably more.

Concerning the samples, it is appropriate to distinguish :

- The method of constant rate injection which requires the measurement of at least five samples and preferably more taken from different points in the flow (see 9.4). This is to verify that a condition of constant concentration has been obtained, and that good mixing has taken place in a transverse direction.

- The method of integration, for which the verification of this same fundamental condition (good mixing) can only be done by carrying out two series of sample abstractions.

To work to a demanded precision, the measurements of each sample and each diluted solution must be repeated a certain number of times.

For the most precise measurement a χ^2 test can first be made, with the help of an additional radioactive source of long half-life, such as Cobalt-60; to check that the counting equipment is operating correctly.

11.6 Checking of half-life

The count rate of one of the diluted solutions (see 11.1) must be measured periodically (at least four times) during the exercise; to check the purity of the tracer and to determine, should the need arise, the actual half-life of the tracer used.

11.7 Presence of sediments and/or organic material

If a deposit of sediments and/or organic matter forms from suspension in the measuring reach, there is a risk of recording too high a measurement; because of the fixation of tracer onto these sediments while in suspension.

If measurements are made in these conditions, by stirring the samples containing sediments and/or organic matter, a homogeneous distribution of particles in suspension may be obtained and maintained, to minimise any error.

12 Determination of uncertainty

12.1 Types of error

Under the term "uncertainty" are included accidental, systematic and random errors to which flow measurements by dilution, like any other measurement, may be subject.

For the detailed analysis of the calculation of uncertainties, reference shall be made to document ISO 5168 and to the annex. The values quoted in the example, while typical, should not be accepted by the user for his particular test without verification. In the earlier chapters of this International Standard are listed the precautions to be taken for minimizing the random and systematic uncertainties due to the equipment or methods.

Five causes of systematic errors peculiar to dilution methods are to be underlined :

a) Error associated with the tracer

The tracer which is introduced into the injection cross-section is likely, in certain cases, to react with the water flowing in the channel, with the suspended sediment, vegetation or the bed and banks of the channel or with the walls of sampling vessels in such a way as to reduce the concentration of tracer detected in the samples. This would lead to an over-estimate of the discharge.

b) Error associated with the duration of the gauging

For a gauging by the constant rate injection method, it is essential that the concentration of tracer at the sampling section has achieved a constant or plateau, value or for gauging by the integration method, that the whole of the radioactive cloud has passed the sampling cross-section. Retention of liquid in pools and in the boundary layer is often significant, and the time taken to achieve these conditions longer than intuition would suggest. Cessation of sampling before these conditions are fulfilled leads to an over-estimate of discharge.

c) Error associated with poor mixing in the gauging reach

The use of the definition of the length of good mixing given in 8.1.1 assumes that stream turbulence will result in a distribution of the concentration at each point around the mean value in the cross-section. If this distribution shows no functional dependence on position, there is no systematic error and the uncertainty can be determined. If there is a systematic variation across the section of the concentration of the integral of the concentration-time curve, then the interaction of this distribution with the distribution of velocity in the section causes a difference between the mean of the samples and the mean concentration of tracer in the section, and a positive or negative systematic error may result.

d) Error associated with change in storage volume in the reach

If the discharge changes appreciably during the measurement, systematic errors of either positive or negative sign are introduced by the change in storage volume between the injection and sampling cross-sections. The error is least when the discharge is varying smoothly and slowly as for example on the recession limb of a hydrograph, and when the mean transit time between injection and sampling cross-sections is least. This implies a choice of the shortest gauging reach consistent with good mixing of the tracer. Similar errors will arise if the storage volume changes for other reasons, for example river management operations.

e) Error associated with sampling and the analysis of samples

Errors are also likely to arise in determining the concentration of tracer in the measuring section and of samples taken from the measuring section. These may be due to sampling techniques, interaction between the walls of the sampling vessel and the tracer and to systematic errors in the tech-

niques used for radiological analysis. Errors may also arise during the process of diluting the injected solution for purposes of activity measurement. Positive and negative errors lead to over or under estimates of the discharge.

12.2 Estimation of uncertainty

Measurement of the flow rate should be presented with the estimated statistical tolerance with 95 % confidence limits; this tolerance is equal for these purposes to twice the overall standard error.

Because of the random nature of the nuclear disintegration process, the possibility $P(n)$ of observing n particles in any time interval t (small compared to half-life) is given by the Poisson distribution curve.

$$P(n) = \frac{N^n}{n!} e^{-N}$$

where N is the true mean (but unknown) number of disintegrations corresponding to time interval t . The estimate of the disintegration rate is n/t and the standard deviation $\sqrt{n/t}$. The result of each individual count rate may be expressed as :

$$\frac{n}{t} \pm 2 \frac{\sqrt{n}}{t}$$

In measurements with radioisotopes therefore, every measurement of net counting rate R should be expressed in the form

$$R = R_s - R_b \pm 2 \sqrt{\frac{n_s}{t_s^2} + \frac{n_b}{t_b^2}}$$

where n_s and n_b are the total counts due to sample (including background) and background alone in time intervals t_s and t_b respectively and R_s and R_b are the corresponding count rates.

When combining independent estimates of the same count rate (or more generally if any independent estimates are to be combined) $A, B, \dots$, with standard errors $s_1, s_2, \dots$ the following relations are used :

1) Addition of several quantities

$$(A \pm 2s_1) + (B \pm 2s_2) + \dots = A + B + \dots \pm 2 \sqrt{(s_1^2 + s_2^2 + \dots)}$$

2) Difference of two quantities

$$(A \pm 2s_1) - (B \pm 2s_2) = A - B \pm 2 \sqrt{(s_1^2 + s_2^2 + \dots)}$$

3) Product

$$(A \pm 2s_1)(B \pm 2s_2)(\dots) = (A \times B \times \dots) \left[1 \pm 2 \sqrt{\left(\frac{s_1^2}{A^2} + \frac{s_2^2}{B^2} + \dots \right)} \right]$$

4) Quotient

$$\frac{(A \pm 2s_1)}{(B \pm 2s_2)} = \frac{A}{B} \left[1 \pm 2 \sqrt{\left(\frac{s_1^2}{A^2} + \frac{s_2^2}{B^2} \right)} \right]$$

These equations are true for all types of uncertainty (for example standard errors or probable errors). It should be noted that for addition and subtraction, the standard errors are used, whilst for multiplication and division it is more convenient to use the relative error (s_1/A), (s_2/B), etc.

Although in principle the standard errors of sums, differences, etc., must always be calculated, in practice this is not necessary if one of the numbers is many times greater than the rest. Experience will indicate which numbers can be omitted from calculations of standard errors without affecting the accuracy of the result.

Another simplification of numerical calculation is possible when calculating the sum of series of almost identical results each of which has its own standard error. In practice it is usually sufficient to replace rule (1) by the approximation

$$\sum_{i=1}^n x_i \pm 2 \sqrt{n s^2}$$

where

n is the number of separate observations;

x_i is the result of an observation; and

$\overline{s^2}$ is the approximate mean of the sequence of variance s_i^2 derived by inspection.

13 Comparison of methods (constant rate injection and integration)

13.1 Advantages of the constant rate injection method

- It is simpler to check for good mixing using a single instrument when it is possible to take samples in different points of the measuring section. On the other hand, several instruments must be used in parallel when this same verification is required in the integration method.
- It is simpler to evaluate the random errors.
- It is not necessary to know the volume of solution injected.

13.2 Advantages of the integration method

- This method calls for a smaller quantity of tracer and a shorter time than the constant rate injection method.
- The method of injection is immaterial and the equipment simple.
- With the same quantity of tracer, it is possible to carry out the measurement over a greater length of channel.

Annex

Example of a numerical calculation with analysis of the uncertainty in the determination of flow rate

The example given illustrates the method used for a flow calculation and the estimation of the limits of error. The treatment of results follows a similar pattern for both the integration and the constant rate injection methods, and will not differ substantially with the isotope used or with the discharge. The example is based on the constant rate injection method using Sodium - 24 as the tracer. In many cases where Sodium - 24 or Bromine - 82 is used, the background can be considered as zero and the following equation can be used :

$$Q = \frac{c_1}{c_2} q$$

where

c_1 is the concentration of the injection solution;

c_2 is the concentration of the samples taken from the river;
and

q is the flow rate of the injection solution.

The injected solution is diluted by a factor, designated F , to obtain a solution of comparable concentration, c_1/F , to that of c_2 . When measuring two solutions of very similar concentration, contamination of the sample vessel is avoided and similar dead times are observed in the counting (dead time corrections are not necessary in this example).

A.1 Preparation of the comparison solution

It is important to note that as the concentration of the injected solution can be 10^8 times higher than that of the samples, great care must be taken to avoid cross contamination. It is prudent to prepare several comparison solutions to avoid errors.

After the injection of the concentrated solution, a syringe is partly filled with a sample of this solution and weighed to 10^{-4} g. About 1 g of the injected solution is emptied into a graduated flask and the syringe weighed again. The graduated flask has a capacity of 250 ml. By partly filling the flask with distilled water and by carefully emptying the syringe, it is possible to prevent the highly active solution coming into contact with the walls of the flask.

The flask is filled with 250 ml of distilled water, weighed and the contents thoroughly mixed. A further dilution by a factor of 3 000 is carried out in the same manner.

The density of the injected solution can be determined after radioactive decay.

A.2 Calculation of the dilution ratio

All measurements are made at 20 °C. Accumulating errors are calculated from the equations of clause 10. All errors are evaluated, even if they are negligible.

A.2.1 Density of the injected solution

Grams

Mass of bottle full of injected solution : 21,709 5 (± 0,000 1)

Mass of bottle empty : 11,492 5 (± 0,000 1)

Mass of solution in bottle : 10,217 0 (± 0,000 14)

Mass of bottle full of distilled water : 21,493 7 (± 0,000 1)

Mass of bottle empty : 11,492 5 (± 0,000 1)

Mass of distilled water in bottle : 10,001 2 (± 0,000 14)

Density of water at 20 °C : 998,2 kg/m³

i.e. : 0,998 2 g/ml

Density of injected solution at 20 °C :

$$\frac{10,217 0 (\pm 0,001 4 \%)}{10,001 2 (\pm 0,001 4 \%)} \times 998,2$$

$$= 1 019, 7 (\pm 0,002 \%) \text{ kg/m}^3$$

i.e. :

$$1,019 7 (\pm 0,002 \%) \text{ g/ml}$$

A.2.2 Preparation of the comparison solution

A.2.2.1 First dilution

Grams

Mass of syringe with injected solution : 5,392 7 (± 0,000 1)

Mass of syringe with remaining solution : 4,186 0 (± 0,000 1)

Mass of dispensed solution : 1,206 7 (± 0,000 14)

Volume of dispensed solution :

$$\frac{1,206 7 (\pm 0,012 \%)}{1,019 7 (\pm 0,002 \%)} = 1,183 4 (\pm 0,012 \%) \text{ ml}$$

This solution is diluted in a 250 ml flask. The mass of the solution, 249,8 ± 0,1 is obtained from a pan balance or from a torsion balance.

Volume of diluted solutions :

$$\begin{aligned} & 1,183\,4 (\pm 0,000\,14) + \\ & \frac{249,8 (\pm 0,1) - 1,206\,7 (\pm 0,000\,14)}{0,998\,2} \\ & = 250,22 (\pm 0,1) \text{ ml} \end{aligned}$$

Dilution factor :

$$\frac{250,22 (\pm 0,04\, \%)}{1,183\,4 (\pm 0,012\, \%)} = 211,44 (\pm 0,04\, \%)$$

A.2.2.2 Second dilution

This solution (actual density 0,998 3 g/ml) is further diluted :

	Grams
Mass of syringe with diluted solution :	5,543 2 ($\pm 0,000\,1$)
Mass of syringe with remaining solution :	4,622 3 ($\pm 0,000\,1$)
Mass of dispensed solution :	0,920 9 ($\pm 0,000\,14$)

Volume of dispensed solutions :

$$\frac{0,920\,9}{0,998\,3} = 0,922\,5 (\pm 0,000\,14) \text{ ml}$$

This volume is diluted to 3 000 ml with distilled water (to which sodium carrier ions have been added to give a total of about 50 mg in 3 000 ml). This is prepared by three weighings each of 1 kg.

Weight of diluted solution : 2 997 ($\pm 0,6$) g

Volume of diluted solution :

$$\frac{2\,997 (\pm 0,6)}{0,998\,2} = 3\,002,4 (\pm 0,6) \text{ ml}$$

Dilution factor :

$$\frac{3\,002,4 (\pm 0,02\, \%)}{0,922\,5 (\pm 0,014\, \%)} = 3\,254,6 (\pm 0,02\, \%)$$

Total dilution factor :

$$\begin{aligned} & 3\,254,6 (\pm 0,02\, \%) \times 211,44 (\pm 0,04\, \%) \\ & = 6,882 \times 10^5 (\pm 0,045\, \%) \end{aligned}$$

Zero time : 10,00 h; half-life of ^{24}Na : 15,00 h;

Background : 572 counts/min.

The counts of 15 samples from the river and three comparison samples are given in the table.

Mean count rate of samples : 15 278 ($\pm 0,18\, \%$) counts/min.

Mean count rate of comparison samples : 14 246 ($\pm 0,22\, \%$) counts/min.

$$\begin{aligned} \frac{c_1}{c_2} &= 6,882 \times 10^5 (\pm 0,045\, \%) \times \frac{14\,246 (\pm 0,22\, \%)}{15\,278 (\pm 0,18\, \%)} \\ &= 6,498 \times 10^5 (\pm 0,29\, \%) \end{aligned}$$

The principal source of error in the count rates is simply the statistical error associated with the total number of counts recorded, but the following effects also contribute and must be reduced as much as possible.

- filling of containers (measured at 0,016 %);
- the counting time;
- uncertainty in the half life (given as 0,02 %);
- positioning of the samples (measured at 0,01 %).

A.3 Calculation of discharge, Q

The injection rate is :

$$\begin{aligned} & 1,314 (\pm 0,2\, \%) \text{ g/s at } 20\, ^\circ\text{C} \\ \text{i.e. } q &= \frac{1,314 (\pm 0,2\, \%)}{0,998\,2} = 1,316 (\pm 0,2\, \%) \text{ ml/s} \end{aligned}$$

The flow of the river :

$$\begin{aligned} Q &= q \frac{c_1}{c_2} \\ &= 1,316 (\pm 0,2\, \%) \times 6,498 \times 10^5 (\pm 0,29\, \%) \text{ ml/s} \end{aligned}$$

i.e. 0,855 ($\pm 0,35\, \%$) m³/s

or, writing the flow and the error in agreement with ISO 5168 :

$$\begin{aligned} Q &= 0,855 \text{ m}^3/\text{s} \\ e_R (95) &= \pm 0,003 \text{ m}^3 \end{aligned}$$

NOTES

1 The dilutions are calculated from the ratio of masses of the solutions. This eliminates the first order correction for buoyancy. The flow from the injection pump is obtained, where possible, by direct weighing, and to obtain the best precision the buoyancy correction should be made. The effect will be to increase by about 0,1 %.

2 The uncertainty that is given is the standard deviation, and characterises the precision with which the laboratory measurements can be made. It does not include the errors arising in the field from causes such as adsorption, imperfect mixing and failure to achieve the condition of constant concentration in the river (see 9.1).

Table

Sample	Start of counting time	Counts for 5 min	Count minus background	Decay correction	Corrected counts	Counts per min
S ₁	10,00	78 810	75 950	0,998 1	76 095	15 219
S ₂	10,06	78 896	76 036	0,993 6	76 526	15 305
S ₃	10,12	78 222	75 362	0,989 0	76 200	15 240
S ₄	10,18	78 259	75 399	0,984 4	76 594	15 319
S ₅	10,24	77 471	74 611	0,979 8	76 149	15 230
S ₆	10,30	77 574	74 714	0,975 3	76 606	15 321
S ₇	10,36	77 392	74 532	0,970 8	76 774	15 355
S ₈	10,42	76 081	73 941	0,966 4	76 512	15 302
S ₉	10,48	76 167	73 307	0,961 9	76 211	15 242
S ₁₀	10,54	76 079	73 219	0,957 6	76 461	15 292
S ₁₁	11,00	75 368	72 508	0,953 2	76 068	15 214
S ₁₂	11,06	75 640	72 708	0,948 8	76 707	15 341
S ₁₃	11,12	74 936	72 076	0,945 9	76 198	15 240
S ₁₄	11,18	74 644	71 784	0,939 9	76 374	15 275
S ₁₅	11,24	74 359	71 499	0,935 7	76 412	15 282
		Counts for 20 min				
Comparison 1	09,30	304 491				
Comparison 2	11,30	279 677				
Comparison 3	11,55	272 488				
			293 051	1,015 4	288 606	14 430
			268 237	0,926 1	289 642	14 482
			261 048	0,908 5	287 340	14 367

NOTE — The counting of the comparison samples is carried out for 20 min.

Mean count rate of samples S₁ to S₁₅ : 15 278 counts/min.

Standard error of mean count rate : $\frac{1}{n} \sqrt{n \times s^2} = 14$ counts/min.

Mean count rate of comparison samples : 14 426 counts/min.

Standard error of mean count rate : $\frac{1}{n} \sqrt{n \times s^2} = 16$ counts/min.

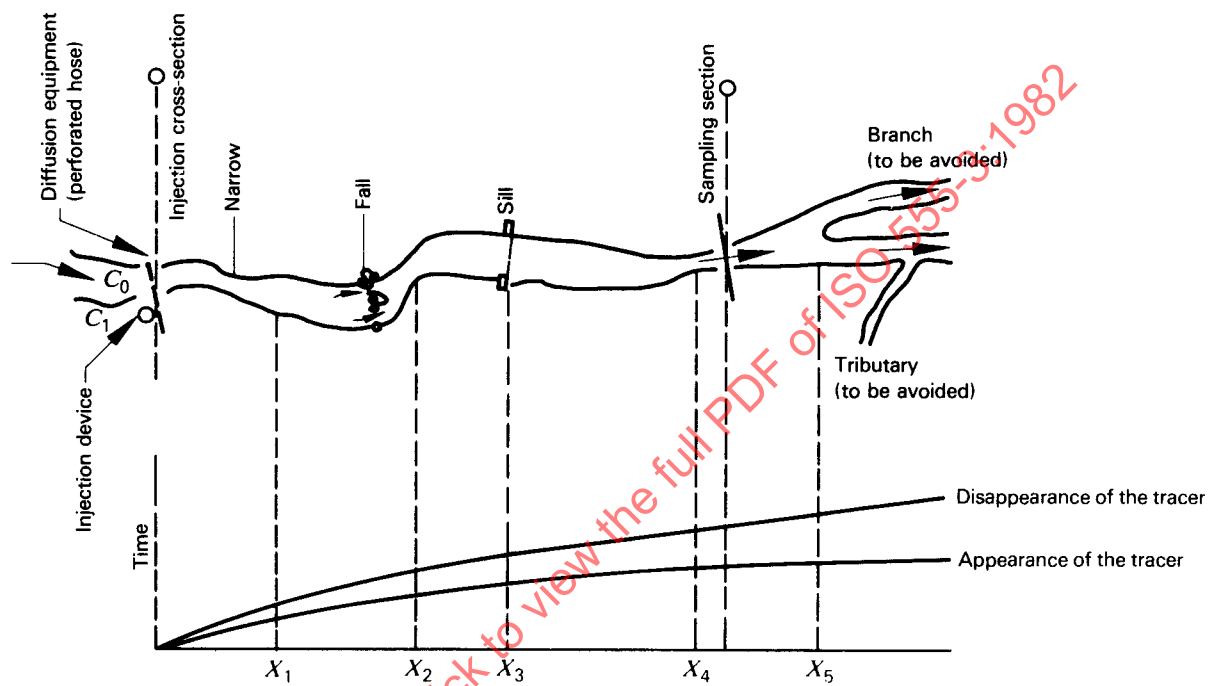


Figure 1 — Diagram of measuring reach