

INTERNATIONAL STANDARD

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Representation of results of particle size analysis —

Part 1: Graphical representation

*Représentation de données obtenues par analyse granulométrique —
Partie 1: Représentation graphique*



Reference number
ISO 9276-1:1990(E)

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 9276-1 was prepared by Technical Committee ISO/TC 24, *Sieves, sieving and other sizing methods*.

ISO 9276 consists of the following parts, under the general title *Representation of results of particle size analysis*:

- Part 1: *Graphical representation*
- Part 2: *Adjustment of an experimental curve to a reference model*

Annex A of this part of ISO 9276 is for information only.

Introduction

The representation of particle size analysis data and the nomenclature used to characterize common entities such as particle size, cumulative and density values of a distribution, etc. vary a great deal between different countries. This International Standard was elaborated therefore to improve mutual understanding and to facilitate the exchange of data of particle size analyses.

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Representation of results of particle size analysis —

Part 1: Graphical representation

1 Scope

This part of ISO 9276 specifies rules for the graphical representation of particle size analysis data in histograms, density distributions and cumulative distributions. It also establishes a standardized nomenclature and specifies the procedures to be followed to obtain the distributions mentioned above from measured data.

This part of ISO 9276 applies to the graphical representation of distributions of solid particles, droplets or gas bubbles covering all possible size ranges.

2 Symbols

k	ratio of geometric progression
n	number of classes
$q_0(x)$	density distribution by number
$q_1(x)$	density distribution by length
$q_2(x)$	density distribution by surface or projected area
$q_3(x)$	density distribution by volume or mass
$q_r(x)$	density distribution (general description)
$\bar{q}_r$	average density distribution of a class (general description)
$\bar{q}_r^*$	average density distribution in a representation with logarithmic abscissa
$Q_0(x)$	cumulative distribution by number
$Q_1(x)$	cumulative distribution by length
$Q_2(x)$	cumulative distribution by surface or projected area

$Q_3(x)$	cumulative distribution by volume or mass
$Q_r(x)$	cumulative distribution (general description)
$Q_{rv}(x)$	cumulative distribution limited to v
S_v	volume-related surface area
x	particle size
$\bar{x}$	average particle size
ΔQ_r	relative quantity
Δx	size of a class
v	integer

3 Particle size, measures and types

In a graphical representation of particle size analysis data the independent variable, i.e. the physical property chosen to characterize the size of the particles, is plotted on the abscissa (see figure 1). The dependent variable, which characterizes the measure and type of quantity, is plotted on the ordinate.

3.1 Particle size x

There is no single definition of particle size x . Different methods of analysis are based on the measurement of different physical properties. In most cases, the particle size is defined as the diameter of a sphere having the same physical properties; this is known as the equivalent spherical diameter. Other definitions are possible such as those based on the opening of a sieve or a statistical diameter, e.g. the Feret diameter, measured by image analysis.

Independently of the particle dimension (e.g. length, area or volume) actually measured, the particle size is reported as a linear dimension x which is called the equivalent diameter. The physical property to which the equivalent diameter refers shall be indicated using a suitable subscript, for example

- x_s equivalent surface area diameter,
- x_v equivalent volume diameter.

Table 1 — Symbols for distributions

Type	Mathematical symbol for density distribution	Mathematical symbol for cumulative distribution
by number	$q_0(x)$	$Q_0(x)$
by length	$q_1(x)$	$Q_1(x)$
by area	$q_2(x)$	$Q_2(x)$
by volume or mass	$q_3(x)$	$Q_3(x)$
General symbol	$q_r(x)$	$Q_r(x)$

3.2 Measures and types

The measures and types are distinguished with respect to the dependent variables by mathematical symbols as shown below.

The different measures are

- cumulative measures, Q , and
- density measures, q .

Each measure can be one of several types as follows:

- number: $r = 0$
- length: $r = 1$
- area: $r = 2$
- volume or mass: $r = 3$

The type is indicated by the general subscript r , or by the appropriate value of r as given above.

A summary of the symbols used to designate density distributions and cumulative distributions is shown in table 1.

4 Graphical representation

Examples of the graphical representation of particle size analysis data are shown in figures 2 to 4.

4.1 Histogram $\bar{q}_r(x_i, x_{i+1})$

Figure 2 shows the normalized histogram of a density distribution $q_r(x)$. It comprises a successive series of rectangular columns, the area of each of which represents the relative quantity $\Delta Q_r(x_i, x_{i+1})$, where

$$\Delta Q_r(x_i, x_{i+1}) = \bar{q}_r(x_i, x_{i+1})\Delta x \quad \dots (1)$$

or

$$\bar{q}_r(x_i, x_{i+1}) = \Delta Q_r(x_i, x_{i+1})/\Delta x \quad \dots (2)$$

The sum of all the relative quantities ΔQ_r forms the area beneath the histogram $\bar{q}_r(x_i, x_{i+1})$, normalized to 100 % or 1 (condition of normalization). Therefore the following equation holds:

$$\sum_{i=0}^n \Delta Q_{ri}(x_i, x_{i+1}) = \sum_{i=0}^n \bar{q}_r(x_i, x_{i+1})\Delta x_i = 100\% \text{ or } 1 \quad \dots (3)$$

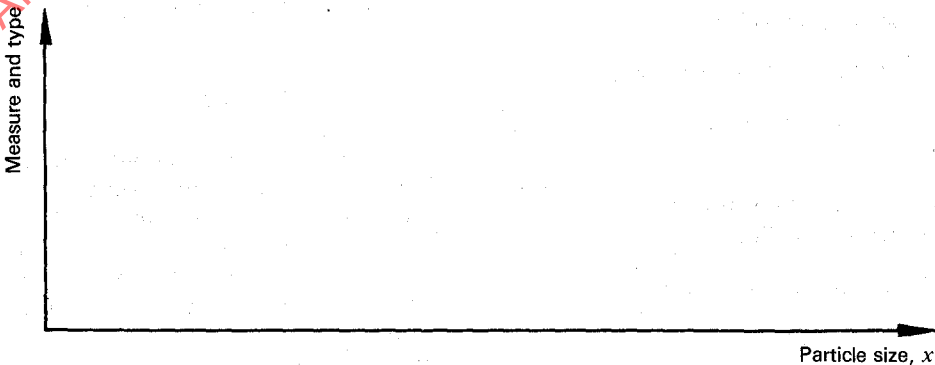


Figure 1 — Co-ordinates for representation of particle size analysis data

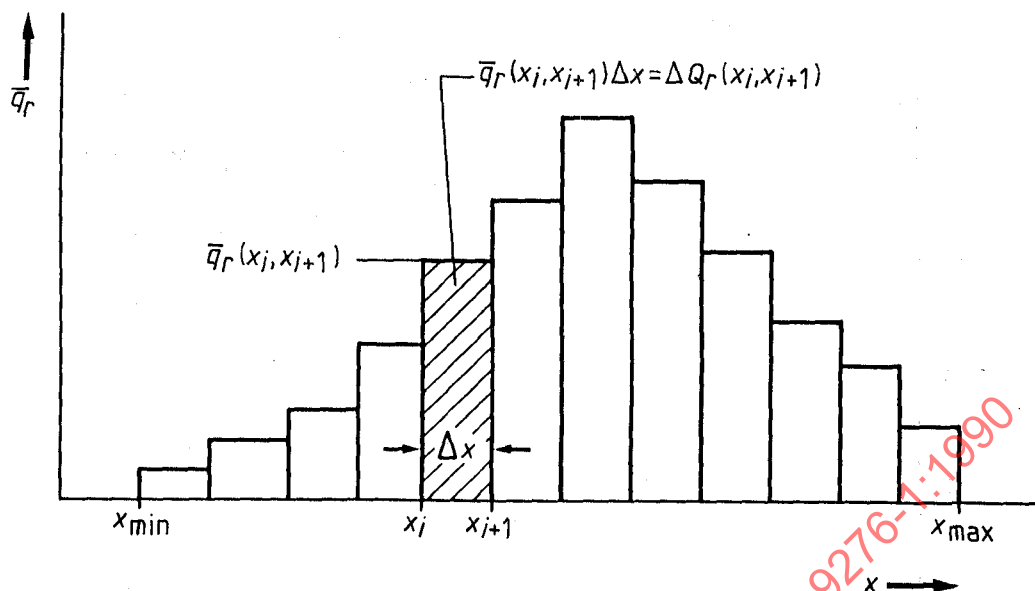


Figure 2 — Histogram $\bar{q}_r(x_i, x_{i+1})$ of a density distribution $q_r(x)$

4.2 Cumulative distribution $Q_r(x)$

Figure 3 shows a typical normalized cumulative distribution $Q_r(x)$. Each individual point of the distribution $Q_r(x)$ defines the relative amount of particles smaller than or equal to a certain equivalent diameter. The normalized cumulative distribution extends between zero and 1, i.e. between 0 and 100 %.

$$Q_{rv} = \sum_{i=0}^v \Delta Q_{ri} = \sum_{i=0}^v \bar{q}_r(x_i, x_{i+1}) \Delta x_i \quad \dots (4)$$

with $0 \leq i \leq v \leq n$.

4.3 Density distribution $q_r(x)$

Under the supposition that the cumulative distribution $Q_r(x)$ is differentiable, the continuous density distribution $q_r(x)$ is obtained from

$$q_r(x) = dQ_r(x)/dx \quad \dots (5)$$

$q_r(x)$ is plotted in figure 4.

Conversely, the cumulative distribution $Q_r(x)$ is obtained from the density distribution $q_r(x)$ by integration:

$$Q_r(x) = \int_{x_{\min}}^x q_r(x) dx \quad \dots (6)$$

5 Graphical representation of cumulative and density distributions on a logarithmic abscissa

Owing to the fact that a size distribution covers between its smallest particle size ($x_{\min}$) and its largest

particle size ($x_{\max}$) several decades, plotting the data on a linear abscissa may not yield the desired information. In such a case, therefore, the results shall be plotted on graph paper with a logarithmic abscissa.

5.1 Representation of the cumulative distribution using graph paper with a logarithmic abscissa

When plotted on graph paper with a logarithmic abscissa the cumulative values Q_{rv} , i.e. the ordinates of a cumulative distribution, do not change. Meanwhile the course of the cumulative distribution curve changes but the relative amounts below a certain particle size remain the same. Therefore

$$Q_{rv}(x) = Q_r(\ln x) \quad \dots (7)$$

5.2 Representation of the density distribution using graph paper with a logarithmic abscissa

The density values $\bar{q}_r(x_i, x_{i+1})$ of a histogram have to be recalculated using equation (8) to fulfil the condition of normalization, i.e. the area under the density distribution remains equal to 1 or 100 %.

$$\begin{aligned} \bar{q}_{ri}^*(\ln x_i, \ln x_{i+1}) &= \\ &= \bar{q}_r(x_i, x_{i+1}) \frac{(x_{i+1} - x_i)}{\ln(x_{i+1}/x_i)} \end{aligned} \quad \dots (8)$$

Substituting $x_{i+1} = kx_i$, one obtains

$$\begin{aligned} \bar{q}_{ri}^*(\ln x_i, \ln x_{i+1}) &= \\ &= x_i \bar{q}_r(x_i, x_{i+1}) \frac{(k - 1)}{\ln k} \end{aligned} \quad \dots (9)$$

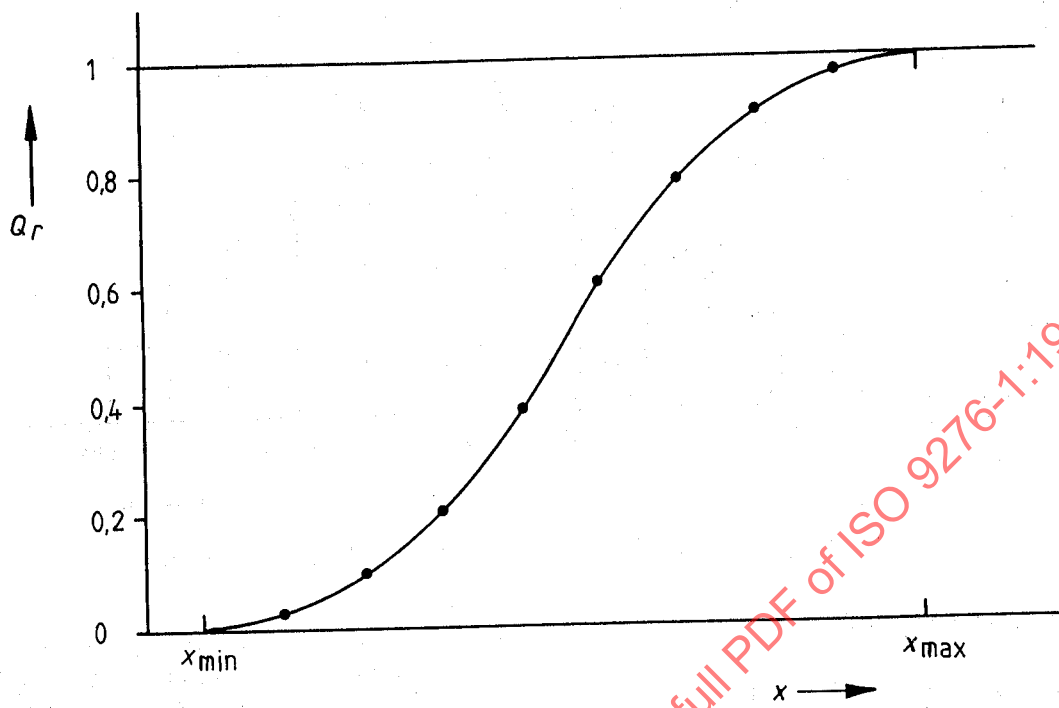


Figure 3 — Cumulative distribution $Q_r(x)$

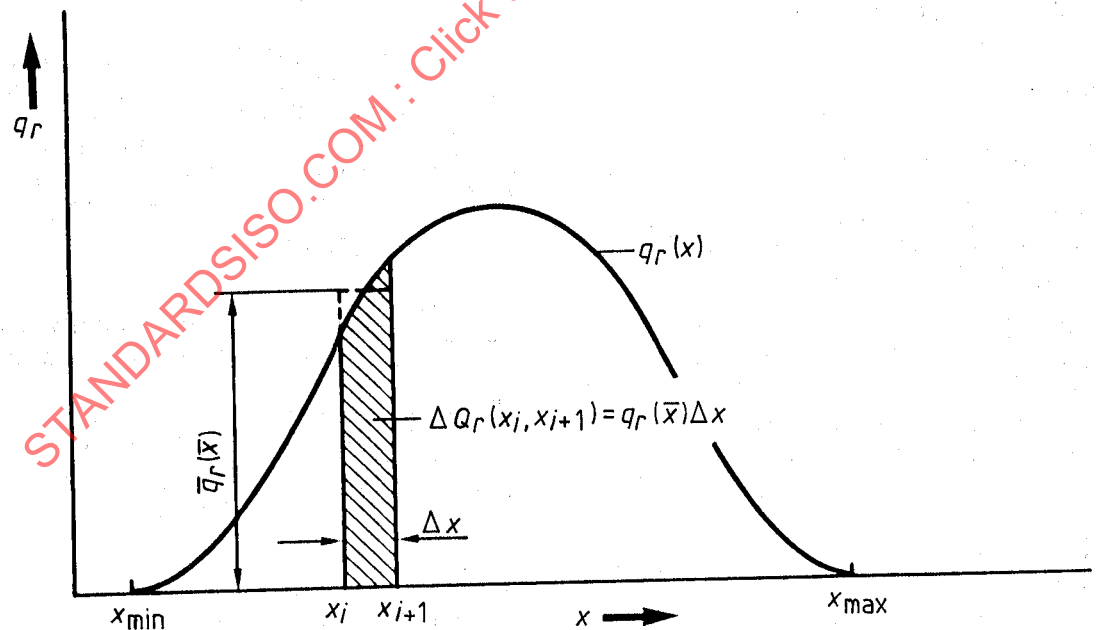


Figure 4 — Density distribution $q_r(x)$

Annex A

(informative)

Example of graphical representation of particle size analysis results

The following example, based on the data obtained by a sieve analysis, illustrates the application of this International Standard.

Table A.1 — Calculation of the histogram and the cumulative distribution from the data of a sieve analysis

1	2	3	4	5	6	7
i	x_i (mm)	ΔQ_{3i} (%)	Δx_i (mm)	$\bar{q}_{3i} = \Delta Q_{3i} / \Delta x_i$ (%/mm)	$Q_3(x)$ (%)	$\bar{q}_{3i}^*$
0	0,063	0,1	0,0275	3,64	0	0,024
1	0,09	0,09	0,035	2,57	0,1	0,0224
2	0,125	0,16	0,055	2,91	0,19	0,0379
3	0,18	0,25	0,07	3,57	0,35	0,0623
4	0,25	0,5	0,105	4,76	0,6	0,1208
5	0,355	1,1	0,145	7,59	1,1	0,2691
6	0,5	1,8	0,21	8,57	2,2	0,435
7	0,71	3,7	0,29	12,76	4	0,9048
8	1	6,1	0,4	15,25	7,7	1,5042
9	1,4	10,2	0,6	17	13,8	2,4444
10	2	16	0,8	20	24	3,9455
11	2,8	21	1,2	17,5	40	5,0327
12	4	24	1,6	15	61	5,9183
13	5,6	12,5	2,4	5,21	85	2,9966
14	8	2,4	3,2	0,75	97,5	0,5918
15	11,2	0,1	4,8	0,02	99,9	0,023
16	16				100	

The values for x_i given in table A.1, column 2, represent the standardized test sieve openings specified in ISO 565:1990, *Test sieves — Metal wire cloth, perforated metal plate and electroformed sheet — Nominal sizes of openings*.

NOTE 1 The notation w used in ISO 565 has been replaced in this International Standard by x_p .

The different amounts of particles retained between two test sieves were obtained by weighing, and the relative weights ΔQ_{3i} are listed in table A.1, column 3, for each particle size interval Δx_i .