
**Calculation of load capacity of spur
and helical gears —**

**Part 3:
Calculation of tooth bending strength**

*Calcul de la capacité de charge des engrenages cylindriques à
dentures droite et hélicoïdale —*

Partie 3: Calcul de la tenue en fatigue à la flexion en pied de dent



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Published in Switzerland

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

This third edition cancels and replaces the second edition (ISO 6336-3:2006), which has been technically revised. It also incorporates the Technical Corrigendum ISO 6336-3:2006/Cor.1:2008.

The main changes compared to the previous edition are as follows:

- modification of the Y_β factor in [Clause 8](#) "Helix angle factor, Y_β ";
- modification of the Y_F factor in [6.2](#) "Calculation of the form factor, Y_F : Method B";
- integration of [6.2.4](#) "Tooth root normal chord, s_{Fn} , radius of root fillet, ρ_F , bending moment arm, h_{Fe} , for external gears generated with a shaper cutter";
- integration of [6.2.5](#) "Tooth root normal chord, s_{Fn} , radius of root fillet, ρ_F , bending moment arm, h_{Fe} , for internal gears generated with a shaper cutter";
- integration of a new [Annex C](#).

A list of all parts in the ISO 6336 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

This corrected version of ISO 6336-3:2019 incorporates the following corrections:

- the indication of the 90° angle in the middle of [Figure 5 b\)](#) has been corrected.

Introduction

ISO 6336 (all parts) consists of International Standards, Technical Specifications (TS) and Technical Reports (TR) under the general title *Calculation of load capacity of spur and helical gears* (see [Table 1](#)).

- International Standards contain calculation methods that are based on widely accepted practices and have been validated.
- Technical Specifications (TS) contain calculation methods that are still subject to further development.
- Technical Reports (TR) contain data that is informative, such as example calculations.

The procedures specified in parts 1 to 19 of the ISO 6336 series cover fatigue analyses for gear rating. The procedures described in parts 20 to 29 of the ISO 6336 series are predominantly related to the tribological behavior of the lubricated flank surface contact. Parts 30 to 39 of the ISO 6336 series include example calculations. The ISO 6336 series allows the addition of new parts under appropriate numbers to reflect knowledge gained in the future.

Requesting standardized calculations according to the ISO 6336 series without referring to specific parts requires the use of only those parts that are currently designated as International Standards (see [Table 1](#) for listing). When requesting further calculations, the relevant part or parts of the ISO 6336 series need to be specified. Use of a Technical Specification as acceptance criteria for a specific design need to be agreed in advance between the manufacturer and the purchaser.

Table 1 — Parts of the ISO 6336 series (status as of DATE OF PUBLICATION)

Calculation of load capacity of spur and helical gears	International Standard	Technical Specification	Technical Report
<i>Part 1: Basic principles, introduction and general influence factors</i>	X		
<i>Part 2: Calculation of surface durability (pitting)</i>	X		
<i>Part 3: Calculation of tooth bending strength</i>	X		
<i>Part 4: Calculation of tooth flank fracture load capacity</i>		X	
<i>Part 5: Strength and quality of materials</i>	X		
<i>Part 6: Calculation of service life under variable load</i>	X		
<i>Part 20: Calculation of scuffing load capacity (also applicable to bevel and hypoid gears) — Flash temperature method</i> (replaces: ISO/TR 13989-1)		X	
<i>Part 21: Calculation of scuffing load capacity (also applicable to bevel and hypoid gears) — Integral temperature method</i> (replaces: ISO/TR 13989-2)		X	
<i>Part 22: Calculation of micropitting load capacity</i> (replaces: ISO/TR 15144-1)		X	
<i>Part 30: Calculation examples for the application of ISO 6336 parts 1, 2, 3, 5</i>			X
<i>Part 31: Calculation examples of micropitting load capacity</i> (replaces: ISO/TR 15144-2)			X

The maximum tensile stress at the tooth root, which may not exceed the permissible bending stress for the material, is the basis for rating the bending strength of gear teeth. The stress occurs in the “tension fillets” of the working tooth flanks. If load-induced cracks are formed, the first of these often appears in the fillets where the compressive stress is generated, i.e. in the “compression fillets”, which are those of the non-working flanks. When the tooth loading is unidirectional and the teeth are of conventional shape, these cracks seldom propagate to failure. Crack propagation ending in failure is most likely to stem from cracks initiated in tension fillets.

The endurable tooth loading of teeth subjected to a reversal of loading during each revolution, such as “idler gears”, is less than the endurable unidirectional loading. The full range of stress in such circumstances is more than twice the tensile stress occurring in the root fillets of the loaded flanks. This is taken into consideration when determining permissible stresses (see ISO 6336-5).

When gear rims are thin and tooth spaces adjacent to the root surface narrow (conditions which can particularly apply to some internal gears), initial cracks commonly occur in the compression fillet. Since, in such circumstances, gear rims themselves can suffer fatigue breakage, special studies are necessary. See [Clause 1](#).

Several methods for calculating the critical tooth root stress and evaluating some of the relevant factors have been approved. See ISO 6336-1.

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Calculation of load capacity of spur and helical gears —

Part 3:

Calculation of tooth bending strength

IMPORTANT — The user of this document is cautioned that when the method specified is used for large helix angles ($\beta > 30^\circ$) and large normal pressure angles ($\alpha_n > 25^\circ$), the calculated results should be confirmed by experience as by Method A.

1 Scope

This document specifies the fundamental formulae for use in tooth bending stress calculations for involute external or internal spur and helical gears with a rim thickness $s_R > 0,5 h_t$ for external gears and $s_R > 1,75 m_n$ for internal gears. In service, internal gears can experience failure modes other than tooth bending fatigue, i.e. fractures starting at the root diameter and progressing radially outward. This document does not provide adequate safety against failure modes other than tooth bending fatigue. All load influences on the tooth root stress are included in so far as they are the result of loads transmitted by the gears and in so far as they can be evaluated quantitatively.

This document includes procedures based on testing and theoretical studies such as those of Hirt^[11], Strasser^[14] and Brossmann^[10]. The results are in good agreement with other methods (References [5], [6], [7] and [12]). The given formulae are valid for spur and helical gears with tooth profiles in accordance with the basic rack standardized in ISO 53. They can also be used for teeth conjugate to other basic racks if the virtual contact ratio $\epsilon_{\alpha m}$ is less than 2,5.

The load capacity determined on the basis of permissible bending stress is termed “tooth bending strength”. The results are in good agreement with other methods for the range, as indicated in the scope of ISO 6336-1.

If this scope does not apply, refer to ISO 6336-1:2019, Clause 4.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile*

ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*

ISO 4287:1997, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Terms, definitions and surface texture parameters*

ISO 4287:1997/Cor 1:1998, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Terms, definitions and surface texture parameters — TECHNICAL CORRIGENDUM 1*

ISO 4287:1997/Cor 2:2005, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Terms, definitions and surface texture parameters — TECHNICAL CORRIGENDUM 2*

ISO 4287:1997/Amd 1:2009, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Terms, definitions and surface texture parameters — AMENDMENT 1: Peak count number*

ISO 4288:1996, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Rules and procedures for the assessment of surface texture*

ISO 6336-1, *Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors*

ISO 6336-5, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of materials*

3 Terms, definitions, symbols and abbreviated terms

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 1122-1:1998 and ISO 6336-1 apply.

ISO and IEC maintain terminological databases for use in standardisation at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://electropedia.org/>

3.2 Symbols and abbreviated terms

For the purposes of this document, the symbols and abbreviated terms given in ISO 1122-1:1998, ISO 6336-1 and [Table 2](#) apply.

Table 2 — Abbreviated terms and symbols used in this document

Abbreviated terms	
Term	Description
Eh	material designation for case-hardened wrought steel
GG	material designation for grey cast iron
GGG	material designation for nodular cast iron (perlitic, bainitic, ferritic structure)
GTS	material designation for black malleable cast iron (perlitic structure)
IF	material designation for flame or induction hardened wrought special steel
M	point
NT	material designation for nitrided wrought steel, nitriding steel
NV	material designation for through-hardened wrought steel, nitrided, nitrocarburized
St	material designation for normalized base steel ($\sigma_B < 800 \text{ N/mm}^2$)
V	material designation for through-hardened wrought special steel, alloy or carbon ($\sigma_B \geq 800 \text{ N/mm}^2$)
X	x-coordinate
Y	y-coordinate
^a For external gears a , d , d_a , z_1 and z_2 are positive; for internal gearing, a , d , d_a and z_2 have a negative sign, z_1 has a positive sign. All calculated diameters have a negative sign for internal gearing.	

Table 2 (continued)

Symbols		
Symbol	Description	Unit
a_0	manufacturing centre distance	mm
b	face width	mm
b_B	face width of one helix on a double helical gear	mm
d	diameter (without subscript, reference diameter ^a)	mm
d_a	tip diameter ^a	mm
d_{an}	tip diameter of virtual gear	mm
d_b	base diameter	mm
d_{bn}	base diameter of virtual gear	mm
d_{b0}	base diameter of the tool	mm
d_{en}	outer single contact diameter of virtual gears	mm
d_n	reference diameter of virtual spur gear	mm
d_{Na}	active tip diameter	mm
d_w	pitch diameter	mm
d_0	reference diameter of the tool	mm
E	auxiliary value	mm
F_b	(nominal) load (normal to the line of contact or transverse to the plane of action)	N
F_{bn}	(nominal) load, normal to the line of contact	N
F_{bt}	(nominal) transverse load in the plane of action (base tangent plane)	N
F_{Rhigh}	load per unit facewidth of the higher loaded flank	N/mm
F_{Rlow}	load per unit facewidth of lower loaded flank	N/mm
F_t	(nominal) transverse tangential load at reference cylinder per mesh	N
F_w	(nominal) tangential load at the pitch cylinder	N
f_ε	load distribution influence factor	—
G	auxiliary value	—
H	auxiliary value	—
h_{aP}	addendum of basic rack of cylindrical gears	mm
h_{aP0}	addendum of tool	mm
h_{Fe}	bending moment arm for tooth root stress relevant to load application at the outer point of single pair tooth contact	mm
h_{fP}	dedendum of basic rack of cylindrical gears (ISO 53:1998 shall apply)	mm
h_t	tooth height	mm
K	distance of point M to the point of contact of the pitch circles	mm

^a For external gears a , d , d_a , z_1 and z_2 are positive; for internal gearing, a , d , d_a and z_2 have a negative sign, z_1 has a positive sign. All calculated diameters have a negative sign for internal gearing.

Table 2 (continued)

Symbols		
Symbol	Description	Unit
K_A	application factor	—
$K_{F\alpha}$	transverse load factor (root stress)	—
$K_{F\beta}$	face load factor (root stress)	—
K_v	dynamic factor	—
K_γ	mesh load factor	—
L	auxiliary value	—
M	mean stress ratio	—
m_n	normal module	mm
N_L	number of load cycles	—
p_{bn}	normal base pitch	mm
pr	protuberance of the tool	mm
q	material allowance for finish machining per flank	mm
q_s	notch parameter, $q_s = s_{Fn}/2\rho_F$	—
q_{sk}	notch parameter of the notched test piece	—
q_{sT}	notch parameter of the standard reference test gear	—
R	stress ratio	—
Rz	mean peak-to-valley roughness (ISO 4287:1997 including ISO 4287:1997/Cor 1:1998, ISO 4287:1997/Cor 2:2005, ISO 4287:1997/Amd 1:2009 and ISO 4288:1996 shall apply)	μm
Rz_k	mean peak-to-valley roughness of the notched, rough test piece	μm
Rz_T	mean peak-to-valley roughness in the fillet of standard reference gears (see ISO/TR 10064-4)	μm
r	radius	mm
r_{a0}	tip radius of tool	mm
r_{b0}	base radius of the tool	mm
r_M	radius for the centre of the tool tip radius	mm
r_w	manufacturing pitch circle radius	mm
r_{w0}	manufacturing pitch circle radius of tool	mm
S	safety factor	—
S_F	safety factor for tooth breakage	—
$S_{F\min}$	minimum required safety factor for tooth root stress	—
s_{Fn}	tooth root chord at the critical section	mm
s_{pr}	residual fillet undercut, $s_{pr} = pr - q$	mm
s_R	rim thickness	mm

^a For external gears a , d , d_a , z_1 and z_2 are positive; for internal gearing, a , d , d_a and z_2 have a negative sign, z_1 has a positive sign. All calculated diameters have a negative sign for internal gearing.

Table 2 (continued)

Symbols		
Symbol	Description	Unit
T	auxiliary value	—
t_g	maximum depth of grinding notch	mm
u_0	manufacturing tooth ratio	—
X_M	x-coordinate of point M	mm
x	profile shift coefficient	—
$x_{E \min}$	smallest generating profile shift	—
x_0	profile shift coefficient of the tool	—
Y_B	rim thickness factor, which adjusts the calculated tooth root stress for thin rimmed gears	—
Y_{DT}	deep tooth factor	—
Y_F	tooth form factor, for the influence on nominal tooth root stress with load applied at the outer point of single pair tooth contact	—
Y_M	mean stress influence factor (see Annex B)	—
Y_M	y-coordinate of point M	mm
Y_{Nk}	life factor for tooth root stress, relevant to the notched test piece	—
Y_{Np}	life factor for tooth root stress, relevant to the plain polished test piece	—
Y_{NT}	life factor for tooth root stress for reference test conditions	—
Y_R	tooth root surface factor (relevant to the plain polished test piece)	—
Y_{Rk}	surface factor	—
Y_{R0}	surface factor of the plain, polished test piece	—
$Y_{R \text{ rel } k}$	relative roughness factor, the quotient of the gear tooth root surface factor of interest divided by the notch test piece factor, $Y_{R \text{ rel } k} = Y_R/Y_{Rk}$	—
$Y_{R \text{ rel } T}$	relative surface factor, the quotient of the gear tooth root surface factor of interest divided by the tooth root surface factor of the reference test gear, $Y_{R \text{ rel } T} = Y_R/Y_{RT}$	—
Y_{RT}	tooth root surface factor of the reference test gears	—
Y_S	stress correction factor, for the conversion of the nominal tooth root stress, determined for application of load at the outer point of single pair tooth contact, to the local tooth root stress	—
Y_{Sg}	stress correction factor, relevant to the notched piece	—
Y_{Sk}	stress correction factor, relevant to the notched test piece	—
Y_{ST}	stress correction factor, relevant to the dimensions of the reference test gears	—
Y_X	size factor (tooth root)	—
Y_β	helix angle factor (tooth root)	—
Y_δ	notch sensitivity factor of the actual gear (relative to a polished test piece)	—

^a For external gears a , d , d_a , z_1 and z_2 are positive; for internal gearing, a , d , d_a and z_2 have a negative sign, z_1 has a positive sign. All calculated diameters have a negative sign for internal gearing.

Table 2 (continued)

Symbols		
Symbol	Description	Unit
$Y_{\delta k}$	notch sensitivity factor of a notched test piece, relative to a smooth polished test piece	—
$Y_{\delta T}$	notch sensitivity factor of the standard reference test gear, relative to the smooth polished test piece	—
$Y_{\delta \text{ rel } T}$	relative notch sensitivity factor, the quotient of the gear notch sensitivity factor of interest divided by the notch sensitivity factor of the standard reference test gear, $Y_{\delta \text{ rel } T} = Y_{\delta} / Y_{\delta T}$	—
y	auxiliary value	° or rad
y'	auxiliary value	—
z	number of teeth ^a	—
z_n	virtual number of teeth of a helical gear	—
z_0	number of teeth of the tool	—
z_{0v}	equivalent number of teeth of the tool	—
α_{en}	profile angle at the outer point of a single pair tooth contact of virtual spur gears	°
α_{Fen}	load direction angle, relevant to direction of application of load at the outer point of single pair tooth contact of virtual spur gears	°
α_M	transverse pressure angle for the radius at the point M	°
α_n	normal pressure angle	°
α_w	working pressure angle	°
α_{w0}	operating pressure angle of the manufacturing pairing	°
α_x	transverse pressure angle of basic rack profile	°
β_b	base helix angle	°
γ	auxiliary angle	°
γ_e	auxiliary angle at the virtual gear	° or rad
$\Delta\alpha$	half angle of thickness at point M	°
Δh	auxiliary value	mm
$\Delta h'$	auxiliary value	mm
δ	auxiliary value	°
ε	contact ratio	—
ε_α	transverse contact ratio	—
$\varepsilon_{\alpha n}$	virtual contact ratio of the virtual spur gear	—
ε_β	overlap ratio	—
θ	tangential angle	° or rad
λ	auxiliary value	—
ξ	auxiliary value	—

^a For external gears a , d , d_a , z_1 and z_2 are positive; for internal gearing, a , d , d_a and z_2 have a negative sign, z_1 has a positive sign. All calculated diameters have a negative sign for internal gearing.

Table 2 (continued)

Symbols		
Symbol	Description	Unit
ρ_{a0}	tool tip corner rounding	mm
ρ_F	tooth root radius at the critical section	mm
ρ_{fP}	tooth root fillet radius of the basic rack for cylindrical gears	mm
ρ_g	radius of grinding notch	mm
ρ'	slip layer thickness	mm
σ	normal stress	N/mm ²
σ_B	tensile strength	N/mm ²
σ_F	tooth root stress	N/mm ²
σ_{FE}	allowable stress number (bending), $\sigma_{FE} = \sigma_{F \lim} Y_{ST}$	N/mm ²
σ_{FG}	tooth root stress limit	N/mm ²
$\sigma_{F \lim}$	nominal stress number (bending)	N/mm ²
σ_{FP}	permissible bending stress	N/mm ²
$\sigma_{FP \text{ stat}}$	permissible bending stress for the static stress	N/mm ²
$\sigma_{FP \text{ ref}}$	permissible bending stress for the reference stress	N/mm ²
σ_{F0}	nominal tooth root stress	N/mm ²
$\sigma_{k \lim}$	nominal notched-bar stress number (bending)	N/mm ²
$\sigma_{p \lim}$	nominal plain-bar stress number (bending)	N/mm ²
σ_S	yield stress	N/mm ²
$\sigma_{0,2}$	proof stress (0,2 % permanent set)	N/mm ²
χ^*	relative stress gradient in the root of a notch	mm ⁻¹
χ_K^*	relative stress gradient in the notch root of the test piece	mm ⁻¹
χ_p^*	relative stress gradient in a smooth polished test piece	mm ⁻¹
χ_T^*	relative stress gradient of the standard reference test gear	mm ⁻¹
ψ	auxiliary angle	° or rad
ω_0	auxiliary angle	°

^a For external gears a , d , d_a , z_1 and z_2 are positive; for internal gearing, a , d , d_a and z_2 have a negative sign, z_1 has a positive sign. All calculated diameters have a negative sign for internal gearing.

4 Tooth breakage and safety factors

Tooth breakage usually ends the service life of a transmission. Sometimes, the destruction of all gears in a transmission can be a consequence of the breakage of one tooth. In some instances, the transmission path between input and output shafts is broken. As a consequence, the chosen value of the safety factor S_F against tooth breakage should be larger than the safety factor against pitting.

General comments on the choice of the minimum safety factor can be found in ISO 6336-1:2019, 4.1.11. It is recommended that the manufacturer and the customer agree on the value of the minimum safety factor.

This document does not apply at stress levels above those permissible for 10^3 cycles, since stresses in this range may exceed the elastic limit of the gear tooth.

5 Basic formulae

5.1 General

The actual tooth root stress σ_F and the permissible (tooth root) bending stress σ_{FP} shall be calculated separately for the pinion and the wheel; σ_F shall be less than σ_{FP} .

5.2 Safety factor for bending strength (safety against tooth breakage), S_F

Calculate S_F separately for the pinion and the wheel:

$$S_{F1} = \frac{\sigma_{FG1}}{\sigma_{F1}} \geq S_{F \min} \quad (1)$$

$$S_{F2} = \frac{\sigma_{FG2}}{\sigma_{F2}} \geq S_{F \min} \quad (2)$$

σ_{F1} and σ_{F2} are derived from [Formulae \(3\) and \(4\)](#). The values of σ_{FG} for reference stress and static stress are calculated in accordance with [5.4.3.2](#) and [5.4.3.3](#), using [Formula \(5\)](#). For a limited life, σ_{FG} is determined in accordance with [5.4.4](#).

The values of the tooth root stress limit σ_{FG} , of the permissible stress σ_{FP} and of the tooth root stress σ_F may each be determined by different methods. The method used for each value shall be stated in the calculation report.

NOTE Safety factors in accordance with this clause are relevant to the transmissible torque.

See ISO 6336-1:2019, 4.1.11 for comments on numerical values for the minimum safety factor and the risk of damage.

5.3 Tooth root stress, σ_F

5.3.1 General

Tooth root stress σ_F is the maximum tensile stress at the surface in the root fillet.

5.3.2 Method A

In principle, the maximum tensile stress can be determined by any appropriate method (finite element analysis, integral formulae, conformal mapping procedures or experimentally by strain measurement, etc.). In order to determine the maximum tooth root stress, the effects of load distribution over two or more engaging teeth and changes of stress with changes of meshing phase shall be taken into consideration.

Method A is only used in special cases and, because of the great effort involved, is only justifiable in such cases.

5.3.3 Method B

According to this document, the local tooth root stress is determined as the product of the nominal tooth root stress and a stress correction factor¹⁾.

This method involves the assumption that the determinant tooth root stress occurs with the application of load at the outer point of the single pair tooth contact of spur gears or of the virtual spur gears of helical gears. However, in the latter case, the “transverse load” shall be replaced by the “normal load”, applied over the facewidth of the actual gear of interest.

For gears having virtual contact ratios in the range $2 \leq \varepsilon_{an} < 2,5$, it is assumed that the determinant stress occurs with the application of load at the inner point of the triple pair tooth contact. In ISO 6336 (all parts), this assumption is taken into consideration by the deep tooth factor, Y_{DT} . In the case of helical gears, the factor, Y_{β} , accounts for deviations from these assumptions.

1) Stresses such as those caused by the shrink-fitting of gear rims, which are superimposed on stresses due to tooth loading, should be taken into consideration in the calculation of permissible tooth root stress σ_{FP} .

Method B is suitable for general calculations and is also appropriate for computer programming and for the analysis of pulsator tests (with a given point of the application of loading).

$$\sigma_F = \sigma_{F0} \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{F\beta} \cdot K_{F\alpha} \quad (3)$$

where

- σ_{F0} is the nominal tooth root stress, which is the maximum local principal stress produced at the tooth root when an error-free gear pair is loaded by the static nominal torque and without any pre-stress such as shrink fitting, i.e. stress ratio $R = 0$ [see [Formula \(4\)](#)];
- σ_{FP} is the permissible bending stress (see [5.4](#));
- K_A is the application factor (ISO 6336-1 shall apply), which takes into account load increments due to externally influenced variations of the input or output torque;
- K_γ is the mesh load factor (ISO 6336-1 shall apply), which takes into account the uneven distribution of the total tangential load between meshes for multiple paths;
- K_v is the dynamic factor (ISO 6336-1 shall apply), which takes into account load increments due to internal dynamic effects;
- $K_{F\beta}$ is the face load factor for tooth root stress (ISO 6336-1 shall apply), which takes into account uneven distribution of load over the facewidth due to mesh-misalignment caused by inaccuracies in manufacture, elastic deformations, etc.;
- $K_{F\alpha}$ is the transverse load factor for tooth root stress (ISO 6336-1 shall apply), which takes into account uneven load distribution in the transverse direction, resulting, for example, from pitch deviations.

NOTE See ISO 6336-1:2019, 4.1.18, for the sequence in which factors K_A , K_v , $K_{F\beta}$ and $K_{F\alpha}$ are calculated.

$$\sigma_{F0} = \frac{F_t}{b \cdot m_n} \cdot Y_F \cdot Y_S \cdot Y_\beta \cdot Y_B \cdot Y_{DT} \quad (4)$$

where

- F_t is the nominal tangential load, the transverse load tangential to the reference cylinder (ISO 6336-1 shall apply);
NOTE In all cases, even when $\varepsilon_{\alpha n} > 2$, it is necessary to substitute the relevant total tangential load as F_t . Reasons for the choice of load application at the reference cylinder are given in [Annex C](#). See ISO 6336-1:2019, 4.2, for definition of F_t and comments on particular characteristics of double helical gears.
- b is the facewidth (for double helical gears $b = 2 b_B$);
NOTE The value b , of mating gears, is the facewidth at the root circle, ignoring any intentional transverse chamfers or tooth-end rounding. If the facewidths of the pinion and the wheel are not equal, it can be assumed that the load bearing width of the wider facewidth is equal to the smaller facewidth plus a maximum of $1 \times$ module at each end.
- m_n is the normal module;
- Y_F is the form factor (see [Clause 6](#)), which takes into account the influence on the nominal tooth root stress of the tooth form with load applied at the outer point of the single pair tooth contact;

Y_S is the stress correction factor (see [Clause 7](#)), which takes into account the influence on the nominal tooth root stress, determined for the application of load at the outer point of the single pair tooth contact, to the local tooth root stress, and thus, by means of which, are taken into account;

- i) the stress amplifying effect of the change of section at the tooth root, and
- ii) the fact that evaluation of the true stress system at the tooth root critical section is more complex than the simple system evaluation presented;

Y_β is the helix angle factor (see [Clause 8](#)), which compensates for the fact that the bending moment intensity at the tooth root of helical gears is, as a consequence of the oblique lines of contact, less than the corresponding values for the virtual spur gears used as bases for calculation;

Y_B is the rim thickness factor (see [Clause 9](#)), which adjusts the calculated tooth root stress for thin rimmed gears;

Y_{DT} is the deep tooth factor (see [Clause 10](#)), which adjusts the calculated tooth root stress for high precision gears with a contact ratio in the range $2 \leq \varepsilon_{\alpha n} < 2,5$.

5.4 Permissible bending stress, σ_{FP}

5.4.1 General

The limit value of tooth root stresses (see [Clause 11](#)) should preferably be derived from material tests using gears as test pieces, since in this way the effects of test piece geometry, such as the effect of the fillet at the tooth roots, are included in the results. The calculation methods provided constitute empirical means for comparing stresses in gears of different dimensions with experimental results. The closer test gears and test conditions resemble the service gears and service conditions, the lesser will be the influence of inaccuracies in the formulation of the calculation expressions.

If the permissible bending stress, σ_{FP} , is obtained from notched, flat, or plain polished test pieces, [Annex A](#) shall apply.

5.4.2 Methods for determination of permissible bending stress, σ_{FP} — Principles, assumptions and application

5.4.2.1 General

Several procedures for the determination of the permissible bending stress σ_{FP} are acceptable. The method adopted shall be validated by carrying out careful comparative studies of well-documented service histories of a number of gears.

5.4.2.2 Method A

By this method, the values for σ_{FP} or for the tooth root stress limit, σ_{FG} , are obtained using [Formulae \(3\)](#) and [\(4\)](#) from the S-N curve or damage curve derived from results of testing facsimiles of the actual gear pair, under the appropriate service conditions.

The cost required for this method is, in general, only justifiable for the development of new products, failure of which would have serious consequences (e.g. for manned space flights).

Similarly, in line with this method, the allowable stress values may be derived from consideration of dimensions, service conditions and performance of carefully monitored reference gears.

5.4.2.3 Method B

Damage curves characterized by the nominal stress number (bending), $\sigma_{F \text{ lim}}$, and the factor Y_{NT} have been determined for a number of common gear materials and heat treatments from results of gear load or pulsator testing of standard reference test gears. Material values so determined are converted to suit the dimensions of the gears of interest, using the relative influence factors for notch sensitivity, $Y_{\delta \text{ rel T}}$, for surface roughness, $Y_{R \text{ rel T}}$, and for size, Y_X .

Method B is recommended for the calculation of reasonably accurate gear ratings whenever bending strength values are available from gear tests, from special tests or, if the material is similar, from ISO 6336-5.

5.4.3 Permissible bending stress, σ_{FP} : Method B

5.4.3.1 General

Subject to the reservations given in 5.4.3.2 and 5.4.3.3, Formula (5) is to be used for this calculation:

$$\sigma_{FP} = \frac{\sigma_{F \text{ lim}} \cdot Y_{ST} \cdot Y_{NT}}{S_{F \text{ min}}} \cdot Y_{\delta \text{ rel T}} \cdot Y_{R \text{ rel T}} \cdot Y_X = \frac{\sigma_{FE} \cdot Y_{NT}}{S_{F \text{ min}}} \cdot Y_{\delta \text{ rel T}} \cdot Y_{R \text{ rel T}} \cdot Y_X = \frac{\sigma_{FG}}{S_{F \text{ min}}} \quad (5)$$

where

- $\sigma_{F \text{ lim}}$ is the nominal stress number (bending) from reference test gears (ISO 6336-5 shall apply), which is the bending stress limit value relevant to the influences of the material, the heat treatment and the surface roughness of the test gear root fillets;
- σ_{FE} is the allowable stress number for bending, which is the basic bending strength of the un-notched test piece, under the assumption that the material condition (including heat treatment) is fully elastic;
 $\sigma_{FE} = (\sigma_{F \text{ lim}} Y_{ST})$;
- Y_{ST} is the stress correction factor, relevant to the dimensions of the reference test gears (see 7.4);
- Y_{NT} is the life factor for tooth root stress, relevant to the dimensions of the reference test gear (see Clause 12), which takes into account the higher load capacity for a limited number of load cycles;
- σ_{FG} is the tooth root stress limit;
 $\sigma_{FG} = (\sigma_{FP} S_{F \text{ min}})$;
- $S_{F \text{ min}}$ is the minimum required safety factor for tooth root stress (see Clause 4 and 5.2);
- $Y_{\delta \text{ rel T}}$ is the relative notch sensitivity factor, which is the quotient of the gear notch sensitivity factor of interest divided by the notch sensitivity factor of the standard reference test gear (see Clause 13) and which enables the influence of the notch sensitivity of the material to be taken into account;
- $Y_{R \text{ rel T}}$ is the relative surface factor, which is the quotient of the surface roughness factor of tooth root fillets of the gear of interest divided by the tooth root fillet factor of the reference test gear (see Clause 14) and which enables the relevant surface roughness of tooth root fillet influences to be taken into account;
- Y_X is the size factor relevant to tooth root strength (see Clause 15), which is used to take into account the influence of tooth dimensions on the tooth bending strength.

5.4.3.2 Permissible bending stress (reference)

The permissible bending stress (reference), $\sigma_{FP\ ref}$, is derived from [Formula \(5\)](#), with $Y_{NT} = 1$ and influence factors $\sigma_{F\ lim}$, Y_{ST} , $Y_{\delta\ rel\ T}$, $Y_{R\ rel\ T}$, Y_X and $S_{F\ min}$ calculated in accordance with the specified Method B.

5.4.3.3 Permissible bending stress (static)

The permissible bending stress (static), $\sigma_{FP\ stat}$, is determined in accordance with [Formula \(5\)](#), with factors $\sigma_{F\ lim}$, Y_{NT} , Y_{ST} , $Y_{\delta\ rel\ T}$, $Y_{R\ rel\ T}$, Y_X and $S_{F\ min}$ calculated in accordance with the specified Method B (for the static stress).

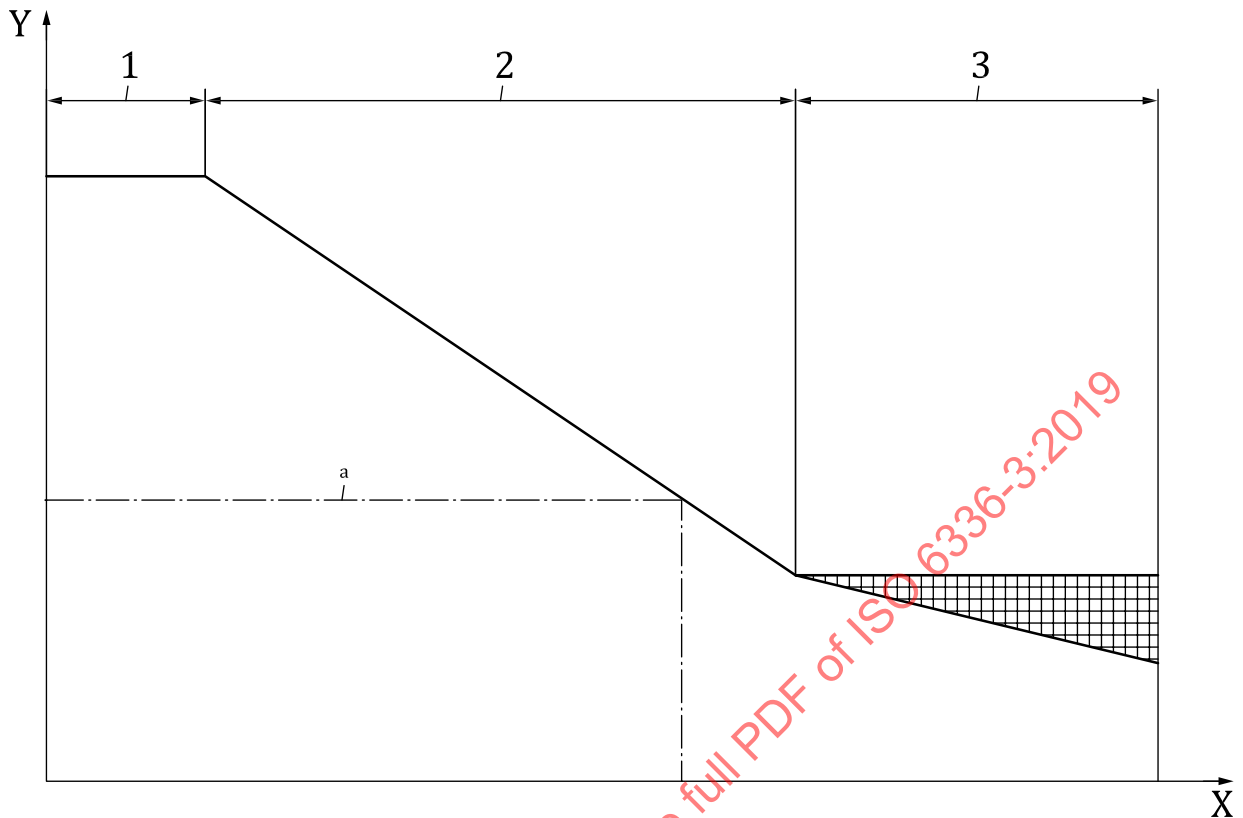
5.4.4 Permissible bending stress, σ_{FP} , for limited and long life: Method B

5.4.4.1 General

σ_{FP} for a given number of load cycles, N_L , is determined by means of graphical or calculated linear interpolation along the S-N curve on a log-log scale, between the value obtained for the reference stress in accordance with [5.4.3.2](#) and the value obtained for the static stress in accordance with [5.4.3.3](#). Also see [Clause 12](#).

5.4.4.2 Graphical values

Calculate $\sigma_{FP\ ref}$ for the reference stress and $\sigma_{FP\ stat}$ for the static stress in accordance with [5.4.3](#) and plot the S-N curve corresponding to the life factor Y_{NT} . See [Figure 1](#) for the principle. σ_{FP} for the relevant number of load cycles N_L can be read from this graph.

**Key**

- X number of load cycles, N_L (log)
 Y permissible bending stress, σ_{FP} (log)
 1 static
 2 limited life
 3 long life
 a Example: Permissible bending stress, σ_{FP} , for a given number of load cycles.

Figure 1 — Graphical determination of permissible bending stress for a limited life, in accordance with Method B

5.4.4.3 Determination by calculation

Calculate $\sigma_{FP\text{ref}}$ for the reference stress and $\sigma_{FP\text{stat}}$ for the static stress in accordance with 5.4.3 and, using these results, determine σ_{FP} for the relevant number of load cycles N_L in the limited life range, as follows.

$$\sigma_{FP} = \sigma_{FP\text{ref}} \cdot Y_N = \sigma_{FP\text{ref}} \cdot \left(\frac{3 \cdot 10^6}{N_L} \right)^{\text{exp}} \quad (6)$$

- a) For St, V, GGG (perl., bai.) or GTS (perl.), the limited life range as shown in Figure 11, $10^4 < N_L \leq 3 \times 10^6$:

$$\text{exp} = 0,4037 \cdot \log \frac{\sigma_{FP\text{stat}}}{\sigma_{FP\text{ref}}} \quad (7)$$

- b) For IF, Eh, NT (nitr.), NV (nitr.), NV (nitrocar.), GGG (ferr.) or GG, the limited life range as shown in Figure 11, $10^3 < N_L \leq 3 \times 10^6$:

$$\exp = 0,2876 \cdot \log \frac{\sigma_{\text{FP stat}}}{\sigma_{\text{FP ref}}} \quad (8)$$

Corresponding calculations may be determined for the range of long life.

6 Form factor, Y_F

6.1 General

Y_F is the factor by which the influence of tooth form on the nominal tooth root stress is taken into account. [Annex C](#) explains the derivation of the factor Y_F . See [5.3.2](#) for principles, assumptions and details of use. Y_F is relevant to the application of load at the outer point of the single pair tooth contact (Method B).

The chord between the points at which the 30° tangents contact the root fillets for external gears, or at which the 60° tangents contact the root fillets for internal gears, defines the section to be used as the basis for calculation (see [Figures 3](#) to [4](#)).

The tooth form factor is sensitive to the tooth thickness. When the manufactured geometry is measured, it should be used. If not, then, based on the tooth thickness tolerance, the smallest generating profile shift, $x_{E \min}$, should be taken into account in the stress calculation.

Because of material allowances for finishing processes such as profile grinding, it is usually the case that the depth setting of the roughing tool, relative to the gear axis, includes the amount of nominal profile shift, $x \cdot m_n$, plus a tolerance designed to ensure that the finishing allowance will be greater instead of less than the requisite minimum. Because of this, calculated values of tooth root stresses usually tend on the side of safety.

The formulae in this document apply to all basic rack profiles (see [Figure 2](#)) with and without undercut, but with the following restrictions:

- the contact point of the 30° (60°) tangent shall lie on the tooth root fillet generated by the root fillet of the basic rack;
- the basic rack profile of the gear shall have a root fillet with $\rho_{\text{FP}} > 0$;
- the teeth shall be generated using tools such as hobs, rack type shaper cutters or pinion type shaper cutters;
- since calculated ratings refer to finished tooth forms, profile grinding and similar allowances, including tooth thickness allowances, can be neglected, and in practice it can be assumed that the dimensions of the basic rack of the tool are the same as those of the counterpart basic rack of the gear;
- for internal gears only the shaper cutter data is used.

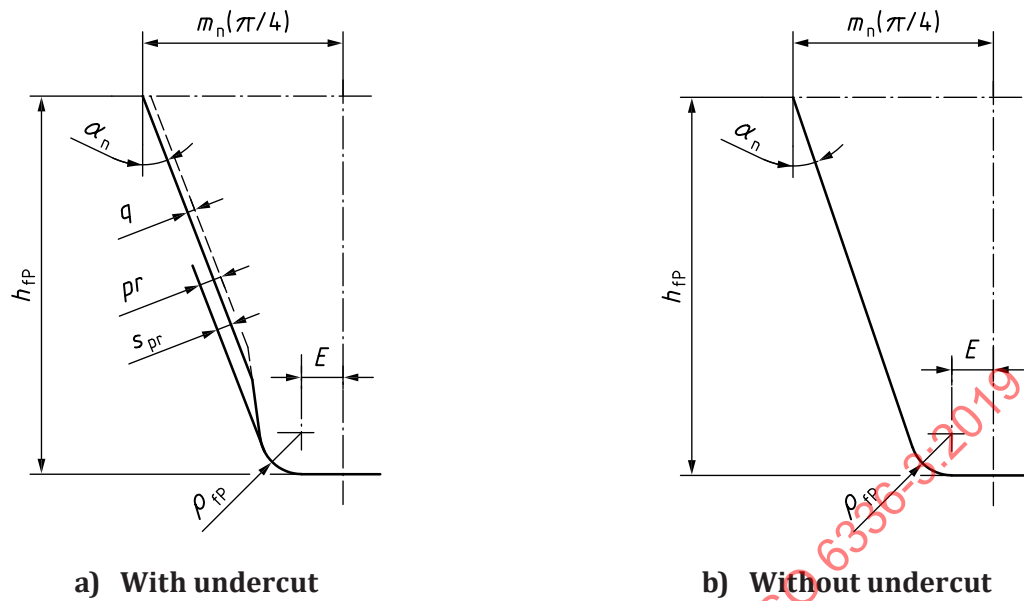


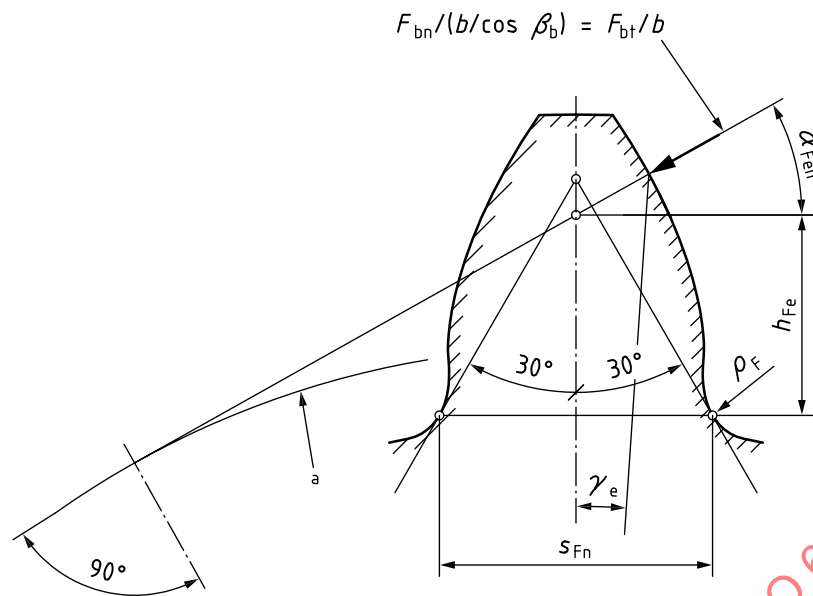
Figure 2 — Dimensions and basic rack profile of the teeth (finished profile)

The above comments apply to straight spur and helical gears. The value Y_F is determined for the virtual spur gears of helical gears; the virtual number of teeth z_n can be determined using [Formula \(16\)](#). Y_F is determined separately for the pinion and the wheel.

6.2 Calculation of the form factor, Y_F : Method B

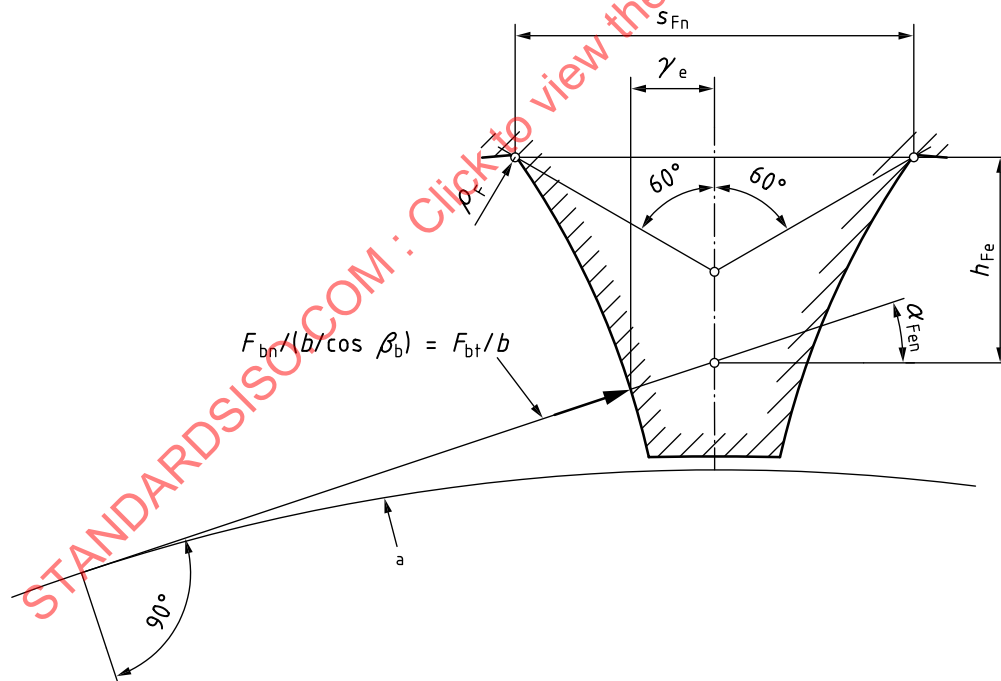
6.2.1 General

The determination of the normal chordal dimension, s_{Fn} , of the tooth root critical section and the bending moment arm, h_{Fe} , relevant to the load application at the outer point of the single pair gear tooth contact for Method B is shown in [Figures 3](#) and [4](#).



a Base circle.

Figure 3 — Determination of normal chordal dimensions of the tooth root critical section for Method B (external gears)



a Base circle.

Figure 4 — Determination of normal chordal dimensions of the tooth root critical section for Method B (internal gears)

The following formula uses the symbols illustrated in [Figures 3](#) and [4](#):

$$Y_F = \frac{\frac{6 \cdot h_{Fe}}{m_n} \cdot \cos \alpha_{Fen}}{\left(\frac{s_{Fn}}{m_n} \right)^2 \cdot \cos \alpha_n} \cdot f_\varepsilon \quad (9)$$

In order to evaluate precise values, s_{Fn} and α_{Fen} , of h_{Fe} it is first necessary to derive a value of θ which is reasonably accurate, usually after five iterations of [Formula \(29\)](#). Determination of Y_F by graphical means is not recommended.

The factor f_ε considers the influence of load distribution between the teeth in the mesh. It provides more accurate results for gears with contact ratios $\varepsilon_{\alpha n} \geq 2,0$. Contact ratios of $\varepsilon_{\alpha n} \geq 2,0$ are calculated for gears with high helix angles, high contact ratios, ε_α , or both.

For spur gears with contact ratios $\varepsilon_{\alpha n} \leq 2,0$ the factor f_ε is equal to one according [Formula \(10\)](#). For helical gears with overlap ratio $\varepsilon_\beta \geq 1$ the factor is calculated according to [Formula \(14\)](#). [Formulae \(12\)](#) and [\(13\)](#) provide a smooth function for f_ε between [Formulae \(10\)](#) and [\(14\)](#).

If $\varepsilon_\beta = 0$ and $\varepsilon_{\alpha n} < 2$ then

$$f_\varepsilon = 1 \quad (10)$$

If $\varepsilon_\beta = 0$ and $\varepsilon_{\alpha n} \geq 2$ then

$$f_\varepsilon = 0,7 \quad (11)$$

If $0 < \varepsilon_\beta < 1$ and $\varepsilon_{\alpha n} < 2$ then

$$f_\varepsilon = \left(1 - \varepsilon_\beta + \frac{\varepsilon_\beta}{\varepsilon_{\alpha n}} \right)^{0,5} \quad (12)$$

If $0 < \varepsilon_\beta < 1$ and $\varepsilon_{\alpha n} \geq 2$ then

$$f_\varepsilon = \left(\frac{1 - \varepsilon_\beta}{2} + \frac{\varepsilon_\beta}{\varepsilon_{\alpha n}} \right)^{0,5} \quad (13)$$

If $\varepsilon_\beta \geq 1$ then

$$f_\varepsilon = \varepsilon_{\alpha n}^{-0,5} \quad (14)$$

6.2.2 Parameters of virtual gears

The parameters of virtual gears are as follows.

$$\beta_b = \arccos \sqrt{1 - (\sin \beta \cdot \cos \alpha_n)^2} = \arcsin (\sin \beta \cdot \cos \alpha_n) \quad (15)$$

$$z_n = \frac{z}{\cos^2 \beta_b \cdot \cos \beta} \quad (16)$$

$$\varepsilon_{\alpha n} = \frac{\varepsilon_\alpha}{\cos^2 \beta_b} \quad (17)$$

$$d_n = \frac{d}{\cos^2 \beta_b} = m_n \cdot z_n \quad (18)$$

$$p_{bn} = \pi \cdot m_n \cdot \cos \alpha_n \quad (19)$$

$$d_{bn} = d_n \cdot \cos \alpha_n \quad (20)$$

$$d_{an} = d_n + d_{Na} - d \quad (21)$$

$$d_{en} = 2 \cdot \frac{z}{|z|} \sqrt{\left[\sqrt{\left(\frac{d_{an}}{2} \right)^2 - \left(\frac{d_{bn}}{2} \right)^2} - \frac{\pi \cdot d \cdot \cos \beta \cdot \cos \alpha_n}{|z|} \cdot (\varepsilon_{\alpha n} - 1) \right]^2 + \left(\frac{d_{bn}}{2} \right)^2} \quad (22)$$

The number of teeth, z , is positive for external gears and negative for internal gears.

$$\alpha_{en} = \arccos \left(\frac{d_{bn}}{d_{en}} \right) \quad (23)$$

$$\gamma_e = \frac{0,5 \cdot \pi + 2 \cdot x \cdot \tan \alpha_n}{z_n} + \text{inv } \alpha_n - \text{inv } \alpha_{en} \quad (24)$$

$$\alpha_{Fen} = \alpha_{en} - \gamma_e = \tan \alpha_{en} - \text{inv } \alpha_n - \frac{0,5 \cdot \pi + 2 \cdot x \cdot \tan \alpha_n}{z_n} \quad (25)$$

6.2.3 Tooth root normal chord, s_{Fn} , radius of root fillet, ρ_F , bending moment arm, h_{Fe} for external gears generated with a hob²⁾

First, determine the auxiliary values for [Formula \(9\)](#):

$$E = \frac{\pi}{4} m_n - h_{fP} \cdot \tan \alpha_n + \frac{s_{pr}}{\cos \alpha_n} - (1 - \sin \alpha_n) \cdot \frac{\rho_{fP}}{\cos \alpha_n} \quad (26)$$

with

$$s_{pr} = pr - q \text{ (see [Figure 2](#));}$$

$$s_{pr} = 0 \text{ when gears are not undercut;}$$

$$G = \frac{\rho_{fP}}{m_n} - \frac{h_{fP}}{m_n} + x \quad (27)$$

$$H = \frac{2}{z_n} \cdot \left(\frac{\pi}{2} - \frac{E}{m_n} \right) - T \quad (28)$$

with

2) If the tip of the tooth has been rounded or chamfered, it is necessary to replace the tip diameter, d_a , in the calculation by d_{Na} the "effective tip diameter".

$T = \pi/3$ for external gears;

$$\theta = \frac{2G}{z_n} \cdot \tan \theta - H \quad (29)$$

The value $\theta = \pi/6$ for external gears may be used as a seed value in the iteration of the transcendental [Formula \(29\)](#). Generally, the function converges after five iterations.

a) Tooth root normal chord, s_{Fn}

$$\frac{s_{Fn}}{m_n} = z_n \cdot \sin\left(\frac{\pi}{3} - \theta\right) + \sqrt{3} \cdot \left(\frac{G}{\cos \theta} - \frac{\rho_{fp}}{m_n}\right) \quad (30)$$

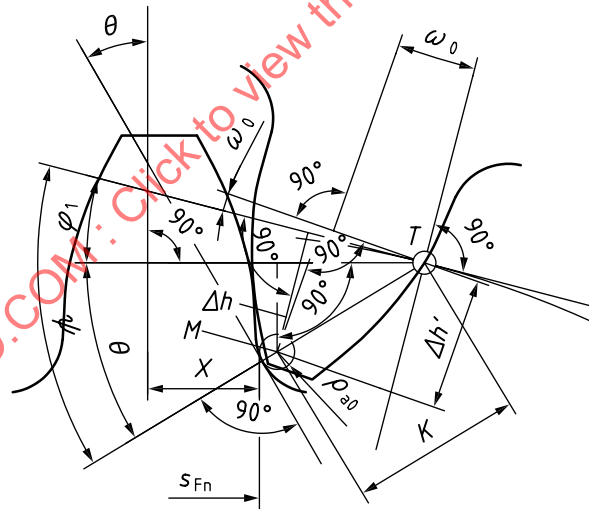
b) Radius of the root fillet, ρ_F (see [Figures 3](#) and [4](#))

$$\frac{\rho_F}{m_n} = \frac{\rho_{fp}}{m_n} + \frac{2 \cdot G^2}{\cos \theta \cdot (z_n \cdot \cos^2 \theta - 2 \cdot G)} \quad (31)$$

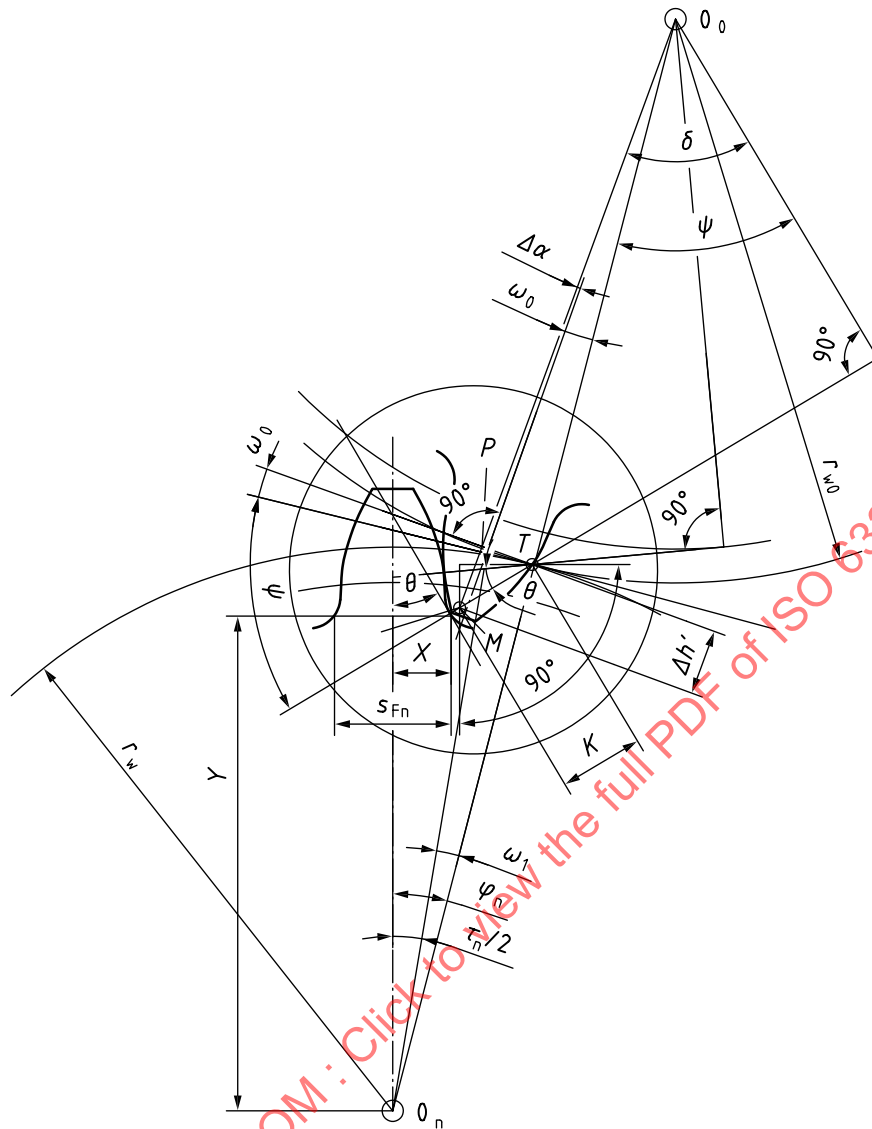
c) Bending moment arm, h_{Fe}

$$\frac{h_{Fe}}{m_n} = \frac{1}{2} \left[\left(\cos \gamma_e - \sin \gamma_e \cdot \tan \alpha_{Fen} \right) \cdot \frac{d_{en}}{m_n} - z_n \cdot \cos\left(\frac{\pi}{3} - \theta\right) - \left(\frac{G}{\cos \theta} - \frac{\rho_{fp}}{m_n} \right) \right] \quad (32)$$

6.2.4 Tooth root normal chord, s_{Fn} , radius of root fillet, ρ_F , bending moment arm, h_{Fe} for external gears generated with a shaper cutter²⁾



a) Shaper cutter



b) Gear blank

Figure 5 — The shaper cutter and the gear blank in a specific meshing position

In the following, formulae for the calculation of the tooth root geometry at a transverse cross section for the equivalent spur gear are given^[9].

Geometrical quantities which shall be known (Figure 5) are:

- z_0 number of teeth of the tool;
- θ tangential angle (30° for external gear, 60° for internal gear);
- x_0 profile shift coefficient of the tool;
- ρ_{a0} tool tip corner rounding (normal plane);
- s_{pr} residual fillet undercut (normal plane).

Initial geometrical quantities to be calculated are:

- equivalent numbers of teeth, z_{0v}

$$z_{0v} = \frac{z_0}{\cos \beta \cdot \cos^2 \beta_b} \quad (33)$$

- operating pressure angle, α_{w0} , of the manufacturing pairing (in general, two iteration steps are necessary)

$$\xi = 2 \cdot \frac{x_0 + x}{z_{0v} + z_n} \tan \alpha_n + \operatorname{inv} \alpha_n \quad (34)$$

Initial value:

$$\alpha_{w0} = \sqrt[3]{3\xi} \quad (35)$$

Iteration:

$$\alpha_{w0} := \alpha_{w0} + \frac{\xi - \operatorname{inv} \alpha_{w0}}{\tan^2 \alpha_{w0}} \quad (36)$$

- manufacturing centre distance, a_0

$$a_0 = m_n \frac{z_{0v} + z_n}{2} \cdot \frac{\cos \alpha_n}{\cos \alpha_{w0}} \quad (37)$$

- manufacturing tooth ratio, u_0

$$u_0 = \frac{z_{0v}}{z_n} \quad (38)$$

- manufacturing pitch circle radii of the gear, r_w , and the tool, r_{w0}

$$r_w = \frac{a_0}{1 + u_0}; r_{w0} = r_w \cdot u_0 \quad (39)$$

- base-circle radius of the tool, r_{b0}

$$r_{b0} = 0,5 \cdot m_n \cdot z_{0v} \cdot \cos \alpha_n \quad (40)$$

Geometrical quantities on the shaper cutter ([Figure 6](#)) are:

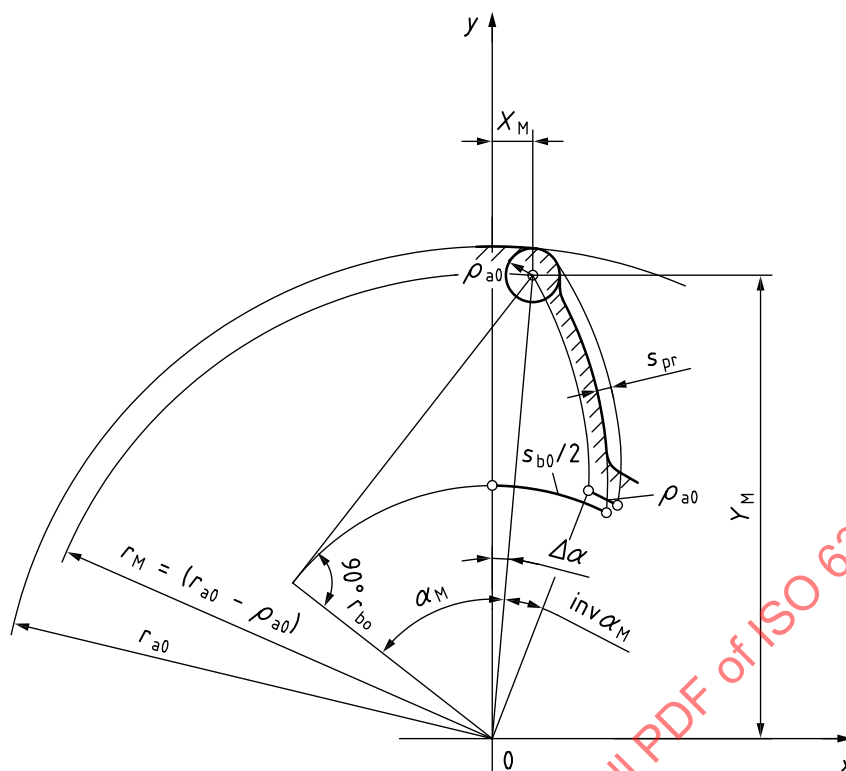


Figure 6 — Quantities at the shaper cutter

- radius, r_M , for the centre of the tool tip fillet radius

$$r_M = r_{a0} - \rho_{a0} \quad (41)$$

$$r_M = m_n \left(\frac{z_{0v}}{2} + \frac{h_{aP0}}{m_n} + x_0 - \frac{\rho_{a0}}{m_n} \right) \quad (42)$$

- transverse pressure angle, α_M , for the radius, r_M

$$\alpha_{\text{M}} = \arccos\left(\frac{r_{\text{b0}}}{r_{\text{M}}}\right) \quad (43)$$

- half angle of thickness, $\Delta\alpha$, at point M

$$\Delta\alpha = \frac{0,5 \cdot \pi + 2x_0 \cdot \tan \alpha_n}{z_{0v}} - \frac{\rho_{a0} - s_{pr}}{r_{b0}} + \text{inv } \alpha_n - \text{inv } \alpha_M \quad (44)$$

- coordinates of the point M

$$X_M = r_M \cdot \sin(\Delta\alpha); Y_M = r_M \cdot \cos(\Delta\alpha) \quad (45)$$

Determination of the auxiliary angle, ψ , through iteration using [Formulae \(48\)](#) to [\(51\)](#) (in general, two iterations steps are sufficient):

$$\psi_0 = \frac{\pi}{z_n} + \theta \quad (46)$$

Initial value:

$$\psi = \psi_0 \quad (47)$$

$$\lambda = \frac{r_{w0}}{r_M} \cdot \cos \psi \quad (48)$$

$$y = \psi - \arccos(\lambda) + \Delta\alpha + \frac{\psi - \psi_0}{u_0} \quad (49)$$

$$y' = 1 + \frac{1}{u_0} - \frac{r_{w0}}{r_M} \cdot \frac{\sin(\psi)}{\sqrt{1 - \lambda^2}} \quad (50)$$

$$\psi := \psi - \frac{y}{y'} \quad (51)$$

end of the successive iteration at $\left| \frac{y}{y'} \right| < 10^{-6}$.

Distance K of the point M to the point of contact of the pitch circles:

$$\omega_0 = \delta - \psi - \Delta\alpha \quad (52)$$

$$\delta = \arccos\left(\frac{r_{w0}}{r_M} \cdot \cos \psi\right) \quad (53)$$

$$\Delta h' = Y_M - r_{w0} \cdot \cos \omega_0 \quad (54)$$

$$\Delta h = \frac{\Delta h' \cdot \sin \psi}{\sin(\psi + \omega_0)} \quad (55)$$

$$K = \frac{\Delta h}{\sin \psi} \quad (56)$$

Characteristic quantities for the root geometry:

— coordinates of the tangent point

$$X = r_w \cdot \sin(\psi - \theta) - (K + \rho_{a0}) \cdot \cos \theta \quad (57)$$

$$Y = r_w \cdot \cos(\psi - \theta) - (K + \rho_{a0}) \cdot \sin \theta \quad (58)$$

— tooth root thickness s_{Fn}

$$s_{Fn} = 2 \cdot X \quad (59)$$

— tooth root fillet radius ρ_F

$$\rho_F = \frac{K^2}{\frac{r_{w0} \cdot r_w}{r_{w0} + r_w} \cdot \sin \psi + K} + \rho_{a0} \quad (60)$$

Bending moment arm, h_{Fe} :

$$h_{Fe} = (\cos \gamma_e - \sin \gamma_e \cdot \tan \alpha_{Fen}) \cdot \frac{d_{en}}{2} - Y \quad (61)$$

6.2.5 Tooth root normal chord, s_{Fn} , radius of root fillet, ρ_F , bending moment arm, h_{Fe} for internal gears generated with a shaper cutter²⁾

The calculation of the tooth root geometry follows [Formulae \(33\)](#) to [\(61\)](#) for virtual spur gears at a transverse cross section^[9]. Use negative sign convention for all diameters of the gear, the manufacturing centre distance, a_0 and the tangential angle, θ (60° for internal gear).

7 Stress correction factor, Y_S

7.1 Basic uses

The stress correction factor, Y_S , is used to convert the nominal tooth root stress to the local tooth root stress and, by means of this factor, the following are taken into account:

- the stress amplifying the effect of the section change at the fillet radius at the tooth root³⁾;
- that evaluation of the true stress system at the tooth root critical section is more complex than the simple system evaluation presented, with evidence indicating that the intensity of the local stress at the tooth root consists of two components, one of which is directly influenced by the value of the bending moment and the other increasing with closer proximity to the critical section of the determinant position of the load application.

Y_S is the factor for the load application at the outer point of the single pair tooth contact (Method B). See [5.3](#) for the principles, assumptions and application of Method B.

The formulae in this clause are based on the data derived from the geometry of external spur gears with 20° pressure angle, by means of measurement and calculations using finite element and integral formula methods. The formulae can also be used to obtain approximate values for internal gears and for gears having other pressure angles.

The present instructions refer to spur and helical gears. See [Clause 6](#) for explanatory comments and information on the calculation of the virtual numbers of teeth relevant to helical gears.

7.2 Stress correction factor, Y_S : Method B

The calculation of the stress correction factor, Y_S , is made in accordance with [Formula \(62\)](#), which is valid in the range: $1 \leq q_s < 8$; symbols are as illustrated in [Figures 3](#) and [4](#).

$$Y_S = (1,2 + 0,13 \cdot L) \cdot q_s^{\left[\frac{1}{1,21 + \frac{2,3}{L}} \right]} \quad (62)$$

where

$$L = \frac{s_{Fn}}{h_{Fe}} \quad (63)$$

with

s_{Fn} from [Formula \(30\)](#) for external gears generated with a hob, [Formula \(59\)](#) for external and internal gears generated with a shaper cutter;

h_{Fe} from [Formula \(32\)](#) for external gears generated with a hob, [Formula \(61\)](#) for external and internal gears generated with a shaper cutter;

3) See [7.3](#) for the procedure to be followed when grinding notches are present in tooth fillets.

$$q_s = \frac{s_{Fn}}{2 \rho_F} \quad (64)$$

with ρ_F from [Formula \(31\)](#) for external gears generated with a hob, [Formula \(60\)](#) for external and internal gears generated with a shaper cutter.

Determination of Y_S by graphical methods is not appropriate.

7.3 Stress correction factor for gears with notches in fillets

A notch such as a grinding notch in the fillet of a gear near the critical section usually engenders a degree of stress concentration exceeding that of the fillet; thus, the stress correction factor is correspondingly greater. A fair estimate of Y_{Sg} , obtainable from [Formula \(65\)](#), can be substituted for Y_S , see [Figure 7](#), if the notch is near the critical section. See also Reference [13].

$$Y_{Sg} = \frac{1,3 \cdot Y_S}{1,3 - 0,6 \cdot \sqrt{\frac{t_g}{\rho_g}}} \quad (65)$$

valid for $\sqrt{\frac{t_g}{\rho_g}} < 2,0$

The effect of the grinding notch is less than that implied in [Formula \(65\)](#) when the notch is above the contact point of the 30° tangent (external gears) or 60° tangent (internal gears).

Y_{Sg} also takes into consideration the reduction in the tooth root thickness.

Deep notches in the fillets of surface hardened steel gears severely reduce the bending strength of their teeth.

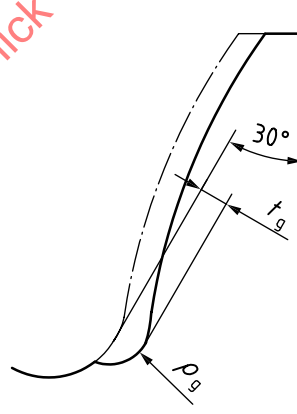


Figure 7 — Notch dimensions if the notch is near the critical section

7.4 Stress correction factor, Y_{ST} , relevant to the dimensions of the standard reference test gears

The tooth root stress limit values for materials, according to ISO 6336-5, were derived from results of tests of standard reference test gears for which either $Y_{ST} = 2,0$ or for which test results were recalculated to this value. See also Reference [13].

8 Helix angle factor, Y_β

8.1 General

The tooth root stress of a virtual spur gear, calculated as a preliminary value, is converted by means of the helix factor, Y_β , to that of the corresponding helical gear. By this means, the oblique orientation of the lines of the mesh contact is taken into account (less tooth root stress).

8.2 Graphical value

Y_β may be read from [Figure 8](#) as a function of the helix angle, β and the overlap ratio, ε_β .

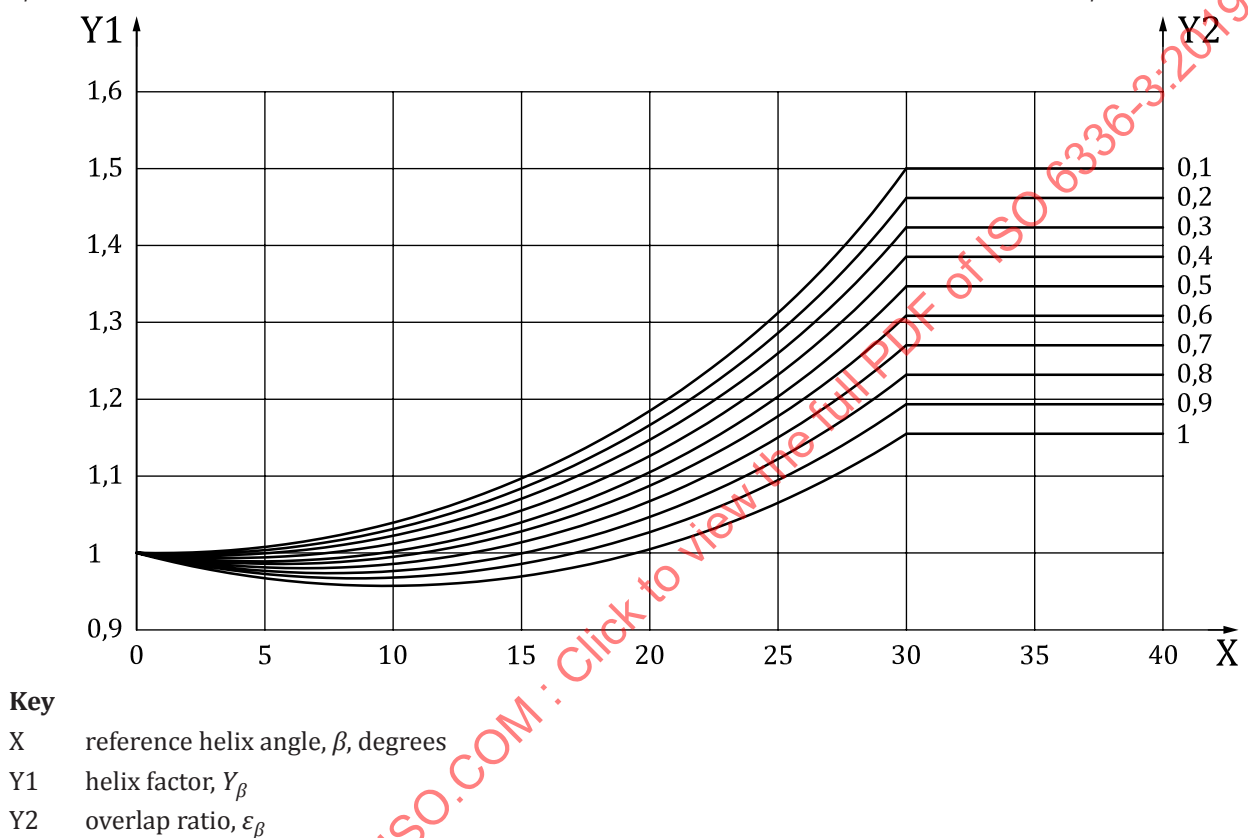


Figure 8 — Helix factor, Y_β

Helix factors Y_β for $\beta > 25^\circ$ shall be confirmed by experience.

8.3 Determination by calculation

The factor Y_β can be calculated using [Formula \(66\)](#) which is consistent with the curves illustrated in [Figure 8](#).

$$Y_\beta = \left(1 - \varepsilon_\beta \cdot \frac{\beta}{120^\circ} \right) \cdot \frac{1}{\cos^3 \beta} \quad (66)$$

where β is the reference helix angle, in degrees and

$$\varepsilon_\beta > 0 \quad (67)$$

The value 1,0 is substituted for ε_β when $\varepsilon_\beta > 1,0$, and 30° is substituted for β when $\beta > 30^\circ$.

9 Rim thickness factor, Y_B

9.1 General

Where the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim, rather than at the root fillet. The rim thickness factor, Y_B , is a simplified factor used to de-rate thin rimmed gears when detailed calculations of stresses in both tension and compression or experience are not available. For critically loaded applications this method should be replaced by a more comprehensive analysis.

9.2 Graphical values

Y_B can be taken from Figure 9 as a function of the backup ratio s_R/h_t for external gears and as a function of the rim thickness s_R/m_n for internal gears.

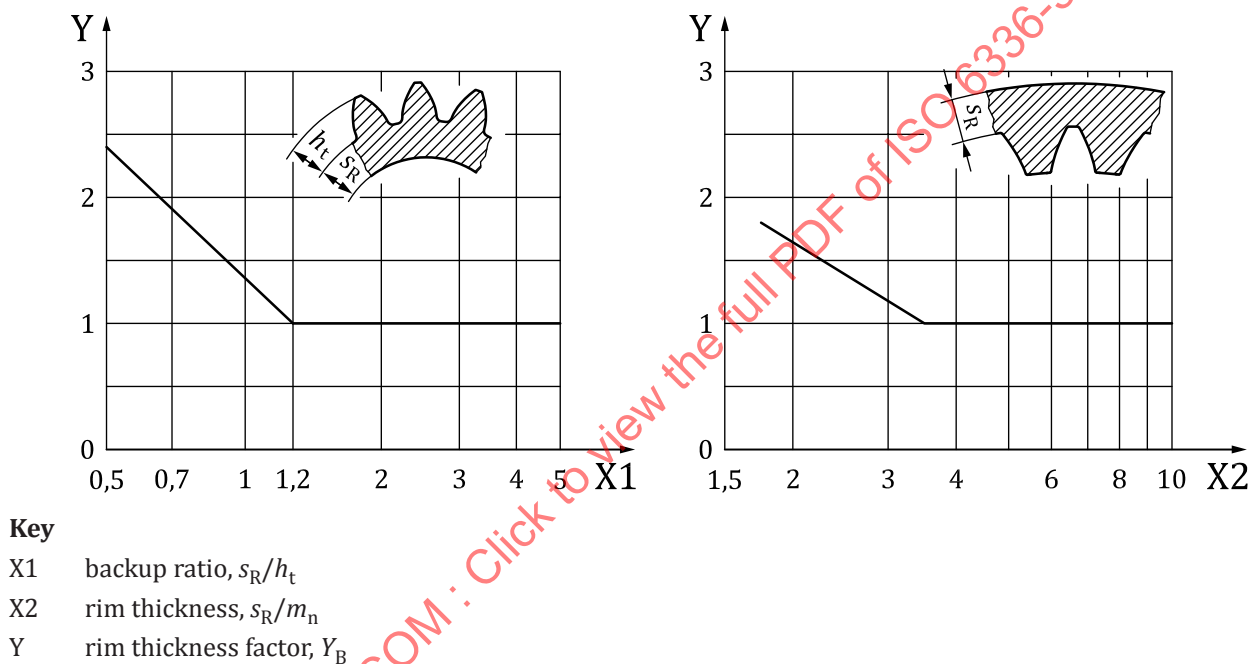


Figure 9 — Rim thickness factor, Y_B

9.3 Determination by calculation

9.3.1 External gears

Y_B can be calculated using Formulae (68) to (69). These are consistent with the curve in Figure 9.

- a) If $s_R/h_t \geq 1,2$, then

$$Y_B = 1,0 \quad (68)$$

- b) If $s_R/h_t > 0,5$ and $s_R/h_t < 1,2$, then

$$Y_B = 1,6 \cdot \ln \left(2,242 \cdot \frac{h_t}{s_R} \right) \quad (69)$$

- c) The case $s_R/h_t \leq 0,5$ shall be avoided.

9.3.2 Internal gears

Y_B can be calculated using [Formulae \(70\)](#) to [\(71\)](#). These are consistent with the curve in [Figure 9](#).

a) If $s_R/m_n \geq 3,5$, then

$$Y_B = 1,0 \quad (70)$$

b) If $s_R/m_n > 1,75$ and $s_R/m_n < 3,5$, then

$$Y_B = 1,15 \cdot \ln \left(8,324 \cdot \frac{m_n}{s_R} \right) \quad (71)$$

c) The case $s_R/m_n \leq 1,75$ shall be avoided.

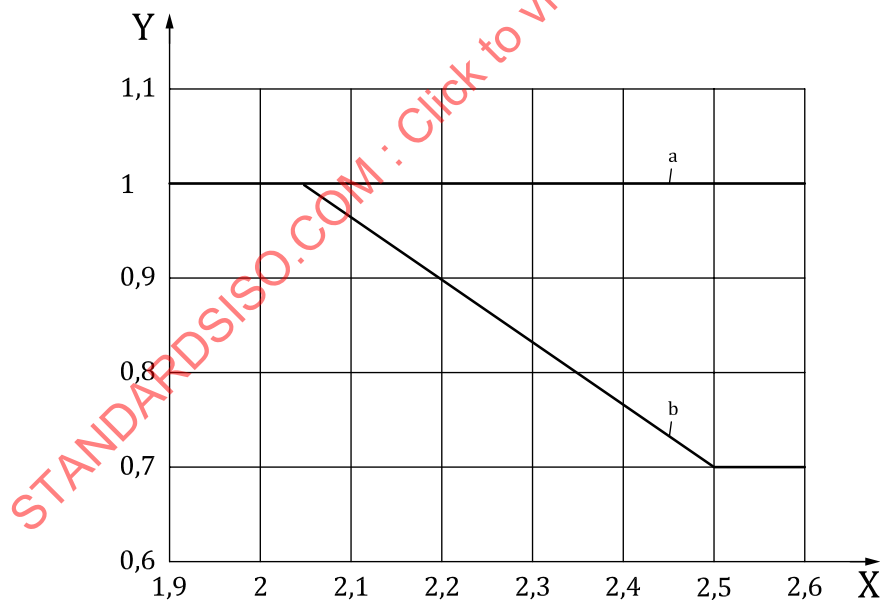
10 Deep tooth factor, Y_{DT}

10.1 General

For gears of high precision (ISO Tolerance Class ≤ 4) with contact ratios in the range of $2 \leq \varepsilon_{\alpha n} < 2,5$ and with applied actual profile modification to obtain a trapezoidal load distribution along the path of contact, the nominal tooth root stress, σ_{F0} , is adjusted by the deep tooth factor, Y_{DT} . If ISO Tolerance Class of the pinion and the wheel is different, the worse one shall be used for both partners.

10.2 Graphical values

Y_{DT} may be read from [Figure 10](#) as a function of the contact ratio, $\varepsilon_{\alpha n}$.



Key

- X virtual contact ratio, $\varepsilon_{\alpha n}$
- Y deep tooth factor, Y_{DT}
- a ISO Tolerance Class > 4 .
- b ISO Tolerance Class ≤ 4 .

Figure 10 — Deep tooth factor, Y_{DT}

10.3 Determination by calculation

Y_{DT} can be calculated using [Formulae \(72\)](#) to [\(74\)](#). These are consistent with the curves in [Figure 10](#).

- a) If $\varepsilon_{\alpha n} \leq 2,05$ or if $\varepsilon_{\alpha n} > 2,05$ and the ISO Tolerance Class > 4 , then

$$Y_{DT} = 1,0 \quad (72)$$

- b) If $2,05 < \varepsilon_{\alpha n} \leq 2,5$ and the ISO Tolerance Class ≤ 4 , then

$$Y_{DT} = -0,666 \cdot \varepsilon_{\alpha n} + 2,366 \quad (73)$$

- c) If $\varepsilon_{\alpha n} > 2,5$ and the ISO Tolerance Class ≤ 4 , then

$$Y_{DT} = 0,7 \quad (74)$$

11 Reference stress for bending

11.1 General

See [5.4](#) for general information on the determination of limit values for the tooth root stress.

11.2 Reference stress for Method A

Method A is consistent with the determination of the tooth root stress reference stress according to [5.4.2.2](#).

11.3 Reference stress, with values $\sigma_{F \lim}$ and σ_{FE} for Method B

See [5.4.2.3](#) and [5.4.3](#) for information. See [Formula \(5\)](#) for definitions of $\sigma_{F \lim}$ and σ_{FE} .

NOTE ISO 6336-5 provides information, derived from the results of testing standard reference test gears, on values of $\sigma_{F \lim}$ and σ_{FE} for the more popular gear materials, heat treatment processes and the influence of the material quality on those values. ISO 6336-5 also includes requirements for quality grades ML, MQ and ME concerning material and heat treatment. Material quality grade MQ is usually chosen for gears unless otherwise agreed upon.

12 Life factor, Y_{NT}

12.1 General

The life factor, Y_{NT} , accounts for the higher tooth root stress, which may be tolerable for a limited life (number of load cycles), as compared with the allowable stress at 3×10^6 cycles. Y_{NT} applies for standard reference use.

The principal influence factors are:

- a) material and heat treatment (see ISO 6336-5),
- b) number of load cycles (service life), N_L ,
- b) failure criteria,
- c) smoothness of operation required,
- d) cleanness of gear material,

- e) material ductility and fracture toughness, and
- f) residual stress.

For the purposes of this document, the number of load cycles, N_L , is defined as the number of mesh contacts, under load, of the gear tooth being analysed. The allowable stress numbers are established for 3×10^6 tooth load cycles at 99 % reliability.

A Y_{NT} value of unity may be used, where justified by experience, beyond 3×10^6 cycles. However, the use of optimum material quality and manufacturing, with selection of an appropriate safety factor, is recommended.

12.2 Life factor, Y_{NT} : Method A

The S-N curve or damage curve derived from facsimiles of the actual gear is determinant for the establishment of the limited life. Y_{NT} can be derived from the S-N curve, which is directly valid for the conditions mentioned, the influences represented by the factors $Y_{\delta \text{ rel T}}$, $Y_{R \text{ rel T}}$, and Y_X are included in the curve.

12.3 Life factor, Y_{NT} : Method B

12.3.1 General

For this method, life factor, Y_{NT} , of the standard reference test gear is used as an aid in the evaluation of the permissible stress for a limited life or reliability (see 5.4).

12.3.2 Graphical values

Y_{NT} may be read from Figure 11 for the static stress and reference stress as a function of material and heat treatment. Values from a large number of tests are presented as typical damage or crack initiation curves for surface-hardened and nitride-hardened steels, or curves of yield stress for structural and through-hardened steels.

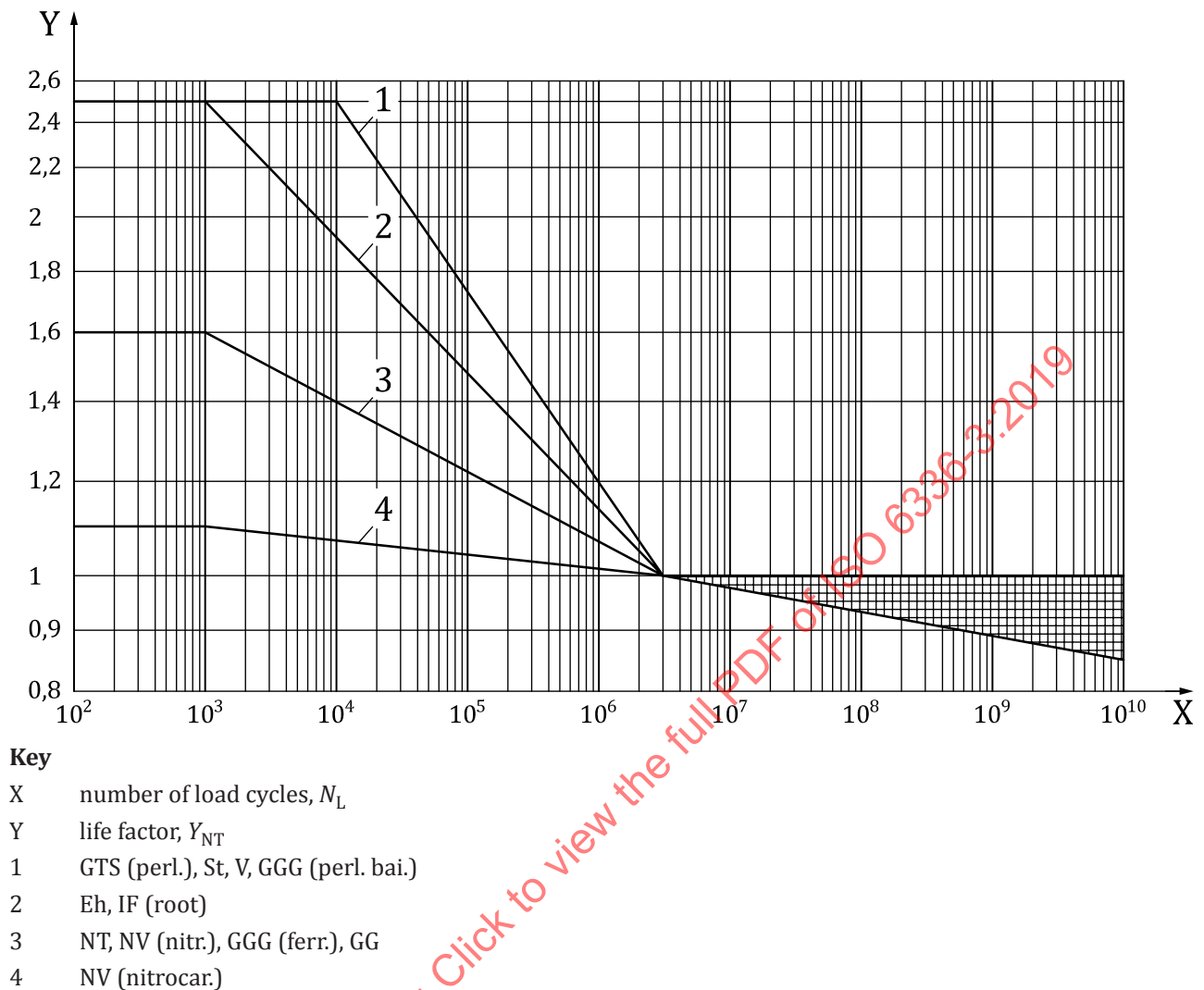


Figure 11 — Life factor, Y_{NT} , for reference test gears

12.3.3 Determination by calculation

The data of Y_{NT} for static stress and reference stress can be taken from [Table 3](#).

Life factor, Y_{NT} , for limited-life stress is determined by means of interpolation on a log-log scale between the values for reference and static stress limits as defined in [5.4.3](#). Evaluation of Y_{NT} is according to [5.4.4](#).

Table 3 — Life factor, Y_{NT}

Material	Number of load cycles, N_L	Life factor, Y_{NT}
St, V, GGG (perl. bai.), GTS (perl.),	$N_L \leq 10^4$, static	2,5
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 ^a
Eh, IF (root)	$N_L \leq 10^3$, static	2,5
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 ^a

^a The lower value of Y_{NT} may be used for critical service, where tooth root breakage shall be minimal. Values between 0,85 and 1,0 may be used for general purpose gearing. With optimum lubrication, material, manufacturing and experience 1,0 may be used.

Table 3 (continued)

Material	Number of load cycles, N_L	Life factor, Y_{NT}
GG, GGG (ferr.), NT, NV (nitr.)	$N_L \leq 10^3$, static	1,6
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 ^a
NV (nitrocar.)	$N_L \leq 10^3$, static	1,1
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 ^a

^a The lower value of Y_{NT} may be used for critical service, where tooth root breakage shall be minimal. Values between 0,85 and 1,0 may be used for general purpose gearing. With optimum lubrication, material, manufacturing and experience 1,0 may be used.

13 Notch sensitivity factor, $Y_{\delta T}$, and relative notch sensitivity factor, $Y_{\delta \text{ rel } T}$

13.1 Basic uses

The extent to which the calculated tooth root stress deemed to have caused fatigue or overload breakage exceeds the relevant material stress limit is indicated by the dynamic or the static sensitivity factor, Y_{δ} . It characterizes the notch sensitivity of the material, and its values depend on the material and the stress gradient. Its values for dynamic stresses are different from its value for static stresses. This applies to $Y_{\delta T}$ in relation to breakage of a standard reference test gear tooth. It applies also to the relative sensitivity factors which relate the sensitivity of a gear of interest to that of a standard reference test gear ($Y_{\delta \text{ rel } T}$).

13.2 Determination of the notch sensitivity factors

13.2.1 General

Comments on these factors given in 5.4 apply in principle.

13.2.2 Method A

The tooth root stress limits are determined by testing facsimiles of the gear of interest (or closely similar test gears), in which case the relative notch sensitivity factor is equal to 1,0. However, a careful analysis — by means of which the relative notch sensitivity factor for the relevant material and relevant tooth form will be established — has yet to be undertaken.

13.2.3 Method B

When the reference and static stress limit values are derived with Method B using reference test gears with notch parameters $q_{sT} = 2,5$, the factor $Y_{\delta \text{ rel } T}$ relevant to the reference and static stress limits of any gear seldom deviates much from 1,0. This is because the value $q_{sT} = 2,5$ is in the mid-range of common gear designs. The reference value $Y_{\delta \text{ rel } T} = 1,0$ for the standard reference test gear coincides with the stress correction factor $Y_S = 2,0$ (see Figures 13 and 15).

13.3 Relative notch sensitivity factor, $Y_{\delta \text{ rel } T}$: Method B

13.3.1 Graphical values

13.3.1.1 $Y_{\delta \text{ rel } T}$ for reference stress

$Y_{\delta \text{ rel } T}$ can be read from Figure 12 as a function of q_s and the material. The curves in this graph for each of the materials were derived from Figure 14 by subtracting from the absolute value Y_{δ} , appropriate to each value of q_s , the value of $Y_{\delta T}$ for that material corresponding to the notch parameter $q_s = 2,5$ (the

notch parameter of the standard reference test gear). For any gear of interest, q_s can be calculated using [Formula \(64\)](#).

13.3.1.2 $Y_{\delta \text{ rel T}}$ for static stress

$Y_{\delta \text{ rel T}}$ may be taken from [Figure 13](#) as a function of the stress correction factor Y_S and the material. The curves in this graph for each of the materials were derived from [Figure 15](#) by subtracting from the absolute value Y_δ appropriate to each value of Y_S , the value of $Y_{\delta T}$ for that material corresponding to $Y_{ST} = 2,0$ (the stress correction factor of the standard reference test gear). For any gear of interest, Y_S can be calculated using [Formula \(62\)](#).

13.3.2 Determination by calculation

13.3.2.1 $Y_{\delta \text{ rel T}}$ for reference stress

$Y_{\delta \text{ rel T}}$ can be calculated using [Formula \(75\)](#). This is consistent with the curves in [Figure 12](#).

$$Y_{\delta \text{ rel T}} = \frac{Y_\delta}{Y_{\delta T}} = \frac{1 + \sqrt{\rho' \cdot \chi^*}}{1 + \sqrt{\rho' \cdot \chi_T^*}} \quad (75)$$

The slip-layer thickness, ρ' , can be taken from [Table 4](#) as a function of the material.

The relative stress gradient can be calculated using [Formula \(76\)](#)⁴⁾:

$$\chi^* = \chi_P^* \cdot (1 + 2q_s) \quad (76)$$

with

$$\chi_P^* = \frac{1}{5} \quad (77)$$

The value of χ_T^* for the standard reference test gear is obtained similarly by substituting $q_{sT} = 2,5$ for q_s in [Formula \(76\)](#).

Table 4 — Values for slip layer thickness ρ'

Item	Material	ρ' [mm] ^a
1	GG $\sigma_B = 150 \text{ N/mm}^2$	0,312 4
2	GG, GGG (ferr.); $\sigma_B = 300 \text{ N/mm}^2$	0,309 5
3	NT, NV; for all hardness	0,100 5
4	St; $\sigma_S = 300 \text{ N/mm}^2$	0,083 3
5	St; $\sigma_S = 400 \text{ N/mm}^2$	0,044 5
6	V, GTS, GGG (perl. bai.); $\sigma_S = 500 \text{ N/mm}^2$	0,028 1
7	V, GTS, GGG (perl. bai.); $\sigma_S = 600 \text{ N/mm}^2$	0,019 4
8	V, GTS, GGG (perl. bai.); $\sigma_{0,2} = 800 \text{ N/mm}^2$	0,006 4
9	V, GTS, GGG (perl. bai.); $\sigma_{0,2} = 1000 \text{ N/mm}^2$	0,001 4
10	Eh, IF (root); for all hardness	0,003 0

^a For the same category of material the given values of ρ' can be interpolated for other values of σ_B , σ_S or $\sigma_{0,2}$.

4) Applies for module $m = 5 \text{ mm}$. The influence of size is covered by the factor Y_X (see [Clause 15](#)).

13.3.2.2 $Y_{\delta \text{ rel T}}$ for static stress

$Y_{\delta \text{ rel T}}$ can be calculated using [Formulae \(78\)](#) to [\(83\)](#). These are consistent with the curves in [Figure 13](#).

a) For St with well-defined yield point:

$$Y_{\delta \text{ rel T}} = \frac{1 + 0,93 \cdot (Y_S - 1) \cdot \sqrt[4]{\frac{200}{\sigma_S}}}{1 + 0,93 \cdot \sqrt[4]{\frac{200}{\sigma_S}}} \quad (78)$$

b) For St with steadily increasing elongation curve and 0,2 % proof stress, V and GGG (perl., bain):

$$Y_{\delta \text{ rel T}} = \frac{1 + 0,82 \cdot (Y_S - 1) \cdot \sqrt[4]{\frac{300}{\sigma_{0,2}}}}{1 + 0,82 \cdot \sqrt[4]{\frac{300}{\sigma_{0,2}}}} \quad (79)$$

These values are only valid if the local stresses do not reach the yield point.

c) For Eh and IF(root) with stress up to crack initiation:

$$Y_{\delta \text{ rel T}} = 0,44 \cdot Y_S + 0,12 \quad (80)$$

d) For NT and NV with stress up to crack initiation:

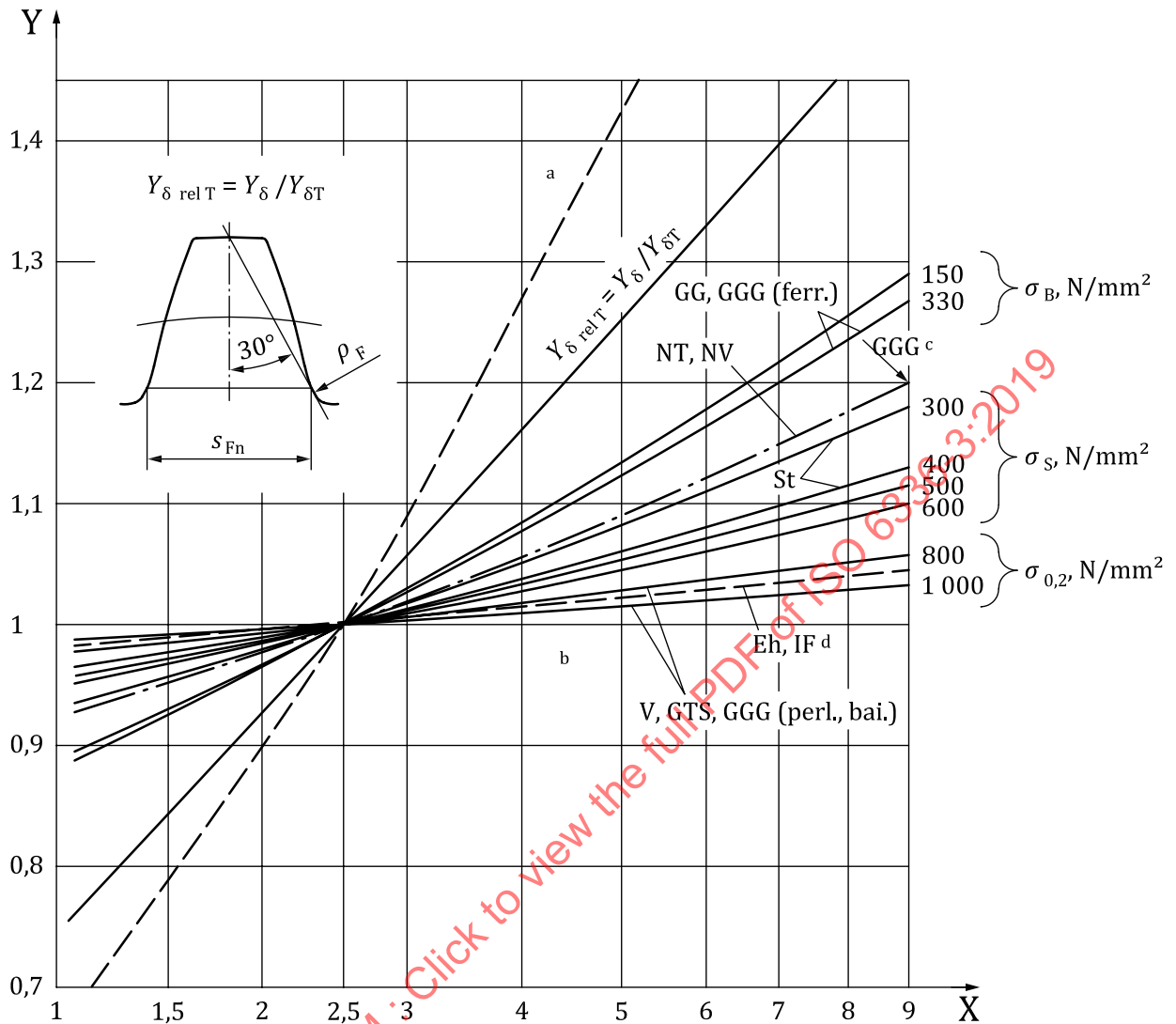
$$Y_{\delta \text{ rel T}} = 0,20 \cdot Y_S + 0,60 \quad (81)$$

e) For GTS with stress up to crack initiation:

$$Y_{\delta \text{ rel T}} = 0,075 \cdot Y_S + 0,85 \quad (82)$$

f) For GG and GGG (ferr.) with stress up to fracture limit:

$$Y_{\delta \text{ rel T}} = 1,0 \quad (83)$$

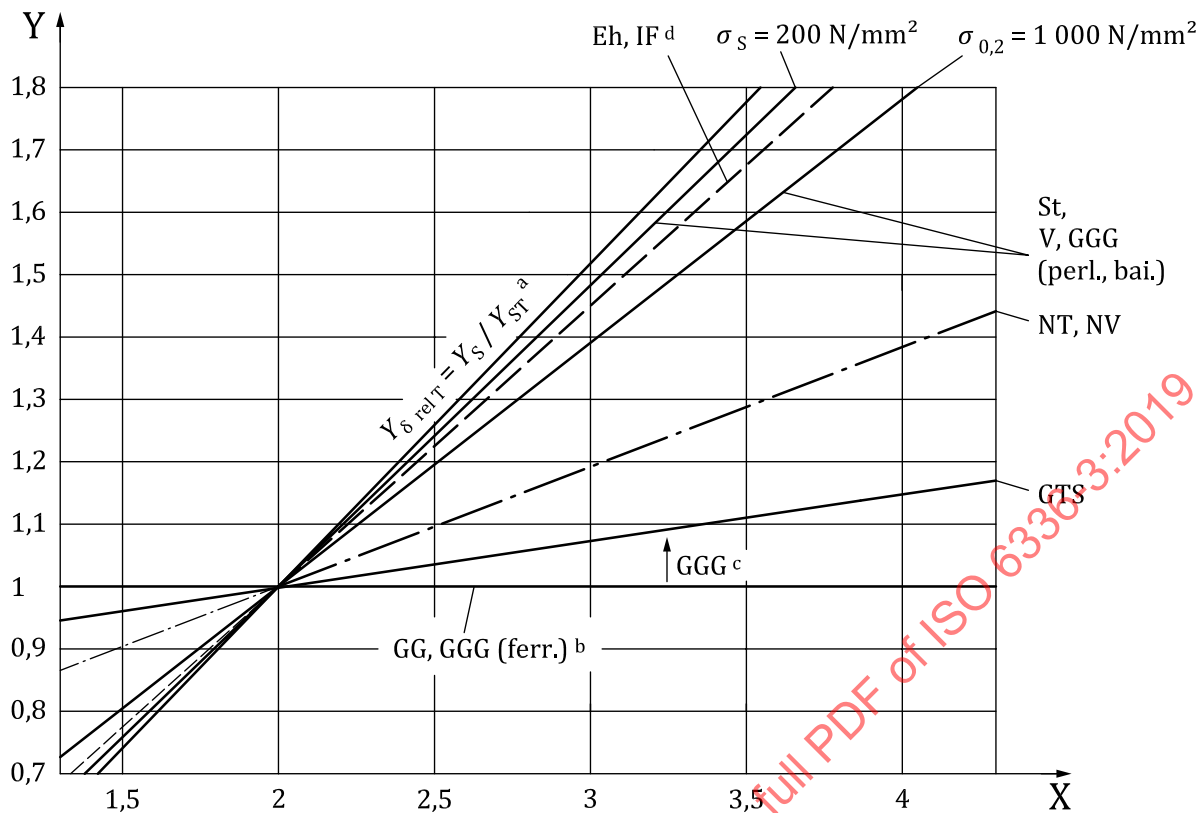
**Key**

- X notch parameter, $q_s = S_{Fn} / 2\rho_F$
 Y relative notch sensitivity factor, $Y_{\delta \text{ rel T}}$, for reference stress
 a Fully insensitive to notches.
 b Fully sensitive to notches.
 c With increasingly pearlitic structure.
 d (root).

NOTE 1 Values of σ are expressed in newton per square millimetre (N/mm²).

NOTE 2 This figure is based on the bending flat bar complying with VDI 2226[8].

Figure 12 — Relative notch sensitivity factor, $Y_{\delta \text{ rel T}}$, for reference stress



Key

- X stress correction factor, Y_s
- Y relative notch sensitivity factor, $Y_{\delta_{rel T}}$, for static stress
- a Fully insensitive to notches.
- b Fully sensitive to notches.
- c With increasingly pearlitic structure.
- d (root).

NOTE This figure is based on the bending flat bar complying with VDI 2226[8].

Figure 13 — Relative notch sensitivity factor, $Y_{\delta_{rel T}}$, for static stress

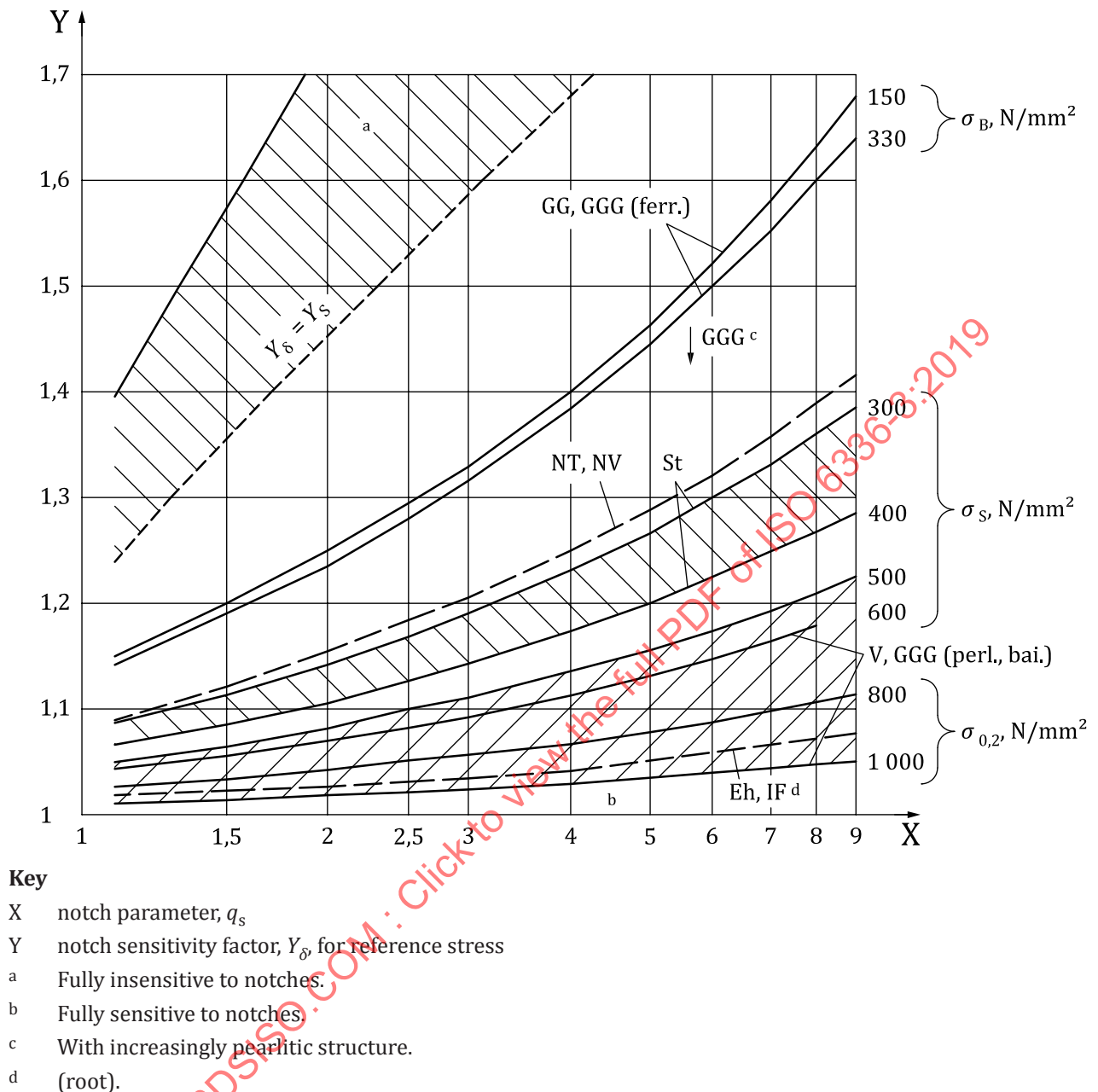
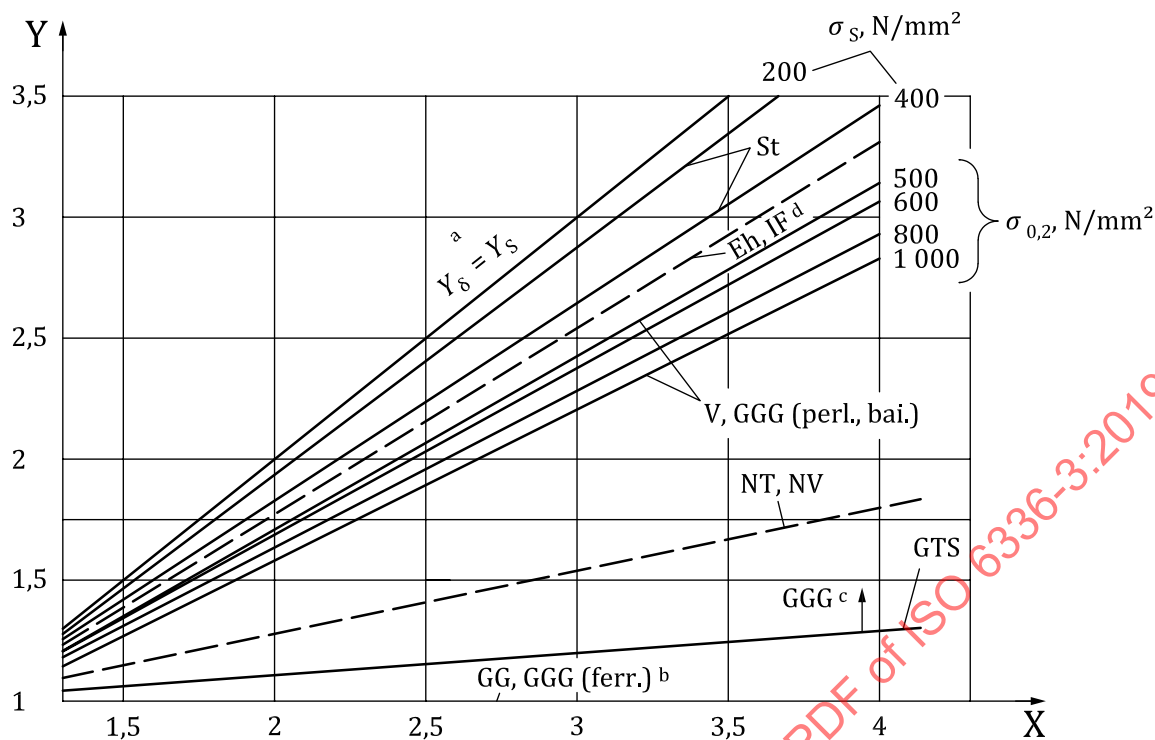


Figure 14 — Notch sensitivity factor, Y_{δ} , for reference stress

**Key**

- X stress correction factor, Y_s
 Y notch sensitivity factor, Y_{δ} , for static stress
 a Fully insensitive to notches.
 b Fully sensitive to notches.
 c With increasingly pearlitic structure.
 d (root).

NOTE This figure is based on the bending flat bar complying with VDI 2226[8].

Figure 15 — Notch sensitivity factor, Y_{δ} , for static stress

14 Surface factors, Y_R , Y_{RT} , and relative surface factor, $Y_{R \text{ rel } T}$

14.1 Influence of surface condition

The surface factor, Y_R , accounts for the influence on the tooth root stress of the surface condition in the tooth roots. This is dependent on the material and the surface roughness in the tooth root fillets (see NOTE below). Y_R for the static stress is different from Y_R for dynamic stresses. This is also true for Y_{RT} , the surface factor of the standard reference test gear. These factors are compared to that of a plain, polished test piece. Relative surface factors represent the relationship of the surface factor of a gear of interest to that of the standard reference test gear ($Y_{R \text{ rel } T}$).

NOTE The influence of the surface condition on the tooth root bending strength does not depend solely on the surface roughness in the tooth root fillets, but also on the size and shape (the problem of “notches within a notch”). This subject has not to date been sufficiently well studied for it to be taken into account in this document. The method applied here is only valid when scratches or similar defects deeper than $2 \times R_z$ are not present ($2 \times R_z$ is a preliminary estimated value).

Besides surface texture, other influences on tooth bending strength are known, and include residual compressive stresses (shot peening), grain boundary oxidation and chemical effects. When fillets are shot-peened and/or are perfectly shaped, a value slightly greater than that obtained from the graph

should be substituted for $Y_{R\ rel\ T}$. When grain boundary oxidation or chemical effects are present, a smaller value than that indicated by the graph should be substituted for $Y_{R\ rel\ T}$.

14.2 Determination of surface factors and relative surface factors

14.2.1 General

The comments in [5.4](#) apply in principle for the determination of these factors.

14.2.2 Method A

In Method A the tooth root stress limit is determined by testing the gear of interest or testing closely similar test gears. By this approach, the relative surface factor is equal, or approximately equal, to 1,0. In order to determine the material surface factor relative to that of the gear tested, a careful analysis shall be undertaken.

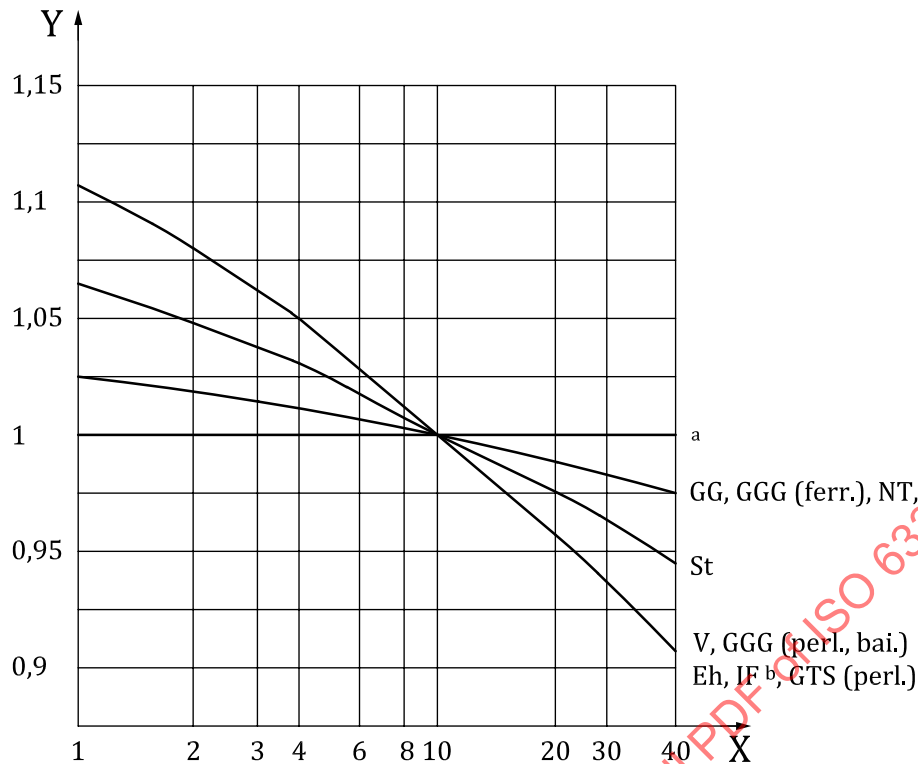
14.2.3 Method B

The material strength values provided are derived in accordance with Method B, from results of tests of standard reference test gears of which $Rz_T = 10\ \mu\text{m}$. In general, the value $Y_{R\ rel\ T}$ relevant to the reference stress of any gear of interest differs little from 1,0, since $Rz_T = 10\ \mu\text{m}$ is a common mean value. $Y_{R\ rel\ T}$ for static stress may also be made equal to 1,0.

14.3 Relative surface factor, $Y_{R\ rel\ T}$: Method B

14.3.1 Graphical values

$Y_{R\ rel\ T}$ can be taken from [Figure 16](#) as a function of the material and Rz , the peak-to-valley roughness in the tooth root fillets of the gear of interest. This graph is derived from [Figure A.1](#).

**Key**X roughness, R_z , μm Y relative surface factor, $Y_{R\text{rel } T}$

a For static stress and all materials.

b (root).

NOTE This figure is derived from [Figure A.1](#).**Figure 16 — Relative surface factor, $Y_{R\text{rel } T}$** **14.3.2 Determination by calculation****14.3.2.1 $Y_{R\text{rel } T}$ for reference stress**

$Y_{R\text{rel } T}$ can be calculated using [Formulae \(84\) to \(90\)](#). These are consistent with the curves in [Figure 16](#).

a) Reference stress in the range $R_z < 1 \mu\text{m}$

— for V, GGG (perl., bai.), Eh, IF (root), and GTS (perl.):

$$Y_{R\text{rel } T} = 1,12 \quad (84)$$

— for St:

$$Y_{R\text{rel } T} = 1,07 \quad (85)$$

— for GG, GGG (ferr.) and NT, NV:

$$Y_{R\text{rel } T} = 1,025 \quad (86)$$

b) Reference stress in the range $1 \mu\text{m} \leq R_z \leq 40 \mu\text{m}$

— for V, GGG (perl., bai.), Eh, IF (root), and GTS (perl.):

$$Y_{\text{RrelT}} = 1,674 - 0,529 \cdot (R_z + 1)^{0,1} \quad (87)$$

— for St:

$$Y_{\text{RrelT}} = 5,306 - 4,203 \cdot (R_z + 1)^{0,01} \quad (88)$$

— for GG, GGG (ferr.) and NT, NV:

$$Y_{\text{RrelT}} = 4,299 - 3,259 \cdot (R_z + 1)^{0,005} \quad (89)$$

14.3.2.2 Y_{RrelT} for static stress

$$Y_{\text{RrelT}} = 1,0 \quad (90)$$

15 Size factor, Y_X

15.1 General

The size factor, Y_X , is used to take into consideration the influence of gear tooth size on the probable distribution of weak points in the structure of the material, on the stress gradients, which, in accordance with the strength of materials theory, decrease with increasing dimensions, on the quality of the material as determined by the extent and effectiveness of forging, on the presence of defects, etc.

The following have significant influence:

- material, its cleanliness, chemistry, and forging process;
- heat-treatment, depth and uniformity of hardening;
- module, in the case of surface-hardening: case depth in relation to tooth size (core support effect).

Size factor Y_X shall be determined separately for the pinion and the wheel.

15.2 Size factor, Y_X : Method A

The value of the size factor Y_X shall be based on reliable experience or testing under the relevant operating conditions of a range of different sizes of gears in each material of interest, appropriately heat-treated. The provisions given in ISO 6336-1:2019, 4.1.16 are relevant.

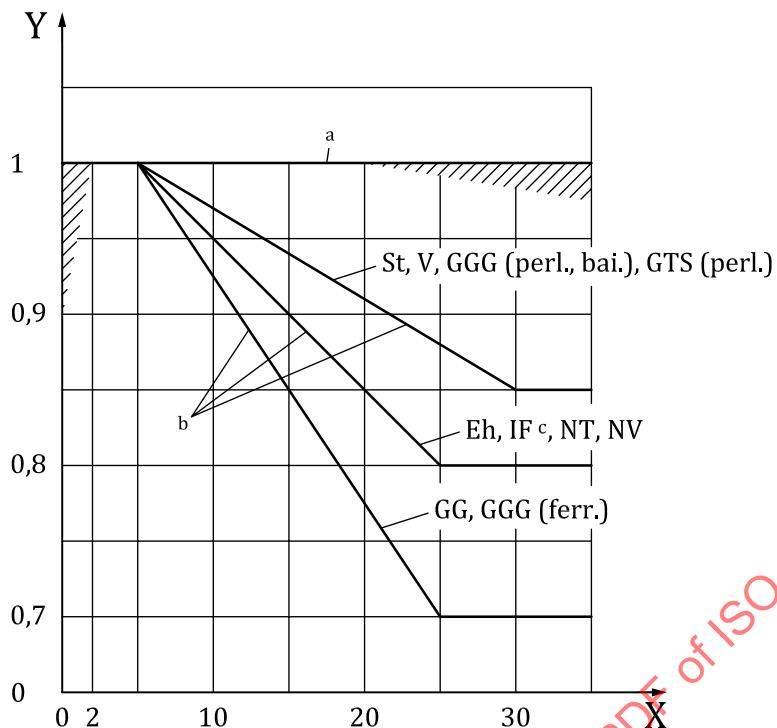
15.3 Size factor, Y_X : Method B

15.3.1 General

The values provided are based on the results of testing gears and bending strength test pieces of different sizes, due regard being paid to the current standards and practices of established heat-treatment practitioners.

15.3.2 Graphical values for reference stress and static stress

The value of Y_X may be taken from [Figure 17](#) as a function of the module, material and heat-treatment.



Key

- X normal module, m_n , mm
- Y size factor, Y_X
- a Static stress (all materials).
- b Reference stress.
- c (root).

NOTE The shaded area is in the range of scatter for the static stress.

Figure 17 — Size factor, Y_X , for tooth bending strength

15.3.3 Determination by calculation

15.3.3.1 Size factor, Y_X , for reference stress

Y_X may be calculated using the formulae in [Table 5](#), which are consistent with the curves given in [Figure 17](#).

Table 5 — Size factor (root), Y_X

Material		Normal module, m_n	Size factor, Y_X
St, V,	For 3×10^6 cycles	$m_n \leq 5$	$Y_X = 1,0$
GGG (perl., bai.),		$5 < m_n < 30$	$Y_X = 1,03 - 0,006 m_n$
GTS (perl.),		$30 \leq m_n$	$Y_X = 0,85$
Eh, IF (root),		$m_n \leq 5$	$Y_X = 1,0$
NT, NV		$5 < m_n < 25$	$Y_X = 1,05 - 0,01 m_n$
		$25 \leq m_n$	$Y_X = 0,8$
GG, GGG (ferr.)		$m_n \leq 5$	$Y_X = 1,0$
		$5 < m_n < 25$	$Y_X = 1,075 - 0,015 m_n$
		$25 \leq m_n$	$Y_X = 0,7$
All materials for static stress		—	$Y_X = 1,0$

15.3.3.2 Size factor, Y_X , for static stress

$$Y_X = 1,0$$

15.3.3.3 Size factor, Y_X , for a limited life

Y_X is obtained by means of linear interpolation between the values for the reference stress and the static stress as determined according to [15.3.3.1](#) and [15.3.3.2](#). This formulation is already included in the determination of the permissible stress for a limited life according to [5.4.4](#). Consequently, if the calculation of the permissible stress for a limited life is done according to [5.4.4](#), additional interpolation of Y_X does not apply.