
**Calculation of load capacity of spur and
helical gears —**

**Part 3:
Calculation of tooth bending strength**

*Calcul de la capacité de charge des engrenages cylindriques à
dentures droite et hélicoïdale —*

Partie 3: Calcul de la résistance à la flexion en pied de dent



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ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 6336-3 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

This second edition cancels and replaces the first edition (ISO 6336-3:1996), Clauses 5 and Clause 9 of which have been technically revised, with a new Clause 8 having been added to this new edition. It also incorporates the Technical Corrigendum ISO 6336-3:1996/Cor.1:1999.

ISO 6336 consists of the following parts, under the general title *Calculation of load capacity of spur and helical gears*:

- *Part 1: Basic principles, introduction and general influence factors*
- *Part 2: Calculation of surface durability (pitting)*
- *Part 3: Calculation of tooth bending strength*
- *Part 5: Strength and quality of materials*
- *Part 6: Calculation of service life under variable load*

This corrected version incorporates the following corrections:

- Figure 3 has been updated;
- in Equation (17), the missing lines denoting the absolute value, Z_n , have been inserted;
- minus signs missing from Equations (18) and (19) have been inserted;
- Equation (50) has been corrected.

Introduction

The maximum tensile stress at the tooth root (in the direction of the tooth height), which may not exceed the permissible bending stress for the material, is the basis for rating the bending strength of gear teeth. The stress occurs in the “tension fillets” of the working tooth flanks. If load-induced cracks are formed, the first of these often appears in the fillets where the compressive stress is generated, i.e. in the “compression fillets”, which are those of the non-working flanks. When the tooth loading is unidirectional and the teeth are of conventional shape, these cracks seldom propagate to failure. Crack propagation ending in failure is most likely to stem from cracks initiated in tension fillets.

The endurable tooth loading of teeth subjected to a reversal of loading during each revolution, such as “idler gears”, is less than the endurable unidirectional loading. The full range of stress in such circumstances is more than twice the tensile stress occurring in the root fillets of the loaded flanks. This is taken into consideration when determining permissible stresses (see ISO 6336-5).

When gear rims are thin and tooth spaces adjacent to the root surface narrow (conditions which can particularly apply to some internal gears), initial cracks commonly occur in the compression fillet. Since, in such circumstances, gear rims themselves can suffer fatigue breakage, special studies are necessary. See Clause 1.

Several methods for calculating the critical tooth root stress and evaluating some of the relevant factors have been approved. See ISO 6336-1.

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Calculation of load capacity of spur and helical gears —

Part 3:

Calculation of tooth bending strength

IMPORTANT — The user of this part of ISO 6336 is cautioned that when the method specified is used for large helix angles and large pressure angles, the calculated results should be confirmed by experience as by Method A.

1 Scope

This part of ISO 6336 specifies the fundamental formulae for use in tooth bending stress calculations for involute external or internal spur and helical gears with a rim thickness $s_R > 0,5 h_t$ for external gears and $s_R > 1,75 m_n$ for internal gears. In service, internal gears can experience failure modes other than tooth bending fatigue, i.e. fractures starting at the root diameter and progressing radially outward. This part of ISO 6336 does not provide adequate safety against failure modes other than tooth bending fatigue. All load influences on tooth stress are included in so far as they are the result of loads transmitted by the gears and in so far as they can be evaluated quantitatively.

The given formulae are valid for spur and helical gears with tooth profiles in accordance with the basic rack standardized in ISO 53. They may also be used for teeth conjugate to other basic racks if the virtual contact ratio $\varepsilon_{\alpha n}$ is less than 2,5.

The load capacity determined on the basis of permissible bending stress is termed “tooth bending strength”. The results are in good agreement with other methods for the range, as indicated in the scope of ISO 6336-1.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile*

ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*

ISO 6336-1:2006, *Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors*

ISO 6336-5:2003, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of material*

3 Terms, definitions, symbols and abbreviated terms

For the purposes of this document, the terms, definitions, symbols and abbreviated terms given in ISO 1122-1 and ISO 6336-1 apply.

4 Tooth breakage and safety factors

Tooth breakage usually ends the service life of a transmission. Sometimes, the destruction of all gears in a transmission can be a consequence of the breakage of one tooth. In some instances, the transmission path between input and output shafts is broken. As a consequence, the chosen value of the safety factor S_F against tooth breakage should be larger than the safety factor against pitting.

General comments on the choice of the minimum safety factor can be found in ISO 6336-1:2006, 4.1.7. It is recommended that manufacturer and customer agree on the value of the minimum safety factor.

This part of ISO 6336 does not apply at stress levels above those permissible for 10^3 cycles, since stresses in this range may exceed the elastic limit of the gear tooth.

5 Basic formulae

The actual tooth root stress σ_F and the permissible (tooth root) bending stress σ_{FP} shall be calculated separately for pinion and wheel; σ_F shall be less than σ_{FP} .

5.1 Safety factor for bending strength (safety against tooth breakage), S_F

Calculate S_F separately for pinion and wheel:

$$S_{F1} = \frac{\sigma_{FG1}}{\sigma_{F1}} \geq S_{Fmin} \quad (1)$$

$$S_{F2} = \frac{\sigma_{FG2}}{\sigma_{F2}} \geq S_{Fmin} \quad (2)$$

σ_{F1} and σ_{F2} are derived from Equations (3) and (4). The values of σ_{FG} for reference stress and static stress are calculated in accordance with 5.3.2.1 and 5.3.2.2, using Equation (5). For limited life, σ_{FG} is determined in accordance with 5.3.3.

The values of tooth root stress limit σ_{FG} , of permissible stress σ_{FP} and of tooth root stress σ_F may each be determined by different methods. The method used for each value shall be stated in the calculation report.

NOTE Safety factors in accordance with the present clause are relevant to transmissible torque.

See ISO 6336-1:2006, 4.1.7 for comments on numerical values for the minimum safety factor and risk of damage.

5.2 Tooth root stress, σ_F

Tooth root stress σ_F is the maximum tensile stress at the surface in the root.

5.2.1 Method A

In principle, the maximum tensile stress can be determined by any appropriate method (finite element analysis, integral equations, conformal mapping procedures or experimentally by strain measurement, etc.). In order to determine the maximum tooth root stress, the effects of load distribution over two or more engaging teeth and changes of stress with changes of meshing phase shall be taken into consideration.

Method A is only used in special cases and, because of the great effort involved, is only justifiable in such cases.

5.2.2 Method B

According to this part of ISO 6336, the local tooth root stress is determined as the product of nominal tooth root stress and a stress correction factor ¹⁾.

This method involves the assumption that the determinant tooth root stress occurs with application of load at the outer point of single pair tooth contact of spur gears or of the virtual spur gears of helical gears. However, in the latter case, the “transverse load” shall be replaced by the “normal load”, applied over the facewidth of the actual gear of interest.

For gears having virtual contact ratios in the range $2 \leq \varepsilon_{\alpha n} < 2.5$, it is assumed that the determinant stress occurs with application of load at the inner point of triple pair tooth contact. In ISO 6336, this assumption is taken into consideration by the deep tooth factor, Y_{DT} . In the case of helical gears, the factor Y_{β} accounts for deviations from these assumptions.

Method B is suitable for general calculations and is also appropriate for computer programming and for the analysis of pulsator tests (with a given point of application of loading).

The total tangential load in the case of gear trains with multiple transmission paths (planetary gear trains, split-path gear trains) is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This is to be taken into consideration by inserting a mesh load factor, K_{γ} , to follow K_A in Equation (3), in order to adjust as necessary the average load per mesh.

$$\sigma_F = \sigma_{F0} K_A K_V K_{F\beta} K_{F\alpha} \quad (3)$$

where

- σ_{F0} is the nominal tooth root stress, which is the maximum local principal stress produced at the tooth root when an error-free gear pair is loaded by the static nominal torque and without any pre-stress such as shrink fitting, i.e. stress ratio $R = 0$ [see Equation (4)];
- σ_{FP} is the permissible bending stress (see 5.3);
- K_A is the application factor (see ISO 6336-6), which takes into account load increments due to externally influenced variations of input or output torque;
- K_V is the dynamic factor (see ISO 6336-1), which takes into account load increments due to internal dynamic effects;
- $K_{F\beta}$ is the face load factor for tooth root stress (see ISO 6336-1), which takes into account uneven distribution of load over the facewidth due to mesh-misalignment caused by inaccuracies in manufacture, elastic deformations, etc.;
- $K_{F\alpha}$ is the transverse load factor for tooth root stress (see ISO 6336-1), which takes into account uneven load distribution in the transverse direction, resulting, for example, from pitch deviations.

NOTE See ISO 6336-1:2006, 4.1.14, for the sequence in which factors K_A , K_V , $K_{F\beta}$ and $K_{F\alpha}$ are calculated.

$$\sigma_{F0} = \frac{F_t}{b_{m_n}} Y_F Y_S Y_{\beta} Y_B Y_{DT} \quad (4)$$

1) Stresses such as those caused by the shrink-fitting of gear rims, which are superimposed on stresses due to tooth loading, should be taken into consideration in the calculation of permissible tooth root stress σ_{FP} .

where

F_t is the nominal tangential load, the transverse load tangential to the reference cylinder²⁾ (see ISO 6336-1);

b is the facewidth (for double helical gears $b = 2 b_B$)³⁾;

m_n is the normal module;

Y_F is the form factor (see Clause 6), which takes into account the influence on nominal tooth root stress of the tooth form with load applied at the outer point of single pair tooth contact;

Y_S is the stress correction factor (see Clause 7), which takes into account the influence on nominal tooth root stress, determined for application of load at the outer point of single pair tooth contact, to the local tooth root stress, and thus, by means of which, are taken into account;

- i) the stress amplifying effect of change of section at the tooth root, and
- ii) the fact that evaluation of the true stress system at the tooth root critical section is more complex than the simple system evaluation presented;

Y_β is the helix angle factor (see Clause 8), which compensates for the fact that the bending moment intensity at the tooth root of helical gears is, as a consequence of the oblique lines of contact, less than the corresponding values for the virtual spur gears used as bases for calculation;

Y_B is the rim thickness factor (see Clause 9), which adjusts the calculated tooth root stress for thin rimmed gears;

Y_{DT} is the deep tooth factor (see Clause 10), which adjusts the calculated tooth root stress for high precision gears with a contact ratio in the range of $2 \leq \varepsilon_{\alpha n} < 2,5$.

5.3 Permissible bending stress, σ_{FP}

The limit value of tooth root stresses (see Clause 11) should preferably be derived from material tests using gears as test pieces, since in this way the effects of test piece geometry, such as the effect of the fillet at the tooth roots, are included in the results. The calculation methods provided constitute empirical means for comparing stresses in gears of different dimensions with experimental results. The closer test gears and test conditions resemble the service gears and service conditions, the lesser will be the influence of inaccuracies in the formulation of the calculation expressions.

5.3.1 Methods for determination of permissible bending stress, σ_{FP} — Principles, assumptions and application

Several procedures for the determination of permissible bending stress σ_{FP} are acceptable. The method adopted shall be validated by carrying out careful comparative studies of well-documented service histories of a number of gears.

2) In all cases, even when $\varepsilon_{\alpha n} > 2$, it is necessary to substitute the relevant total tangential load as F_t . Reasons for the choice of load application at the reference cylinder are given in 6.3. See ISO 6336-1, 4.2, for definition of F_t and comments on particular characteristics of double helical gears.

3) The value b , of mating gears, is the facewidth at the root circle, ignoring any intentional transverse chamfers or tooth-end rounding. If the facewidths of the pinion and wheel are not equal, it can be assumed that the load bearing width of the wider facewidth is equal to the smaller facewidth plus such extension of the wider that does not exceed $1 \times$ the module at each end of the teeth.

5.3.1.1 Method A

By this method, the values for σ_{FP} or for the tooth root stress limit, σ_{FG} , are obtained using Equations (3) and (4) from the S-N curve or damage curve derived from results of testing facsimiles of the actual gear pair, under the appropriate service conditions.

The cost required for this method is, in general, only justifiable for the development of new products, failure of which would have serious consequences (e.g. for manned space flights).

Similarly, in line with this method, the allowable stress values may be derived from consideration of dimensions, service conditions and performance of carefully monitored reference gears.

5.3.1.2 Method B

Damage curves characterized by the nominal stress number (bending), $\sigma_{F \text{ lim}}$, and the factor Y_{NT} have been determined for a number of common gear materials and heat treatments from results of gear load or pulsator testing of standard reference test gears. Material values so determined are converted to suit the dimensions of the gears of interest, using the relative influence factors for notch sensitivity, $Y_{\delta \text{ rel T}}$, for surface roughness, $Y_{R \text{ rel T}}$, and for size, Y_X .

Method B is recommended for the calculation of reasonably accurate gear ratings whenever bending strength values are available from gear tests, from special tests or, if the material is similar, from ISO 6336-5.

5.3.2 Permissible bending stress, σ_{FP} : Method B

Subject to the reservations given in 5.3.2.1 and 5.3.2.2, Equation (5) is to be used for this calculation:

$$\sigma_{FP} = \frac{\sigma_{F \text{ lim}} Y_{ST} Y_{NT}}{S_{F \text{ min}}} Y_{\delta \text{ rel T}} Y_{R \text{ rel T}} Y_X = \frac{\sigma_{FE} Y_{NT}}{S_{F \text{ min}}} Y_{\delta \text{ rel T}} Y_{R \text{ rel T}} Y_X = \frac{\sigma_{FG}}{S_{F \text{ min}}} \quad (5)$$

where

$\sigma_{F \text{ lim}}$ is the nominal stress number (bending) from reference test gears (see ISO 6336-5), which is the bending stress limit value relevant to the influences of the material, the heat treatment and the surface roughness of the test gear root fillets;

σ_{FE} is the allowable stress number for bending, corresponding to the basic bending strength of the un-notched test piece, under the assumption that the material condition (including heat treatment) is fully elastic

$$\sigma_{FE} = (\sigma_{F \text{ lim}} Y_{ST});$$

Y_{ST} is the stress correction factor, relevant to the dimensions of the reference test gears (see 7.4);

Y_{NT} is the life factor for tooth root stress, relevant to the dimensions of the reference test gear (see Clause 12), which takes into account the higher load capacity for a limited number of load cycles;

σ_{FG} is the tooth root stress limit;

$$\sigma_{FG} = (\sigma_{FP} S_{F \text{ min}});$$

$S_{F \text{ min}}$ is the minimum required safety factor for tooth root stress (see Clause 4 and 5.1);

$Y_{\delta \text{ rel T}}$ is the relative notch sensitivity factor, which is the quotient of the notch sensitivity factor of the gear of interest divided by the standard test gear factor (see Clause 13) and which enables the influence of the notch sensitivity of the material to be taken into account;

$Y_{R\ rel\ T}$ is the relative surface factor, which is the quotient of the surface roughness factor of tooth root fillets of the gear of interest divided by the tooth root fillet factor of the reference test gear (see Clause 14) and which enables the relevant surface roughness of tooth root fillet influences to be taken into account;

Y_X is the size factor relevant to tooth root strength (see Clause 15), which is used to take into account the influence of tooth dimensions on tooth bending strength.

5.3.2.1 Permissible bending stress (reference)

The permissible bending stress (reference), $\sigma_{FP\ ref}$, is derived from Equation (5), with $Y_{NT} = 1$ and influence factors $\sigma_{F\ lim}$, Y_{ST} , $Y_{\delta\ rel\ T}$, $Y_{R\ rel\ T}$, Y_X and $S_{F\ min}$ calculated in accordance with the specified Method B.

5.3.2.2 Permissible bending stress (static)

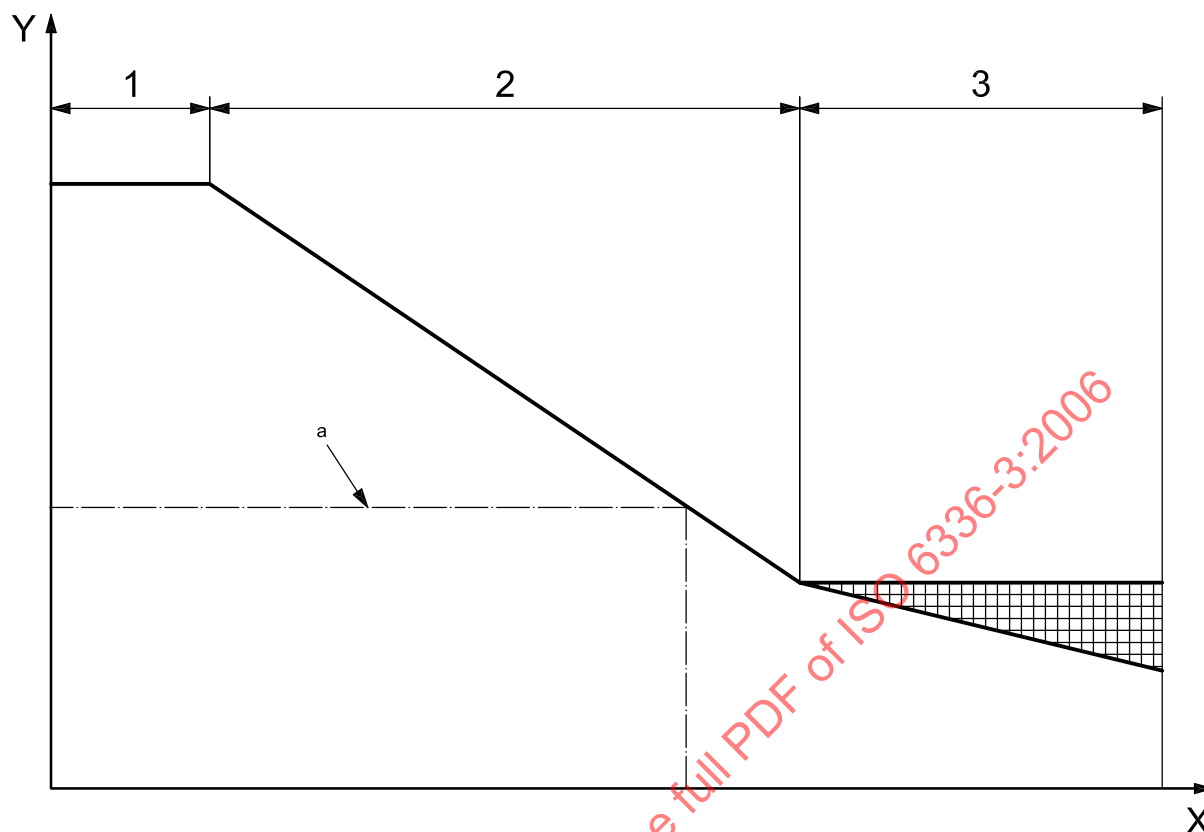
The permissible bending stress (static), $\sigma_{FP\ stat}$, is determined in accordance with Equation (5), with factors $\sigma_{F\ lim}$, Y_{NT} , Y_{ST} , $Y_{\delta\ rel\ T}$, $Y_{R\ rel\ T}$, Y_X and $S_{F\ min}$ calculated in accordance with the specified Method B (for static stress).

5.3.3 Permissible bending stress, σ_{FP} , for limited and long life: Method B

σ_{FP} for a given number of load cycles, N_L , is determined by means of graphical or calculated linear interpolation along the S-N curve on a log-log scale, between the value obtained for reference stress in accordance with 5.3.2.1 and the value obtained for static stress in accordance with 5.3.2.2. Also see Clause 12.

5.3.3.1 Graphical values

Calculate $\sigma_{FP\ ref}$ for the reference stress and $\sigma_{FP\ stat}$ for the static stress in accordance with 5.3.2 and plot the S-N curve corresponding to life factor Y_{NT} . See Figure 1 for the principle. σ_{FP} for the relevant number of load cycles N_L can be read from this graph.

**Key**X number of load cycles, N_L (log)Y permissible bending stress, σ_{FP} (log)

1 static

2 limited life

3 long life

a Example: permissible bending stress, σ_{FP} , for a given number of load cycles.

Figure 1 — Graphical determination of permissible bending stress for limited life, in accordance with Method B

5.3.3.2 Determination by calculation

Calculate $\sigma_{FP \text{ ref}}$ for the reference stress and $\sigma_{FP \text{ stat}}$ for the static stress in accordance with 5.3.2 and, using these results, determine σ_{FP} for the relevant number of load cycles N_L in the limited life range, as follows (see ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used).

$$\sigma_{FP} = \sigma_{FP \text{ ref}} Y_N = \sigma_{FP \text{ ref}} \left(\frac{3 \times 10^6}{N_L} \right)^{\text{exp}} \quad (6)$$

a) For St, V, GGG (perl., bai.) or GTS (perl.), limited life range as shown in Figure 9, $10^4 < N_L \leq 3 \times 10^6$:

$$\text{exp} = 0,4037 \log \frac{\sigma_{FP \text{ stat}}}{\sigma_{FP \text{ ref}}} \quad (7)$$

- b) For IF, Eh, NT (nitr.), NV (nitr.), NV (nitrocar.), GGG (ferr.) or GG, limited life range as shown in Figure 9, $10^3 < N_L \leq 3 \times 10^6$:

$$\exp = 0,287\ 6 \log \frac{\sigma_{\text{FP stat}}}{\sigma_{\text{FP ref}}} \quad (8)$$

Corresponding calculations may be determined for the range of long life.

6 Form factor, Y_F

6.1 General

Y_F is the factor by which the influence of tooth form on nominal tooth root stress is taken into account. See 5.2.1 for principles, assumptions and details of use. Y_F is relevant to application of load at the outer point of single pair tooth contact (Method B).

The chord between the points at which the 30° tangents contact the root fillets for external gears, or at which the 60° tangents contact the root fillets for internal gears, defines the section to be used as the basis for calculation (see Figures 3 to 4).

Determination of the values Y_F and Y_S is based on the nominal tooth form with the profile shift coefficient x . In general, the effect of reduction of tooth thickness on the tooth bending strength of finished-cut cylindrical gears may be ignored. Since the tooth roots of ground or shaved gear teeth are usually generated by cutting tools such as hobs, their shapes and dimensions are usually determined by the cutting depth settings.

Because of material allowances for finishing processes such as profile grinding, it is usually the case that the depth setting of the roughing tool, relative to the gear axis, includes the amount of nominal profile shift, xm_n , plus a tolerance designed to ensure that the finishing allowance will be greater instead of less than the requisite minimum. Because of this, calculated values of tooth root stresses usually err on the side of safety.

If the tooth thickness deviation near the root results in a thickness reduction of more than $0,05 m_n$, this shall be taken into account in the stress calculation, by taking the generated profile, x_E , relative to rack shift amount m_n instead of the nominal profile.

The equations in this part of ISO 6336 apply to all basic rack profiles (see Figure 2) with and without undercut, but with the following restrictions:

- the contact point of the 30° (60°) tangent shall lie on the tooth root fillet generated by the root fillet of the basic rack;
- the basic rack profile of the gear shall have a root fillet with $\rho_{\text{FP}} > 0$;
- the teeth shall be generated using tools such as hobs or rack type cutters;
- since calculated ratings refer to finished tooth forms, profile grinding and similar allowances, including tooth thickness allowances, can be neglected, and in practice it can be assumed that the dimensions of the basic rack of the tool are the same as those of the counterpart basic rack of the gear;
- for internal gears, a virtual basic rack profile is used which differs from the basic rack profile in the root radius ρ_{FP} [see Equation (11)].

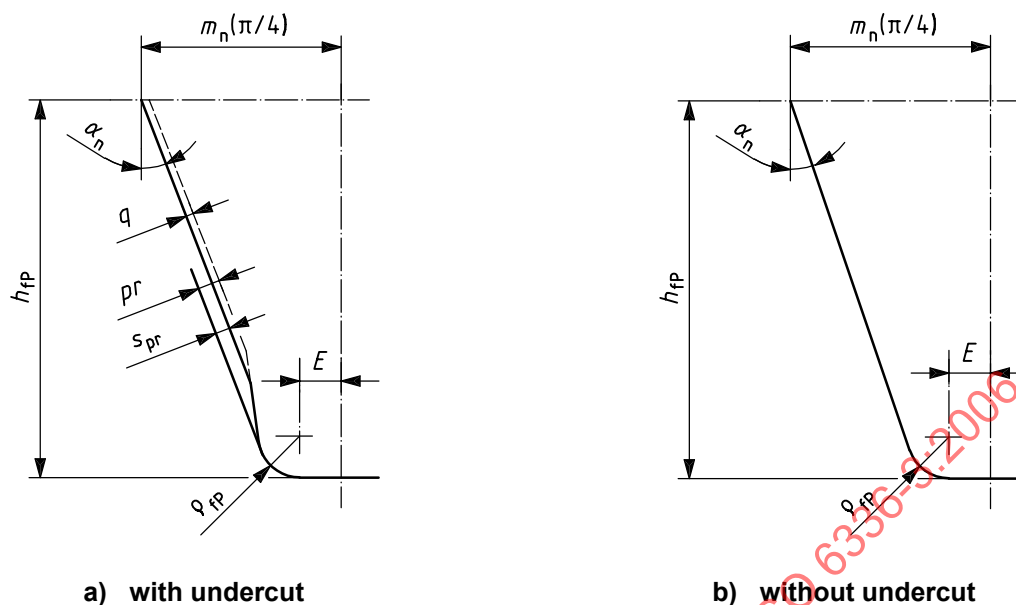


Figure 2 — Dimensions and basic rack profile of the teeth (finished profile)

The above comments apply to straight spur and helical gears. The value Y_F is determined for the virtual spur gears of helical gears; the virtual number of teeth z_n can be determined using Equation (21) or (22). Y_F is determined separately for the pinion and the wheel.

NOTE For a description of symbols and abbreviations, see ISO 6336-1:2006, Table 1.

6.2 Calculation of the form factor, Y_F : Method B

The determination of the normal chordal dimension s_{Fn} of the tooth root critical section and the bending moment arm h_{Fe} relevant to load application at the outer point of single pair gear tooth contact for Method B is shown in Figures 3 and 4.

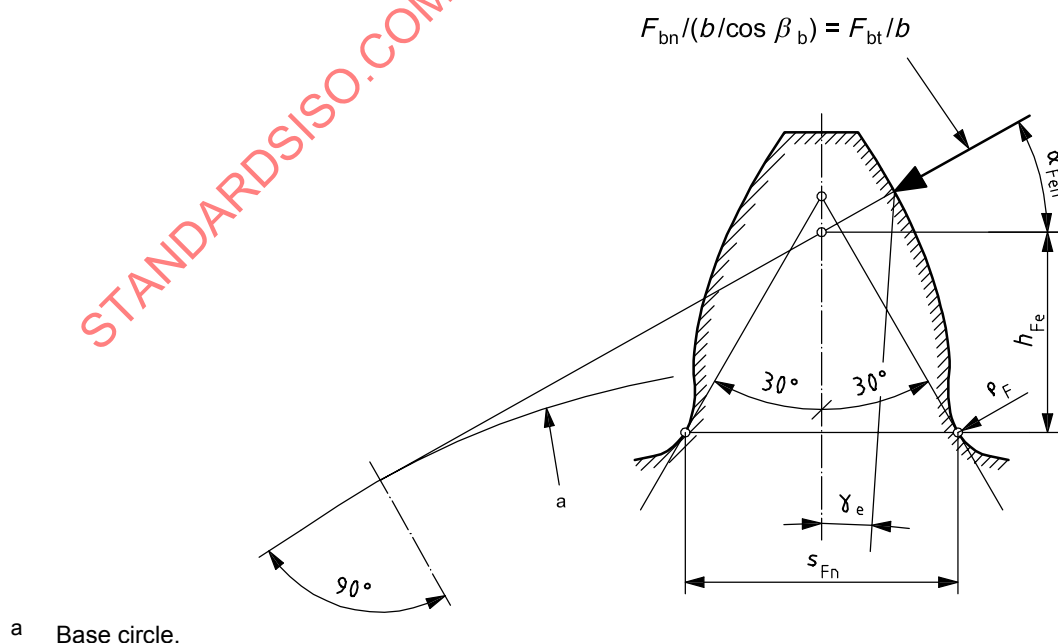
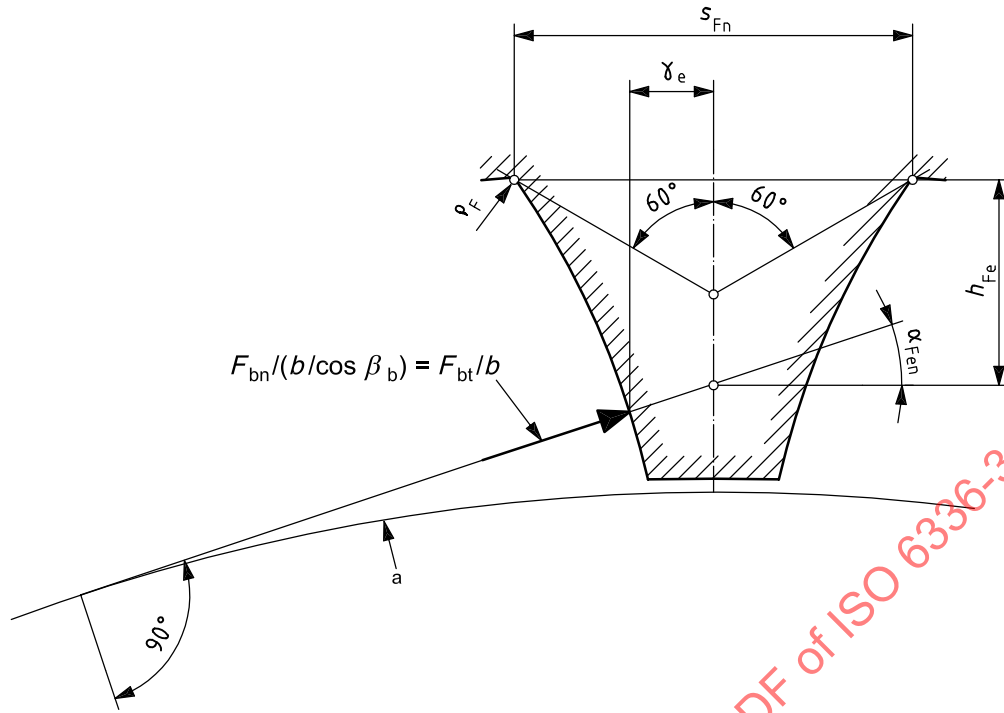


Figure 3 — Determination of normal chordal dimensions of tooth root critical section for Method B (external gears)



a Base circle.

Figure 4 — Determination of normal chordal dimensions of tooth root critical section for Method B (internal gears)

The following equation uses the symbols illustrated in Figures 3 and 4:

$$Y_F = \frac{6 h_{Fe} \cos \alpha_{Fen}}{m_n} \frac{1}{\left(\frac{s_{Fn}}{m_n} \right)^2 \cos \alpha_n} \quad (9)$$

In order to evaluate precise values, s_{Fn} and α_{Fen} , of h_{Fe} it is first necessary to derive a value of θ which is reasonably accurate, usually after five iterations of Equation (14). Determination of Y_F by graphical means is not recommended.

6.2.1 Tooth root normal chord, s_{Fn} , radius of root fillet, ρ_F , bending moment arm, h_{Fe} ⁴⁾

First, determine the auxiliary values for Equation (9):

$$E = \frac{\pi}{4} m_n - h_{fP} \tan \alpha_n + \frac{s_{pr}}{\cos \alpha_n} - (1 - \sin \alpha_n) \frac{\rho_{fP}}{\cos \alpha_n} \quad (10)$$

4) If the tip of the tooth has been rounded or chamfered, it is necessary to replace the tip diameter d_a in the calculation by d_{Na} the "effective tip diameter". d_{Na} is the diameter of a circle near the tip cylinder, containing limits of the usable gear flanks.

with

$$s_{pr} = pr - q \text{ (see Figure 2);}$$

$$s_{pr} = 0 \text{ when gears are not undercut;}$$

$$\rho_{fPv} = \rho_{fP} \text{ for external gears, or}$$

$$\rho_{fPv} \approx \rho_{fP} + m_n \frac{(x_0 + h_{fP}/m_n - \rho_{fP}/m_n)^{1,95}}{3,156 \cdot 1,036^{z_0}} \text{ for internal gears} \quad (11)$$

where

x_0 is the pinion-cutter shift coefficient;

z_0 is the number of teeth of the pinion cutter;

$$G = \frac{\rho_{fPv}}{m_n} - \frac{h_{fP}}{m_n} + x \quad (12)$$

$$H = \frac{2}{z_n} \left(\frac{\pi}{2} - \frac{E}{m_n} \right) - T \quad (13)$$

with

$$T = \pi/3 \text{ for external gears;}$$

$$T = \pi/6 \text{ for internal gears;}$$

$$\theta = \frac{2G}{z_n} \tan \theta - H \quad (14)$$

The value $\theta = \pi/6$ for external gears and $\theta = \pi/3$ for internal gears may be used as a seed value in the iteration of the transcendental Equation (14). Generally, the function converges after five iterations.

a) Tooth root normal chord s_{Fn}

— For external gears:

$$\frac{s_{Fn}}{m_n} = z_n \sin \left(\frac{\pi}{3} - \theta \right) + \sqrt{3} \left(\frac{G}{\cos \theta} - \frac{\rho_{fPv}}{m_n} \right) \quad (15)$$

— For internal gears:

$$\frac{s_{Fn}}{m_n} = z_n \sin \left(\frac{\pi}{6} - \theta \right) + \left(\frac{G}{\cos \theta} - \frac{\rho_{fPv}}{m_n} \right) \quad (16)$$

b) Radius of root fillet ρ_F (see Figures 3 and 4)

$$\frac{\rho_F}{m_n} = \frac{\rho_{fPv}}{m_n} + \frac{2G^2}{\cos \theta (|z_n| \cos^2 \theta - 2G)} \quad (17)$$

c) Bending moment arm h_{Fe}

— For external gears:

$$\frac{h_{Fe}}{m_n} = \frac{1}{2} \left[(\cos \gamma_e - \sin \gamma_e \tan \alpha_{Fen}) \frac{d_{en}}{m_n} - z_n \cos \left(\frac{\pi}{3} - \theta \right) - \left(\frac{G}{\cos \theta} - \frac{\rho_{fPv}}{m_n} \right) \right] \quad (18)$$

— For internal gears:

$$\frac{h_{Fe}}{m_n} = \frac{1}{2} \left[(\cos \gamma_e - \sin \gamma_e \tan \alpha_{Fen}) \frac{d_{en}}{m_n} - z_n \cos \left(\frac{\pi}{6} - \theta \right) - \sqrt{3} \left(\frac{G}{\cos \theta} - \frac{\rho_{fPv}}{m_n} \right) \right] \quad (19)$$

6.2.2 Parameters of virtual gears

These are as follows.

$$\beta_b = \arccos \sqrt{1 - (\sin \beta \cos \alpha_n)^2} = \arcsin (\sin \beta \cos \alpha_n) \quad (20)$$

$$z_n = \frac{z}{\cos^2 \beta_b \cos \beta} \quad (21)$$

Approximation:

$$z_n \approx \frac{z}{\cos^3 \beta} \quad (22)$$

$$\varepsilon_{\alpha n} = \frac{\varepsilon_\alpha}{\cos^2 \beta_b} \quad (23)$$

$$d_n = \frac{d}{\cos^2 \beta_b} = m_n z_n \quad (24)$$

$$p_{bn} = \pi m_n \cos \alpha_n \quad (25)$$

$$d_{bn} = d_n \cos \alpha_n \quad (26)$$

$$d_{an} = d_n + d_a - d \quad (27)$$

$$d_{en} = 2 \frac{z}{|z|} \sqrt{\left[\sqrt{\left(\frac{d_{an}}{2} \right)^2 - \left(\frac{d_{bn}}{2} \right)^2} - \frac{\pi d \cos \beta \cos \alpha_n}{|z|} (\varepsilon_{\alpha n} - 1) \right]^2 + \left(\frac{d_{bn}}{2} \right)^2} \quad (28)$$

The number of teeth, z , is positive for external gears and negative for internal gears.

$$\alpha_{en} = \arccos \left(\frac{d_{bn}}{d_{en}} \right) \quad (29)$$

$$\gamma_e = \frac{0,5 \pi + 2 \tan \alpha_n x}{z_n} + \text{inv } \alpha_n - \text{inv } \alpha_{en} \quad (30)$$

$$\alpha_{Fen} = \alpha_{en} - \gamma_e = \tan \alpha_{en} - \operatorname{inv} \alpha_n - \frac{0,5 \pi + 2 \tan \alpha_n x}{z_n} \quad (31)$$

6.3 Derivations of determinant normal tooth load for spur gears

Nominal bending stress = $\frac{\text{bending moment}}{\text{section modulus of gear at } s_{Fn}}$ in accordance with the following equation, with symbols in accordance with Figures 3 and 4.

$$\sigma = \frac{F_b \cos \alpha_{Fen}}{\frac{1}{6} (b s_{Fn}^2)} h_{Fe} \quad (32)$$

$$F_b \frac{d_b}{2} = F_t \frac{d}{2} = F_w \frac{d_w}{2} \quad (33)$$

where

d_b is the base diameter;

d is the reference diameter;

d_w is the pitch diameter;

F_t is the nominal tangential load at the reference cylinder;

F_w is the nominal tangential load at the pitch cylinder.

$$F_b = \frac{F_t}{\cos \alpha} = \frac{F_w}{\cos \alpha_w} \quad (34)$$

$$\sigma = \left[\frac{\frac{h_{Fe}}{m} \cos \alpha_{Fen}}{\frac{1}{6} \left(\frac{s_{Fn}}{m} \right)^2 \cos \alpha} \right] \frac{F_t}{b m} = \frac{F_t}{b m} Y_F \quad (35)$$

where

α is the pressure angle of the basic rack profile;

α_w is the working pressure angle.

When σ is expressed as a function of F_t , a form factor, Y_F , can be derived from Equation (35).

7 Stress correction factor, Y_S

7.1 Basic uses

The stress correction factor, Y_S , is used to convert the nominal tooth root stress to local tooth root stress and, by means of this factor, the following are taken into account:

- a) the stress amplifying effect of section change at the fillet radius at the tooth root⁵⁾;
- b) that evaluation of the true stress system at the tooth root critical section is more complex than the simple system evaluation presented, with evidence indicating that the intensity of the local stress at the tooth root consists of two components, one of which is directly influenced by the value of the bending moment and the other increasing with closer proximity to the critical section of the determinant position of load application.

Y_S is the factor for load application at the outer point of single pair tooth contact (Method B). See 5.2 for the principles, assumptions and application of Method B.

The formulae in this clause are based on data derived from the geometry of external spur gears with 20° pressure angle, by means of measurement and calculations using finite element and integral equation methods. The formulae can also be used to obtain approximate values for internal gears and for gears having other pressure angles.

The present instructions refer to spur and helical gears. See Clause 6 for explanatory notes and information on the calculation of the virtual numbers of teeth relevant to helical gears.

7.2 Stress correction factor, Y_S : Method B

The calculation of the stress correction factor, Y_S , is made in accordance with Equation (36), which is valid in the range: $1 \leq q_s < 8$; symbols are as illustrated in Figures 3 and 4.

$$Y_S = (1,2 + 0,13 L) q_s^{\left[\frac{1}{1,21 + \frac{2,3}{L}} \right]} \quad (36)$$

where

$$L = \frac{s_{Fn}}{h_{Fe}} \quad (37)$$

with

s_{Fn} from Equation (15) for external gears, Equation (16) for internal gears;

h_{Fe} from Equation (18) for external gears, Equation (19) for internal gears;

$$q_s = \frac{s_{Fn}}{2\rho_F} \quad (38)$$

with ρ_F from Equation (17).

Determination of Y_S by graphical methods is not appropriate.

5) See 7.3 for the procedure to be followed when grinding notches are present in tooth fillets.

7.3 Stress correction factor for gears with notches in fillets

A notch such as a grinding notch in the fillet of a gear near the critical section usually engenders a degree of stress concentration exceeding that of the fillet; thus, the stress correction factor is correspondingly greater. A fair estimate of Y_{Sg} , obtainable from Equation (39), can be substituted for Y_S , see Figure 5, if the notch is near the critical section. See also Reference [6].

$$Y_{Sg} = \frac{1,3 Y_S}{1,3 - 0,6 \sqrt{\frac{t_g}{\rho_g}}} \quad (39)$$

valid for $\sqrt{\frac{t_g}{\rho_g}} < 2,0$

The effect of the grinding notch is less than that implied in Equation (39) when the notch is above the contact point of the 30° tangent (external gears) or 60° tangent (internal gears).

Y_{Sg} also takes into consideration the reduction in the tooth root thickness.

Deep notches in the fillets of surface hardened steel gears severely reduce the bending strength of their teeth.

7.4 Stress correction factor, Y_{ST} , relevant to the dimensions of the standard reference test gears

The tooth root stress limit values for materials, according to ISO 6336-5, were derived from results of tests of standard reference test gears for which either $Y_{ST} = 2,0$ or for which test results were recalculated to this value. See also Reference [6].

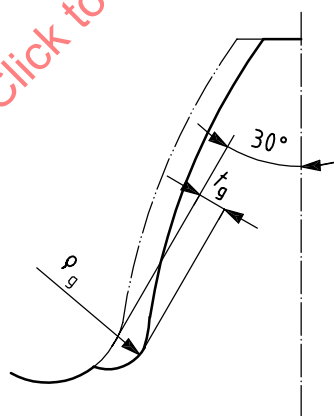


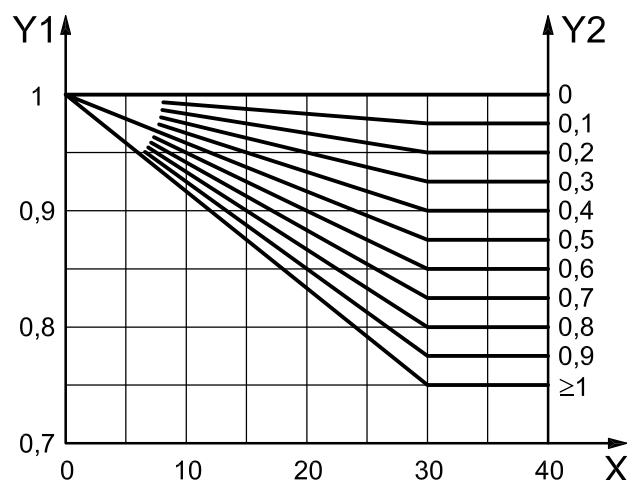
Figure 5 — Notch dimensions

8 Helix angle factor, Y_β

The tooth root stress of a virtual spur gear, calculated as a preliminary value, is converted by means of the helix factor Y_β to that of the corresponding helical gear. By this means, the oblique orientation of the lines of mesh contact is taken into account (less tooth root stress).

8.1 Graphical value

Y_β may be read from Figure 6 as a function of the helix angle, β , and the overlap ratio, ε_β .



Key

X reference helix angle, β , degrees

Y1 helix factor, Y_β

Y2 overlap ratio, ε_β

Helix factors $Y_\beta > 25^\circ$ shall be confirmed by experience.

Figure 6 — Helix factor, Y_β

8.2 Determination by calculation

The factor Y_β can be calculated using Equation (40), which is consistent with the curves illustrated in Figure 6.

$$Y_\beta = 1 - \varepsilon_\beta \frac{\beta}{120^\circ} \quad (40)$$

where β is the reference helix angle, in degrees.

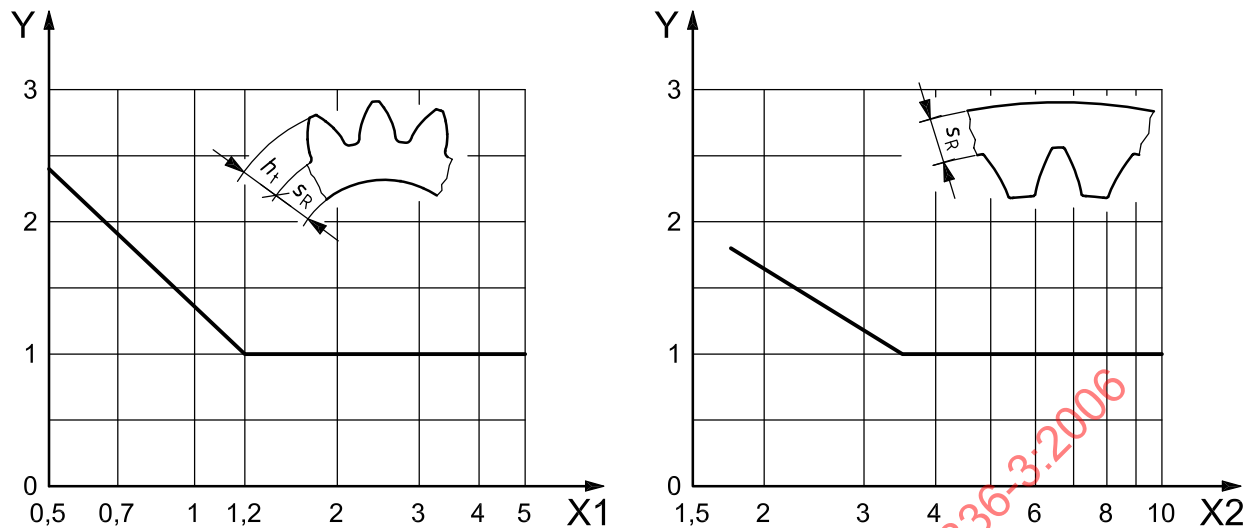
The value 1,0 is substituted for ε_β when $\varepsilon_\beta > 1,0$, and 30° is substituted for β when $\beta > 30^\circ$.

9 Rim thickness factor, Y_B

Where the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim, rather than at the root fillet. The rim thickness factor Y_B is a simplified factor used to de-rate thin rimmed gears when detailed calculations of stresses in both tension and compression or experience are not available. For critically loaded applications this method should be replaced by a more comprehensive analysis.

9.1 Graphical values

Y_B can be taken from Figure 7 as a function of the backup ratio s_R/h_t for external gears and as a function of the rim thickness s_R/m_n for internal gears.

**Key**X1 backup ratio, s_R/h_t X2 rim thickness, s_R/m_n Y rim thickness factor, Y_B **Figure 7 — Rim thickness factor, Y_B** **9.2 Determination by calculation****9.2.1 External gears**

Y_B can be calculated using Equations (41) to (42). These are consistent with the curve in Figure 7.

- a) If $s_R/h_t \geq 1,2$, then

$$Y_B = 1,0 \quad (41)$$

- b) If $s_R/h_t > 0,5$ and $s_R/h_t < 1,2$, then

$$Y_B = 1,6 \ln \left(2,242 \frac{h_t}{s_R} \right) \quad (42)$$

- c) The case $s_R/h_t \leq 0,5$ shall be avoided.

9.2.2 Internal gears

Y_B can be calculated using Equations (43) to (44). These are consistent with the curve in Figure 7.

- a) If $s_R/m_n \geq 3,5$, then

$$Y_B = 1,0 \quad (43)$$

- b) If $s_R/m_n > 1,75$ and $s_R/m_n < 3,5$, then

$$Y_B = 1,15 \ln \left(8,324 \frac{m_n}{s_R} \right) \quad (44)$$

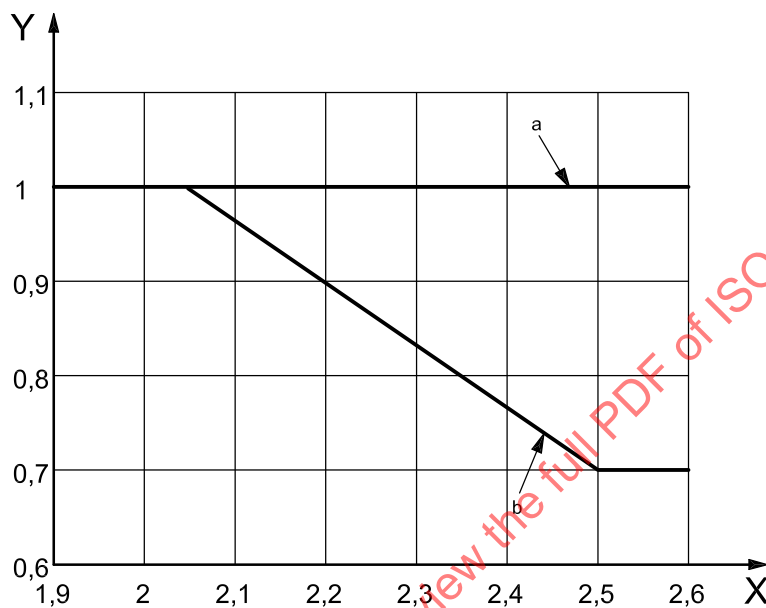
- c) The case $s_R/m_n \leq 1,75$ shall be avoided.

10 Deep tooth factor, Y_{DT}

For gears of high precision (accuracy grade ≤ 4) with contact ratios in the range of $2 \leq \varepsilon_{\alpha n} < 2,5$ and with applied actual profile modification to obtain a trapezoidal load distribution along the path of contact, the nominal tooth root stress σ_{F0} is adjusted by the deep tooth factor Y_{DT} .

10.1 Graphical values

Y_{DT} may be read from Figure 8 as a function of the contact ratio $\varepsilon_{\alpha n}$.



Key

X virtual contact ratio, $\varepsilon_{\alpha n}$

Y deep tooth factor, Y_{DT}

a Accuracy grade > 4 .

b Accuracy grade ≤ 4 .

Figure 8 — Deep tooth factor, Y_{DT}

10.2 Determination by calculation

Y_{DT} can be calculated using Equations (45) to (47). These are consistent with the curves in Figure 8.

a) If $\varepsilon_{\alpha n} \leq 2,05$ or if $\varepsilon_{\alpha n} > 2,05$ and the accuracy grade > 4 , then

$$Y_{DT} = 1,0 \quad (45)$$

b) If $2,05 < \varepsilon_{\alpha n} \leq 2,5$ and the accuracy grade ≤ 4 , then

$$Y_{DT} = -0,666 \varepsilon_{\alpha n} + 2,366 \quad (46)$$

c) If $\varepsilon_{\alpha n} > 2,5$ and the accuracy grade ≤ 4 , then

$$Y_{DT} = 0,7 \quad (47)$$

11 Reference stress for bending

See 5.3 for general notes on the determination of limit values for tooth root stress.

11.1 Reference stress for Method A

Method A is consistent with the determination of tooth root stress reference stress according to 5.3.1.1.

11.2 Reference stress, with values $\sigma_{F \text{ lim}}$ and σ_{FE} for Method B

See 5.3.1.2 and 5.3.2 for information. See Equation (5) for definitions of $\sigma_{F \text{ lim}}$ and σ_{FE} .

NOTE ISO 6336-5 provides information, derived from the results of testing standard reference test gears, on values of $\sigma_{F \text{ lim}}$ and σ_{FE} for the more popular gear materials, heat treatment processes and the influence of the material quality on those values. ISO 6336-5 also includes requirements for quality grades ML, MQ and ME concerning material and heat treatment. Material quality grade MQ is usually chosen for gears unless otherwise agreed upon.

12 Life factor, Y_{NT}

The life factor, Y_{NT} , accounts for the higher tooth root stress, which may be tolerable for a limited life (number of load cycles), as compared with the allowable stress at 3×10^6 cycles.

The principal influence factors are:

- a) material and heat treatment (see ISO 6336-5),
- b) number of load cycles (service life), N_L ,
- c) failure criteria,
- d) smoothness of operation required,
- e) cleanness of gear material,
- f) material ductility and fracture toughness, and
- g) residual stress.

For the purposes of this part of ISO 6336, the number of load cycles, N_L , is defined as the number of mesh contacts, under load, of the gear tooth being analysed. The allowable stress numbers are established for 3×10^6 tooth load cycles at 99 % reliability.

A Y_{NT} value of unity may be used, where justified by experience, beyond 3×10^6 cycles. However, consideration should be given to the use of optimum material quality and manufacturing, with selection of an appropriate safety factor.

12.1 Life factor, Y_{NT} : Method A

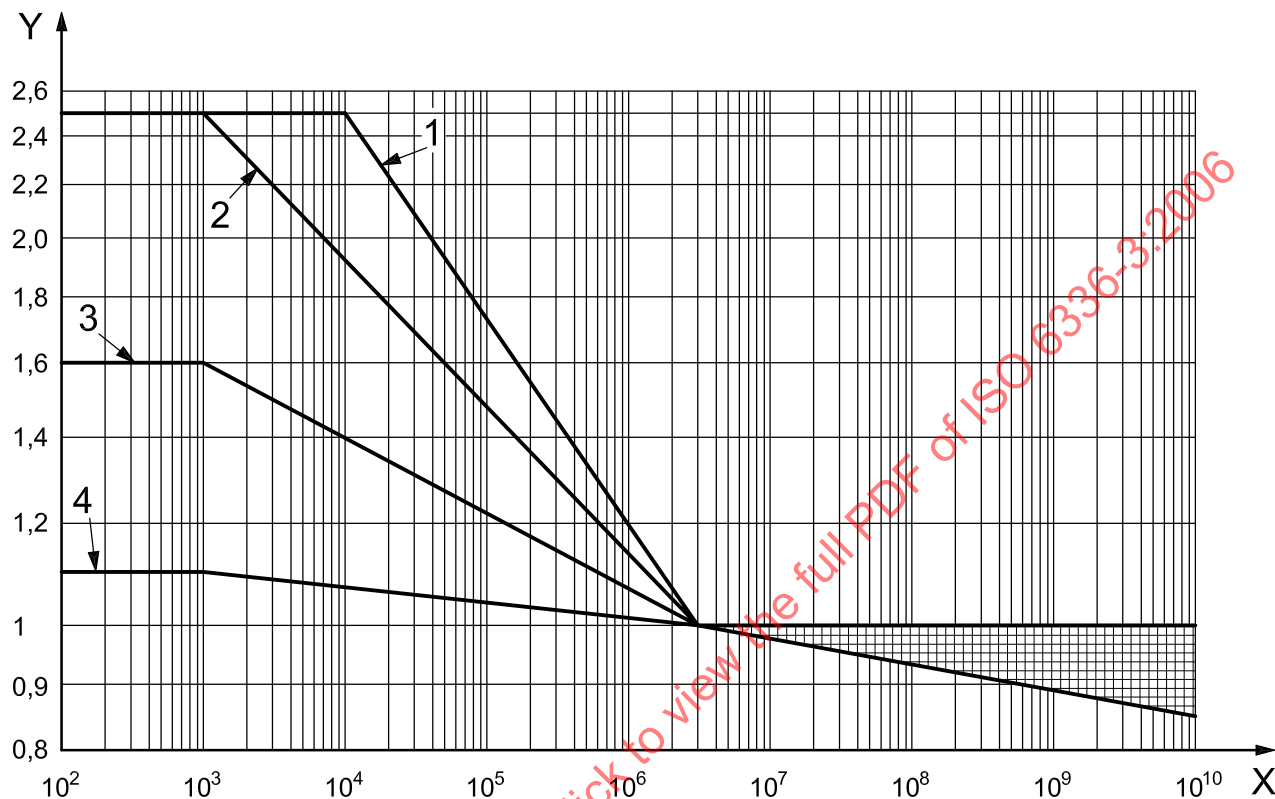
The S-N curve or damage curve derived from facsimiles of the actual gear is determinant for the establishment of the limited life. Since under such circumstances factors $Y_{\delta \text{ rel T}}$, $Y_{R \text{ rel T}}$ and Y_X are in effect already included in the S-N or damage curves, the value 1,0 is substituted for each in the calculation of permissible stress.

12.2 Life factor, Y_{NT} : Method B

For this method, life factor Y_{NT} of the standard reference test gear is used as an aid in the evaluation of permissible stress for limited life or reliability (see 5.3).

12.2.1 Graphical values

Y_{NT} may be read from Figure 9 for the static stress and reference stress as a function of material and heat treatment. Values from a large number of tests are presented as typical damage or crack initiation curves for surface-hardened and nitride-hardened steels, or curves of yield stress for structural and through-hardened steels.



Key

X number of load cycles, N_L
Y life factor, Y_{NT}

- 1 GTS (perl.), St, V, GGG (perl. bal.)
- 2 Eh, IF (root)
- 3 NT, NV (nitr.), GGG (ferr.), GG
- 4 NV (nitrocar.)

Figure 9 — Life factor, Y_{NT} , for reference test gears
(see ISO 6336-1:2006, Table 2, for explanation of abbreviations used)

12.2.2 Determination by calculation

The data of Y_{NT} for static stress and reference stress can be taken from Table 1.

Life factor Y_{NT} for limited-life stress is determined by means of interpolation on a log-log scale between the values for reference and static stress limits as defined in 5.3.2. Evaluation of Y_{NT} is according to 5.3.3.

Table 1 — Life factor, Y_{NT}

Material ^a	Number of load cycles, N_L	Life Factor, Y_{NT}
St, V, GGG (perl. bai.), GTS (perl.),	$N_L \leq 10^4$, static	2,5
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 ^b
Eh, IF (root)	$N_L \leq 10^3$, static	2,5
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 ^b
GG, GGG (ferr.), NT, NV (nitr.)	$N_L \leq 10^3$, static	1,6
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 ^b
NV (nitrocar.)	$N_L \leq 10^3$, static	1,1
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 ^b

^a See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used.

^b The lower value of Z_{NT} may be used for critical service, where pitting must be minimal. Values between 0,85 and 1,0 may be used for general purpose gearing. With optimum lubrication, material, manufacturing and experience 1,0 may be used.

13 Sensitivity factor, $Y_{\delta T}$, and relative notch sensitivity factor, $Y_{\delta rel T}$

13.1 Basic uses

The extent to which the calculated tooth root stress deemed to have caused fatigue or overload breakage exceeds the relevant material stress limit is indicated by the dynamic or the static sensitivity factor, Y_{δ} . It characterizes the notch sensitivity of the material, and its values depend on the material and the stress gradient. Its values for dynamic stresses are different from its value for static stress. This applies to $Y_{\delta T}$ in relation to breakage of a standard reference test gear tooth. It applies also to the relative sensitivity factors which relate the sensitivity of a gear of interest to that of a standard reference test gear ($Y_{\delta rel T}$).

13.2 Determination of the sensitivity factors

Comments on these factors given in 5.3 apply in principle.

13.2.1 Method A

The tooth root stress limits are determined by testing facsimiles of the gear of interest (or closely similar test gears), in which case the relative sensitivity factor is equal to 1,0. However, a careful analysis — by means of which the relative sensitivity factor for the relevant material and relevant tooth form will be established — has yet to be undertaken.

13.2.2 Method B

When the reference and static stress limit values are derived with Method B using reference test gears with notch parameters $q_{sT} = 2,5$, the factor $Y_{\delta rel T}$ relevant to the reference and static stress limits of any gear seldom deviates much from 1,0. This is because the value $q_{sT} = 2,5$ is in the mid-range of common gear designs. The reference value $Y_{\delta rel T} = 1,0$ for the standard reference test gear coincides with the stress correction factor $Y_S = 2,0$ (see Figures 11 and 13).

13.3 Relative notch sensitivity factor, $Y_{\delta \text{rel T}}$: Method B

13.3.1 Graphical values

13.3.1.1 $Y_{\delta \text{rel T}}$ for reference stress

$Y_{\delta \text{rel T}}$ can be read from Figure 10 as a function of q_s or Y_S and the material. The curves in this graph for each of the materials were derived from Figure 12 by subtracting from the absolute value Y_{δ} , appropriate to each value of q_s , the value of $Y_{\delta T}$ for that material corresponding to the notch parameter $q_s = 2,5$ (the notch parameter of the standard reference test gear). For any gear of interest, q_s can be calculated using Equation (38).

13.3.1.2 $Y_{\delta \text{rel T}}$ for static stress

$Y_{\delta \text{rel T}}$ may be taken from Figure 11 as a function of stress correction factor Y_S and the material. The curves in this graph for each of the materials were derived from Figure 13 by subtracting from the absolute value Y_{δ} , appropriate to each value of Y_S , the value of $Y_{\delta T}$ for that material corresponding to $Y_{ST} = 2,0$ (the stress correction factor of the standard reference test gear). For any gear of interest, Y_S can be calculated using Equation (36).

13.3.2 Determination by calculation

13.3.2.1 $Y_{\delta \text{rel T}}$ for reference stress

$Y_{\delta \text{rel T}}$ can be calculated using Equation (48). This is consistent with the curves in Figure 10.

$$Y_{\delta \text{rel T}} = \frac{Y_{\delta}}{Y_{\delta T}} = \frac{1 + \sqrt{\rho' \chi^*}}{1 + \sqrt{\rho' \chi_T^*}} \quad (48)$$

The slip-layer thickness ρ' can be taken from Table 2 as a function of the material.

The relative stress gradient can be calculated using the Equation (49) ⁶⁾:

$$\chi^* = \chi_P^* (1 + 2 q_s)$$

with

$$\chi_P^* = \frac{1}{5} \quad (49)$$

The value of χ_T^* for the standard reference test gear is obtained similarly by substituting $q_{sT} = 2,5$ for q_s in Equation (49).

6) Applies for module $m = 5$ mm. The influence of size is covered by the factor Y_X (see Clause 15).

Table 2 — Values for slip-layer thickness ρ'

Item	Material ^a	ρ' [mm]
1	GG $\sigma_B = 150 \text{ N/mm}^2$	0,312 4
2	GG, GGG (ferr.); $\sigma_B = 300 \text{ N/mm}^2$	0,309 5
3	NT, NV; for all hardness	0,100 5
4	St; $\sigma_S = 300 \text{ N/mm}^2$	0,083 3
5	St; $\sigma_S = 400 \text{ N/mm}^2$	0,044 5
6	V, GTS, GGG (perl. bai.); $\sigma_S = 500 \text{ N/mm}^2$	0,028 1
7	V, GTS, GGG (perl. bai.); $\sigma_S = 600 \text{ N/mm}^2$	0,019 4
8	V, GTS, GGG (perl. bai.); $\sigma_{0,2} = 800 \text{ N/mm}^2$	0,006 4
9	V, GTS, GGG (perl. bai.); $\sigma_{0,2} = 1000 \text{ N/mm}^2$	0,001 4
10	Eh, IF (root); for all hardness	0,003 0
^a See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used.		

13.3.2.1.1 $Y_{\delta \text{rel T}}$ for static stress

$Y_{\delta \text{rel T}}$ can be calculated using Equations (50) to (54). These are consistent with the curves in Figure 11 (see ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used).

- a) For St with well defined yield point:

$$Y_{\delta \text{rel T}} = \frac{1 + 0,93 (Y_S - 1) \sqrt[4]{\frac{200}{\sigma_S}}}{1 + 0,93 \sqrt[4]{\frac{200}{\sigma_S}}} \quad (50)$$

- b) For St with steadily increasing elongation curve and 0,2 % proof stress, V and GGG (perl., bai.):

$$Y_{\delta \text{rel T}} = \frac{1 + 0,82 (Y_S - 1) \sqrt[4]{\frac{300}{\sigma_{0,2}}}}{1 + 0,82 \sqrt[4]{\frac{300}{\sigma_{0,2}}}} \quad (51)$$

These values are only valid if the local stresses do not reach the yield point.

- c) For Eh and IF(root) with stress up to crack initiation:

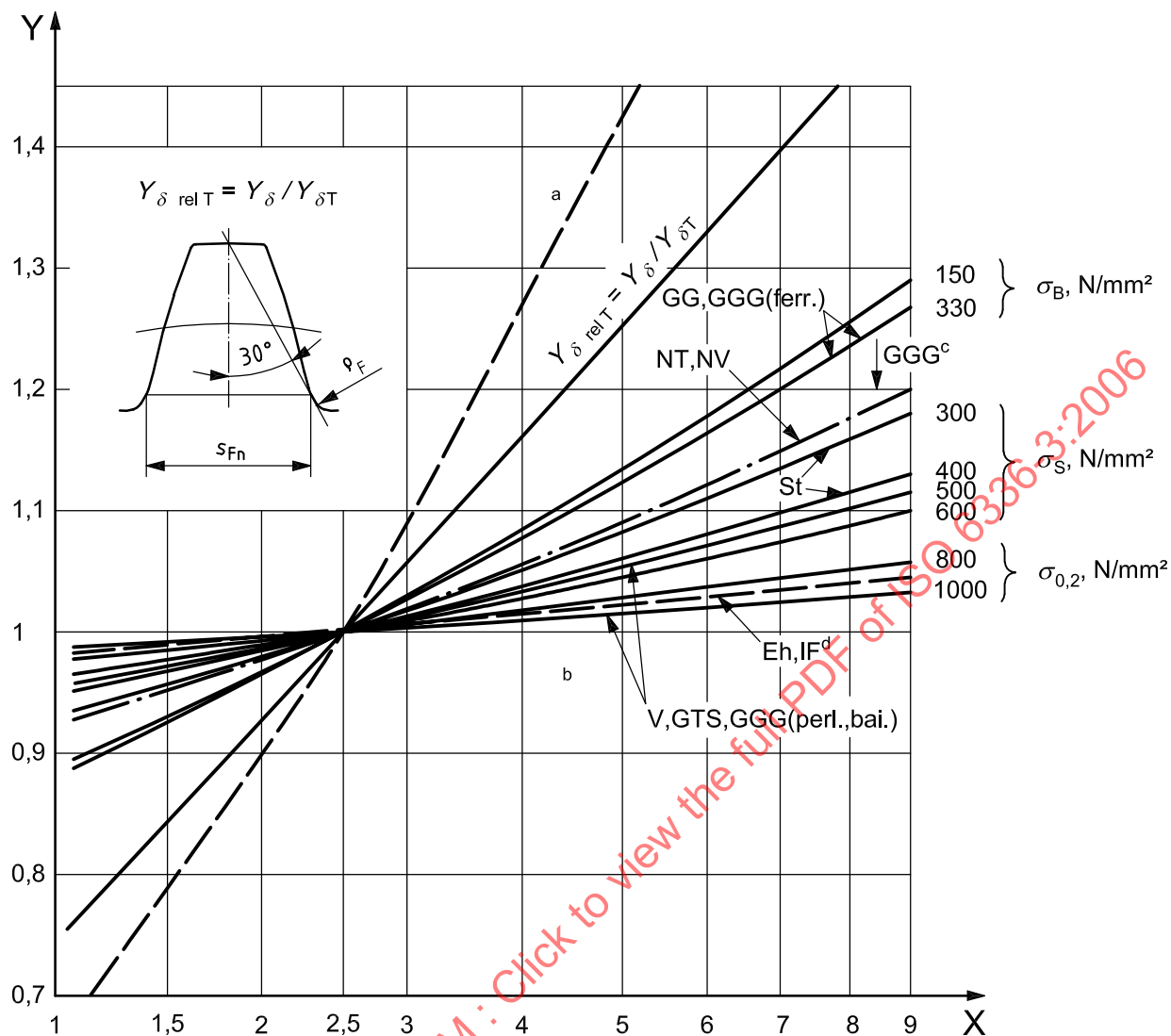
$$Y_{\delta \text{rel T}} = 0,44 Y_S + 0,12 \quad (52)$$

- d) For NT and NV with stress up to crack initiation:

$$Y_{\delta \text{rel T}} = 0,20 Y_S + 0,60 \quad (53)$$

- e) For GG and GGG (ferr.) with stress up to fracture limit:

$$Y_{\delta \text{rel T}} = 1,0 \quad (54)$$



Key

- X notch parameter, $q_s = s_{Fn} / 2\rho_F$
Y relative notch sensitivity factor, $Y_{\delta_{rel T}}$, for reference stress

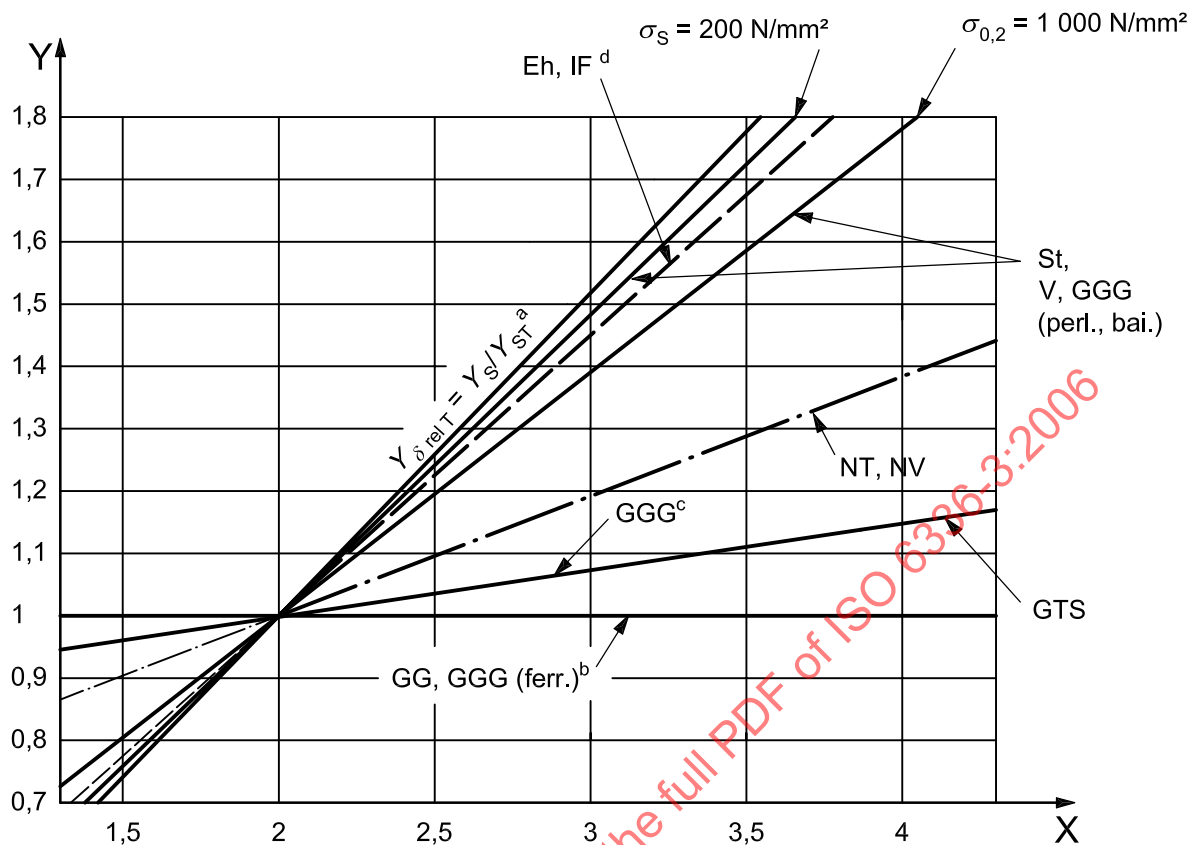
NOTE 1 Values of σ in newtons per square millimetre (N/mm²).

NOTE 2 See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used.

NOTE 3 Based on bending flat bar complying with VDI 2226^[7].

- a Fully insensitive to notches.
b Fully sensitive to notches.
c With increasingly pearlitic structure.
d (root).

Figure 10 — Relative notch sensitivity factor, $Y_{\delta_{rel T}}$, for reference stress

**Key**

X stress correction factor, Y_S

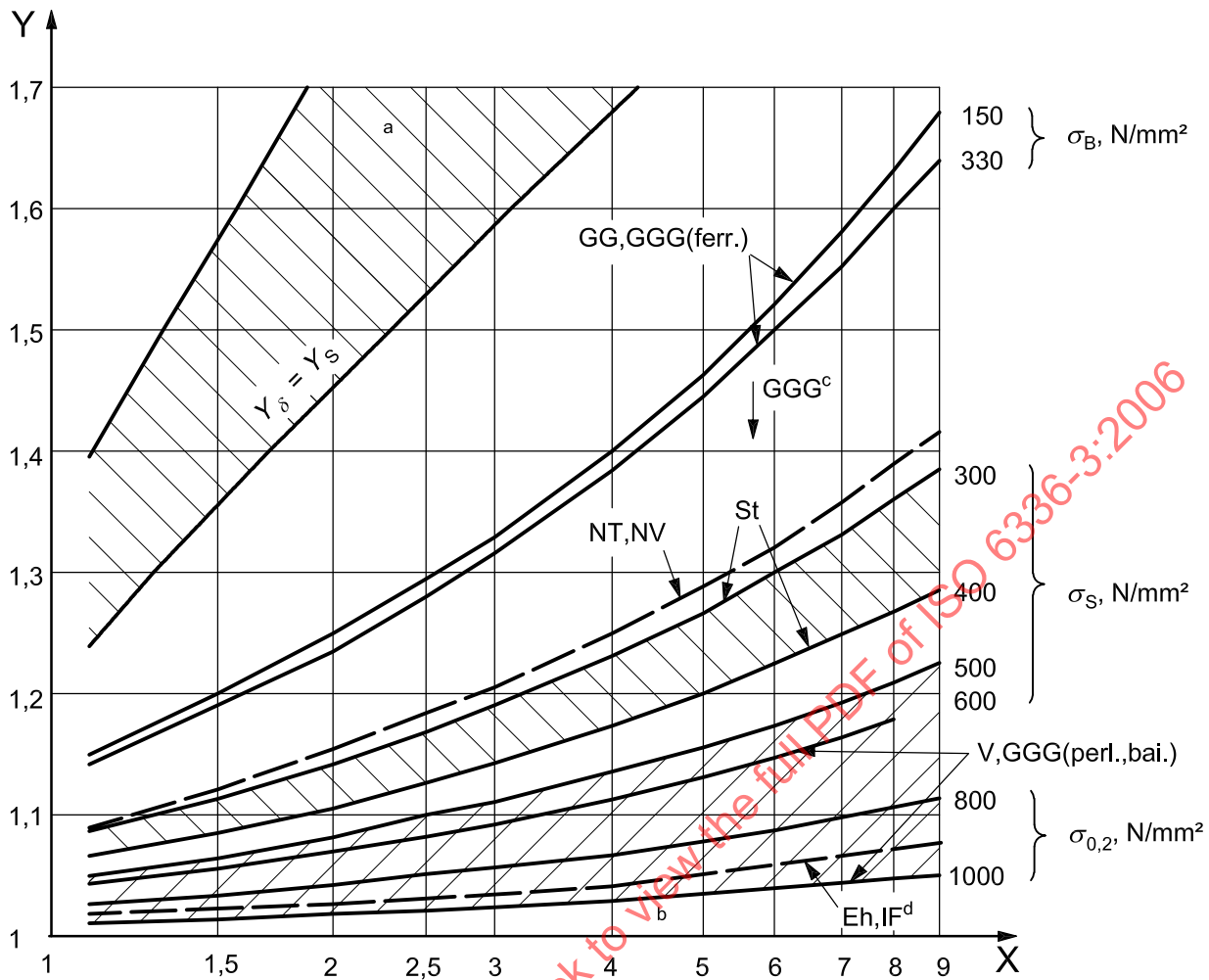
Y relative notch sensitivity factor, $Y_{\delta_{rel T}}$, for static stress

NOTE 1 See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used.

NOTE 2 Based on bending flat bar complying with VDI 2226^[7].

- a Fully insensitive to notches.
- b Fully sensitive to notches.
- c With increasingly pearlitic structure.
- d (root).

Figure 11 — Relative notch sensitivity factor, $Y_{\delta_{rel T}}$, for static stress



Key

X notch parameter, q_s

Y sensitivity factor, Y_δ , for reference stress

NOTE 1 See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used.

NOTE 2 Based on bending flat bar complying with VDI 2226^[7].

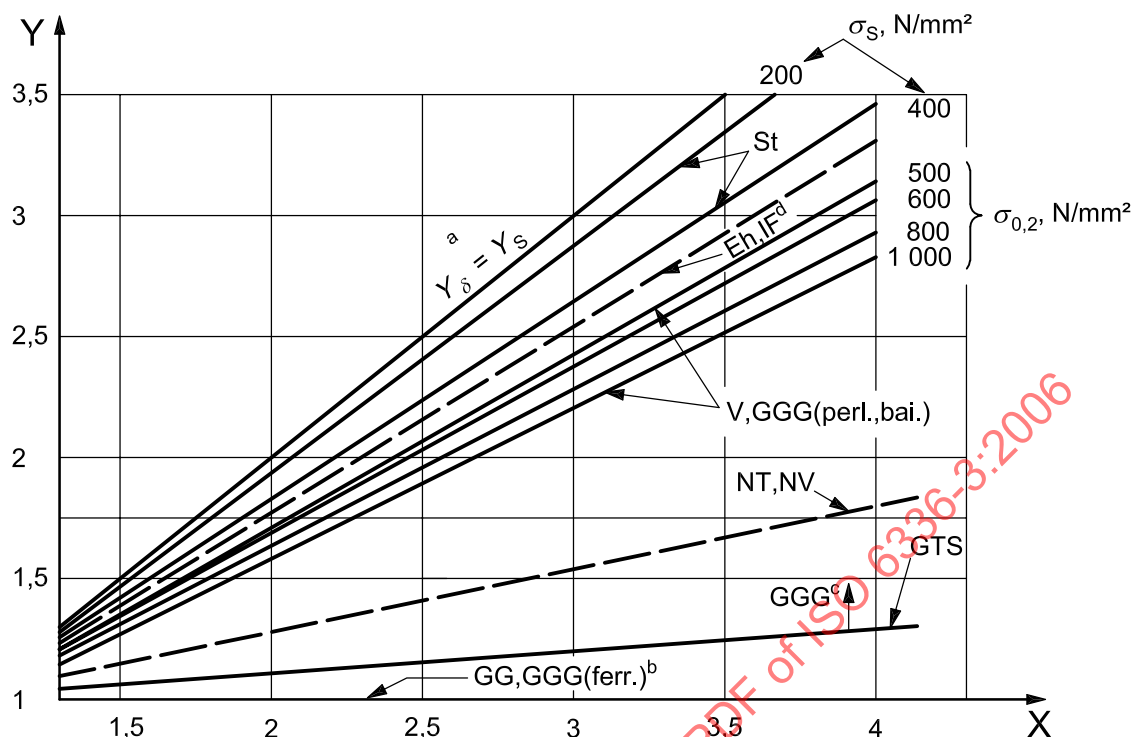
a Fully insensitive to notches.

b Fully sensitive to notches.

c With increasingly pearlitic structure.

d (root).

Figure 12 — Relative notch sensitivity factor, Y_δ , for reference stress

**Key**

- X stress correction factor, Y_S
 Y sensitivity factor, Y_{δ} for static stress

NOTE 1 See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used.

NOTE 2 Based on bending flat bar complying with VDI 2226^[7].

- a Fully insensitive to notches.
 b Fully sensitive to notches.
 c With increasingly pearlitic structure.
 d (root).

Figure 13 — Relative notch sensitivity factor, Y_{δ} for static stress

14 Surface factors, Y_R , Y_{RT} , and relative surface factor, $Y_{R \text{ rel } T}$

14.1 Influence of surface condition

The surface factor, Y_R , accounts for the influence on tooth root stress of the surface condition in the tooth roots. This is dependent on the material and the surface roughness in the tooth root fillets (see Note, below). Y_R for static stress is different from Y_R for dynamic stresses. This is also true for Y_{RT} , the surface factor of the standard reference test gear. These factors are compared to that of a plain, polished test piece. Relative surface factors represent the relationship of the surface factor of a gear of interest to that of the standard reference test gear ($Y_{R \text{ rel } T}$).

NOTE The influence of surface condition on tooth root bending strength does not depend solely on the surface roughness in the tooth root fillets, but also on the size and shape (the problem of “notches within a notch”). This subject has not to date been sufficiently well studied for it to be taken into account in this part of ISO 6336. The method applied here is only valid when scratches or similar defects deeper than $2R_z$ are not present ($2R_z$ is a preliminary estimated value).

Besides surface texture, other influences on tooth bending strength are known, and include residual compressive stresses (shot peening), grain boundary oxidation and chemical effects. When fillets are shot-peened and/or are perfectly shaped, a value slightly greater than that obtained from the graph should be substituted for $Y_{R\ rel\ T}$. When grain boundary oxidation or chemical effects are present, a smaller value than that indicated by the graph should be substituted for $Y_{R\ rel\ T}$.

14.2 Determination of surface factors and relative surface factors

The comments in 5.3 apply in principle for the determination of these factors.

14.2.1 Method A

In Method A the tooth root stress limit is determined by testing the gear of interest or testing closely similar test gears. By this approach, the relative surface factor is equal, or approximately equal, to 1,0. In order to determine the material surface factor relative to that of the gear tested, a careful analysis shall be undertaken.

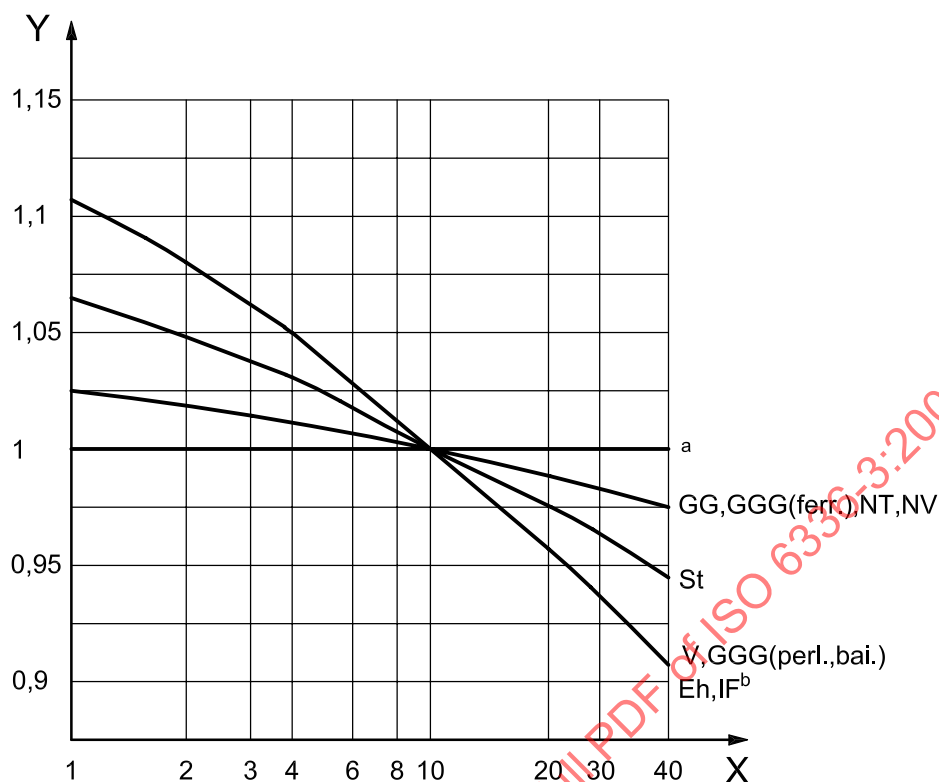
14.2.2 Method B

The material strength values provided are derived in accordance with Method B, from results of tests of standard reference test gears of which $R_{zT} = 10\ \mu\text{m}$. In general, the value $Y_{R\ rel\ T}$ relevant to the reference stress of any gear of interest differs little from 1,0, since $R_{zT} = 10\ \mu\text{m}$ is a common mean value. $Y_{R\ rel\ T}$ for static stress may also be made equal to 1,0.

14.3 Relative surface factor, $Y_{R\ rel\ T}$: Method B

14.3.1 Graphical values

$Y_{R\ rel\ T}$ can be taken from Figure 14 as a function of the material and R_z , the peak-to-valley roughness in the tooth root fillets of the gear of interest. This graph is derived from Figure A.1.

**Key**X roughness, R_z , μm Y relative surface factor, $Y_{R \text{ rel } T}$

NOTE See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used.

a For static stress and all materials.

b (root).

Figure 14 — Relative surface factor, $Y_{R \text{ rel } T}$
(derived from Figure A.1)

14.3.2 Determination by calculation**14.3.2.1 $Y_{R \text{ rel } T}$ for reference stress**

$Y_{R \text{ rel } T}$ can be calculated using Equations (55) to (61). These are consistent with the curves in Figure 14 (see ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used).

a) Reference stress in the range $R_z < 1 \mu\text{m}$

— for V, GGG (perl., bai.), Eh and IF (root):

$$Y_{R \text{ rel } T} = 1,12 \quad (55)$$

— for St:

$$Y_{R \text{ rel } T} = 1,07 \quad (56)$$

— for GG, GGG (ferr.) and NT, NV:

$$Y_{R \text{ rel } T} = 1,025 \quad (57)$$

b) Reference stress in the range $1 \mu\text{m} \leq R_z \leq 40 \mu\text{m}$

— for V, GGG (perl., bai.), Eh and IF (root):

$$Y_{R \text{ rel } T} = 1,674 - 0,529 (R_z + 1)^{0,1} \quad (58)$$

— for St:

$$Y_{R \text{ rel } T} = 5,306 - 4,203 (R_z + 1)^{0,01} \quad (59)$$

— for GG, GGG (ferr.) and NT, NV:

$$Y_{R \text{ rel } T} = 4,299 - 3,259 (R_z + 1)^{0,0058} \quad (60)$$

14.3.2.1.1 $Y_{R \text{ rel } T}$ for static stress

$$Y_{R \text{ rel } T} = 1,0. \quad (61)$$

15 Size factor, Y_X

The size factor, Y_X , is used to take into consideration the influence of size on the probable distribution of weak points in the structure of the material, the stress gradients, which, in accordance with strength of materials theory, decrease with increasing dimensions, the quality of the material as determined by the extent and effectiveness of forging, the presence of defects, etc.

The following have significant influence:

- a) material, its cleanliness, chemistry, and forging process;
- b) heat-treatment, depth and uniformity of hardening;
- c) module, in the case of surface-hardening: case depth in relation to tooth size (core support effect).

Size factor Y_X shall be determined separately for the pinion and wheel.

15.1 Size factor, Y_X : Method A

The value of size factor Y_X shall be based on reliable experience or testing under the relevant operating conditions of a range of different sizes of gears in each material of interest, appropriately heat-treated. The provisions given in ISO 6336-1:2006, 4.1.12, are relevant.

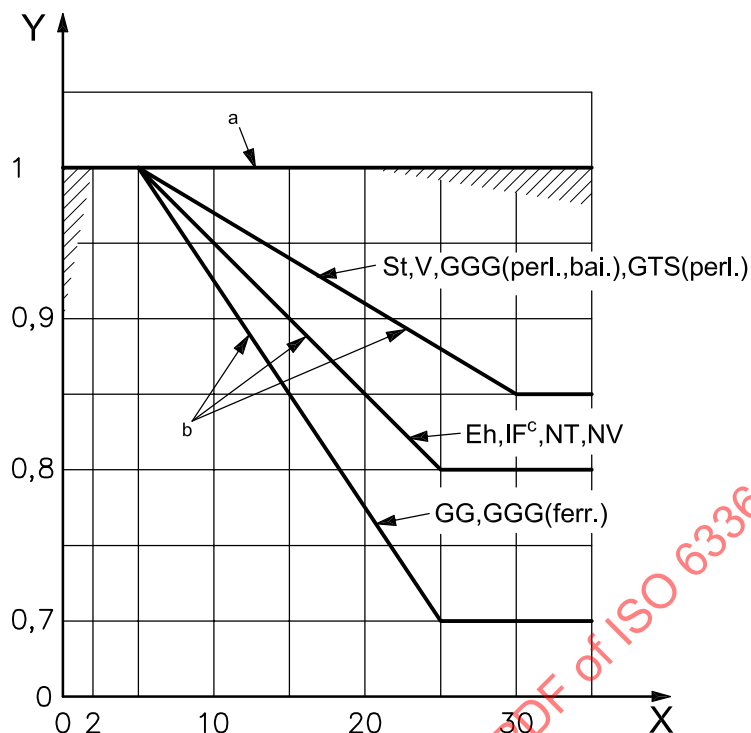
15.2 Size factor, Y_X : Method B

The values provided are based on the results of testing gears and bending strength test pieces of different sizes, due regard being paid to the current standards and practices of established heat-treatment practitioners.

15.2.1 Size factor, Y_X , for reference stress and static stress

15.2.1.1 Graphical values

The value of Y_X may be taken from Figure 15 as a function of the module, material and heat-treatment.

**Key**

X normal module, m_n , mm

Y size factor, Y_X

NOTE 1 See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used.

NOTE 2 Shaded area is in the range of scatter for static stress.

a Static stress (all materials).

b Reference stress.

c (root).

Figure 15 — Size factor, Y_X , for tooth bending strength

15.2.1.2 Determination by calculation

Y_X may be calculated using the equations in Table 3, which are consistent with the curves given in Figure 15.

15.2.2 Size factor, Y_X , for limited life

Y_X is obtained by means of linear interpolation between the values for the reference stress and the static stress as determined in accordance with 5.3.2. This formulation is included in the determination of permissible stress for limited life, according to 5.3.3.

Table 3 — Size factor (root), Y_X

Material ^a		Normal module, m_n	Size factor, Y_X
St, V, GGG (perl., bai.), GTS (perl.),	For 3×10^6 cycles	$m_n \leq 5$ $5 < m_n < 30$ $30 \leq m_n$	$Y_X = 1,0$ $Y_X = 1,03 - 0,006 m_n$ $Y_X = 0,85$
Eh, IF (root), NT, NV		$m_n \leq 5$ $5 < m_n < 25$ $25 \leq m_n$	$Y_X = 1,0$ $Y_X = 1,05 - 0,01 m_n$ $Y_X = 0,8$
GG, GGG (ferr.)		$m_n \leq 5$ $5 < m_n < 25$ $25 \leq m_n$	$Y_X = 1,0$ $Y_X = 1,075 - 0,015 m_n$ $Y_X = 0,7$
All materials for static stress		—	$Y_X = 1,0$

^a See ISO 6336-1:2006, Table 2 for an explanation of the abbreviations used.

Annex A (normative)

Permissible bending stress, σ_{FP} , obtained from notched, flat or plain polished test pieces

A.1 Methods for determining permissible bending stress, σ_{FP} — Principles, assumptions and application

A.1.1 Method B_k

The permissible bending stress is to be derived from the bending stress number $\sigma_{k \text{ lim}}$ and life factor Y_{Nk} results, usually presented as S-N or damage curves, of the pulsator fatigue testing of notched, flat test pieces. As in the case for Method B, the test data shall be transformed to suit the gears of interest, using the influence factors appropriate to both the method and the test piece: $Y_{\delta \text{ rel } k}$ for notch sensitivity, $Y_{R \text{ rel } k}$ for surface roughness and size factor Y_X in accordance with Method B.

This method can be applied when values obtained from test gears are not available, and is particularly suitable for evaluating, relative to one another, the tooth root strength values for different materials.

A.1.2 Method B_p

The permissible bending stresses are to be derived from the bending stress number $\sigma_{p \text{ lim}}$ and life factor Y_{Np} results, usually presented as S-N or damage curves, of the pulsator fatigue testing of plain, polished test pieces. As in the case for Method B, the test data shall be transformed to suit the gears of interest, using the (absolute) influence factors appropriate to the method and the test piece: Y_{δ} for notch sensitivity, Y_R for surface roughness, and size factor Y_X in accordance with Method B.

This method can be applied when values obtained from either gears or notched test pieces are not available, and is particularly suitable for evaluating, relative to one another, the tooth root strength values for different materials.

A.2 Permissible bending stress, σ_{FP} : Method B_k

A.2.1 σ_{FP} for static stress and reference stress

Using these methods, the permissible bending stress is calculated on the basis of the strength of a notched test piece from Equation (A.1):

$$\sigma_{FP} = \frac{\sigma_{k \text{ lim}} Y_{Sk} Y_{Nk}}{S_{Fmin}} Y_{\delta \text{ rel } k} Y_{R \text{ rel } k} Y_X = \frac{\sigma_{FG}}{S_{Fmin}} \quad (\text{A.1})$$

where

- $\sigma_{k \text{ lim}}$ is the nominal notched-bar stress number (bending), which is the bending stress limit value of the notched-bar test piece relevant to its material, heat treatment and surface condition in relation to its dimensions (see recommendation below);
- Y_{Sk} is the stress correction factor, relevant to the notched test piece;
- Y_{Nk} is the life factor for tooth root stress, relevant to the notched test piece, which is used to take into account the higher load bearing capacity for a limited number of load cycles;

$Y_{\delta \text{ rel } k}$ is the relative notch sensitivity factor, which is the quotient of the notch sensitivity factor of the gear of interest divided by the notched test piece factor (see A.6), and which enables the influence of the notch sensitivity of the material to be taken into account;

$Y_{R \text{ rel } k}$ is the relative roughness factor, which is the quotient of the tooth root fillet roughness factor of the gear of interest divided by the notched test piece factor (see A.7), and which enables relevant surface roughness of tooth root fillet influences to be taken into account.

For $\sigma_{k \text{ lim}}$, differences between the properties of the heat treated materials, application of stresses and sections of test piece and gear of interest due to conditions of manufacture should be taken into consideration.

Other relevant symbols are defined in 5.3.2.

The values of the factors related to the notched test piece ($\sigma_{k \text{ lim}}$, Y_{Sk} and Y_{Nk}) shall be determined by tests or to be taken from literature (see A.5). Evaluations of $\sigma_{k \text{ lim}}$, and all corresponding influence factors shall be based on values of static stress and reference stress appropriate to the notched test piece.

The influence factors shall be determined in accordance with 5.3.2.

A.2.2 σ_{FP} for limited life

The value of σ_{FP} shall be determined in accordance with 5.3.3.

A.3 Permissible bending stress, σ_{FP} : Method B_p

For these methods the permissible bending stress is calculated on the basis of the strength of a plain, polished test piece from Equation (A.2):

$$\sigma_{FP} = \frac{\sigma_{P \text{ lim}} Y_{Np}}{S_{Fmin}} Y_{\delta} Y_R Y_X = \frac{\sigma_{FG}}{S_{Fmin}} \quad (\text{A.2})$$

where

$\sigma_{p \text{ lim}}$ is the nominal plain-bar stress number (bending), which is the bending stress limit value of the plain-bar test piece relevant to its material and heat treatment in relation to its dimensions (see recommendations, below);

Y_{Np} is the life factor for tooth root stress, relevant to the plain, polished test piece which is used in order to take into account the higher load capacity for a limited number of cycles;

Y_{δ} is the notch sensitivity factor of the gear of interest, as related to a plain, polished test piece, which enables the influence of the notch sensitivity of the material to be taken into account;

Y_R is the surface factor of the gear of interest, as related to the plain, polished test piece, which enables relevant surface roughness influences to be taken into account.

For $\sigma_{k \text{ lim}}$, differences between the properties of the heat treated materials of the test piece and gear of interest due to conditions of manufacture should be taken into consideration.

Other relevant terms and symbols are defined in 5.3.2.

Evaluations of $\sigma_{p \text{ lim}}$ and Y_{Np} for plain test pieces shall be based on tests or obtained from the literature (see A.5). Evaluations of $\sigma_{p \text{ lim}}$ and all corresponding influence factors shall be based on values of static stress and reference stress.

The influence factors shall be determined in accordance with 5.3.2 and 5.3.3.