
**Representation of results of particle size
analysis —**

Part 6:
**Descriptive and quantitative
representation of particle shape and
morphology**

Représentation de données obtenues par analyse granulométrique

*Partie 6: Description et représentation quantitative de la forme et de la
morphologie des particules*



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 9276-6 was prepared by Technical Committee ISO/TC 24, *Particle characterization including sieving*, Subcommittee SC 4, *Sizing by methods other than sieving*.

ISO 9276 consists of the following parts, under the general title *Representation of results of particle size analysis*:

- *Part 1: Graphical representation*
- *Part 2: Calculation of average particle sizes/diameters and moments from particle size distributions*
- *Part 3: Adjustment of an experimental curve to a reference model*
- *Part 4: Characterization of a classification process*
- *Part 5: Methods of calculation relating to particle size analyses using logarithmic normal probability distribution*
- *Part 6: Descriptive and quantitative representation of particle shape and morphology*

Introduction

A variety of different methods for the descriptive and quantitative representation of particle shape and morphology are known. Even for the term particle size, there is no single definition. Different methods of size analysis are based on the measurement of different physical properties. In ISO 9276-1, the particle size is defined as the diameter of a sphere having the same physical property. This is known as the equivalent spherical diameter. So-called property functions help to correlate it with the property of primary interest, which may, for instance, be flowability, taste or dissolution time.

Broad application of sizing methods in particle characterization shows that particle size is often an important factor. But particle size alone is not sufficient to allow particle phenomena such as powder flow, mixing, abrasion or biological response to be understood. Particle shape and morphology play an important role in particle systems and therefore it is also necessary to characterize and describe these characteristics quantitatively.

Including additional shape parameters in property functions is supposed to give a better correlation with the particular property of the particle system. For instance, knowledge of the size of grinding particles and of the sharpness of their edges will make it possible not only to distinguish between fresh and used grinding particles but also to predict their abrasive effect quantitatively by means of a property function.

ISO 13322-1 and ISO 13322-2 give guidance on the measurement, description and validation methodologies used when determining particle sizes by static and dynamic image analysis, respectively. Broad industrial use of image analysis techniques requires standardized methods of measurement for the characterization of the size, geometrical shape and morphology of particles.

A particle's shape is the envelope formed by all the points on the surface of the particle. Particle morphology represents the extension of a simple shape description of this kind to more complex descriptions including characteristics such as porosity, roughness and texture.

Various glossaries of terms giving descriptions, in words, of particle shape and morphology already exist (see Clause 5). These descriptions may be useful for the classification or identification of particles but, at the moment, there is insufficient consensus on the definition of particle shape and morphology in the quantitative terms necessary for them to be implemented in software routines. A future revision of this part of ISO 9276 may cover this.

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Representation of results of particle size analysis —

Part 6:

Descriptive and quantitative representation of particle shape and morphology

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1 Scope

This part of ISO 9276 specifies rules and nomenclature for the description and quantitative representation of particle shape and morphology. To achieve a more comprehensive description of a particle or particle system, particle size information can be used together with other information but, in most cases, the particle size information cannot be replaced.

The averaging of shape over all particles in a sample has been shown to be an ineffective approach. Distributions of other particle characteristics are required in addition to particle size distributions (see ISO 9276-1).

The relevance, to technological applications, of any method of representing particle shape is the deciding factor in its use. Therefore this part of ISO 9276 is restricted to methods which can be correlated with physical properties in industrial applications.

The aim of particle analysis is to determine the most appropriate characterization method for a particular application. This implies a profound understanding of the relationship between particle characteristics and macroscopic product and process properties (or at least a database of broad empirical data).

Problems of shape and morphology would normally be three-dimensional problems, but most definitions in this part of ISO 9276 are in fact given for two dimensions because of the widespread use of image analysis methods.

With the help of the evaluation criteria given in Clause 4, a minimum set of shape descriptors is derived in Clause 8 from the various descriptors and methods in Clause 5, enabling a direct comparison of different shape analysis equipment or methods to be made within the limits discussed in Clause 6.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 9276-1:1998, *Representation of results of particle size analysis — Part 1: Graphical representation* (and its Technical Corrigendum ISO 9276-1:1998/Cor.1:2004)

ISO 13322-1:2004, *Particle size analysis — Image analysis methods — Part 1: Static image analysis methods*

3 Symbols and abbreviated terms

For the purposes of this document, the symbols given in ISO 13322-1 and ISO 9276-1 and the following apply.

In ISO 9276-1, the symbol x is used to denote the particle size or the diameter of a sphere. However, it is recognized that the symbol d is also widely used to designate these values. Therefore, in this part of ISO 9276, the symbol x may be replaced by d wherever it appears.

Symbols for the particle size other than x or d shall not be used.

A	projection area
A_{box}	Feret box area
A_{c}	area of the convex hull (envelope) bounding the particle
b	intercept on graph for fractal dimension
C	circularity
CI	global surface concavity index
D_{F}	fractal dimension
d_{cmin}	diameter of the minimum circumscribed circle
d_{imax}	diameter of the maximum inscribed circle
d_{L}	spacing of a series of parallel lines [for use in the Cauchy-Crofton formula (see Clause A.1)]
E	thickness
I_{α}	number of intercepts [for use in the Cauchy-Crofton formula (see Clause A.1)]
L_{G}	geodesic length
N	number
P	length of perimeter
P_{C}	length of the perimeter of the convex hull (envelope) bounding the particle
Rn	roundness
S	surface area
V	volume
x_{A}	area-equivalent diameter of particle
x_{E}	thickness of a very long particle
x_{Fmax}	maximum Feret diameter
x_{Fmin}	minimum Feret diameter
x_{LF}	Feret diameter perpendicular to the minimum Feret diameter, normally known as "length"
x_{LG}	geodesic length of a very long particle

x_{Lmax}	length of major axis of Lengendre ellipse of inertia
x_{Lmin}	length of minor axis of Lengendre ellipse
x_P	perimeter-equivalent diameter of particle
x_S	surface-equivalent diameter of particle
x_V	volume-equivalent diameter of particle
α	angle or direction
Ω_1	robustness
Ω_2	largest concavity index
Ω_3	concavity/robustness ratio
ω	number of erosions
Ψ	Wadell's sphericity
Ψ_{FP}	average concavity

4 Criteria for the evaluation of shape description methods

A common problem in shape description is how to judge the quality of a shape description method. Not all methods are suitable for every kind of shape and application. Until now, consistent evaluation criteria have not existed for shape description methods.

Criteria for the evaluation of shape description methods:

- *accessibility*, which describes how easy it is to compute a shape descriptor in terms of memory requirements and computation time;
- *scope*, which refers to the classes of shape that can be described by the method;
- *uniqueness*, which describes whether a one-to-one mapping relationship exists between shapes and shape descriptors;
- *stability* and *sensitivity*, which describe how sensitive a shape description is to “small” changes in shape.

Each method shall use descriptors with a specific degree of complexity. In general, descriptors can be described as sets of numbers that are produced to describe a given shape. The shape may not be entirely reconstructable from these descriptors, but the descriptors for different shapes shall be sufficiently different to make it possible to discriminate between the shapes.

Criteria for shape descriptors:

- *invariance with respect to rotation and reflection* — for a given shape, the values of the descriptors shall be the same irrespective of the orientation of the particle;
- *invariance with respect to scale* — for a given shape, the values of the descriptors shall be the same irrespective of the size of the particle;
- *independence* — if the elements of the descriptors are independent, some can be discarded without the need to recalculate the others;

- *economy* — it is desirable that the descriptors be economical in the number of terms used to describe a shape.

The above three invariance conditions (concerning rotation, reflection and scale) guarantee that the result of a shape analysis is not affected by the parameters of the analysis and is independent of the particle size. It should, however, be stressed that the particle size at which certain shape information is obtained may be of practical relevance, as in the case of surface roughness, and size shall therefore be included in the shape analysis.

The robustness of shape descriptors with respect to the density, translation and rotation of the sampling grid can indicate whether it is acceptable to compare measurement results from different algorithms or different image analysers ^[1].

5 Classification of methods and descriptors

5.1 General classification

Methods of shape description, as well as the various shape descriptors, can be classified according to different criteria. An obvious way of classifying shape descriptors is to determine whether they are qualitative or quantitative in nature:

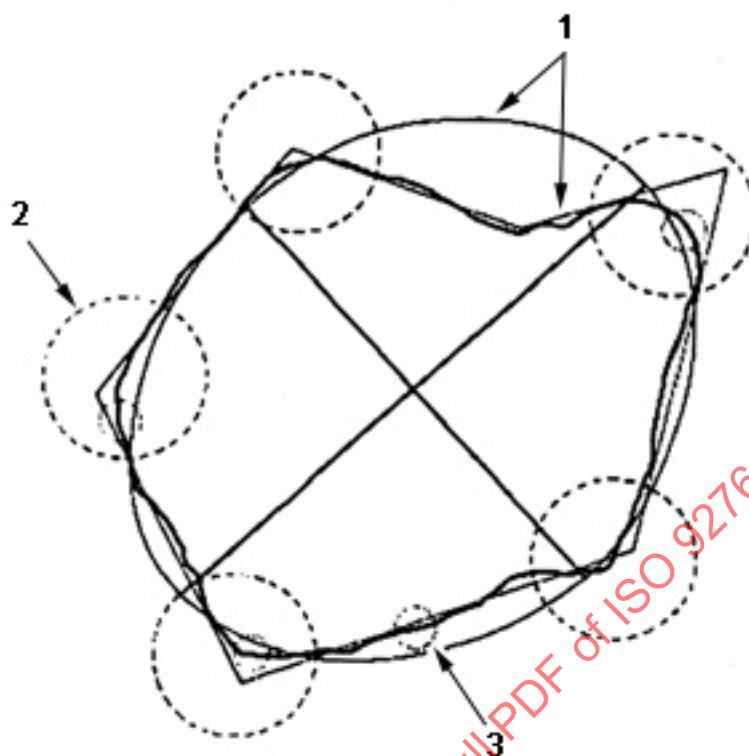
- a) Qualitative description, i.e. in words: expressions such as “needlelike particles” and “oblate shape”. Examples of this type of shape characterization are given in the US Pharmacopoeial Convention ^[2], in ASTM F 1877 ^[3] and in the glossary made available by the NIST Center for Analytical Chemistry ^[4].
- b) Quantitative description: in the following text, shape descriptors will be understood as numbers that can be calculated from particle images or physical particle properties via mathematical or numerical operations.

5.2 Levels of shape

For a better understanding of shape description, it is important to establish definitions regarding the basic characteristics of an arbitrary object. The shape of an arbitrary object can be defined in many ways. One such definition describes shape as a binary image representing the extent of the particle. This can be understood as the silhouette of the particle. Barrett ^[5] recognizes three potentially independent particle shape properties (see Figure 1):

- *form*, which reflects the geometrical proportions of a particle;
- *roundness*, which expresses the radius of curvature at the particle corners;
- *surface texture*, which is taken as defining local roughness features at corners and at edges between corners only.

These particle shape properties may not suffice for a complete description of the shape of a particular particle and may be defined differently by different authors. But they give us a good idea of how shape parameters can be measured at different levels of size. Three corresponding levels of shape can thus be distinguished: macroshape, mesoshape and microshape.

**Key**

- 1 form
- 2 roundness
- 3 surface texture

Figure 1 — Illustration of form, roundness and surface texture ^[6]

Macrodescription is a description of the overall form of a particle defined in terms of the geometrical proportions of the particle. In general, simple geometrical descriptors calculated from size measurements made on the particle silhouette are used. Low-order Fourier descriptors can also be regarded as macrodescriptors.

Mesodescription provides information about details of the particle shape and/or surface structure that are in a size range not much smaller than the particle proportions, like Barrett's ^[5] roundness and concavity.

The following mesodescriptors can be defined:

- a) morphological mathematical descriptors, computing robustness and largest concavity index;
- b) a concavity tree, providing general insight into the organization of concavities and their complexity;
- c) angularity descriptors, determining those parts of the boundary that are active in the abrasion process:
 - 1) an angularity factor, selecting the apices of corners which are coincident with the convex hull because it is these points that will make contact with the surface of another particle,
 - 2) a quadratic spike parameter, taking into account those spikes that are outside a circle, of area equal to that of the particle, centred over the particle centroid,
 - 3) slip chording, generating information on the number of cutting edges and their sharpness in the facet signature waveform;

- d) fractal dimension, providing data on the overall structural complexity by consideration of a larger measurement step;
- e) Fourier descriptors, of higher order than macrodescriptors, specifying the smaller-scale components of morphology;
- f) bending energy, measuring the overall complexity of contour lines.

Microdescription determines the roughness of shape boundaries using two of the descriptors mentioned above:

- fractal dimension, measured using a measurement step smaller than that used for structural description;
- higher-order Fourier descriptors/coefficients for surface-textural analysis.

5.3 Principles for deriving shape descriptors

The level of inspection used in a method is a very practical criterion for the classification of the method, since many methods provide shape information at different size levels. Another convenient way of classifying methods is to differentiate between those which derive shape descriptors from particle images and those which derive shape descriptors from physical properties:

- a) Calculation of geometrical descriptors/shape factors:

Geometrical shape factors are ratios between two different geometrical properties, such properties usually being some measure of the proportions of the image of the whole particle or some measure of the proportions of an ideal geometrical body that envelops, or forms an envelope around, the particle. These results are macroshape descriptors similar to an aspect ratio.

- b) Calculation of dynamic shape factors from physically equivalent diameters:

These shape factors are similar to geometrical shape factors except that at least one physical property is considered in the comparison. Usually, the results are expressed as a ratio of equivalent diameters, e.g. Stokes sedimentation velocity to volume-equivalent diameter x_{Stokes}/x_V .

- c) Morphological analysis:

Morphological analysis descriptors give mean values of particle shape that are not much smaller than the proportions of the whole particle. A typical example is concavity analysis.

- d) Analysis of the contour line (shape boundary):

Multiple operations on the contour line of a particle can produce a set of shape descriptors. This set of shape descriptors contains information on the particle shape at different size levels. Methods belonging to this group include fractal dimension analysis and the use of Fourier analysis.

- e) Analysis of grey-level images:

Multiple operations on the grey-level pixel image of a particle can produce a set of shape descriptors which can be correlated with the topology or surface texture of the particle.

- f) Analysis of physical spectra:

Multiple operations on, or the mathematical treatment of, the physical spectra of a single particle can extract the shape information as a set of descriptors. Such a procedure has been described for shape analysis by azimuthal light scattering and diffraction spectroscopy.

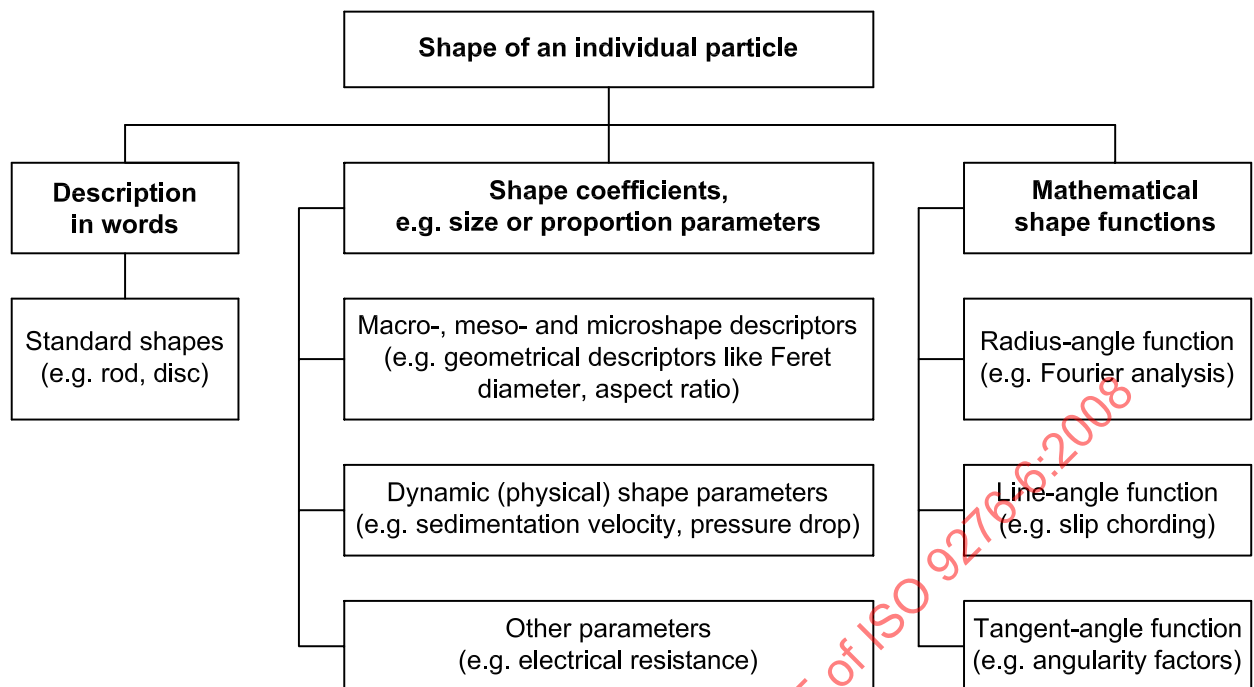


Figure 2 — Classification of some methods for particle shape description

The choice of an appropriate shape description method depends on the measurement technique available and the particle system under examination (in particular its size range). Methods based on mathematical operations on contour lines (e.g. fractal dimension analysis or Fourier analysis) require a relatively high resolution of particle images. This may be obtained by using a scanning electron or light microscope. Apart from such factors, the results of shape analysis may also be significantly affected by sample preparation (e.g. by the sample size and whether the sample is representative, by particle orientation in 2D-analysis).

This part of ISO 9276 defines the set of parameters necessary for the comparison of shape analysis methods and shape analysis instruments. Any other shape descriptors used shall be clearly defined.

6 Errors which can occur in the analysis of a single image

6.1 Generation of shape descriptors

Problems associated with image analysis are manifold and errors can be introduced in the generation of shape descriptors. These errors can exist at many levels, but most of them are fundamentally different from those observed in the more traditional techniques used for the characterization of dispersed matter. Such shape descriptor errors are usually introduced by the protocols necessary to perform calculations on any given image (see ISO 13322-1:2004, Annex D). The common sources of errors which occur when performing image analysis and in the comparison of image analysis protocols are specified in 6.2 to 6.4.

6.2 Image resolution

The optimum resolution shall not and cannot be stated absolutely, but shall rather be related to the size of the element features to be determined (e.g. agglomerate branches, roughness scale). Analysis of image parameters is generally based on a digitized image. The process of digitizing an image can result in information loss because of the transformation of the continuous features innate to the particle into discrete elements of finite size — the finite resolution. For pixel errors smaller than 5 % for a circle, the necessary pixel numbers per particle range from 100 to 200 for robust parameters, like projection area and ellipse ratio, up to 5 000 for parameters using the perimeter ^{[1], [7], [8]}.

The resolution of a particle in terms of pixels per unit length cannot be greater than the optical resolution of the same particle. This limitation is due to the sharpness of the particle image generated by the focal depths achievable with optical microscopy and the numerical aperture of the objective lenses (see ISO 13322-1:2004, Annex C).

6.3 Binarization

To discriminate between particles of irregular shape against a background, several techniques can be employed. One example is the application of a threshold limit for the colour or grey levels within the image. All pixels with colour levels or a specific grey level on one side of a threshold may be considered as part of the background, whilst all other pixels are deemed to be part of the particle. The threshold value selected requires a criterion representing an appropriate value. This criterion can be chosen using an image analysis algorithm or can be selected manually, in which case it will be subject to individual perception.

Systematic errors of both calibration and binarization can be determined experimentally by use of reference particles with a known size or area.

6.4 Algorithms for calculating shape descriptors

Information gathered from a digitized image by an image analysis software programme to determine features such as lengths and areas includes only colour and discrete locational information. Therefore, to determine a feature such as the diameter of a particle, the image analysis system and software has to provide an algorithm capable of extracting this desired feature.

Consequently, no single particle parameter, be it a length, a fractal dimension or an area, is "natural" to image analysis. In every image analysis, logic has to be applied to extract the desired feature from the lower-level information limits of colour and discrete locational information.

The intention of image analysis algorithms is to be as close as possible to the original meaning of a feature, or to any other (non-image-processing tool) methods, regardless of the small basis of lower-level information which can occur in the process of analysis. In order to achieve this intention, several algorithms may be applied for the determination of each feature, each based on a different set of assumptions. These assumptions should be congruent with each other, but often are not. In some cases, parameters within the algorithm need to be adjusted.

If basic pixel routines are not sufficiently documented, image analysis software can deliver non-explainable differences in shape descriptors.

7 Size parameters for normalization of shape descriptors

Volume V	3D descriptor	
	Ideal normalization parameter (no orientation dependence)	
Volume-equivalent diameter x_V	Diameter of a sphere having the same volume as the particle: $x_V = \sqrt[3]{6V/\pi}$ Difficult to obtain directly	See Reference [9] See Reference [10] (electrical sensing zone method)
Projection area A	2D descriptor	
	Ideal normalization parameter	
Area-equivalent diameter x_A	Diameter of a sphere having the same projection area as the particle: $x_A = \sqrt{4A/\pi}$ (Also used: equivalent circle diameter, ECD)	See Reference [9] See Reference [3]
Surface area S	3D descriptor	
	Not suitable for form analysis normalization	
	Value depends on method of measurement used	
	Cauchy's theorem for convex particles if particle is projected in average position: $S = 4\bar{A}$	See Reference [9]
Surface-equivalent diameter x_S	Diameter of a sphere having the same surface area as the particle: $x_S = \sqrt{S/\pi}$	
Perimeter P	2D descriptor	
	Not suitable for form analysis normalization	
	Value very sensitive to measurement errors, especially concerning the corresponding pixels (resolution and magnification) and orientation, therefore the Cauchy-Crofton formula is recommended (see Clause A.1)	See Reference [11] See Reference [1]
Perimeter-equivalent diameter x_P	Diameter of a circle having the same perimeter as the projection area of the particle: $x_P = P/\pi$	

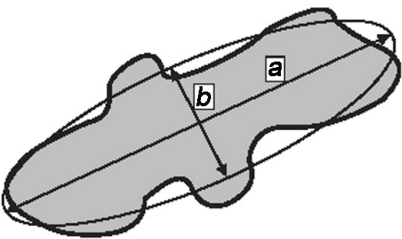
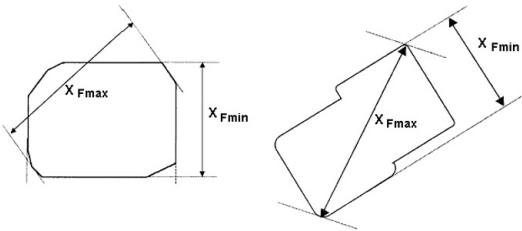
8 Shape descriptors

8.1 Macroshape descriptors

8.1.1 General

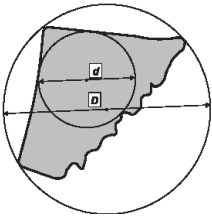
Macroshape descriptors represent the geometrical proportions of particles. Most of them are ratios of descriptors of different geometrical properties.

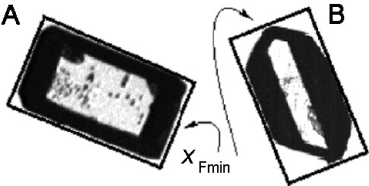
8.1.2 Geometrical descriptors

Legendre ellipse of inertia 	An ellipse with its centre at the particle's centroid and with the same geometrical moments, up to the second order, as the original particle area The major and minor axes are given by x_{Lmax} and x_{Lmin}, respectively Robust measurement For equations, see Clause A.2	See Reference [17] See Reference [12]
Feret diameters x_{Fmax}, x_{Fmin} 	Distances between parallel tangents Maximum diameter x_{Fmax} corresponds to the "length" of the particle Minimum diameter x_{Fmin} corresponds to the "breadth" of the particle	See Reference [3] See Reference [13]
Length x_{LF}	Feret diameter perpendicular to the minimum Feret diameter	See Reference [14]
Geodesic length x_{LG}, thickness x_E 	Better approximations for very long and concave particles, such as fibres Robust method of determining x_{LG} as an approximation for geodesic length and x_E, using the following equations for an area- and perimeter-equivalent rectangle: $A = x_E \cdot x_{LG} \quad P = 2(x_E + x_{LG})$	See Reference [18] See Reference [15]

8.1.3 Proportion descriptors

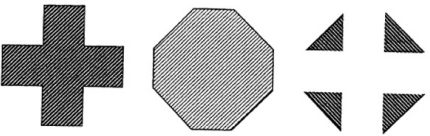
In this part of ISO 9276, ratio definitions which give values between 0 and 1 are preferred.

Ellipse ratio	Ellipse ratio = x_{Lmin} / x_{Lmax} where x_{Lmin} and x_{Lmax} are the lengths of the axes of the Legendre ellipse (Also used: elliptical shape factor) More robust parameter than aspect ratio	See References [12] and [16] See References [7] and [17]
Aspect ratio	For not very elongated particles: Aspect ratio = x_{Fmin} / x_{Fmax}	See References [11] and [3]
Elongation	For very elongated particles, such as fibres: Elongation = x_E / x_{LG} (Also used: eccentricity)	See References [11] and [3]
Straightness	For very elongated particles (reciprocal of curl): Straightness = x_{Fmax} / x_{LG}	See Reference [11]
Irregularity (modification ratio)	 Relationship between the diameter of the maximum inscribed circle d_{imax} and that of the minimum circumscribed circle d_{cmin}: Irregularity = d_{imax} / d_{cmin} (Also used: modification ratio)	See References [11] and [12]
Compactness	Degree to which the particle (or its projection area) is similar to a circle, considering the overall form of the particle: $\text{Compactness} = \frac{\sqrt{4A / \pi}}{x_{Fmax}}$ Roundness Rn is also used, but is less robust: $Rn = 4A / \pi x_{Fmax}^2$	See References [11] and [18] See Reference [3]
Extent	Extent = $\frac{A}{x_{Fmax} \cdot x_{Fmin}}$ (Also used: bulkiness)	See Reference [11]

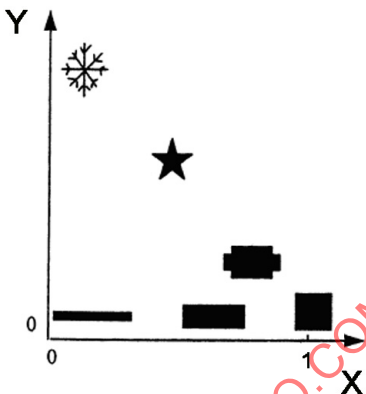
Box ratio 	Ratio of the Feret box area to the projected area: $\text{Box ratio} = A / A_{\text{box}}$ $A_{\text{box}} = x_{\text{Fmin}} \cdot x_{\text{LF}}$ Very sensitive to orientation	See References [13] and [19]
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8.2 Mesoshape descriptors

Wadell's sphericity ψ	$\psi = (x_V / x_S)^2 = \pi \cdot x_V^2 / S$	See Reference [20]
Circularity C	Degree to which the particle (or its projection area) is similar to a circle, considering the smoothness of the perimeter: $C = \sqrt{\frac{4\pi A}{P^2}} = \frac{x_A}{x_P}$ (Term under square root sign is called the form factor, FF)	See References [11] and [18] See Reference [3]
Solidity	Measure of the overall concavity of a particle: $\text{Solidity} = A / A_C$ where A_C is the area of the convex hull (envelope) bounding the particle Global surface concavity index (CI) and concavity are also used: $\text{CI} = \frac{A_C - A}{A} \quad \text{Concavity} = \frac{A_C - A}{A_C}$	See References [11] and [13]
Convexity	$\text{Convexity} = P_C / P$ where P_C is the length of the perimeter of the convex hull (envelope) bounding the particle	See Reference [11]
Average concavity	$\psi_{\text{FP}} = \frac{\bar{x}_F}{x_P}$ where the angle-average Feret diameter $\bar{x}_F$ is given by: $\bar{x}_F = \frac{1}{\pi} \int_0^\pi x_F(\alpha) d\alpha$	See References [21] and [22]

Particle robustness Ω_1  Object	$\Omega_1 = \frac{2\omega_1}{\sqrt{A}}$ where ω_1 is the number of erosions necessary to make the silhouette disappear completely	See References [18] and [23]
Largest concavity index Ω_2  Object Convex hull Complement B to convex hull	$\Omega_2 = \frac{2\omega_2}{\sqrt{A}}$ where ω_2 is the number of erosions necessary to make the residual of the silhouette, set with respect to the convex hull of area A_C, disappear completely	See References [18] and [23]

8.3 Combination of shape descriptors

Concavity/robustness ratio Ω_3  Key X robustness Ω_1 Y largest concavity index Ω_2	Secondary mesoshape descriptor: $\Omega_3 = \frac{\Omega_2}{\Omega_1} = \frac{\omega_2}{\omega_1}$	See Reference [23]
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8.4 Roughness descriptor

This descriptor represents microshape properties. Fractal dimensions are necessary to distinguish between mesoshape (concavity) and microshape descriptors.

Fractal dimension D_F 	The relationship between the length of the perimeter $P(\lambda)$ and the length λ of the step is linear on a log-log plot, known as a Richardson plot The data are first normalized by dividing by the maximum Feret diameter The upper limit for the step size is given by: $\lambda = 0,3x_{Fmax}$ The equation of the straight line is: $\log P(\lambda) = (1 - D_F)\log \lambda + \log b$	See Reference [24]
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Annex A (normative)

Some computation equations

A.1 Estimation of the perimeter of a disc (Cauchy-Crofton formula)

The length of the perimeter P is calculated from the number of intercepts I_α formed by a series of parallel lines with spacing d_L exploring N directions α from 0 to π [1].

$$P = \frac{\pi}{N} \cdot \sum_{\alpha} I_{\alpha} \cdot d_L$$

A.2 Legendre ellipse of inertia

The calculation of the equation of this ellipse is carried out by determining the real moments of inertia of the particle and deriving mathematically from them an ellipse having the same inertial properties. The ellipse can be characterized by its major and minor diameters, the position of its centre of gravity and its orientation [17]:

- a) Determination of the moments of inertia of the shape coordinates:

$$\sigma_{xx} = \frac{1}{n} \sum (x_i - \bar{x})^2$$

$$\sigma_{yy} = \frac{1}{n} \sum (y_i - \bar{y})^2$$

$$\sigma_{xy} = \frac{1}{n} \sum (y_i - \bar{y})(x_i - \bar{x})$$

- b) Definition of intermediate terms:

$$\alpha = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) \quad \beta = \sqrt{\alpha^2 - \sigma_{xx}\sigma_{yy} + \sigma_{xy}^2}$$

- c) Determination of the lengths of the axes of an ellipse with equivalent inertia:

$$x_{Lmax} = 4\sqrt{\alpha + \beta} \quad x_{Lmin} = 4\sqrt{\alpha - \beta}$$

where

x_{Lmax} is the length of the major axis;

x_{Lmin} is the length of the minor axis.

- d) Determination of the orientation θ , in degrees, of the major axis:

$$\theta = 90 - \frac{180}{\pi} \arctan \left(\frac{\sigma_{xx} - \alpha - \beta}{\sigma_{xy}} \right)$$

Annex B (informative)

Examples of methods of presentation of shape and size distribution data

B.1 Example of a data matrix

Table B.1 gives the results of a classification, by size and shape, of catalyst particles in the size range from 2,5 µm to 142,5 µm and with aspect ratios from 0 to 1. To calculate the probability density distribution function values, the numbers of particles are divided by the sum of all the particles in that size class (or with that aspect ratio) and the corresponding class width of 5 µm (for size) or 0,1 (for aspect ratio) as in ISO 9276-1.

Table B.1 — Particles classified by size (from top to bottom) and aspect ratio (from left to right)

Particle size class	Aspect ratio									
	0,05	0,15	0,25	0,35	0,45	0,55	0,65	0,75	0,85	0,95
	Number of particles									
5	0	0	0	0	0	82	964	2 300	15 000	25 000
10	0	0	0	0	0	1	198	4 780	20 000	29 774
15	0	0	0	0	0	4	495	8 661	25 100	32 478
20	0	0	0	0	0	15	1 584	16 772	35 100	29 730
25	0	0	0	0	0	48	4 196	27 103	40 100	26 654
30	0	0	0	0	1	274	9 528	36 130	44 954	21 284
35	0	0	0	2	5	1 088	17 281	40 092	35 056	14 714
40	0	0	0	4	43	2 597	23 429	36 754	24 175	9 484
45	0	0	0	7	156	4 695	26 889	31 055	16 378	5 955
50	0	0	0	11	367	6 710	26 136	23 156	10 117	3 560
55	0	0	0	13	812	8 616	24 295	17 276	6 348	2 051
60	0	0	0	46	1 579	10 209	21 627	12 361	3 809	1 175
65	0	0	0	128	2 566	11 352	18 173	8 530	2 239	703
70	0	0	1	361	3 883	12 540	15 446	5 831	1 240	346
75	2	6	56	1 587	6 006	12 107	10 827	3 369	658	389
80	0	1	28	2 382	7 335	10 799	7 541	1 714	259	177
85	1	33	208	4 566	7 842	8 341	4 521	860	172	256
90	7	73	479	6 614	7 708	6 163	2 706	391	132	259
95	15	139	978	8 093	6 030	3 570	1 048	138	89	252
100	24	292	1 629	8 015	3 788	1 601	322	47	69	227
105	56	624	2 226	6 979	2 213	682	104	33	66	240
110	152	1 060	2 487	5 448	1 203	309	64	36	63	247
115	217	1 222	2 146	3 156	299	65	17	28	24	129
120	500	1 498	1 740	1 863	151	37	20	29	23	128
125	400	1 100	1 081	889	56	18	18	27	19	88
130	200	800	631	100	25	16	19	32	18	65
135	50	100	367	50	15	13	27	27	14	52
140	50	60	50	5	7	8	23	18	7	26

B.2 Examples of two-dimensional graphical presentation

In Figures B.1 and B.2, x is particle size, q_0 is the probability density distribution value and AR is the aspect ratio.

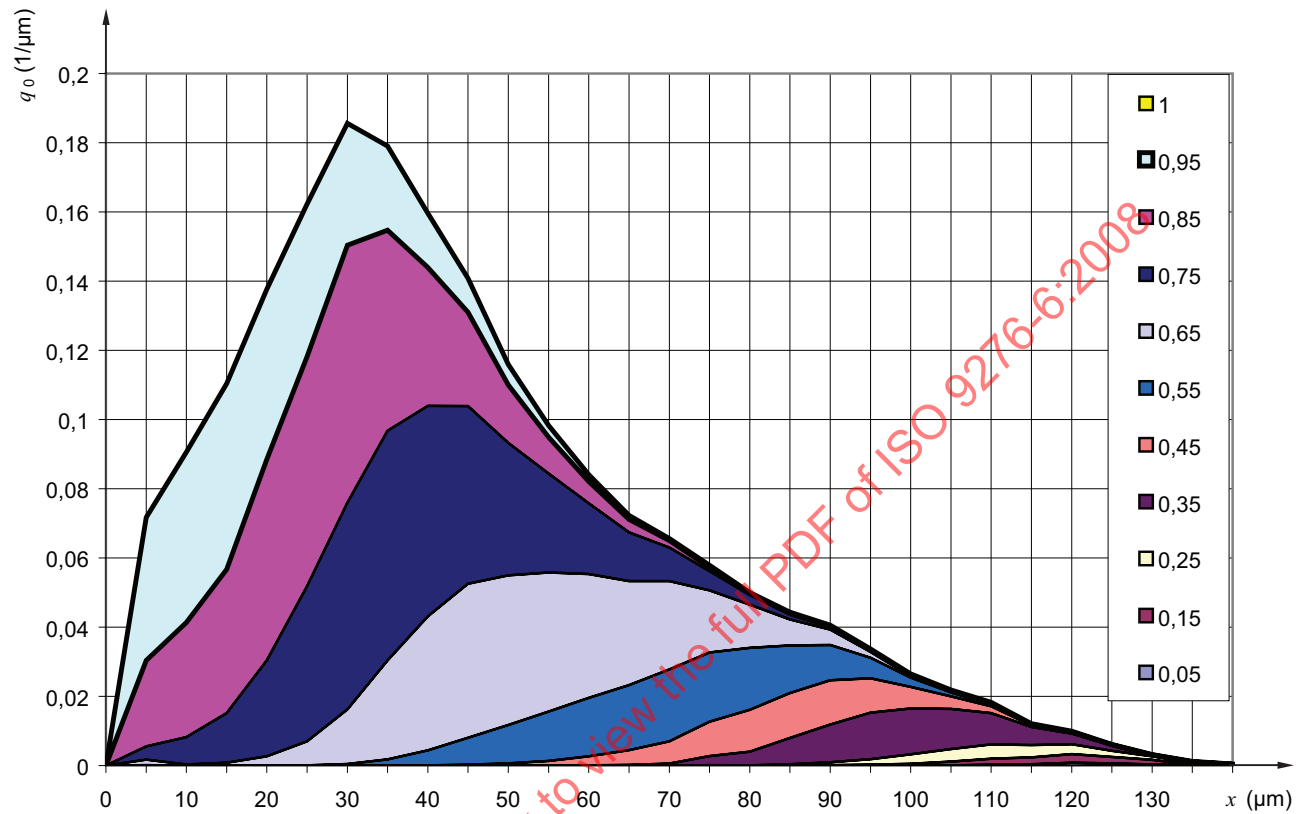


Figure B.1 — Particle size density distributions cumulated with increasing aspect ratio

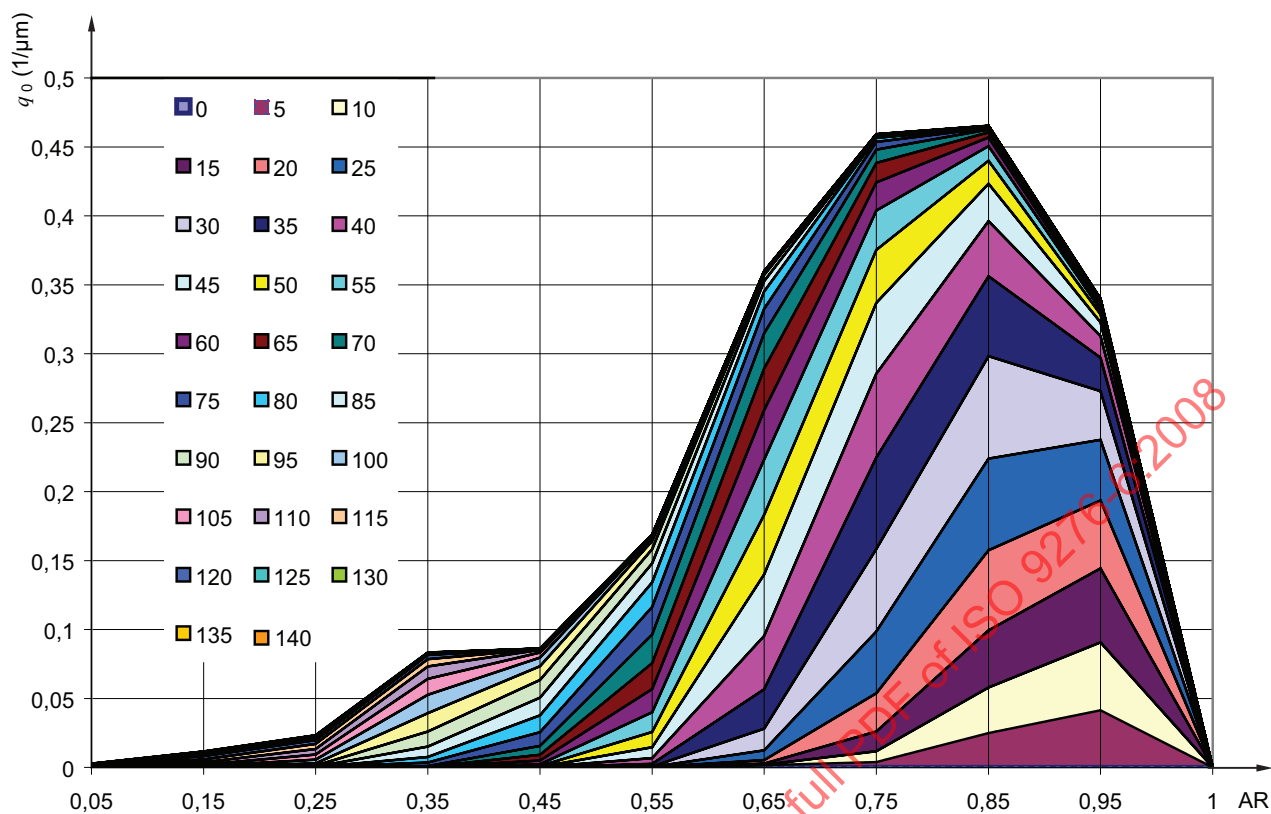


Figure B.2 — Particle aspect ratio density distributions cumulated with increasing particle size

B.3 Examples of three-dimensional graphical presentation

In Figures B.3 to B.8, x is particle size, q_0 is the probability density distribution value and AR is the aspect ratio.

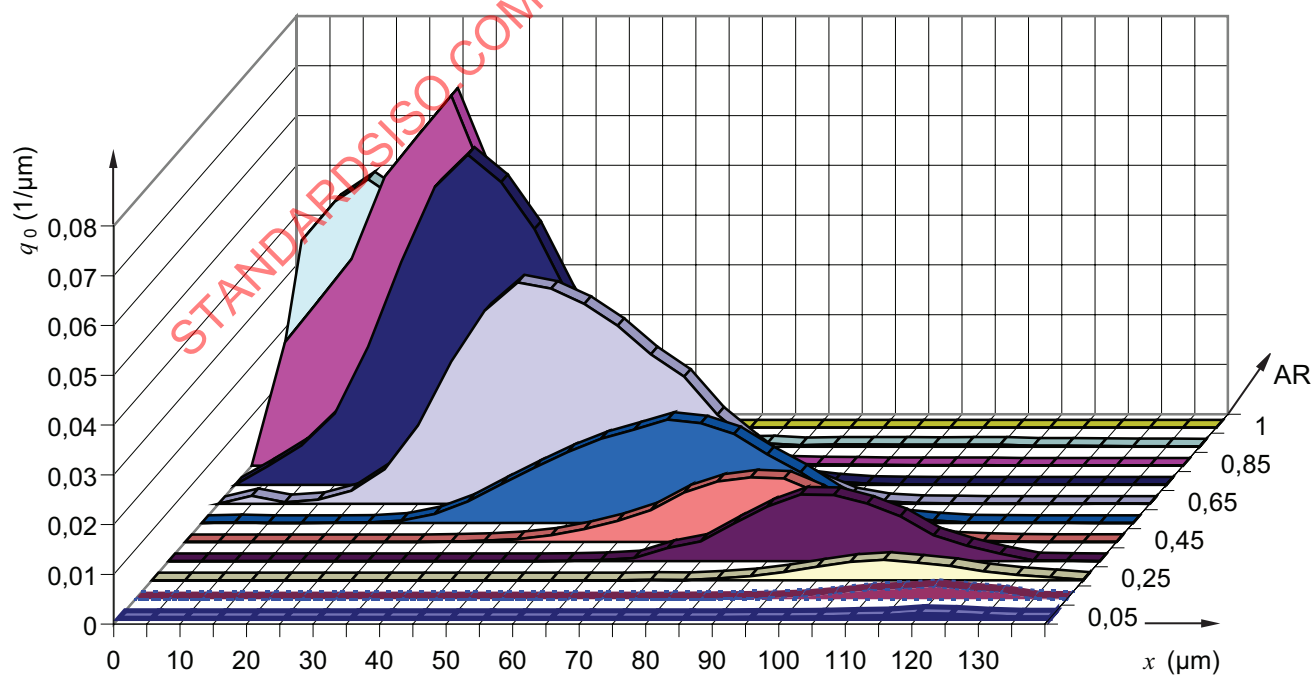


Figure B.3 — Particle size density distributions for different aspect ratios