
**Information technology — Programming
languages, their environments and
system software interfaces —
Programming language Extended APL**

*Technologies de l'information — Langues de programmation, leurs
environnements et interfaces logiciel système — Langage de
programmation APL étendu*

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Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

In the field of information technology, ISO and IEC have established a joint technical committee, ISO/IEC JTC 1. Draft International Standards adopted by the joint technical committee are circulated to national bodies for voting. Publication as an International Standard requires approval by at least 75 % of the national bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO and IEC shall not be held responsible for identifying any or all such patent rights.

International Standard ISO/IEC 13751 was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology*, Subcommittee SC 22, *Programming languages, their environments and system software interfaces*.

Annex A forms a normative part of this International Standard.

Introduction

APL stands for **A Programming Language**. It is a notation invented by K. E. Iverson in the late 1950s for the description of algorithms, and expanded on and made into the programming system *APL\360* by Iverson and his colleagues Adin Falkoff, Larry Breed, Dick Lathwell, and Roger Moore in the mid-1960s.

This document, **Programming Language APL, Extended**, is a sequel to **Programming Language APL**, ISO 8485 (1989).

The principal differences that the reader will find here have to do with new features that have been added. These topics are:

- without
- greatest common divisor
- least common multiple
- duplicate
- commute
- table
- join along first axis
- mixed arrays
- overbar in names
- underbar in names
- replicate
- character grades
- grades of arrays greater than rank one
- unique
- alpha as a name
- omega as a name
- ambivalent defined functions
- event handling
- n-wise reduction
- complex arithmetic
- left
- right
- function rank operator
- defined operators
- component file system

enclose
disclose
enlist
pick
depth
identical
each
first

An entry for each of these topics will be found in the index. Some new system commands have been added. Shared variable extensions have been added. Workspace Interchange Standard 2 is given, in which canonical representation vectors of type “E” are used to represent generalised arrays.

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Information technology — Programming languages, their environments and system software interfaces — Programming language Extended APL

1 Scope

This International Standard defines the programming language APL and the environment in which APL programs are executed. Its purpose is to facilitate interchange and promote portability of APL programs and programming skills. This International Standard specifies the syntax and semantics of APL programs and the characteristics of the environment in which APL programs are executed.

It also specifies requirements for conformance to this International Standard, including the publication of values and characteristics of implementation properties so that conforming implementations can be meaningfully compared.

This International Standard does not specify:

- implementation properties that are likely to vary with the particular equipment or operating system used;
- required values for implementation limits such as APL workspace size or numeric precision;
- the data structures used to represent APL objects;
- the facilities available through shared variables.

2 Normative References

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO/IEC 2382-15:1999, *Information technology – Vocabulary – Part 15: Programming languages*.

ISO 8485:1989, *Programming languages – APL*.

International Register of Coded Character Sets To Be Used With Escape Sequences, Registered character set 68. (<http://www.itscj.ipsj.or.jp/ISO-IR/068.pdf>)

3 Form of this International Standard

This International Standard is a formal model of an APL machine, specified as a collection of finite sets, diagrams, and evaluation sequences, and objects constructed from finite sets, diagrams, and evaluation sequences.

The finite sets are the **implementation-defined character-set**, the **implementation-defined** set of **numbers**, and the enumerated sets **array-type**, **class-names**, **keyboard-states**, **mode-names**, **required-character-set**, and **workspace-presence**.

Diagrams are directed graphs used to designate syntactic forms.

Evaluation sequences are formal procedures that operate on finite sets, diagrams, other evaluation sequences and objects defined in the International Standard.

Objects are entities consisting of enumerated set members and other objects. The objects are **list**, **array**, **defined-function**, **defined-operator**, **token**, **symbol**, **context**, **workspace**, **session**, **shared-variable**, and **system**.

Each object has attributes describing its state. The attributes of an array, for example, are its typical element, its shape, and its ravel.

Objects often have defined properties derived from their attributes. The rank of an array, for example, is the shape of the shape of the array.

3.1 Form of Definitions

Defined terms in this International Standard are always set in **bold** and indexed. The index entry begins with the page number of the definition followed by the page numbers of all references to the term. If the definition and a use of a term occur on the same page, that page number will occur twice in the Index.

The following terms occur throughout the document and are not cross-indexed: **character**, **content**, **class**, **item**, and **number**.

Note: Terms in this document include both phrases such as **implementation-parameter** and words

such as **nil**.

Each definition in this International Standard takes one of four forms.

1. Regular definitions consist of the term being defined followed by a colon and the body of the definition. The term **Boolean** is defined in this way.
2. Members of an enumerated set are defined simply by being listed in the definition of the enumerated set. The term **nil** is defined in this way, as a member of the enumerated set **class-names**.
3. Diagrams are defined by directed graphs. The term **expression** is defined in this way.
4. Definitions of terms that designate evaluation sequences take the following form:
 - The term being defined, such as **scan**.
 - The forms that the evaluation mechanism used in the International Standard recognises as designating this term, such as $Z \leftarrow f \setminus B$.
 - An informal introduction indicating the purpose of the procedure. The informal introduction is considered commentary on the International Standard.
 - An evaluation sequence expressed in a formal, though English-like, language defined in the subsection **Evaluation Sequences**. A **conforming-implementation** is required to emulate the behaviour described in the evaluation sequence as modified by the **additional-requirements**, if any.
 - **Examples**, which show effects of the procedure specified by the evaluation sequence. Examples are considered commentary on the International Standard.
 - **Additional Requirements**, giving aspects of the behaviour required of this operation that cannot conveniently be expressed in the evaluation sequence.

3.2 Named Arrays in Examples

In the examples in this International Standard, APL identifiers beginning with *N*, such as *N234*, represent numeric arrays whose shape and content are specified by the digits in the identifier. Each digit in the identifier specifies an element of the shape vector; each element of the array, when broken down into digits, gives the index of that element.

For example,

Example:

```

      N4
    1 2 3 4

      N23
    11 12 13
    21 22 23
    
```

N_{234}
 111 112 113 114
 121 122 123 124
 131 132 133 134

 211 212 213 214
 221 222 223 224
 231 232 233 234

 $N_{234}[2; 3; 1]$
 231

3.3 Notes

This International Standard contains notes that comment on the text of the International Standard, pointing out the significance of definitions, noting relationships between definitions, and otherwise making the text approachable. These notes are set in a different type style than the text of the International Standard, and are prefixed with the word “Note:”. The following is an example of a note.

Note: *This is an example of a note.*

Notes never set requirements for conformance. They may suggest desired properties, but such suggestions are not mandatory for conformance.

3.4 Cross-References

The heading levels in this International Standard are chapter, section, and subsection. When cross-references are given, they are always to a subsection title. In the Index, subsections are treated like definitions: the page on which a subsection begins is always the first entry in the Index; subsequent page numbers in the index show where references are made to the subsection.

3.5 General Definitions

For the purpose of this International Standard, the definitions given in ISO/DIS 2382/15 shall apply, together with the following:

– **Program:** An application.

Note: *The term is used in this International Standard to include everything from an APL expression to a collection of workspaces communicating via shared variables.*

- **Implementation:** A combination of computer hardware and software that processes (APL) **programs**.

Note: *An implementation is an instance of the object **system** specified by this International Standard.*

- **Facility** (of an implementation): A unit of behaviour. Every **facility** is one of:
 - **Defined-Facility:** A **facility** fully specified in this International Standard and not designated **optional** or **implementation-defined**.
 - **Optional-Facility:** A **facility** fully specified in this International Standard and designated **optional**.
 - **Implementation-Defined-Facility:** A **facility** not fully specified by this International Standard that is designated **implementation-defined**.
 - **Consistent-Extension:** A **facility** not specified in this International Standard that, for a construct this International Standard specifies as producing an error, gives some effect other than signalling the specified error.

4 Compliance

4.1 Conforming Implementations

An APL **implementation** conforms to this International Standard if it meets the following requirements for both behaviour and documentation.

4.1.1 Required Behaviour for Conforming Implementations

A **conforming-implementation** shall provide all **defined-facilities** and **implementation-defined-facilities**. Each such **facility** shall behave as specified by this International Standard.

A **conforming-implementation** may provide **optional-facilities**. If provided, an **optional-facility** shall behave as specified by this International Standard. Attempted use of an **optional-facility** that is not provided shall cause the **conforming-implementation** to signal an error. A **conforming-implementation** shall not replace the error signalled by a missing **optional-facility** with other behaviour.

A **conforming-implementation** may provide **consistent-extensions**. The presence of a **consistent-extension** shall not affect the behaviour of a **conforming-program**.

A **conforming-implementation** shall use algorithms that produce results that are the same as those produced by the evaluation sequences. Mathematical function algorithms shall have at least the accuracy that the algorithms given in the evaluation sequences would produce.

Note: The evaluation sequences used in this International Standard are intended to specify results, not implementation techniques.

The errors produced by the absence of an **optional-facility** cannot be replaced by **consistent-extensions** in a **conforming-implementation**, since this would affect the behaviour of **conforming-programs** that use the **optional-facility**.

4.1.2 Required Documentation for Conforming Implementations

A **conforming-implementation** shall provide a reference document that satisfies the following requirements for the documentation of **optional-facilities**, **implementation-defined-facilities**, and **consistent-extensions**:

4.1.2.1 Documentation of Optional-Facilities

A **conforming-implementation** shall document the presence or absence of each of the following **optional-facilities**:

- **Shared-Variable-Protocol**. A mechanism that permits one session to exchange data with other autonomous sessions.
- **Statement-Separator-Facility**. A mechanism for placing more than one statement on a line.
- **Trace-and-Stop-Control**. A mechanism that assists the testing and correction of defined functions.
- **Complex-Arithmetic-Facility**. A mechanism that permits the generation and manipulation of complex-arithmetic results.

4.1.2.2 Documentation of Implementation-Defined-Facilities

A **conforming-implementation** shall document the following aspects of **implementation-defined-facilities**:

- A description of the **character-set**. This shall include a table showing the correspondence between index positions in the **atomic-vector** and the members of the **required-character-set**. If the graphic symbols used in a **conforming-implementation** are dissimilar to those in **Table 1**, the correspondence between the graphic symbols used in the implementation and those in **Table 1** shall be given.
- A description of the **numbers**. This shall include a characterisation of the internal representation used for **numbers**.
- Descriptions of the characteristics of each **implementation-algorithm**.
- The value of each **implementation-parameter**.
- A description of each **internal-value-set**.
- A description of additional **event-types**.

4.1.2.3 Consistent Extensions

A **conforming-implementation** shall document all **consistent-extensions** it provides. The documentation shall clearly indicate that the use of a **consistent-extension** prevents a **program** from conforming with this International Standard.

Note: Implementers of **conforming-implementations** should, in general, be wary of replacing **limit-errors** with **consistent-extensions**, since these errors are the only safeguards a **conforming-program** has when attempting to operate in a **conforming-implementation** whose **implementation-parameters** are inadequate to support it.

For example, if the **limit-error** on **identifier-length-limit** were not signalled, a **conforming-program** with identifiers longer than the local **identifier-length-limit** could malfunction without warning.

4.2 Conforming Programs

A **program** conforms to this International Standard if it meets the following requirements for both behaviour and documentation.

4.2.1 Required Behaviour for Conforming Programs

A **conforming-program** shall use only those **facilities** specified in this International Standard.

A **conforming-program** shall not use **consistent-extensions**.

A **conforming-program** shall not depend on the signalling of any error by a **conforming-implementation**.

Note: **Conforming-programs** cannot depend on error behaviour, since **consistent-extensions** that replace errors are permitted in **conforming-implementations**. A conforming program may make use of certain event handling features unreservedly (for example, $\square ES$), and others with some restrictions. Consider the following example:

$$Z \leftarrow '1' \quad \square EA \quad 'A \quad \square SVO \quad B'$$

$$Z \leftarrow (0 \ 1 \ 2 \ \neg 1) [0 \ 1 \ 2 \neg Z]$$

This is conforming because it only depends on the values 0, 1, and 2, which have defined meanings when returned as the result of $\square SVO$. The first line can return anything, but the second line filters out all but 0, 1, and 2, hence eliminating any subtle dependencies on error behaviour or consistent extensions.

4.2.2 Required Documentation for Conforming Programs

A **conforming-program** shall document which **optional-facilities** it requires.

A **conforming-program** shall document any specific requirements it has for **implementation-parameters**.

Note: A **conforming-program** may or may not work, and may or may not produce identical results on all **conforming-implementations**, because of inherent dependencies on **implementation-parameters** or **implementation-algorithms** such as the algorithm used for matrix inversion.

It is suggested that **conforming-programs** provide documentation detailing the values of **implementation-parameters** they require so that their suitability for a given **conforming-implementation** can be determined readily.

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5 Definitions

5.1 Characters

- **Character-Set**: An **implementation-defined** finite set.
- **Character**: A member of the **implementation-defined** finite set **character-set**.
- **Required-Character-Set**: An enumerated set which is the union of a set of alphabets with the sets of Numeric Characters and Special Characters designated in **Table 1**. The alphabets may be any alphabet; this International Standard uses the Alphabetic Characters shown in **Table 1** to represent the dual-case Latin alphabet. The **character-set** provided by a **conforming-implementation** shall contain all members of the **required-character-set** which results from the choice of alphabet.

Additional Requirement:

Members of the **required-character-set** are represented in this International Standard by graphic symbols in a particular typeface, as shown in **Table 1**. The graphic symbols associated with the **required-character-set** in a **conforming-implementation** are **implementation-defined**.

Note: There are conformance requirements associated with the **required-character-set**. A **conforming-implementation** must publish, as part of its required documentation, a table giving the correspondence between the graphic symbols in **Table 1** and the **atomic-vector**. Where the graphic symbols provided are not similar in appearance to those used in this International Standard, the correspondence between the graphic symbols in this International Standard and those provided by the implementation must also be provided.

Some of the graphic symbols given in **Table 1** are not used to designate APL constructs in this International Standard. They are present to provide all keys on common terminal keyboards with corresponding symbols.

The names in **Table 1** are not to be considered part of this International Standard.

Alphabetic Characters

A B C D E F G H I J K L M N O P Q R S T U V W
X Y Z
A B C D E F G H I J K L M N O P Q R S T U V W
X Y Z

Numeric Characters

0 1 2 3 4 5 6 7 8 9

Special Characters

α alpha	∇ del tilde	⌘ quote quad
↓ down arrow	Δ delta	ρ rho
← left arrow	⏞ delta stile	⋮ semicolon
→ right arrow	⏟ delta underbar	⌋ down shoe
↑ up arrow	¨ diaeresis	⌋ left shoe
– bar	◇ diamond	⌋ right shoe
blank	÷ divide	⌋ up shoe
{ left brace	\$ dollar sign	⌋ up shoe jot
} right brace	· dot	/ slash
[left bracket	ε epsilon	\ back slash
] right bracket	= equal	/ slash bar
∨ down caret	≥ greater-than or equal	\ back slash bar
⋈ down caret tilde	ι iota	* star
< left caret	◊ jot	stile
> right caret	≤ less-than or equal	⌋ down stile
^ up caret	× multiply	⌋ up stile
⋈ up caret tilde	≠ not equal	⊥ up tack
○ circle	— overbar	⊥ up tack jot
⌘ circle backslash	(left parenthesis	⌋ right tack
⊖ circle bar	) right parenthesis	⌋ left tack
⊗ circle star	+ plus	⌋ down tack
⊘ circle stile	□ quad	⌋ down tack jot
: colon	⊞ quad divide	~ tilde
, comma	? query	— underbar
⌘ comma bar	¨ diaeresis jot	¨ diaeresis tilde
∇ del	‘ quote	ω omega
∇ del stile	! quote dot	≡ equal underbar

Table 1: The Required Character Set

5.2 Numbers

- **Number-Set:** An **implementation-defined** finite set used to represent arithmetic quantities.
- **Number:** A member of the **number-set**.

Note: The **number-set** is an abstraction used in this document to represent the floating-point arithmetic quantities of an arbitrary computer.

5.2.1 Elementary Operations

Elementary-Operation: One of the following four **implementation-algorithms**.

- *A Plus B.*

Note: **Plus** implements addition.

- *A Minus B.*

Note: **Minus** implements subtraction.

- *A Times B.*

Note: **Times** implements multiplication.

- *A Divided-by B.*

Note: **Divided-by** implements division.

Note: Each **elementary-operation** is a mapping from the Cartesian product of the **number-set** with itself back onto the union of the **number-set** and the **metaclass error**, as described in the subsection **Implementation-Algorithms**. It is assumed that members of the **metaclass error** are returned by **elementary-operations** when results cannot be represented as **numbers**. In particular, **exponent-overflow** is returned when the result of an **elementary-operation** is too large in magnitude to be represented by a **number**, **exponent-underflow** is returned when the result of an **elementary-operation** is non-zero but too small in magnitude to be represented by a **number** other than **zero**, and **domain-error** is returned by an **elementary-operation** called with arguments for which its mathematical counterpart is undefined, such as **one divided-by zero**.

The treatment of **exponent-overflow** and **exponent-underflow** is not specified by this International Standard, except as suggestions to the implementer. Implementers should avoid having **exponent-overflow** and **exponent-underflow** occur in the intermediate results of **implementation-algorithms** wherever possible. In any case, **exponent-underflow** should not cause a **limit-error**.

The fact that these fundamental algorithms are effectively undefined in the International Standard is intentional. A definition general enough to cover all known and possible floating-point systems was felt to be less useful than the requirement that the internal representation for **numbers** in a **conforming-implementation** be described in the **required-documentation** for the implementation.

5.2.2 Number Constants

Note: The following **numbers**, used in this International Standard, are defined here as terms to make clear the distinction between APL constant arrays represented as strings of APL digits, such as 1, and members of the the **implementation-defined** finite set containing **numbers**, such as **one**.

These terms are always set in **bold** when they are used in this formal sense. They are not cross-indexed.

- **Zero:** A **number** such that, for any **number** A , A **plus zero** is A .
- **One:** A **number** such that, for any **number** A , A **times one** is A .
- **Negative-One:** **Zero minus one**.
- **Two:** **One plus one**.
- **Three:** **Two plus one**.
- **Four:** **Three plus one**.
- **Five:** **Four plus one**.
- **Six:** **Five plus one**.
- **One-half:** **One divided-by two**.
- **Imaginary-One:** **Negative-One to-the-power one-half**.

5.2.3 Subsets of the Set of Numbers

Note: The following **subsets** of the numbers are used to define the domains of operations.

- **Boolean:** **zero** or **one**.

Note: *False* is represented by **zero**, *true* by **one**.

Note: A **counting-number** is a member of the subset of the **numbers** accessible under a successor function.

- **Positive-Counting-Number:** **one** or any **number** that can be generated by A **plus one**, where A is a **positive-counting-number**.
- **Negative-Counting-Number:** The **number negative-one** or any **number** that can be generated by N **plus negative-one**, where N is a **negative-counting-number**.
- **Nonnegative-Counting-Number:** A **positive-counting-number** or **zero**.

- **Counting-Number:** A **negative-counting-number** or a **nonnegative-counting-number**.

Note: An **integer** is a member of the subset of the **numbers** accessible from **zero** and **one** through addition and subtraction.

- **Positive-Integer:** A **positive-counting-number** or a **number** that can be generated by $A \text{ plus } B$, where A and B are **positive-integers**.
- **Negative-Integer:** A **negative-counting-number** or a **number** that can be generated by $A \text{ plus } B$, where A and B are **negative-integers**.
- **Nonnegative-Integer:** A **positive-integer** or **zero**.
- **Integer:** A **negative-integer** or a **nonnegative-integer**.
- **Positive-Number:** A **positive-integer**, or a **number** other than **zero** that can be generated by $A \text{ times } B$, by $A \text{ divided-by } B$, or by $A \text{ plus } B$, where A and B are **positive-numbers**.
- **Negative-Number:** A **negative-integer**, or a **number** other than **zero** that can be generated by $A \text{ times } B$, by $A \text{ divided-by } B$, or by $C \text{ plus } D$, where A is a **positive-number** and B , C and D are **negative-numbers**.
- **Nonnegative-Number:** A **positive-number** or **zero**.
- **Real-Number:** A **negative-number** or a **nonnegative-number**.
- **Complex-Number:** A **real-number**, or a **number** that can be generated by $A \text{ plus } (B \text{ times imaginary-one})$, where A and B are **real-numbers**.
- **Complex-Integer:** A **complex-number** that can be generated by $A \text{ plus } (\text{imaginary-one times } B)$, where A and B are **integers**.
- **Half-plane:** Two numbers are in the same **half-plane** if the signs of either their **real-parts** or **imaginary-parts** are nonzero and are equal.
- **Unit-square:** A **unit-square** contains all numbers for which the integral part of the real parts are the same and the integral parts of the imaginary parts are the same.
- **Arc:** The **arc** of a number is **zero** if the number is **zero**, and otherwise is the **imaginary-part** of the **natural-logarithm** of the number, and has a value greater than minus pi and less than or equal to pi.
- **Fraction:** A **number** lying in the rectangle of the **complex-plane** bounded by the two parallel lines passing through **zero** and **one** at an angle of 135 degrees with the real axis, and the two parallel lines passing through **one** and **negative-one** at an angle of forty-five degrees with the real axis, including the **numbers** on the lines going through **zero** and **negative-one**, but not including the **numbers** on the two lines through **one**.

- **First-Quadrant-Associate:** Either a **nonnegative-number**, or a **number** with both **real-part** and **imaginary-part positive-numbers**, and having the same **magnitude** as a given nonzero **number**, and separated from it by an angle of either 0, 90, 180, or 270 degrees.

Note: The **numbers** are assumed to be identical to the **complex-numbers**.

Example

Consider a normalised base-2 floating point number system with a two position exponent field and a two position mantissa field with the radix point between the digits. The decimal values representable with this system and the categories into which each falls are as follows:

Exponent in Base 2	Mantissa in Base 2	Value in Base 10	Boolean	Counting- Number	Integer
00	00	0	Y	Y	Y
00	10	1	Y	Y	Y
00	11	1.5			
01	10	2		Y	Y
01	11	3		Y	Y
10	10	4		Y	Y
10	11	6			Y
11	10	8			Y
11	11	12			Y

As can be seen in this example, **counting-numbers** form a dense subset of the **integers**; the largest **counting-number** is typically a power of the number system base.

A given number, such as a **counting-number**, may have several hardware representations. Except for their effect on system resources, these representations should be indistinguishable to a **conforming-program**.

5.2.4 Implementation Algorithms

An **Implementation-Algorithm** is an algorithm used in this International Standard whose behaviour is **implementation-defined**. The following implementation algorithms are used in this International Standard.

- **Conjugate**
- **Cosine**
- **Current-Time**
- **Deal**
- **Display**
- **Divided-by**

- Exponential
- Function-Display
- Gamma-Function
- Greatest-Common-Divisor
- Hyperbolic-Cosine
- Hyperbolic-Sine
- Hyperbolic-Tangent
- Inverse-Cosine
- Inverse-Hyperbolic-Cosine
- Inverse-Hyperbolic-Sine
- Inverse-Hyperbolic-Tangent
- Inverse-Sine
- Inverse-Tangent
- Magnitude
- Matrix-Divide
- Minus
- Modulo
- Natural-Logarithm
- Next-Definition-Line
- Numeric-Input-Conversion
- Numeric-Output-Conversion
- Pi-Times
- Plus
- Produce-Canonical-Representation-Vector
- Pseudorandom-Number-Generator
- Read-Keyboard
- Sine
- Tangent
- Times

- Time-Stamp
- To-the-Power
- Trace-Display
- Typical-Element-For-Mixed

The following **implementation-algorithms** take a **number** as an argument and return either a **number** or an **error**: **Cosine**, **Exponential**, **Gamma-Function**, **Hyperbolic-Cosine**, **Hyperbolic-Sine**, **Hyperbolic-Tangent**, **Inverse-Cosine**, **Inverse-Hyperbolic-Cosine**, **Inverse-Hyperbolic-Sine**, **Inverse-Hyperbolic-Tangent**, **Inverse-Sine**, **Inverse-Tangent**, **Magnitude**, **Natural-Logarithm**, **Pi-Times**, **Sine**, **Tangent**.

The following **implementation-algorithms** take two **numbers** as arguments and return either a **number** or an **error**: **Divided-by**, **Minus**, **Modulo**, **Plus**, **Times**, **To-the-Power**.

The properties of the remaining **implementation-algorithms** are described in the chapters in which they are used.

5.2.5 Defined Operations

- ***A* Equals *B***: An operation that, for *A* and *B* **numbers**, returns **one** if *A* and *B* are the same **number**, and **zero** otherwise.
- **Direction of *A***: An operation that, for *A* a **number**, returns **zero** if *A* is **zero**, and otherwise returns the **number** determined by the radial projection of *A* onto the **unit-circle**.

Note: For *A* a **real-number**, the **direction** of *A* is either **negative-one** or **zero** or **one**.

- ***A* is Greater-Than *B***: An operation that, for *A* and *B* **real-numbers** returns **one** if *A* is a **positive-number** and *B* is a **negative-number**, returns **one** if *A* minus *B* is a **positive-number**, and returns **zero** otherwise.
- ***A* is Less-Than *B***: An operation that, for *A* and *B* **real-numbers**, returns *B* **greater-than** *A*.
- **Negation of *A***: An operation that, for any **number** *A*, returns **zero minus** *A*.
- **Magnitude of *A***: An operation that, for *A* a **number**, returns the **non-negative real-number** determined by rotating *A* onto the **nonnegative real-axis**.
- **Open-Interval-Between *A* and *B***: An operation that, for any **numbers** *A* and *B*, returns a subset of the **numbers** as follows: if *A* is not **greater-than** *B*, the set of all **numbers** **greater-than** *A* and **less-than** *B*; otherwise, the **open-interval-between** *B* and *A*.
- **Closed-Interval-Between *A* and *B***: An operation that, for any **numbers** *A* and *B*, returns a subset of the **numbers** consisting of *A*, *B*, and the **open-interval-between** *A* and *B*.
- **Larger-Magnitude of *A* and *B***: An operation that, for any **numbers** *A* and *B*, returns the **magnitude** of *A* if it is **greater-than** that of *B*, and the **magnitude** of *B* otherwise.

- **Distance-Between** A and B : An operation that, for any two **numbers** A and B , returns the **magnitude** of A **minus** B .
- A is **Tolerantly-Equal** to B **Within** C : An operation that, given three **numbers** A , B , and C , with C greater than or equal to zero, returns a **Boolean** Z determined as follows:
 If A **equals** B , then Z is **one**.
 If A and B are not in the same **half-plane**, then Z is **zero**.
 If the **distance-between** A and B is **less-than** or **equals** C **times** the **larger-magnitude** of A and B , then Z is **one**.
 Otherwise, Z is **zero**.
- **Tolerant-Floor** of A **Within** B : An operation that, for A a **number** and B a **nonnegative-number**, returns a **complex-integer** Z determined as follows:
 Let A be a member of the set of **numbers** in the **unit-square** at the **complex-integer** C , and let D be A **minus** C .
 If the sum of the real and imaginary parts of D is **tolerantly-less-than one** **within** B , then Z is C .
 Otherwise, if the **imaginary-part** of D is **greater-than** the **real-part** of D , then Z is C **plus imaginary-one**.
 Otherwise, Z is C **plus one**.
- A is **Integral-Within** B : An operation that, for a **number** A and a **positive-number** B , returns a **Boolean** Z determined as follows:
 Let C stand for the **negation** of A .
 Z is **one** if the **tolerant-floor** of C **within** B **equals** the **negation** of the **tolerant-floor** of A **within** B , and **zero** otherwise.
- A is a **Near-Integer**: An operation that, for a **number** A , returns **one** if A is **integral-within integer-tolerance**, and **zero** otherwise.

Note: The foregoing definition contains a forward reference to **integer-tolerance**.

- **Integer-Nearest-to** A : An operation that, for a **near-integer** A , returns the **tolerant-floor** of A **within integer-tolerance**.
- A is **Near-Boolean**: An operation that, for a **near-integer** A , returns **one** if the **integer-nearest-to** A is a **Boolean**, and **zero** otherwise.

Note: Near-integers and near-Booleans include **numbers** whose **magnitude** is **less-than integer-tolerance**. The operation **integer-nearest-to** maps such numbers to **zero**.

- A is **Real-Within** B : An operation that, for a **number** A and a **positive-number** B , returns a **Boolean** Z determined as follows:
 Let R stand for the **magnitude** of the **real-part** of A , and I for the **magnitude** of the **imaginary-part** of A .
 Z is **one** if I is **less-than-or-equal** either to B , or to R **times** B , and is **zero** otherwise.
- A is **Near-Real**: An operation that, for a **number** A , returns **one** if A is **real-within real-tolerance**, and **zero** otherwise.

5.3 Objects

Note: The description of APL in this document is based on objects—abstract data structures that are described in terms of characters, numbers, and elementary set theory.

Each object has a small set of named attributes that take on values that are characters, numbers, or other objects.

Each object has additionally a small set of operations that can be performed upon it.

All the objects in the description are defined here, as are all the attributes of each object. The operations are introduced as they are needed. All objects, attributes, and operations are cross-referenced in the index.

5.3.1 Lists

Index: A **nonnegative-counting-number** less than or equal to **index-limit**.

List: An object with the following attributes:

- **Index-Set:** A finite set I of **positive-counting-numbers** chosen so that for every subset of I , except the empty set, the cardinality of that subset is in I .
- **Value-Set:** A finite set in a specific correspondence with the **index-set** of the **list**.

Empty-List: A **list**, the cardinality of whose **index-set** is **zero**.

Nonempty-List: A **list**, the cardinality of whose **index-set** is **greater-than zero**.

Number-of-Items in L : The cardinality of the **index-set** of the **list** L .

Item X of L : An operation that, for X a member of the **index-set** of the **list** L , returns the member of the **value-set** of L associated with X by the correspondence between the **value-set** of L and the **index-set** of L .

Note: This form of indexing is always in origin one.

First-Item in L : An operation that for a **nonempty-list** L returns **item one** of L .

Last-Item in L : An operation that for a **nonempty-list** L , and for C the **number-of-items** in L , returns **item C** of L .

Rest-of L : An operation that, for a **nonempty-list** L whose **index-set** has cardinality C , returns a second **list** R whose **index-set** has cardinality C **minus one**, such that for each **item** J in the **index-set** of R , **item** J of R is **item** (J **plus one**) of L .

Product-of L : An operation defined on a **list of numbers** L as follows:

If the **number-of-items** in L is **zero**, **one**.

If the **number-of-items** in L is **one**, the **first-item** in L .

If the **number-of-items** in L is **greater-than one**, **first-item** in L **times** the **product-of** the **rest-of** L .

Prefix: A (possibly empty) **list** P is a **prefix** of a **list** L if the **index-set** of P is a subset of the **index-set** of L , and each **item** of P is the same as the corresponding **item** of L .

5.3.2 Arrays

Array-Type: An enumerated set containing the members **character**, **numeric**, and **mixed**.

Array: An object with the following attributes:

– **Shape-list:** A **list** of **nonnegative-counting-numbers**.

– **Ravel-list:** A **list**. Each **item** of the **list** is either a **character**, a **number**, or an **array**.

The **number-of-items** in the **ravel-list** of an **array** A is the same as the **product-of** the **shape-list** of A .

– **Type:** A member of the enumerated set **array-type**.

If the **type** of A is **character**, the **ravel-list** of A is a **list** of **characters**; if the **type** of A is **numeric**, the **ravel-list** of A is a **list** of **numbers**.

Sufficient-Type of L under T : the following operation on a **type** T and a **list** L .

If T is **character** or **numeric**, return T

else if L is a **nonempty-list**

if all items of L are **numbers**, return the **type numeric**

if all items of L are **characters**, return the **type character**

otherwise, return the **type mixed**

else return the **type** of the **typical-element** of L .

Note: The foregoing definition is intended to provide for the cases where a *mixed-array* becomes a *non-mixed array*, and vice-versa.

Rank of A : An operation that, for an **array** or **array-of-vectors** A , returns the **number-of-items** in the **shape-list** of A .

Simple: An **array** is **simple** if each item of its **ravel-list** is either a **character** or a **number**.

Scalar: An **array** whose **rank** is **zero**.

Simple-scalar: A **simple array** whose **rank** is **zero**.

Max-shape-of B An operation that, for B , an **arrays** defined as follows:

Set $B1$ to the **first-thingy** of B .
 Set $M1$ to $\rho B1$.
 If the **count-of** B is **one**, return $M1$.
 Set $B2$ to the **remainder-of** B .
 Set $M2$ to the **max-shape-of** $B2$.
 If the **shape-list** of $M1$ differs from the **shape-list** of $M2$, signal **rank-error**.
 Return $M1 \uparrow M2$

First-thingy in A : An operation that for A , an **array**, returns an **array** B , defined as follows:
 If A is empty, set $B1$ to the **typical-element** of A .
 Otherwise, set $B1$ to the **first-item** of the **ravel-list** of A .
 If $B1$ is a **number** or a **character**, set B to an array, whose **ravel-list** contains the single item $B1$, and whose **shape-list** is empty.
 Otherwise, set B to $B1$.

Numeric-Scalar with value I : For I a **number**, the **scalar** Z such that the **type** of Z is **numeric** and the **ravel-list** of Z is the **list** L such that the **number-of-items** in L is **one** and the **first-item** of L is I .

Count of A : For A an **array**, the **product-of** the **shape-list** of A .

Vector: An **array** whose **rank** is **one**.

Matrix: An **array** whose **rank** is **two**.

Length of A : For A a **vector**, the **count** of A .

Typical-Element of A : For A an **array**, if the **type** of A is **character**, the **character** blank; if the **type** of A is **numeric**, the **number** zero; if the **type** of A is **mixed**, the **typical-element** is determined by an **implementation-defined-algorithm**.

Empty: Of an **array**, the **count** of which is **zero**.

One-Element-Vector: A **vector** A is a **one-element-vector** if the **length** of A is **one**.

K is a **Valid-Axis** of A : An operation that, for A and K **arrays**, returns **one** if K is a **scalar** or **one-element-vector**, and the **first-item** in the **ravel-list** of K is a **near-integer**, the **integer-nearest-to** which is a member of the **index-set** of the **shape-list** of A ; and returns **zero** otherwise.

Axis K of A : An operation that, for A an **array** and K a **valid-axis** of A , returns **item** K of the **shape-list** of A .

Array-of-vectors: An object with the following attributes:

- **Shape-list**: A list of **nonnegative-counting-numbers**.
- **Ravel-list**: Each **vector** V in the **ravel-list** of an **array-of-vectors** A has the **type** **sufficient-type** of the **ravel-list** of V under the **type** of A .

– **Type:** A member of the enumerated set **array-type**.

Note: The term **array-of-vectors** is used only as an expository device.

Along-Axis K of A : An operation that, for an **array** A with non-zero **rank** N , produces Z , an **array-of-vectors** of **rank** $N-1$ such that the **shape-list** of Z is the **shape-list** of A with **item** K omitted. Each **item** of the **ravel-list** of Z is a **vector** whose **length** is the same as **axis** K of A .

Note: Some operations on vectors extend to arrays of greater rank in a manner similar to **scalar-extension**. **Along-axis** is an expository device used in the definition of these operations.

Vector-Item I of A : An operation that, for an **array-of-vectors** A , returns **item** I of the **ravel-list** of A .

Ravel-Along-Axis K of A : An operation that, for an **array** A with non-zero **rank** N , produces an **array-of-vectors** Z such that the **shape-list** of Z is the **product-of** the **shape-list** of A with **item** K omitted, and the **ravel-list** of Z is the **ravel-list of along-axis K of A** .

First-Scalar in A : An operation that, for A a nonempty **array**, returns a **scalar** Z such that the **ravel-list** of Z is the **first-item** in the **ravel-list** of A and the **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of A .

Remainder-of A : An operation that, for A a non-empty **vector**, returns a **vector** Z such that the **length** of Z is **negative-one plus the length** of A , the **ravel-list** of Z is the **rest-of** the **ravel-list** of A and the **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of A .

Row I of A : An operation that, for A an **array** of **rank two** and I a **number**, returns **vector-item I along-axis two** of A .

Number-of-Rows in A : An operation that, for A an **array** of **rank two**, returns **item one** of the **shape-list** of A .

Integer-Array-Nearest-to A : An operation that, for A an **array** having the property that each **item** of the **ravel-list** of A is a **near-integer**, returns a **numeric array** Z such that the **shape-list** of Z is the same as the **shape-list** of A and each **item** of the **ravel-list** of Z is the **integer-nearest-to** the corresponding **item** of the **ravel-list** of A .

Boolean-Array-Nearest-to A : An operation that, for A an **array** having the property that each **item** of the **ravel-list** of A is a **near-Boolean**, returns the **integer-array-nearest-to** A .

5.3.3 Defined-Functions

Note: A **defined-function** represents a function defined by a user. The attributes of a **defined-function** are given here. Operations are described in the **Defined Functions** chapter. **Defined-operators** are a subclass of **defined functions**.

Defined-Function: An object with the following attributes:

- **Canonical-Representation:** A character array whose rank is two.
- **Stop-Vector:** A numeric vector.
- **Trace-Vector:** A numeric vector.

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5.3.4 Tokens

Class-Names: A set containing the following members:

Assignment-Arrow	Local-Name
Axis-Error	Monadic-Operator
Branch	Nil
Branch-Arrow	Niladic-Defined-Function
Character-Literal	Niladic-Defined-Function-Name
Clear-State-Indicator	Niladic-System-Function-Name
Colon	Not-Copied
Command-Complete	Not-Erased
Committed-Value	Not-Found
Complete-Index-List	Not-Saved
Constant	Numeric-Literal
Defined-Function	Partial-Index-List
Defined-Function-Name	Primitive
Defined-Dyadic-Operator	Primitive-Function
Defined-Dyadic-Operator-Name	Rank-Error
Defined-Monadic-Operator	Result-Name
Defined-Monadic-Operator-Name	Right-Argument-Name
Definition-Error	Right-Axis-Bracket
Distinguished-Identifier	Right-End-of-Statement
Domain-Error	Right-Index-Bracket
Dyadic-Operator	Right-Operand-Name
Elided-Index-Marker	Right-Parenthesis
Escape	Semicolon
Implicit-Error	Shared-Variable
Incorrect-Command	Shared-Variable-Name
Index-Error	Simple-Identifier
Index-Separator	Small-Circle
Interrupt	Syntax-Error
Label	System-Function-Name
Label-Name	System-Variable-Name
Left-Argument-Name	Unwind
Left-Axis-Bracket	Value-Error
Left-End-of-Statement	Variable
Left-Index-Bracket	Variable-Name
Left-Operand-Name	
Left-Parenthesis	
Length-Error	
Limit-Error	

Token: An object with the following attributes:

- **Class:** A member of the set **Class-Names**.

- **Content:** A **number**, a **character**, or an object, according to the **class** of the **token**, as indicated in **Table 2**.

Note: **Tokens** represent the internal objects manipulated by an implementation of APL.

The **class** of a **token** is a straightforward indication of the sort of object it represents. A **token** of **class nil**, for example, is used to return a **value-error** from a defined function that does not set its result name. Note that **class-names** is an enumerated set. This means that **nil** has no definition other than its literal appearance on this page. Its significance, like the significance of APL characters in this document, lies in the use of its name in evaluation sequences.

The **content** of a **token** varies with the **class** of the **token**. If specified, the content is a **character**, a **list of characters**, a **list of numbers**, an **array**, an **index-list**, a **shared-variable**, or a **defined-function**.

5.3.4.1 Metaclasses

Note: Metaclasses are subsets of the enumerated set **class-names**. They are used to shorten evaluation sequences by abstracting a sequence of tests on the **class** of a **token** into a single test for membership in a metaclass. “If *B* is an **error**,” is equivalent to “If the **class** of *B* is in the metaclass **error**.”

- **Metaclass:** A subset of the enumerated set **class-names**.
- **Identifier:** A metaclass containing **simple-identifier** and **distinguished-identifier**.
- **Literal:** A metaclass containing **character-literal** and **numeric-literal**.
- **Lexical-Unit:** A metaclass containing **primitive**, **literal**, and **identifier**.
- **Value:** A metaclass containing **committed-value** and **constant**.
- **Delimiter:** A metaclass containing **primitive-function**, **branch-arrow**, **assignment-arrow**, **left-end-of-statement**, **right-end-of-statement**, **left-index-bracket**, **right-index-bracket**, **elided-index-marker**, **left-axis-bracket**, **right-axis-bracket**, **left-parenthesis**, **right-parenthesis**, **small-circle**, and **semicolon**.
- **Defined-Name:** A metaclass containing **shared-variable-name**, **variable-name**, **defined-function-name**, **defined-dyadic-operator-name**, **defined-monadic-operator-name**, **niladic-defined-function-name**, **result-name**, **left-argument-name**, **right-argument-name**, **label-name**, and **local-name**.
- **Defined-Operator:** A metaclass containing **defined-dyadic-operator** and **defined-monadic-operator**.
- **System-Name:** A metaclass containing **system-variable-name**, **system-function-name**, and **niladic-system-function-name**.
- **Classified-Name:** A metaclass containing the members of **system-name** and **defined-name**.
- **Syntactic-Unit:** A metaclass containing the members of **classified-name** and **delimiter**.

Class-Name	Content
Assignment-Arrow	
Axis-Error	
Branch	An array
Branch-Arrow	
Character-Literal	A list of characters
Clear-State-Indicator	
Colon	
Command-Complete	
Committed-Value	An array
Complete-Index-List	An index-list
Constant	An array
Defined-Function	A defined-function
Defined-Function-Name	A list of characters
Defined-Dyadic-Operator	A defined-function
Defined-Dyadic-Operator-Name	A list of characters
Defined-Monadic-Operator	A defined-function
Defined-Monadic-Operator-Name	A list of characters
Definition-Error	
Distinguished-Identifier	A list of characters
Domain-Error	
Dyadic-Operator	A character
Elided-Index-Marker	
Escape	
Implicit-Error	
Incorrect-Command	
Index-Error	
Index-Separator	
Interrupt	
Label	An array
Label-Name	A list of characters
Left-Argument-Name	A list of characters
Left-Axis-Bracket	
Left-End-of-Statement	
Left-Index-Bracket	
Left-Operand-Name	A list of characters
Left-Parenthesis	
Length-Error	
Limit-Error	

Table 2: Relationship between Class-Name and Content

Class-Name	Content
Local-Name	A list of characters
Monadic-Operator	A character
Nil	
Niladic-Defined-Function	A defined-function
Niladic-Defined-Function-Name	A list of characters
Niladic-System-Function-Name	A list of characters
Not-Copied	
Not-Erased	
Not-Found	
Not-Saved	
Numeric-Literal	A list of numbers
Partial-Index-List	An index-list
Primitive	A character
Primitive-Function	A character
Rank-Error	
Result-Name	A list of characters
Right-Argument-Name	A list of characters
Right-Axis-Bracket	
Right-End-of-Statement	
Right-Index-Bracket	
Right-Operand-Name	A list of characters
Right-Parenthesis	
Semicolon	
Shared-Variable	A shared-variable
Shared-Variable-Name	A list of characters
Simple-Identifier	A list of characters
Small-Circle	
Syntax-Error	
System-Function-Name	A list of characters
System-Variable-Name	A list of characters
Unwind	
Value-Error	
Variable	An array
Variable-Name	A list of characters

Table 2: (Continued)

- **Error:** A metaclass containing **axis-error**, **domain-error**, **implicit-error**, **index-error**, **length-error**, **limit-error**, **rank-error**, **syntax-error**, **value-error**, and **interrupt**.

Note: This metaclass includes only errors that occur in the evaluation of APL statements.

- **Report:** A metaclass containing **incorrect-command**, **not-copied**, **not-erased**, **not-found**, and **not-saved**.
- **Exception:** A metaclass containing **branch**, **escape**, **clear-state-indicator**, **unwind**, and the members of **error** and **report**.
- **Result:** A metaclass containing **nil** and the members of **exception** and **value**.

5.3.4.2 Index-List

- **Index-List:** A (possibly empty) **list** consisting of **tokens** whose **class** is either **constant** or **elided-index-marker**.

5.3.5 Symbols

Symbol: An object with the following attributes:

- **Name:** A **list** of **characters**.
- **Referent-List:** A **list** of **tokens**.

5.3.6 Contexts

- **Mode-Names:** An enumerated set containing the members **immediate-execution**, **execute**, **function-definition**, **quad-input**, and **defined-function**.

Context: An object with the following attributes:

- **Mode:** An element of the enumerated set **mode-names**.
- **Stack:** A **list** of **tokens**.
- **Current-Line:** A **list** of **characters**.
- **Current-Statement:** A **list** of **tokens**.
- **Current-Function:** If **mode** is **defined-function**, a **defined-function**; otherwise undefined.
- **Current-Line-Number:** If **mode** is **defined-function**, an **index**; otherwise undefined.

5.3.7 Workspaces

Note: *The workspace is the basic organisational unit in an APL system. A workspace contains data, programs, execution status, and environmental information.*

- **Workspace-Presence:** An enumerated set containing **absent** and **present**.

Workspace: An object with the following attributes:

- **Owner:** A **user-identification**.
- **Workspace-Name:** A **list** of **characters**.
- **Symbol-Table:** A **list** of all **symbols** whose **names** are distinct.
- **State-Indicator:** A **list** of **contexts**.
- **Existential-Property:** A member of the enumerated set **workspace-presence**.

Clear-Workspace: A **workspace** with the following values:

- **Owner:** **this-owner**.
- **Workspace-Name:** The **clear-workspace-identifier**.
- **Symbol-Table:** A **list** of all **symbols** whose **names** are distinct and whose **referent-lists** each consist of the **list** whose only member is **nil**.
- **State-Indicator:** An empty **list** of **contexts**.
- **Existential-Property:** **absent**.

5.3.8 Sessions

Note: *A user interacts with an APL system through a session, an abstraction that represents a hypothetical machine capable of carrying out the evaluation sequences in the International Standard.*

- **Session-Identification:** Either a **number** or a **list** of **characters**, depending upon the **implementation-parameter session-identification-type**.
- **User-Identification:** Either a **number** or a **list** of **characters**, depending upon the **implementation-parameter user-identification-type**.

- **Keyboard-States:** An enumerated set containing the members **open-keyboard** and **locked-keyboard**.

Session: An object with the following attributes:

- **Active-Workspace:** A **workspace**.
- **This-Session:** A **session-identification**.
- **This-Owner:** A **user-identification**.
- **Attention-Flag:** A member of the enumerated set **Boolean**.
- **Keyboard-State:** A member of the enumerated set **keyboard-states**.
- **Current-Prompt:** A **character vector**.
- **Quote-Quad-Prompt:** A **character vector**.
- **Event-Time:** A **nonnegative-number**.
- **Event-Message:** A **character array** giving, in an **implementation-defined** form, the **event-report** and any other information the implementation deems helpful in locating the error's source.
- **Event-Type:** A two-element integer vector corresponding to the most recent event. Defined values are:

- 0 0 — No error
- 0 1 — Undefined event

There may be additional **implementation-defined** values as well as values supplied by a program via $\square ES$.

- **Current-Context:** The **first-item** in the **state-indicator** of the **active-workspace**.
- **Current-Stack:** The **stack** of the **current-context**.
- **Symbol-Named-By T :** The **symbol** in the **symbol-table** of the **active-workspace** whose **name** is the same as the **content** of the **token T** .
- **Current-Referent of T :** The **first-item** in the **referent-list** of the **symbol-named-by T** .
- **Current-Class of T :** The **class** of the **current-referent** of T .
- **Current-Content of T :** The **content** of the **current-referent** of T .
- **Comparison-Tolerance:** The **current-content** of $\square CT$.
- **Random-Link:** The **current-content** of $\square RL$.

- **Print-Precision:** The **current-content** of $\square PP$.
- **Index-Origin:** The **current-content** of $\square IO$.
- **Latent-Expression:** The **current-content** of $\square LX$.

System-Parameter: Any of **comparison-tolerance**, **random-link**, **print-precision**, **index-origin**, or **latent-expression**.

The initial values of **system-parameters** in a **clear-workspace** are **implementation-defined**.

5.3.9 Shared-Variables

Note: A **shared-variable** is a variable shared between two **sessions**.

Shared Variables are an **optional-facility**.

Shared-Variable: An object with the following attributes:

- **Session-A:** A **session-identification**.
- **Session-A-Active:** A **Boolean**.
- **Session-A-ACV:** A **Boolean vector** of **length four**.

Note: ACV stands for *access control vector*.

- **Session-B:** A **session-identification**.
- **Session-B-Active:** A **Boolean**.
- **Session-B-ACV:** A **Boolean vector** of **length four**.
- **Shared-Name:** An **identifier**.
- **Shared-Value:** A **token**, either a **constant** or **nil**.

State: An **integer**, either **zero**, **one**, or **two**.

- **Session-A-Event:** A **Boolean**.
- **Session-B-Event:** A **Boolean**.

Note: The operations in the chapter **Shared Variables** use the **shared-variable** attributes as follows:

- **Session-A:** The **session-identification** of the first session to offer the **shared-variable**.

- **Session-A-Active:** A Boolean; **one** if session-A is currently sharing this **shared-variable**, **zero** if not.
- **Session-A-ACV:** The contribution of session-A to the ACV.
- **Session-B:** The **session-identification** of the **session** with which session-A offered to share the **shared-variable**; this may be the **general-offer** while **state** is **one**.
- **Session-B-Active:** A Boolean; **one** if session-B is currently sharing this **shared-variable**, **zero** if not.
- **Session-B-ACV:** The contribution of session-B to the ACV.
- **Shared-Name:** An **identifier**, the name session-A designated for this **shared-variable**.
- **Shared-Value:** A **token** of class **constant** or **nil**; the current value of this **shared-variable**.
- **State:** An **integer**, either **zero**, **one**, or **two**, used in combination with the ACV to determine which operations must be delayed.

5.3.10 Systems

Note: A **system** represents a set of active APL users, a library, and, optionally, a shared variable facility.

System: An object with the following attributes:

- **Library:** A list of **workspaces** in which each combination of possible values for the attributes **owner** and **workspace-name** occurs exactly once.
- **Active-Users:** A list of **sessions**.
- **Shared-Variable-List:** A list of **shared-variables**.
- **Implementation-Parameters:** The following quantities, referred to in this International Standard by name, whose values are **implementation-defined**:
 - **Atomic-Vector:** An **implementation-defined character vector** containing every member of the **required-character-set** exactly once.
 - **Initial-Comparison-Tolerance:** A member of the **internal-value-set** for **comparison-tolerance** that is the value of **comparison-tolerance** in a **clear-workspace**.
 - **Initial-Index-Origin:** A member of the **internal-value-set** for **index-origin** that is the value of **index-origin** in a **clear-workspace**.
 - **Initial-Latent-Expression:** A member of the **internal-value-set** for **latent-expression** that is the value of **latent-expression** in a **clear-workspace**.
 - **Initial-Print-Precision:** A member of the **internal-value-set** for **print-precision** that is the value of **print-precision** in a **clear-workspace**.
 - **Initial-Random-Link:** A member of the **internal-value-set** for **random-link** that is the value of **random-link** in a **clear-workspace**.

- **Initial-Event-Message:** The **empty-event-message**.
- **Reduction-Style:** One of the two symbolics **Enclose-Reduction-Style** or **Insert-Reduction-Style**, indicating which of the two styles of reduction the system uses.
- **Initial-Event-Type:** The two-element integer vector $(0, 0)$.
- **Clear-Workspace-Identifier:** A list of characters.
- **Positive-Number-Limit:** The **real-number** greater-than all other **real-numbers**.
- **Negative-Number-Limit:** The **real-number** less-than all other **real-numbers**.
- **Positive-Counting-Number-Limit:** The **counting-number** greater than all other **counting-numbers**.
- **Negative-Counting-Number-Limit:** The **counting-number** less-than all other **counting-numbers**.
- **Index-Limit:** The **index** greater than all other indices. This value specifies the maximum value of any **item** of the **shape-list** of any **array**, ignoring storage limitations.
- **Count-Limit:** An **index** not greater than **index-limit** that specifies the maximum value for the **count** of an array, ignoring storage limitations.
- **Rank-Limit:** An **index** not greater than **count-limit** that specifies the maximum value for the **rank** of any array, ignoring storage limitations.
- **Workspace-Name-Length-Limit:** A **positive-counting-number** that specifies the maximum number of **characters** in a **workspace-name**.
- **Identifier-Length-Limit:** A **positive-counting-number** not greater than **count-limit** that specifies the maximum number of **characters** in an **identifier**.
- **Quote-Quad-Output-Limit:** A **nonnegative-counting-number** that specifies the maximum number of **characters** that can be used in a prompt set by **Quote Quad Output**.
- **Comparison-Tolerance-Limit:** The largest **real-number** permitted by the implementation for the system parameter **comparison-tolerance**.
- **Integer-Tolerance:** A **real-number** not greater than **comparison-tolerance-limit** used to determine whether a given number is to be considered integral.
- **Real-Tolerance:** A **nonnegative-number** not greater than **comparison-tolerance-limit** used to determine whether a given number is to be considered real.
- **Full-Print-Precision:** The smallest **positive-counting-number** which, when used as the value of **print-precision**, causes **numeric-output-conversion** to produce a unique **numeric-scalar-literal** for every **number**.
- **Print-Precision-Limit:** The largest **positive-counting-number** permitted by the implementation for the system parameter **print-precision**.

Note: **Print-precision-limit** must be at least **full-print-precision** if every unique **number** is to have a unique decimal representation.

- **Exponent-Field-Width:** A **positive-counting-number** giving the number of places, including sign and trailing blanks, used for the exponent field in the result of **dyadic format**.

- **Session-Identification-Type**: A member of the enumerated set **array-type**, either **character** or **numeric**.
- **User-Identification-Type**: A member of the enumerated set **array-type**, either **character** or **numeric**.
- **Indent-Prompt**: A list of **characters** used to indicate to the user that the **session** is in **immediate-execution** mode.
- **Quad-Prompt**: A list of **characters** used to indicate to the user that the **session** is in **quad-input** mode.
- **Function-Definition-Prompt**: A list of **characters** used to indicate to the user that the **session** is in **function-definition** mode.
- **Line-Limit**: A **positive-counting-number** that specifies the maximum number of lines permitted in a defined function, ignoring storage limitations.
- **Definition-Line-Limit**: A **positive-number** that specifies the maximum value of a line number in **function-definition** mode.
- **General-Offer**: A reserved **session-identification** used to indicate that the offerer of a **shared-variable** is willing to share the proffered variable with any other **session**. This **implementation-parameter** is required only if the **optional-facility shared-variable-protocol** is present.

Any action that would cause a limit specified by an implementation parameter to be exceeded shall signal a **limit-error**.

5.4 Evaluation Sequences

The evaluation sequences that define APL operations in the remainder of this International Standard are written in English in the imperative mood. The English phrases used in evaluation sequences are restricted to the set specified in this subsection.

Indentation is used in evaluation sequences to indicate scope, typically of the consequent of an implication.

For example, in the evaluation sequence fragment below, the indented text is evaluated only if both *A* and *B* are **vectors**; the “otherwise” clause is evaluated only if at least one of *A* or *B* is not a **vector**.

```
...
  If A is a vector and B is a vector,
    If A is empty and ...
  ...
  Otherwise, return ...
```

Expressions in APL are used in evaluation sequences. A given evaluation sequence uses only APL operations that have been specified earlier in the International Standard. Where indices are generated or used by APL expressions in evaluation sequences, they are evaluated with origin **one**. The APL relational operations are never used in evaluation sequences unless they are qualified with the value to be used for **comparison-tolerance**.

5.4.1 Evaluation Sequence Phrases

Note: The following phrases are used in evaluation sequences. They are not set in **bold** type nor cross-referenced when employed in evaluation sequences.

- **For all** A, C : An evaluation sequence phrase used to specify that the action or condition specified by the consequent C is to be performed or checked for every value in the antecedent A .

Example:

...
 For all I in the **index-set** of A ,
 Set **item** I of the **ravel-list** of A to **zero**.
 ...

- **For form** F, C :
- **For pattern** F, C :

An evaluation sequence phrase used to specify that the actions listed in the consequent C are to be performed only if the pattern or form F is the one that caused this evaluation sequence to be selected.

Example:

For form $A \ \phi B$
 If B is a **scalar** ...
 ...
 For form $A \ \ominus B$
 If ...

- **If** A, C : An evaluation sequence phrase used to specify that the actions listed in the consequent C are to be performed only if the value of the antecedent A is **one**.
- **If** T is an **mc**, C : For T a **token** and **mc** a **metaclass**, an evaluation sequence phrase equivalent to “If the **class** of T is in the **metaclass** **mc**, C .”
- **Let** A **stand for** B : An evaluation sequence phrase used to indicate that the name A is to be an abbreviation for the phrase B in subsequent evaluation sequence lines.
- **Otherwise**, C : An evaluation sequence phrase used to indicate that the actions listed in consequent C are to be performed only if the antecedent in the immediately preceding **if** phrase was **zero**. If a consequent is an indented paragraph, the immediately preceding **if** statement is the one at the same level of indentation as the **otherwise** phrase.
- **Repeat**: An evaluation sequence phrase used to indicate that the block of text indented after the **repeat** is to be executed repeatedly until a **return** or **signal** phrase is encountered. The end of a repeated block is indicated by the parenthetic remark “(End of repeated block).”

- **Return** X : An evaluation sequence phrase used to specify that evaluation of this evaluation sequence is to stop and that a **token** is to be returned to the caller of the evaluation sequence. If X is a **token**, then X is returned; if X is an **array**, a **token** of class **constant** and content X is returned.
- **Set** A **to** B : An evaluation sequence phrase used to specify that the referent of A is to be assigned the value B .
- **Signal** X : An evaluation sequence phrase used to specify that evaluation of this evaluation sequence is to stop and that a **token** whose **class** is X , where X is an **error**, is to be returned.
- **Using** O , C : An evaluation sequence phrase used to indicate that the consequent C is to be evaluated against the specific object O . This construct is used, for example, in the description of shared variables to indicate which shared variable is to be changed.
- **Wait until**: An evaluation sequence phrase that indicates that the **session** is waiting for a condition to hold before continuing.

5.4.2 Diagrams

Note: **Diagrams** are used in this International Standard to indicate permissible sequences of characters or of tokens.

- **Character-Diagram**: A graph that designates a subset of the set of all **lists** of **characters**.
- **Token-Diagram**: A graph that designates a subset of the set of all **lists** of **tokens**.
- **Thread** D **with** A : An evaluation sequence phrase used to indicate that a search is to be made for a route through the diagram D that corresponds to the list or part of a list A . A **character-diagram** is **threaded** with a **list** of **characters** by setting a list-cursor to the first item in the **list**, and a diagram-cursor to the arrow-tail $\rightarrow$, in the diagram, then progressing along paths in the diagram to the arrow-head $\rightarrow$. At a junction, the diagram-cursor may enter the alternate path only if it can do so by making an acute-angle turn from its current direction. It may not back up into the alternate path.

For the diagram-cursor to advance along a path labelled with a graphic symbol such as $\div$, that graphic symbol must be pointed to by the list-cursor, which then also advances. For the diagram-cursor to advance along a path labelled with a diagram name, that diagram must itself be threaded, using the same list-cursor and a new diagram-cursor. If in either case the diagram-cursor cannot advance, the diagram-cursor is set back to the previous junction, the list-cursor is set back correspondingly, and a new path is tried; or if the diagram-cursor is now at the arrow-tail, the diagram cannot be threaded with the list.

If a route is found through a diagram for a given **list** of **characters**, it is unique. In this case, the diagram can be **rethreaded**, following the same route without entering blind alleys. For example, characters are collected into identifier tokens through actions performed while a **character-diagram** is being **rethreaded**.

Token-diagrams are **threaded** exactly like **character-diagrams**, except that the items pointed to by the list-cursor are **tokens** and are matched either by their **class** or by both their **class** and their **content**.

- **Rethread** *D* **with** *A*: An evaluation sequence phrase used to indicate that the route found by a previous **threaded** phrase is to be threaded again, in order to effect certain actions through **when** phrases.
- **When** *A*, *C*: An evaluation sequence phrase used during a **rethread** phrase to indicate that when the antecedent *A* is true, the consequent *C* is to be performed.
- *A* **Matches** *D*: An operation that, for diagram *D* and **list** *A*, is true if *A* is a member of the set of **lists** designated by *D*, and false otherwise.

A **list** matches a diagram if it can be **threaded** in such a way that there are no items remaining in the **list** when the final exit path is taken.

Note: The question of whether a **list matches** a diagram is different from the question of whether a **list can thread** a diagram, since there will normally be items left over in a **list** once a diagram has been **threaded**—in collecting the digits in a number, for example, the **digit** diagram removes only one digit at a time from a **list** of **characters**.

5.5 Other Terms

- **Side-effect:** Any effect an operation has other than returning a result.
- **Atomic:** The property of an operation with **side-effects** to produce its **side-effects** only if it completes without error.

Note: For example, the specification of items 3, 4, and 5 of *A* in the APL line

$B \leftarrow A[3 \ 4 \ 5] \leftarrow 2$

is a side effect, since the result placed in *B* is the scalar 2; further, since **indexed assignment** is specified to be **atomic**, no change to *A* will occur if the indexed assignment fails with, for example, an index error.

6 Syntax and Evaluation

6.1 Introduction

Note: This chapter specifies the rules for evaluating lines. These rules are used by the subsections **execute**, **quad input**, **immediate-execution**, and **defined-function-control**.

The rules are described in three subsections, **evaluate-line**, **evaluate-statement**, and **reduce-statement**.

The data structures and procedures used to describe syntax and evaluation in this International Standard are strictly expository devices; they are not intended to represent required or desirable implementation techniques. The order of reporting errors implied by the diagram-matching actions in the model is not required.

6.1.1 Evaluate-Line

Form: **Evaluate-Line**

Informal Description: **Evaluate-line** is the principal procedure in the evaluation of APL lines. It decomposes the **list of characters** named **current-line** into **lexical-units** according to the **character-diagram** named **line**.

Some diagrams referred to in **line** have their exit paths flagged with asterisks. Once the diagram **line** has been threaded, the rethreading pass is used to gather certain character sequences (diagrams ending in two asterisks), and either create tokens (diagrams ending in one asterisk) or discard the character sequences gathered (diagrams ending in three asterisks).

Evaluate-line calls **evaluate-statement** to convert the **lexical-units** into a **result**.

Note that the **result** can be a **constant** (from, for example, $2+2$), **committed-value** (from $A\leftarrow 1$), **nil** ($\emptyset$ ' '), **branch** ($\emptyset$ ' $\rightarrow 3$ '), **escape** ($\rightarrow$), **unwind** ($\rightarrow$), **error** ($2-$), **clear-state-indicator** (from $)SIC$) or **interrupt** (from **signal-interrupt**).

Note: The handling of these result classes by the different callers of **evaluate-line** specifies the treatment of exceptions, and should be analysed carefully.

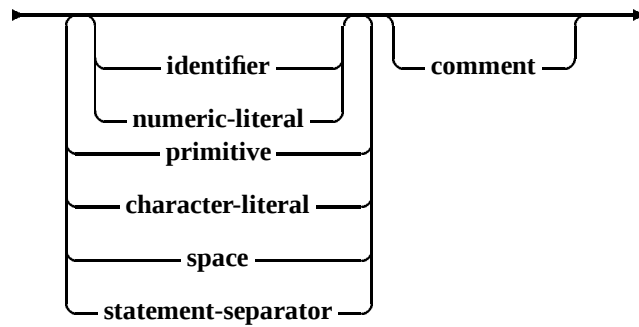
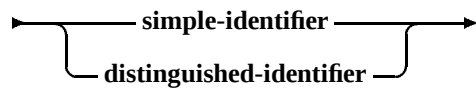
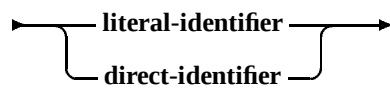
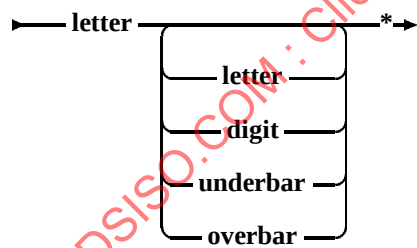
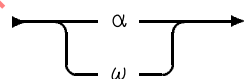
Evaluate-line is called from evaluation sequences in the subsections **defined-function-control**, **execute**, **execute-alternate**, **immediate-execution**, and **quad-input**. If the **optional-facility statement-separator-facility** is implemented, **evaluate-line** evaluates successive statements in **current-line** beginning with the leftmost statement and continuing until evaluation produces an **exception** or the rightmost statement has been evaluated.

Evaluation Sequence:

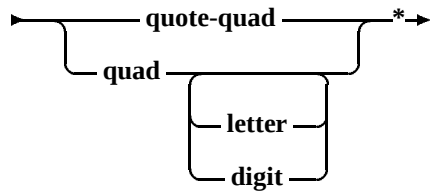
Set *C* to the **empty-list** of **characters**.
 Set **current-statement** to the **empty-list** of **tokens**.
 Thread **line** with **current-line**; if **current-line** does not match **line**, signal **syntax-error**.
 Rethread **line** with **current-line**, taking the following actions:
 When a **character-diagram** ending in '***→' is recognised, set *C* to the **empty-list** of **characters**.
 When a **character-diagram** ending in '—**→' is recognised, append to *C* as a new last **item** the **character** just passed.
 When a **character-diagram** ending in '—*→' is recognised,
 Append to **current-statement** as a new last **item** a **token** with **class** given by the name of the **character-diagram** and with **content** *C*.
 Set *C* to the **empty-list** of **characters**.
 When the **optional-facility statement-separator-facility** is implemented and the **character-diagram statement-separator** is recognised,
 Set *Z* to **evaluate-statement**.
 If *Z* is an **exception**, return *Z*.
 If *Z* is a **constant**, display *Z*.
 Set **current-statement** to the **empty-list** of **tokens**.
 Set *C* to the **empty-list** of **characters**.
 When the **character-diagram line** is matched,
 Set *Z* to **evaluate-statement**.
 Return *Z*.

6.1.2 Character-Diagrams

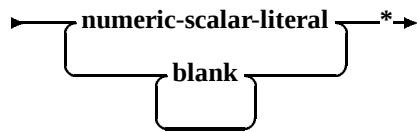
The diagrams in this subsection are **character-diagrams**: the APL Graphic Symbols such as ∇ in these **character-diagrams** match corresponding **characters** in the **required-character-set**.

Line**Identifier****Simple-identifier****Literal-identifier****Direct-identifier**

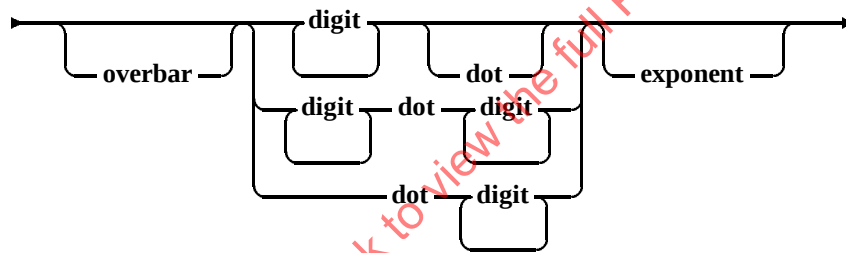
Distinguished-identifier



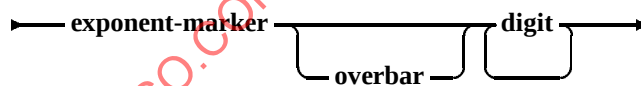
Numeric-Literal



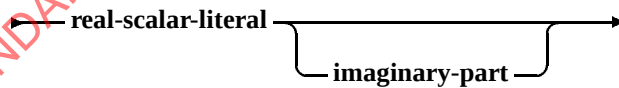
Real-scalar-literal



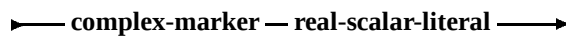
Exponent



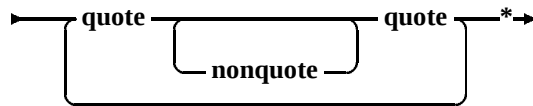
Numeric-scalar-literal



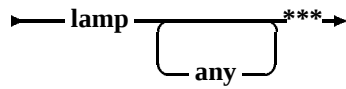
Imaginary-part



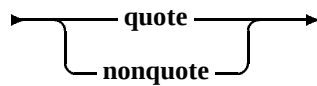
Character-literal



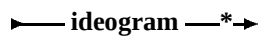
Comment



Any



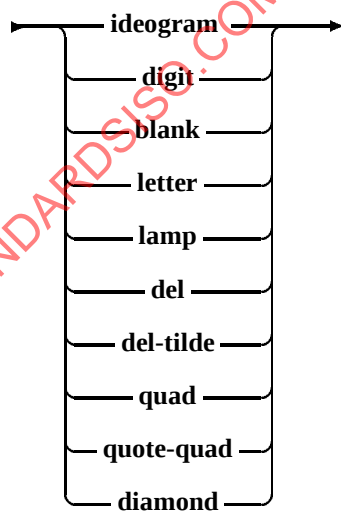
Primitive



Space



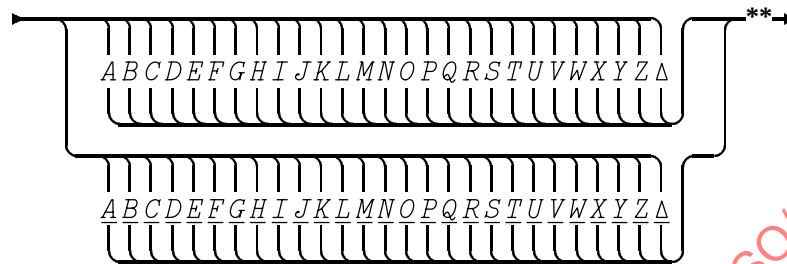
Nonquote



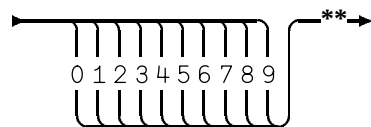
Statement-separator

— diamond —

Letter

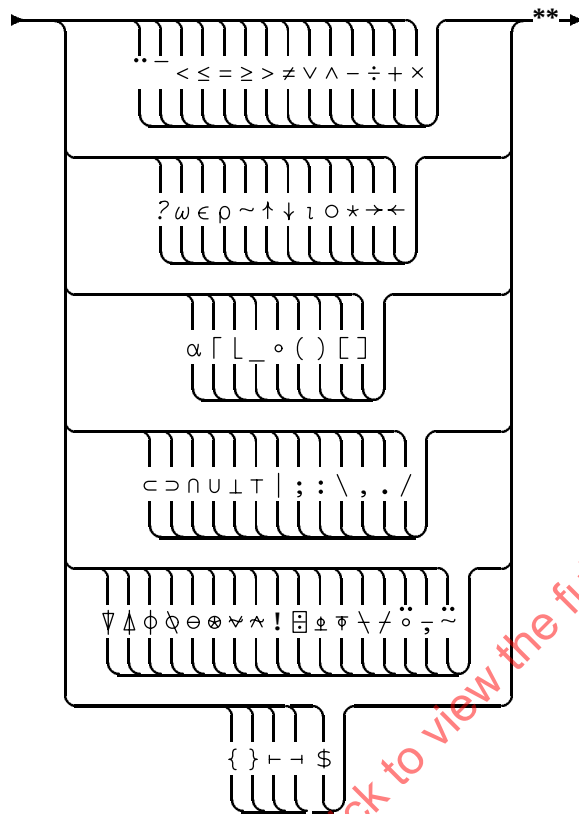


Digit



STANDARDSISO.COM : Click to view the full PDF of ISO/IEC 13751:2001

Ideogram



Quote

! —**→

Exponent-marker

E —**→

Complex-marker

J —**→

Dot

. —**→

Underbar

— _ —**→

Overbar

— ¯ —**→

Blank

— —**→

Del

— ∇ —**→

Del-tilde

— ∇̃ —**→

Lamp

— ϱ —**→

Quad

— □ —**→

Quote-quad

— □ —**→

Diamond

— ◇ —**→

Example

The example introduced in this subsection is continued through the syntax analysis portion of this International Standard.

After **evaluate-line** has processed the **current-line**

ABC←FN □φ[1+0] DEF[1;5 6]×3.45E4,ρ'ABC' ϱCOMMENT

current-statement looks like this:

Identification	Content	Class
T01	'ABC'	simple-identifier
T02	'←'	primitive
T03	'FN'	simple-identifier
T04	'□'	distinguished-identifier
T05	'φ'	primitive
T06	'['	primitive
T07	'1'	numeric-literal
T08	'+'	primitive
T09	'0'	numeric-literal
T10	']'	primitive
T11	'DEF'	simple-identifier
T12	'['	primitive
T13	'1'	numeric-literal
T14	','	primitive
T15	'5 6'	numeric-literal
T16	']'	primitive
T17	'×'	primitive
T18	'3.45E4'	numeric-literal
T19	','	primitive
T20	'ρ'	primitive
T21	' 'ABC' '	character-literal

Each **token** has a **content**, which is a **list** of **characters**, and a **class**, which is in the metaclass **lexical-unit**. The **tokens** produced are numbered T01 to T21 for later reference. Note that comments and blanks between tokens are discarded during this tokenisation process.

Note: As the diagram **line** shows, identifiers and numeric literals are separated by one or more spaces, character-literals, or primitives. 1 ABC' A' is a line, 1ABC' A' is not. No such separation rule applies to primitive function symbols. 1ρ 'AB' is a line.

The sequence 3+.4 is parsed **numeric-literal**, **primitive**, **numeric-literal** but the sequence 3+.× is parsed **numeric-literal**, **primitive**, **primitive**, **primitive**.

A comment may appear at the end of a line or alone on a line.

The **statement-separator-facility** is an **optional-facility**.

Parsing an **identifier** token signals a **limit-error** if the **number-of-items** in the **list** of **characters** is greater than **identifier-length-limit**.

6.1.3 Evaluate-Statement

Evaluate-Statement

Informal Description: Evaluate-statement is performed on **current-statement**, a **list** of

tokens found in the **current-context**.

It uses **bind-token-class** to convert **identifiers** to **classified-names** and **constants**.

It uses **literal-conversion** to convert **literals** to **constants**.

It uses the **token-diagram** named **statement** to verify that the statement is properly formed and to resolve certain ambiguous **tokens**. For example, the **token** containing **]** is resolved to either **right-axis-bracket** or **right-index-bracket**.

It calls the evaluation sequence in the subsection **reduce-statement** to convert the **current-statement** to a **result**, which it returns to its caller.

Evaluate-statement is called by **evaluate-line**.

Evaluation Sequence:

For every **index** I in the **index-set** of **current-statement**,

Let T stand for **item** I of **current-statement**.

If T is an **identifier**, set Q to **bind-token-class** of T .

If T is a **literal**, set Q to the **literal-conversion** of T .

If T is a **primitive**, set Q to T .

If Q is an **exception**, return Q .

Otherwise, set T to Q .

Thread **statement** with **current-statement**; if **statement** cannot be matched, signal **syntax-error**.

Rethread **statement** with **current-statement**, taking the following action:

When any **token-diagram** ending in **—*→** is threaded,

Replace the **token** in **current-statement** that matched the diagram with a **token** having the same **content**, but having a **class** given by the name of the **token-diagram**.

Append to **current-statement** as a new first **item** a **left-end-of-statement token**.

Append to **current-statement** as a new last **item** a **right-end-of-statement token**.

Set Z to **reduce-statement**.

If **mode** is **defined-function** and **current-line-number** is in **current-trace-vector**, set Z to **trace-display** of Z .

Return Z .

Additional Requirement:

This International Standard permits two orders of evaluation for expressions in brackets, as follows:

The syntax evaluator used here distinguishes axis brackets from index brackets in order to classify a function immediately to the right as monadic or dyadic. For example, the evaluation of $\phi[\rho X] + X$ is specified to proceed by first calling monadic plus, then evaluating monadic rho.

The permitted alternative is to evaluate any expression in brackets before determining whether a function immediately to the right is monadic or dyadic. In the example given, this alternative behaviour would evaluate monadic rho before discovering that plus is used

monadically.

Note: *The distinction can be observed only through side effects.*

6.1.4 Bind-Token-Class

Bind-Token-Class of T

Informal Description: **Bind-token-class** is used to bind each **identifier** in **current-statement** to its current **syntactic-unit**; if the **class** of a **token** changes between this point and the time the **token** is used (as it would for example in $F \rightarrow EX' E'$, assuming F were initially a **defined-function**), the change will be detected and reported as a **syntax-error** in the appropriate **phrase-evaluator**.

This prebinding limits the International Standard to defining the meaning of statements only when the syntax class of their tokens does not change in mid-statement. **Conforming-implementations** may, of course, relax these rules. **Conforming-programs** must abide by them.

Evaluation Sequence:

Let f stand for the **content** of T .

If T is a **simple-identifier**,

If the **current-class** of T is

defined-monadic-operator, return a **token** of class **defined-monadic-operator-name** and **content** f .

defined-dyadic-operator, return a **token** of class **defined-dyadic-operator-name** and **content** f .

defined-function, return a **token** of class **defined-function-name** and **content** f .

niladic-defined-function, return a **token** of class **niladic-defined-function-name** and **content** f .

nil or **variable**, return a **token** of class **variable-name** and **content** f .

shared-variable, return a **token** of class **shared-variable-name** and **content** f .

label, return a **token** of class **constant** and **content** the **current-content** of T .

Note: *Other cases cannot occur.*

If T is a **distinguished-identifier**,

If both forms $Z \leftarrow f$ and $Z \leftarrow f \leftarrow B$ occur in the **form-table**, return a **token** of class **system-variable-name** and **content** f .

If the form $Z \leftarrow f$ occurs in the **form-table**, but the form

$Z \leftarrow f \leftarrow B$ does not, return a **token** of class **niladic-system-function-name** and **content** f .

If either form $Z \leftarrow f \leftarrow B$ or form $Z \leftarrow A \leftarrow f \leftarrow B$ occurs in the **form-table**, return a **token** of class **system-function-name** with **content** f .

Otherwise, signal **syntax-error**.

6.1.5 Literal-Conversion

Literal-Conversion of T .

Informal Description: **Literal-conversion** converts T , a **token** of **class literal**, to a **token** of **class constant**. The **content** of such a **token** is converted from a **list** of **characters** to an **array**.

Evaluation Sequence:

If T is a **character-literal**, generate Z , a **character vector** such that the **ravel-list** of Z is the **content** of T with the initial and final quotes removed, and each pair of adjacent quotes in T replaced by a single quote.

If T is a **numeric-literal**, generate Z , a **numeric vector** such that the **ravel-list** of Z is a **list** of **numbers**, each of which is obtained by calling the **implementation-algorithm numeric-input-conversion** for the corresponding **numeric-scalar-literal** in the **numeric-literal**.

If the **length** of Z is **greater-than one**, return Z .
Otherwise, return **first-scalar** in Z .

Note: A quote character is represented in a **character-literal** by two adjacent quote characters. A single character between quotes is a scalar. All other cases are **character vector literals**. For example, the **character literal** `' '` is the empty **character vector literal** and the character literal `' ' ' '` is the **character scalar** "quote".

A numeric literal containing a single number is a scalar. A numeric literal containing two or more numbers is a vector.

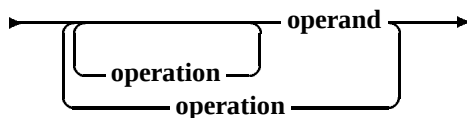
6.1.6 Statement-Analysis Token-Diagrams

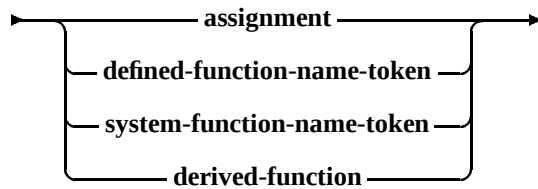
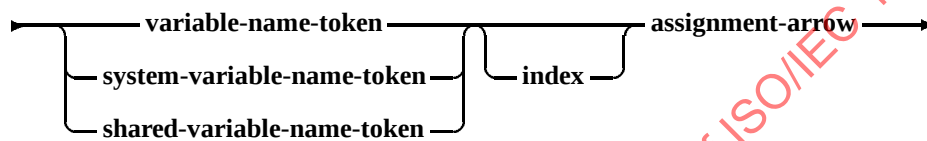
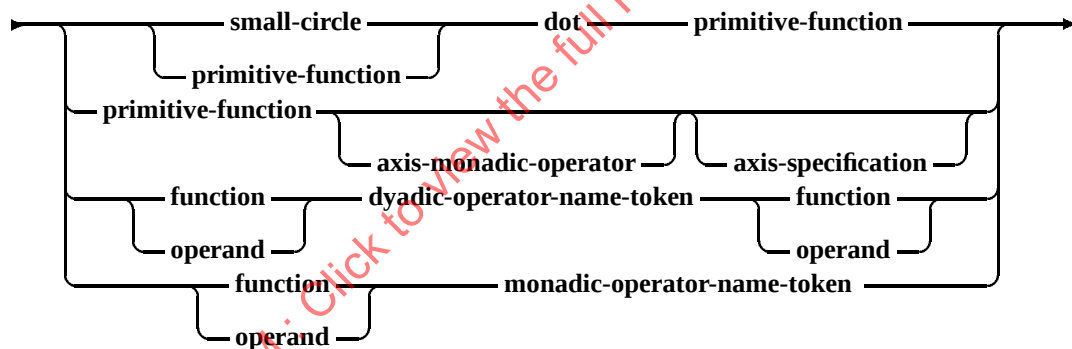
Paths containing **ideograms** such as $\otimes$ in these **token-diagrams** match **tokens** whose **class** is **primitive** and whose **content** is the ideogram. Paths containing the word **token**, such as **shared-variable-name token** in **operand**, match **tokens** with the given **class**.

Statement

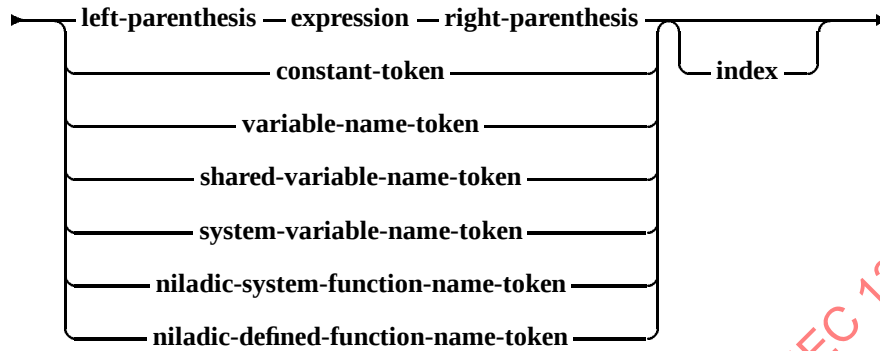


Expression



Operation**Assignment****Derived-Function****Axis-Specification**

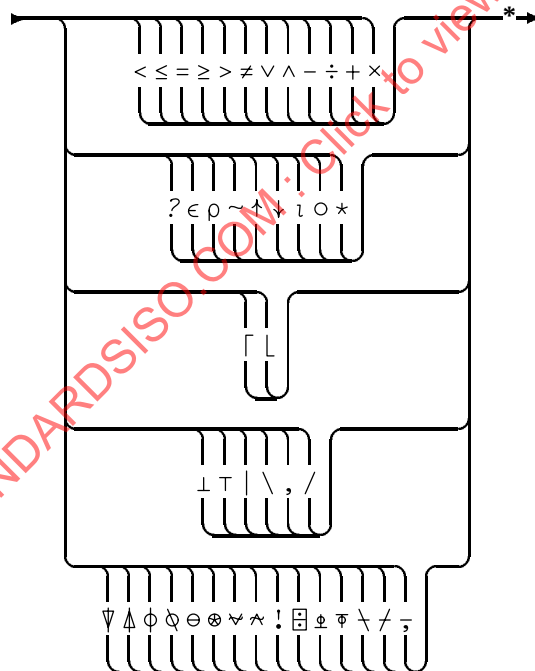
Operand



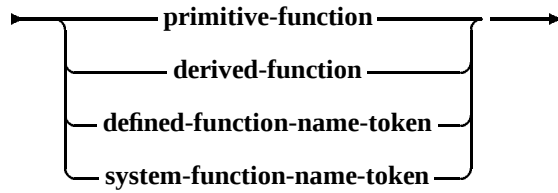
Index



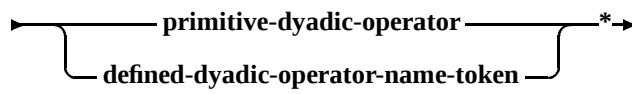
Primitive-Function



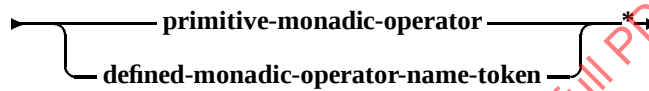
Function



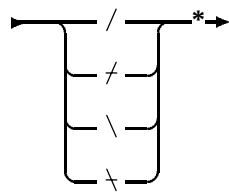
Dyadic-Operator



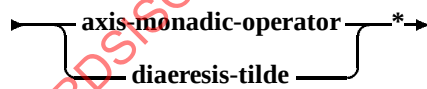
Monadic-Operator



Axis-Monadic-Operator



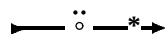
Primitive-Monadic-Operator



Primitive-Dyadic-Operator



Diaeresis-Jot



Diaeresis-Tilde

— ¨ —*→

Left-Parenthesis

— (—*→

Right-Parenthesis

—) —*→

Left-Axis-Bracket

— [—*→

Right-Axis-Bracket

—] —*→

Branch-Arrow

— → —*→

Assignment-Arrow

— ← —*→

Left-Index-Bracket

— [—*→

Right-Index-Bracket

—] —*→

Index-Separator

— ; —*→

Small-Circle

— ◦ —*→

Example

From the **list of characters** in **current-line**,

ABC←FN □φ[1+0] DEF[1;5 6]×3.45E4,ρ'ABC' ρCOMMENT

evaluate-line generated a **list of tokens** and stored it in **current-statement**. Here, **evaluate-statement** has replaced that **list** with a new **list of tokens**.

Identification	Content	Class
T00		left-end-of-statement
T01	'ABC'	variable-name
T02		assignment-arrow
T03	'FN'	defined-function-name
T04	'□'	system-variable-name
T05	'φ'	primitive-function
T06		left-axis-bracket
T07	1	constant
T08	'+'	primitive-function
T09	0	constant
T10		right-axis-bracket
T11	'DEF'	variable-name
T12		left-index-bracket
T13	1	constant
T14		index-separator
T15	5 6	constant
T16		right-index-bracket
T17	'×'	primitive-function
T18	34500	constant
T19	','	primitive-function
T20	'ρ'	primitive-function
T21	'ABC'	constant
T22		right-end-of-statement

The new **list**, shown above, begins with a **left-end-of-statement token** and ends with a **right-end-of-statement token**. Old **identifier tokens** have a new **class** and the same **content**; old **literal tokens** are now **constants** whose **content** is an appropriate **array**; old **primitive tokens** are now either **primitive-functions** whose **content** is the **character** identifying the primitive function, or are grouping signs such as **right-axis-bracket** with no specified **content**.

6.2 Reduce-Statement

Reduce-Statement

Informal Description: **Reduce-statement** converts **current-statement**, a list of **syntactic-units**, to a **result** by decomposing it into shorter lists of **syntactic-units** called **phrases**, then calling procedures termed **phrase-evaluators** to convert these phrases into **tokens**.

The letters *Z*, *K*, and *J* in this evaluation sequence refer to the graphic symbols found in the **resultant-prefix** column of **Table 3**.

Evaluation Sequence:

Set **current-stack** to the **empty-list** of **tokens**.

Repeat:

Find the first entry in the **phrase-table** whose **pattern** matches a **prefix** of **current-stack**.

If there is no matching entry,

If **current-statement** is empty, signal **syntax-error**.

Otherwise,

Remove the last **token** from **current-statement**.

Append it to **current-stack** as a new first **item**.

If there is a matching entry,

Let **p** stand for the **prefix** of the **current-stack** that matched the entry.

Let **r** stand for the **resultant-prefix** of the entry.

Let **s** stand for the **phrase-evaluator** of the entry.

Call **s** and set **y** to the **token** it returns.

If **s** is **process-end-of-statement**, return **y**.

If **y** is an **exception**, return **y**.

Otherwise, replace **p** with **r** in which **y** has been substituted for *Z*, *K* or *J* according to whether **y** is a **result**, a **complete-index-list** or a **partial-index-list**.

(End of repeated block)

The graphic symbols in the **Pattern** and **Resultant-Prefix** columns of **Table 3** designate lists of **syntactic-units**. Each graphic symbol matches **tokens** of the designated **classes** or **metaclasses**.

Note: The graphic symbols employed are chosen to be suggestive of the list of characters that give rise to such phrases.

The **build-index-list** entries are called with *B* bound to a **value** and *I* bound to a **partial-index-list**; they return either another **partial-index-list** (*J*) or a **complete-index-list** (*K*). Brackets are not passed as arguments or returned by **build-index-list**; they are preserved by **reduce-statement** to make the patterns more obvious.

Example

This example is continued from **evaluate-statement**.

$$ABC \leftarrow FN \square \phi [1+0] DEF [1; 56] \times 34500, \rho 'ABC'$$

T_{00} is **left-end-of-statement**, and T_{22} is **right-end-of-statement**.

Consider step 5: the line fragment $\rho'ABC'$ matches the phrase XFB as follows: X matches **token** T_{19} because ρ is a **primitive-function**. F matches **token** T_{20} because ρ is a **primitive-function**. B matches **token** T_{21} because ABC is a **constant**. **Token** T_{22} , **right-end-of-statement**, is not considered because the pattern XFB is concerned with only the first three **tokens in current-stack**.

This becomes Z in the **resultant-prefix** column for $X \ F \ B$. **Current-stack** now holds three **tokens**: (**primitive-function**; $,$), (**constant**; $, 3$) and (**right-end-of-statement**;). Since no entry in the **phrase-table** has an entry whose **pattern** matches a **prefix** of **current-stack**, the **token** $T18$ (**constant**; 34500) is added to **current-stack**. A **prefix** of **current-stack** now matches a pattern ($A \ F \ B$ matches (**constant**; 34500), (**primitive-function**; $,$), (**constant**; $, 3$)), so the corresponding **phrase-evaluator** (**evaluate-dyadic-function**) is called.

Pattern	Phrase Evaluator	Resultant-Prefix
(<i>B</i>)	Remove-Parentheses	<i>Z</i>
<i>N</i>	Evaluate-Niladic-Function	<i>Z</i>
<i>X F B</i>	Evaluate-Monadic-Function	<i>X Z</i>
<i>X F [C] B</i>	Evaluate-Monadic-Function	<i>X Z</i>
<i>X F M B</i>	Evaluate-Monadic-Operator	<i>X Z</i>
<i>X F M [C] B</i>	Evaluate-Monadic-Operator	<i>X Z</i>
<i>A F M B</i>	Evaluate-Monadic-Operator	<i>X Z</i>
<i>A F M [C] B</i>	Evaluate-Monadic-Operator	<i>X Z</i>
<i>A F B</i>	Evaluate-Dyadic-Function	<i>Z</i>
<i>A F [C] B</i>	Evaluate-Dyadic-Function	<i>Z</i>
<i>X F D G B</i>	Evaluate-Dyadic-Operator	<i>Z</i>
<i>A F D G B</i>	Evaluate-Dyadic-Operator	<i>Z</i>
<i>A ◦ D G B</i>	Evaluate-Dyadic-Operator	<i>Z</i>
<i>A [K]</i>	Evaluate-Indexed-Reference	<i>Z</i>
<i>V [K] ← B</i>	Evaluate-Indexed-Assignment	<i>Z</i>
<i>V ← B</i>	Evaluate-Assignment	<i>Z</i>
<i>V</i>	Evaluate-Variable	<i>Z</i>
<i>]</i>	Build-Index-List	<i>J]</i>
<i>;</i> <i>I</i>	Build-Index-List	<i>J</i>
<i>;</i> <i>B I</i>	Build-Index-List	<i>J</i>
<i>[I</i>	Build-Index-List	<i>[K</i>
<i>[B I</i>	Build-Index-List	<i>[K</i>
<i>L R</i>	Process-End-of-Statement	–
<i>L B R</i>	Process-End-of-Statement	–
<i>L → B R</i>	Process-End-of-Statement	–
<i>L → R</i>	Process-End-of-Statement	–

Legend:

A, B, Z match **result**.

D matches **dyadic-operator**.

F, G match **defined-function-name, primitive-function, or system-function-name**.

I, J match **partial-index-list**.

C, K match **complete-index-list**.

L matches **left-end-of-statement**.

M matches **monadic-operator**.

N matches **niladic-defined-function-name or niladic-system-function-name**.

R matches **right-end-of-statement**.

V matches **variable-name, system-variable-name, or shared-variable-name**.

X matches **assignment-arrow, branch-arrow, defined-function-name, index-separator, left-axis-bracket, left-end-of-statement, left-index-bracket, left-parenthesis, primitive-function, system-function-name, or right-axis-bracket**.

(matches **left-parenthesis**.

) matches **right-parenthesis**.

[matches **left-axis-bracket or left-index-bracket**.

] matches **right-axis-bracket or right-index-bracket**.

◦ matches **small-circle**.

; matches **index-separator**.

← matches **assignment-arrow**.

→ matches **branch-arrow**.

Table 3: The Phrase Table.

1 GET NEXT TOKEN T22 ()
 2 GET NEXT TOKEN T21 (R)
 3 GET NEXT TOKEN T20 (B R)
 4 GET NEXT TOKEN T19 (F B R)
 5 EVALUATE MONADIC FUNCTION (<X F B>R)
 6 GET NEXT TOKEN T18 (<X Z> R)
 7 EVALUATE DYADIC FUNCTION (<A F B> R)
 8 GET NEXT TOKEN T17 (Z R)
 9 GET NEXT TOKEN T16 (F B R)
 10 BUILD INDEX LIST (<]> F B R)
 11 GET NEXT TOKEN T15 (J] F B R)
 12 GET NEXT TOKEN T14 (B I] F B R)
 13 BUILD INDEX LIST (<; B I>] F B R)
 14 GET NEXT TOKEN T13 (J] F B R)
 15 GET NEXT TOKEN T12 (B I] F B R)
 16 BUILD INDEX LIST (<[B I>] F B R)
 17 GET NEXT TOKEN T11 ([K] F B R)
 18 EVALUATE VARIABLE (<V>[K] F B R)
 19 EVALUATE INDEXED REFERENCE (<A [K]> F B R)
 20 EVALUATE DYADIC FUNCTION (<A F B> F B R)
 21 GET NEXT TOKEN T10 Z R)
 22 BUILD INDEX LIST (<]>B R)
 23 GET NEXT TOKEN T09 (I] B R)
 24 GET NEXT TOKEN T08 (B I] B R)
 25 GET NEXT TOKEN T07 (F B I] B R)
 26 EVALUATE DYADIC (<A F B>I] B FUNCTION R)
 27 GET NEXT (Z I] B TOKEN T06 R)
 28 BUILD INDEX (<[B I>] B LIST R)
 29 GET NEXT ([C] B TOKEN T05 R)
 30 GET NEXT (F [C] B TOKEN T04 R)
 31 EVALUATE (<V>F [C] B VARIABLE R)
 32 EVALUATE (<A F [C] B> DYADIC FUNCTION R)
 33 GET NEXT (Z TOKEN T03 R)
 34 (F B GET NEXT TOKEN T02 R)
 35 (<X F B> EVALUATE MONADIC FUNCTION R)
 36 (X Z GET NEXT TOKEN T01 R)
 37 (<V ← B> EVALUATE ASSIGNMENT R)
 38 (Z GET NEXT TOKEN T00 R)
 39 * (<L B END OF STATEMENT R>)

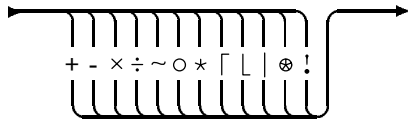
Figure 1: Statement Evaluation.

6.3 The Phrase Evaluators

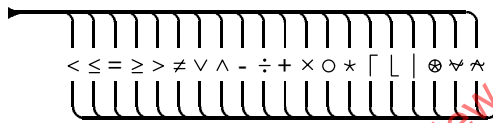
Informal Description: Each **phrase-evaluator** takes a **phrase** and reduces it to a single **token**. The **form-table** used by the **phrase-evaluators** is given as Table 4.

6.3.1 Diagrams

Primitive-Monadic-Scalar-Function



Primitive-Dyadic-Scalar-Function



6.3.2 Remove-Parentheses

Pattern (B)

Evaluation Sequence:

If *B* is **nil**, signal **value-error**.
 If *B* is a **branch**, signal **value-error**.
 Otherwise, return *B*.

Note: (Φ ' ') and (Φ ' $\rightarrow$ 3 ') fail with a **value-error**. ($A \leftarrow 3$) and (Φ ' $A \leftarrow 3$ ') do not display, since the **token** returned by **remove-parentheses** is a **committed-value**, not a **constant**; for the same reason, ($\square \leftarrow 3$) displays the value **three** only once. ($\rightarrow 3$) fails with a **syntax-error** in **evaluate-statement**.

($\rightarrow$) also fails in **evaluate-statement**, but (Φ ' $\rightarrow$ ') succeeds; **evaluate-statement** threads **line** successfully, and **remove-parentheses** receives and returns an **escape token**.

6.3.3 Evaluate-Niladic-Function

Pattern *N*

Evaluation Sequence:

Let **n** stand for the **content** of *N*.
 If *N* is a **niladic-defined-function-name**,
 If the **current-class** of *N* is **niladic-defined-function**,
 Search the **form-table** for $Z \leftarrow DFN$.
 Call the corresponding evaluation sequence, passing **n** as the value of *DFN*.
 Return the **token** it returns.
 Otherwise, signal **syntax-error**.
 If *N* is a **niladic-system-function-name**,
 Search the **form-table** for $Z \leftarrow n$.
 If it is not found, signal **syntax-error**.
 Otherwise, call the corresponding evaluation sequence.
 Return the **token** it returns.

Note: This phrase evaluator checks for a change in syntax class.

6.3.4 Evaluate-Monadic-Function

Pattern $X \ F \ B$

Pattern $X \ F[C] \ B$

Evaluation Sequence:

If *B* is not a **value**, signal **value-error**.
 Let **f** stand for the **content** of *F*.
 For pattern $X \ F \ B$
 If *F* is a **defined-function-name**,
 If the **current-class** of *F* is **defined-function**,
 Search the **form-table** for $Z \leftarrow DFN \ B$.
 Call the corresponding evaluation sequence, passing **f** as the value of *DFN*.
 Return the **token** it returns.
 Otherwise, signal **syntax-error**.
 If *F* is a **primitive-function** or a **system-function-name**,
 If *F* matches **primitive-monadic-scalar-function** and *B* is not a **scalar**, perform **monadic-scalar-extension** as follows:
 Return a **numeric array** *Z* such that the **shape-list** of *Z* is the **shape-list** of *B*
 and for all *I* in the **index-set** of the **ravel-list** of *Z*, **item** *I* of the **ravel-list** of
 Z is **f** (**item** *I* of the **ravel-list** of *B*).
 Otherwise,
 Search the **form-table** for $Z \leftarrow f \ B$.
 If it is not found, signal **valence-error**.
 Otherwise, call the corresponding evaluation sequence.
 Return the **token** it returns.
 For pattern $X \ F[C] \ B$

If F is not a **primitive-function**, signal **syntax-error**.
 If **index-origin** is **nil**, signal **implicit-error**.
 Let $C1$ stand for the **first-item** in the **index-list** C .
 If $C1$ is not a **scalar** or **one-element-vector**, signal **axis-error**.
 If any **item** of the **ravel-list** of $C1$ is not a **number**, signal **axis-error**.
 Set K to $C1$ **plus (one minus index-origin)**.
 Search the **form-table** for $Z \leftarrow f[K] B$.
 If it is not found, signal **syntax-error**.
 Otherwise, call the corresponding evaluation sequence and return the **token** it returns.

Additional Requirement:

The order in which the individual items of Z are produced during **monadic-scalar-extension** is not specified by this International Standard.

If any call of f signals an error during **monadic-scalar-extension**, that error is returned as the result of **evaluate-monadic-function**.

Note: This phrase evaluator checks for a change in *syntax class*.

6.3.5 Evaluate-Monadic-Operator

Pattern $X F M B$

Pattern $A F M B$

Pattern $X F M[C] B$

Pattern $A F M[C] B$

Evaluation Sequence:

If B is not a **value**, signal **value-error**.
 Let m stand for the **content** of M .
 Let f stand for the **content** of F .
 For pattern $X F M B$
 If M is a **primitive-monadic-operator**
 Search the **form-table** for $Z \leftarrow f m B$
 If it is not found, signal **syntax-error**.
 Otherwise, call the corresponding evaluation sequence, passing token F as the value of f .
 Return the **token** it returns.
 Otherwise,
 Search the **form-table** for $Z \leftarrow f DFN B$.
 If it is not found, signal **syntax-error**.

Otherwise, call the corresponding evaluation sequence, passing **token** F as the value of **f** and the **token** DFN as the value of **m**.
 Return the **token** it returns.

For pattern $A \ F \ M \ B$
 If A is not a **value**, signal **value-error**.
 If **m** is a **primitive-monadic-operator**
 Search the **form-table** for $Z \leftarrow A \ f \ m \ B$.
 If it is not found, signal **syntax-error**.
 Call the corresponding evaluation sequence, passing **token** F as the value of **f**.
 Return the **token** it returns.

Otherwise,
 Search the **form-table** for $Z \leftarrow A \ F \ DFN \ B$.
 If it is not found, signal **syntax-error**.
 Call the corresponding evaluation sequence, passing **token** F as the value of **f** and the **token** DFN as the value of **m**.
 Return the **token** it returns.

For pattern $X \ F \ M[C] \ B$ or $A \ F \ M[C] \ B$
 If M is not an **axis-operator**, signal **syntax-error**.
 If F is not a **primitive-function**, signal **syntax-error**.
 If **index-origin** is **nil**, signal **implicit-error**.
 Let $C1$ stand for the **first-item** in the **index-list** C .
 If $C1$ is not a **scalar** or **one-element-vector**, signal **axis-error**.
 If any **item** of the **ravel-list** of $C1$ is not a **number**, signal **axis-error**.
 Set K to $C1$ **plus** (**one minus index-origin**).
 For pattern $X \ F \ M[C] \ B$
 Search the **form-table** for $Z \leftarrow f \ m \ [K] \ B$.
 If it is not found, signal **syntax-error**.
 Otherwise, call the corresponding evaluation sequence, passing **token** F as the value of **f**.
 Return the **token** it returns.

For pattern $A \ F \ M[C] \ B$
 If A is not a **value**, signal **value-error**.
 Search the **form-table** for $Z \leftarrow A \ f \ m \ [K] \ B$.
 If it is not found, signal **syntax-error**.
 Otherwise, call the corresponding evaluation sequence, passing **token** F as the value of **f**.
 Return the **token** it returns.

6.3.6 Evaluate-Dyadic-Function

Pattern $A \ F \ B$

Pattern $A \ F[C] \ B$

Evaluation Sequence:

If A is not a **value**, signal **value-error**.
 If B is not a **value**, signal **value-error**.
 Let f stand for the **content** of F .
 For pattern $A \ F \ B$
 If F is a **defined-function-name**,
 If the **current-class** of F is **defined-function**,
 Search the **form-table** for $Z \leftarrow A \ DFN \ B$.
 Call the corresponding evaluation sequence, passing f as the value of DFN .
 Return the **token** it returns.
 Otherwise, signal **syntax-error**.
 If F is a **primitive-function** or a **system-function-name**,
 If F matches **primitive-dyadic-scalar-function** and A and B are not both **scalars**,
 perform **dyadic-scalar-extension** as follows:
 If the **rank** of A differs from the **rank** of B ,
 If A is a **scalar** or **one-element-vector** and B is not a **scalar**, return
 $((\rho B) \rho A) \ f \ B$.
 If B is a **scalar** or **one-element-vector**, return $A \ f \ (\rho A) \rho B$.
 Otherwise, signal **rank-error**.
 If the **shape-list** of A differs from the **shape-list** of B , signal **length-error**.
 Return Z , an array such that the **shape-list** of Z is the same as the **shape-list**
 of A , the **type** of Z is **numeric**, and the **ravel-list** of Z is such that, for all I
 in the **index-set** of the **ravel-list** of Z , **item** I of the **ravel-list** of Z is (**item** I
 of the **ravel-list** of A) f (**item** I of the **ravel-list** of B).
 Otherwise, search the **form-table** for $Z \leftarrow A \ f \ B$.
 If it is not found, signal **valence-error**.
 Call the corresponding evaluation sequence and return the **token** it returns.
 For pattern $A \ F[C] \ B$
 If F is not a **primitive-function**, signal **syntax-error**.
 If **index-origin** is **nil**, signal **implicit-error**.
 Let $C1$ stand for the **first-item** in the **index-list** C .
 If $C1$ is not a **scalar** or **one-element-vector**, signal **axis-error**.
 If any **item** of the **ravel-list** of $C1$ is not a **number**, signal **axis-error**.
 Set K to $C1$ **plus (one minus index-origin)**.
 Search the **form-table** for $Z \leftarrow A \ f[K] \ B$.
 If it is not found, signal **syntax-error**.
 Otherwise, call the corresponding evaluation sequence, passing **token** F as the value
 of f .
 Return the **token** it returns.

Additional Requirement:

There is an intentional forward reference to **shape** and **reshape** in the description of **dyadic-scalar-extension**.

The order in which the individual items of Z are produced during **dyadic-scalar-extension** is not specified by this International Standard.

If any call of **f** signals an error during **dyadic-scalar-extension**, that error is returned as the result of **evaluate-dyadic-function**.

Note: **Dyadic-scalar-extension** is intentionally stricter than it is in existing systems. For example, $(1\ 1\rho 1)+13$ signals a **rank-error** and $1\ 2\ +\ ,1$ signals a **length-error**.

This phrase evaluator checks for a change in syntax class.

An ambivalent function must be called either monadically or dyadically; hence, during syntax analysis and execution of a line, there is never a reference to an ambivalent function as any given instance is either monadic or dyadic.

6.3.7 Evaluate-Dyadic-Operator

Pattern $X\ F\ D\ G\ B$

Pattern $A\ F\ D\ G\ B$

Pattern $A\ \circ\ D\ G\ B$

Evaluation Sequence:

If B is not a **value**, signal **value-error**.

Let **d** stand for the **content** of D .

Let **f** stand for the **content** of F .

Let **g** stand for the **content** of G .

For pattern $X\ F\ D\ G\ B$

If D is a **primitive-dyadic-operator**

Search the **form-table** for $Z\ \leftarrow f\ d\ g\ B$.

If it is not found, signal **syntax-error**.

If it is found, call the corresponding evaluation sequence, passing **token f** as the value of F and **token g** as the value of G .

Return the **token** it returns.

Otherwise,

Search the **form-table** for $Z\ \leftarrow f\ DEN\ g\ B$.

If it is not found, signal **syntax-error**.

If it is found, call the corresponding evaluation sequence, passing **token f** as the value of F , **token g** as the value of G , and **token DEN** as the value of d .

Return the **token** it returns.

For pattern $A\ F\ D\ G\ B$

If A is not a **value**, signal **value-error**.

If **d** is a **primitive-dyadic-operator**,

Search the **form-table** for $Z\ \leftarrow A\ f\ d\ g\ B$.

If it is not found, signal **syntax-error**.

If it is found, call the corresponding evaluation sequence, passing **token f** as the value of F and **token g** as the value of G .

Return the **token** it returns.

Otherwise,

Search the **form-table** for $Z \leftarrow A \text{ f } DFN \text{ g } B$.
 If it is not found, signal **syntax-error**.
 If it is found, call the corresponding evaluation sequence, passing **token f** as the value of F and **token g** as the value of G , and **token DFN** as the value of d .
 Return the **token** it returns.

For pattern $A \circ D \text{ G } B$
 If D is not **dot**, signal **syntax-error**.
 Otherwise,
 Search the **form-table** for $Z \leftarrow A \circ d \text{ g } B$.
 If it is not found, signal **syntax-error**.
 If it is found, call the corresponding evaluation sequence, passing **token g** as the value of G .
 Return the **token** it returns.

6.3.8 Evaluate-Indexed-Reference

Pattern $A[K]$

Evaluation Sequence:

If A is not a **value**, signal **value-error**.
 If **index-origin** is **nil**, signal **implicit-error**.
 If the **number-of-items** in the **index-list** K differs from the **rank** of A , signal **rank-error**.
 If the **rank** of A is **greater-than one**,
 Search the **form-table** for $Z \leftarrow A[I]$.
 Call the corresponding evaluation sequence, passing K as the value of I .
 Return the **token** it returns.
 Otherwise,
 If **first-item** in K is an **elided-index-marker**, return A .
 Otherwise,
 Set X to **first-item** in the **index-list** K .
 If any **item** of the **ravel-list** of X is not a **near-integer**, signal **domain-error**.
 Generate $X1$, a **numeric array** with the **shape-list** of X such that each **item** of the **ravel-list** of $X1$ is **(one minus index-origin) plus the integer-nearest-to** X .
 If any **item** of the **ravel-list** of $X1$ is not in the **index-set** of A , signal **index-error**.
 Return Z , an array with the **shape-list** of $X1$, such that for each **integer** I in the **index-set** of Z , **item** I of the **ravel-list** of Z is **item** J of the **ravel-list** of A , where J is **item** I of the **ravel-list** of $X1$. The **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of A .

Note: Since an **index-list** will never have zero **items**, indexing will always signal a **rank-error** when argument A is a **scalar**.

6.3.9 Evaluate-Assignment

Pattern $V \leftarrow B$

Evaluation Sequence:

- If B is not a **value**, signal **value-error**.
- If V is a **shared-variable-name**,
 - If the **current-class** of V is **shared-variable**,
 - Search the **form-table** for $Z \leftarrow SHV \leftarrow B$.
 - Call the corresponding evaluation sequence, passing **token** V as the value of SHV .
 - Return the **token** it returns.
 - Otherwise, signal **syntax-error**.
- If V is a **system-variable-name**,
 - Search the **form-table** for $Z \leftarrow q \leftarrow B$, where q is the **content** of V .
 - If it is not found, signal **syntax-error**.
 - Otherwise,
 - Call the corresponding evaluation sequence
 - Return the **token** it returns.
- If V is a **variable-name**,
 - If the **current-class** of V is **nil** or **variable**,
 - Set the **current-referent** of V to a **token** whose **class** is **variable** and whose **content** is the **content** of B .
 - Return a **token** whose **class** is **committed-value** and whose **content** is B .
 - Otherwise, signal **syntax-error**.

Note: The phrase $ABC \leftarrow 1 \rightarrow 3$ yields **value-error**. The phrase $V \leftarrow SVR \rightarrow V$ where V was a **shared-variable** yields **syntax-error**.

6.3.10 Evaluate-Indexed-Assignment

Pattern $V[K] \leftarrow B$

Evaluation Sequence:

- If **index-origin** is **nil**, signal **implicit-error**.
- If B is not a **value**, signal **value-error**.
- If V is a **shared-variable-name**,
 - If the **current-class** of V is **shared-variable**,
 - Search the **form-table** for $Z \leftarrow SHV[I] \leftarrow B$.
 - Call the corresponding evaluation sequence, passing **token** V as the value of SHV , and K as the value of I .
 - Return the **token** it returns.
 - Otherwise, signal **syntax-error**.
- If V is a **system-variable-name**,
 - Search the **form-table** for $Z \leftarrow q[I] \leftarrow B$, where q is the **content** of V .

If it is not found, signal **syntax-error**.
 Otherwise,
 Call the corresponding evaluation sequence, passing K as the value of I .
 Return the **token** it returns.
 If V is a **variable-name**,
 If the **current-class** of V is **nil**, signal **value-error**.
 If the **current-class** of V is a **variable**,
 Search the **form-table** for $Z \leftarrow V[I] \leftarrow B$.
 Call the corresponding evaluation sequence, passing K as the value of I .
 Return the **token** it returns.
 Otherwise, signal **syntax-error**.

6.3.11 Evaluate-Variable

Pattern V

Evaluation Sequence:

If V is a **shared-variable-name**,
 If the **current-class** of V is **shared-variable**,
 Search the **form-table** for $Z \leftarrow SHV$.
 Call the corresponding evaluation sequence, passing **token** V as the value of SHV .
 Return the **token** it returns.
 Otherwise, signal **syntax-error**.
 If V is a **system-variable-name**,
 If the **current-class** of V is **nil**, signal **value-error**.
 Search the **form-table** for $Z \leftarrow q$, where q is the **content** of V .
 Call the corresponding evaluation sequence.
 Return the **token** it returns.
 If V is a **variable-name**,
 If the **current-class** of V is **nil**, signal **value-error**.
 If the **current-class** of V is **variable**, return the **current-content** of V .
 Otherwise, signal **syntax-error**.

6.3.12 Build-Index-List

Pattern $]$

Pattern $; I$

Pattern $; B I$

Pattern $[I$

Pattern $[B I$

Evaluation Sequence:

For pattern $\square$
 Return J , a **partial-index-list** with **content** the **index-list** of length **zero**.

For pattern $\square I$
 Return J , a **partial-index-list** with **content** Z , an **index-list** such that the **first-item** in Z is an **elided-index-marker** and the **rest-of** Z is I .

For pattern $\square B I$
 Return J , a **partial-index-list** with **content** Z , an **index-list** such that the **first-item** in Z is B and the **rest-of** Z is I .

For pattern $\square I$
 Return J , a **complete-index-list** with **content** Z , an **index-list** such that the **first-item** in Z is an **elided-index-marker** and the **rest-of** Z is I .

For pattern $\square B I$
 Return J , a **complete-index-list** with **content** Z , an **index-list** such that the **first-item** in Z is B and the **rest-of** Z is I .

6.3.13 Process-End-of-Statement

Pattern $L R$

Pattern $L A R$

Pattern $L \rightarrow R$

Pattern $L \rightarrow A R$

Evaluation Sequence:

For pattern $L R$
 Return a **token** whose **class** is **nil**.

For pattern $L A R$
 Return A .

For pattern $L \rightarrow R$ Process-End-of-Statement
 Return a **token** whose **class** is **escape**.

Process-End-of-Statement

For pattern $L \rightarrow A R$
 If the **rank** of A is **greater-than one**, signal **rank-error**.
 If A is **empty**, return a **token** whose **class** is **nil**.
 Otherwise, set $A1$ to the **first-scalar** in A .
 If $A1$ is not a **near-integer**, signal **domain-error**.
 Return a **token** whose **class** is **branch** and whose **content** is the **numeric-scalar** with value the **integer-nearest-to** $A1$.

6.4 The Form Table

The **form-table** is the **list** of all **lists** of **syntactic-units** for which evaluation sequences exist.

The following matching rules apply in the **form-table**.

A, B, Z match **constant**.

I, K match **complete-index-list**.

f, g match **primitive-function** or **defined-function**, but not **defined-operator**.

A given ideogram, such as $\otimes$, matches a **primitive-function token** that contains it.

A given **distinguished-identifier**, such as $\square$ IO, matches any **system-variable-name token** or **system-function-name token** that contains it.

The behaviour of operations in the **form-table** that do not create new **contexts** is **atomic**.

This behaviour is observable only for those operations that have **side-effects**. For example, if any of the elements of an argument array is not in the domain of **roll**, the value of the system parameter **random-link** following execution will be as it was when **roll** was called.

Form	Operation Name	Page
$Z \leftarrow + B$	Conjugate	76
$Z \leftarrow - B$	Negative	76
$Z \leftarrow \times B$	Direction	77
$Z \leftarrow \div B$	Reciprocal	77
$Z \leftarrow \lfloor B$	Floor	78
$Z \leftarrow \lceil B$	Ceiling	78
$Z \leftarrow * B$	Exponential	79
$Z \leftarrow \otimes B$	Natural Logarithm	79
$Z \leftarrow B$	Magnitude	80
$Z \leftarrow ! B$	Factorial	81
$Z \leftarrow \circ B$	Pi times	82
$Z \leftarrow \sim B$	Not	83
$Z \leftarrow A + B$	Plus	84
$Z \leftarrow A - B$	Minus	84
$Z \leftarrow A \times B$	Times	85
$Z \leftarrow A \div B$	Divide	85
$Z \leftarrow A \lceil B$	Maximum	86
$Z \leftarrow A \lfloor B$	Minimum	86
$Z \leftarrow A * B$	Power	87
$Z \leftarrow A \otimes B$	Logarithm	88
$Z \leftarrow A B$	Residue	89
$Z \leftarrow A ! B$	Binomial	90
$Z \leftarrow A \circ B$	Circular Functions	91
$Z \leftarrow A \wedge B$	And/LCM	93
$Z \leftarrow A \vee B$	Or/GCD	94
$Z \leftarrow A \nwarrow B$	Nand	94
$Z \leftarrow A \nwarrow B$	Nor	95
$Z \leftarrow A = B$	Equal	96
$Z \leftarrow A < B$	Less than	97
$Z \leftarrow A \leq B$	Less than or equal to	98
$Z \leftarrow A \neq B$	Not equal	99
$Z \leftarrow A \geq B$	Greater than or equal to	100
$Z \leftarrow A > B$	Greater than	101
$Z \leftarrow , B$	Ravel	102
$Z \leftarrow \rho B$	Shape	103
$Z \leftarrow \imath B$	Index Generator	104
$Z \leftarrow \imath B$	Table	105
$Z \leftarrow A \rho B$	Reshape	107
$Z \leftarrow A , B$	Join	109
$Z \leftarrow A \imath B$	Join	109

Table 4: The Form Table

Form	Operation Name	Page
$Z \leftarrow \mathbf{f} / B$	Reduction	110
$Z \leftarrow \mathbf{f} / [K] B$	Reduction	110
$Z \leftarrow \mathbf{f} \neq B$	Reduction	110
$Z \leftarrow \mathbf{f} \neq [K] B$	Reduction	110
$Z \leftarrow \mathbf{f} \setminus B$	Scan	113
$Z \leftarrow \mathbf{f} \setminus [K] B$	Scan	113
$Z \leftarrow \mathbf{f} \setminus B$	Scan	113
$Z \leftarrow \mathbf{f} \setminus [K] B$	Scan	113
$Z \leftarrow N \mathbf{f} / B$	N-wise Reduction	115
$Z \leftarrow N \mathbf{f} / [K] B$	N-wise Reduction	115
$Z \leftarrow N \mathbf{f} \neq B$	N-wise Reduction	115
$Z \leftarrow N \mathbf{f} \neq [K] B$	N-wise Reduction	115
$Z \leftarrow \mathbf{f} \ddot{\sim} B$	Duplicate	118
$Z \leftarrow A \mathbf{f} \ddot{\sim} B$	Commute	118
$Z \leftarrow A \circ . \mathbf{f} B$	Outer Product	120
$Z \leftarrow A \mathbf{f} . \mathbf{g} B$	Inner Product	121
$Z \leftarrow \mathbf{f} \ddot{\circ} \mathbf{y} B$	Rank	124
$Z \leftarrow A \mathbf{f} \ddot{\circ} \mathbf{y} B$	Rank	125
$Z \leftarrow ? B$	Roll	127
$Z \leftarrow \mathbb{A} B$	Grade Up	129
$Z \leftarrow \mathbb{V} B$	Grade Down	131
$Z \leftarrow \phi B$	Reverse	132
$Z \leftarrow \ominus B$	Reverse	132
$Z \leftarrow \phi [K] B$	Reverse	132
$Z \leftarrow \ominus [K] B$	Reverse	132
$Z \leftarrow \mathbb{Q} B$	Monadic Transpose	133
$Z \leftarrow \mathbb{E} B$	Matrix Inverse	134
$Z \leftarrow \mathbb{E} B$	Execute	135
$Z \leftarrow \cup B$	Unique	136
$Z \leftarrow A , [K] B$	Join Along an Axis	137
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7 Scalar Functions

Note: The primitive functions described in this chapter are called **scalar-functions**. All **scalar-functions** have uniform behaviour with respect to the structure of their argument arrays. The shape of the result of a **scalar-function** is determined solely by the shapes of its arguments.

This section defines **scalar-functions** individually for scalar arguments. Their common behaviour is described in this informal description by the expository device of an implicit operator, called the **scalar-extension-operator**.

If the argument to a monadic scalar function is not a scalar, a monadic scalar extension operator can be thought of as being invoked to produce a derived function which, in turn, applies the monadic scalar function to every element of the argument array, producing a result array of the same shape as the argument. The order in which the elements of the argument array are presented to the monadic scalar function is not specified by this International Standard. Monadic scalar functions never signal **rank-error** or **length-error**.

If either of the arguments of a dyadic scalar function is not a scalar, a dyadic scalar extension operator can be thought of as being invoked to produce a derived function, which provides pairs of scalars to the scalar function as follows:

The dyadic scalar extension operator first tests whether the two argument arrays have the same shape. Arguments to a dyadic scalar function must have the same shape. If they do not, and the argument of lesser rank is a scalar or one-element vector, the argument of lesser rank is reshaped to the shape of the argument of greater rank.

If the arguments cannot be made to have the same shape, the dyadic scalar extension operator signals a **rank-error** if the arguments are of different ranks and a **length-error** otherwise.

When the dyadic scalar extension operator succeeds, it produces a derived function which generates a scalar for each position in its result **array** by applying the subject **scalar-function** to pairs of scalars selected from corresponding positions in the argument arrays. The order in which the elements of the result **array** are produced is not specified by this International Standard.

Because the derived function produced by either scalar extension operator never calls its **scalar-function** argument for empty arrays, **domain-error** can never be signalled for empty array arguments or for arrays reshaped by scalar extension to empty.

The type of all empty results produced by the functions derived from monadic and dyadic scalar extension is a property of the function to scalar extension. Since all scalar functions specified in this International Standard produce **numeric** results, the type of all empty results produced by scalar extension is specified as **numeric**.

For example, $1 \wedge 5$ and -1 return empty **numeric** results rather than signalling an error.

7.1 Monadic Scalar Functions

Note: The definitions in this section cover only scalar arguments. The **phrase-evaluator evaluate-monadic-function** handles non-scalar cases. All **scalar-functions** yield scalar results when applied to scalar arguments.

Note that, in this International Standard, **roll** is not a **scalar-function**.

7.1.1 Conjugate

$$Z \leftarrow + B$$

Informal Description: Z is the conjugate of B . Geometrically, it is the reflection of the number about the real axis.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return B with its **imaginary-part** negated.

Example:

$$\begin{array}{r} + 3 \sqrt{4} 0 0.5 3\sqrt{4} \sqrt{3}\sqrt{4} 3\sqrt{4} \\ 3 \sqrt{4} 0 0.5 3\sqrt{4} \sqrt{3}\sqrt{4} 3\sqrt{4} \end{array}$$

7.1.2 Negative

$$Z \leftarrow - B$$

Informal Description: Z is the **negation** of B .

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return **zero** minus B .

Example:

$$\begin{array}{r} - 7 0 \sqrt{7} \\ \sqrt{7} 0 7 \end{array}$$

7.1.3 Direction

$$Z \leftarrow \times B$$

Informal Description: Z is **zero** if B is **zero**, and otherwise is the **number** with **magnitude one** that has the same **direction** as B . Geometrically, it is the number determined by the radial projection of B onto the **unit-circle**. For real numbers, this is identical with the **sign** or **signum** function, and takes on only the values **one**, **zero**, and **negative one**.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.

If B is **zero**, return **zero**; otherwise, return B **divided-by** the **magnitude** of B .

Examples:

```

      ×1 - .5 .33 0 -1E-20
1 -1 1 0 -1
      ×3J4 3J-4 -3J4 -3J-4
0.6J0.8 0.6J-0.8 -0.6J0.8 -0.6J-0.8

```

7.1.4 Reciprocal

$$Z \leftarrow \div B$$

Informal Description: Z is $1 \div B$.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.

If B is **zero**, signal **domain-error**.

Return **one** **divided-by** B .

Examples:

```

      ÷ - .25 .5 1 2 -4
-4 2 1 0.5 -0.25

```

÷0

domain-error

7.1.5 Floor

$$Z \leftarrow \lfloor B$$

Informal Description: Z is the **complex-integer** U associated with the **unit-square** containing B , unless the sum of the **fractional-parts** of the **real-part** and **imaginary-part** of B is **greater-than-or-equal-to one**, in which case it is the nearer of U **plus one** or U **plus imaginary-one**. In case of a tie, it is U **plus one**. For **real-numbers**, Z is the greatest **integer** tolerantly less than or equal to B . Uses **comparison-tolerance**

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
 If B is not a **number**, signal **domain-error**.
 Return the **tolerant-floor** of B **within comparison-tolerance**.

Examples:

- In the following, **comparison-tolerance** is $1E^{-10}$.

```

⌊ 3.1416 3.1416 .9999999999 5E20 ⌊ 0.5E-10
⌊ 4 3 1 5E20 0
⌊ 0.3J0.6 0.6J0.8 0.8J0.6 0.6J0.3 0.8J0.2 0.5J0.5
0 0J1 1 0 1 1

```

Note: The following article describes the design of the floor function for complex arguments:
 McDonnell, E. E., "Complex Floor", **APL Congress 73**, North Holland Publishing Co., 1973

7.1.6 Ceiling

$$Z \leftarrow \lceil B$$

Informal Description: Z is the **negation** of the **floor** of the **negation** of B . For B a **real-number**, Z is the least **integer** tolerantly greater than or equal to B . Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
 If B is not a **number**, signal **domain-error**.
 Return $-\lfloor -B$.

Example:

- In the following, **comparison-tolerance** is $1E^{-10}$.

```

⌈ 3.1416 3.1416 5.00000000001
⌈ 3 4 5

```

7.1.7 Exponential

$$Z \leftarrow * B$$

Informal Description: Z is e raised to the power B , where e is the base of the natural logarithms.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return the **exponential** of B .

Examples:

```
* 1E50 2 1 0 1 2
0 0.135335 0.367879 1 2.71828 7.38906
* .693147
2
```

7.1.8 Natural Logarithm

$$Z \leftarrow \otimes B$$

Informal Description: Z is the natural logarithm of B .

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
If B is **zero**, signal **domain-error**.
Return the **natural logarithm** of B .

Examples:

```
\otimes 1828459045 2 1E-50 1E50
1 0.693147 115.129 115.129
\otimes * 1
1
```

7.1.9 Magnitude

$Z \leftarrow | B$

Informal Description: Z is the magnitude of B .

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return the **magnitude** of B .

Example:

```
| 1 -0.5 0.33 -0.25 0 1E-20
1 0.5 0.33 0.25 0 1E-20
```

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7.1.10 Factorial

$$Z \leftarrow ! B$$

Informal Description: Z is the **gamma-function** of $B+1$. If B is a **nonnegative-integer**, this is factorial B .

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.

If B is a **negative-integer**, signal **domain-error**.

Set $B1$ to B **plus one**.

Return **gamma-function** of $B1$.

Examples:

```

      ! 0 1 2 3 4 5 6 7 8 9
1 1 2 6 24 120 720 5040 40320 362880

      ! .5
1.77245

      5 1 p! = 1.502 1.503 1.504 1.505 1.506
-3.54471
-3.54466
-3.54464
-3.54466
-3.5447

```

Note: The **gamma-function** is defined in, for example, the **National Bureau of Standards Handbook of Mathematical Functions**, U.S. Government Printing Office, Washington D.C., 1964.

See also Hart, J. F., **Computer Approximations**, Robert C. Krieger Publishing Company, Huntington, NY, 1978.

7.1.11 Pi times

$Z \leftarrow \circ B$

Informal Description: Z is π times B .

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return **pi-times** B .

Example:

$\circ 1 \ 10 \ 100$
3.14159 31.4159 314.159

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7.1.12 Not

$$Z \leftarrow \sim B$$

Informal Description: Z is the Boolean complement of B .

Evaluation Sequence:

If B is not **near-Boolean**, signal **domain-error**.
 If the **integer-nearest-to** B is **one**, return **zero**.
 Otherwise, return **one**.

Example:

- For the following, the **implementation-parameter integer-tolerance** is $1E-10$.

```

~0 1 1E-11 .999999999999
1 0 1 0

```

7.2 Dyadic Scalar Functions

Note: The definitions in this section cover only scalar arguments. The **phrase-evaluator evaluate-dyadic-function** handles non-scalar cases. All **scalar-functions** yield scalar results when applied to scalar arguments.

The outer product operator, which has not yet been formally introduced at this point in the document, is used in the examples in this section as a convenient way of generating tables. The use of outer product in this section is limited to vector arguments. The same results could be obtained from each example, although not so compactly, by supplying the elements of the left argument one at a time, starting from the leftmost, as left arguments to the scalar function.

For example,

Example:

```

0 1 • = 0 1 2
1 0 0
0 1 0

```

is equivalent to

Example:

```

0 = 0 1 2
1 0 0
1 = 0 1 2
0 1 0

```

7.2.1 Plus

$$Z \leftarrow A + B$$

Informal Description: Z is A plus B .

Evaluation Sequence:

If either of A or B is not a **number**, signal **domain-error**.
Return A **plus** B .

Example:

$$\begin{array}{cccc} & -2 & -1 & 0 & 1 \\ -4 & -3 & -2 & -1 & \\ -3 & -2 & -1 & 0 & \\ -2 & -1 & 0 & 1 & \\ -1 & 0 & 1 & 2 & \end{array} \circ . + \begin{array}{cccc} -2 & -1 & 0 & 1 \end{array}$$

7.2.2 Minus

$$Z \leftarrow A - B$$

Informal Description: Z is A minus B .

Evaluation Sequence:

If either of A or B is not a **number**, signal **domain-error**.
Return A **minus** B .

Example:

$$\begin{array}{cccc} & -2 & -1 & 0 & 1 \\ 0 & -1 & -2 & -3 & \\ 1 & 0 & -1 & -2 & \\ 2 & 1 & 0 & -1 & \\ 3 & 2 & 1 & 0 & \end{array} \circ . - \begin{array}{cccc} -2 & -1 & 0 & 1 \end{array}$$

7.2.3 Times

$$Z \leftarrow A \times B$$

Informal Description: Z is A times B .

Evaluation Sequence:

If either of A or B is not a **number**, signal **domain-error**.
Return A **times** B .

Example:

	⁻²	⁻¹	0	1	∘.×	⁻²	⁻¹	0	1
4	2	0	⁻²						
2	1	0	⁻¹						
0	0	0	0						
⁻²	⁻¹	0	1						

7.2.4 Divide

$$Z \leftarrow A \div B$$

Informal Description: Z is A divided by B .

Evaluation Sequence:

If either of A or B is not a **number**, signal **domain-error**.
If B is **zero** and A is not **zero**, signal **domain-error**.
If B is **zero** and A is **zero**, return **one**.
Otherwise, return A **divided-by** B .

Example:

	0	1	2	3	4	∘.÷	1	2	3	4
0	0		0							0
1	0.5		0.333333							0.25
2	1		0.666666							0.5
3	1.5		1							0.75
4	2		1.33333							1

7.2.5 Maximum

$$Z \leftarrow A \upharpoonright B$$

Informal Description: Z is the larger of A and B .

Evaluation Sequence:

If either of A or B is not a **near-real number**, signal **domain-error**.
 Set $A1$ to the **real-number** nearest to A .
 Set $B1$ to the **real-number** nearest to B .
 If $A1$ is **greater-than** $B1$, return $A1$.
 Otherwise, return $B1$.

Example:

$$\begin{array}{cccc} & -2 & -1 & 0 & 1 \\ -2 & -1 & 0 & 1 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{array} \circ . \begin{array}{cccc} & -2 & -1 & 0 & 1 \\ -2 & -1 & 0 & 1 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{array}$$

7.2.6 Minimum

$$Z \leftarrow A \downharpoonright B$$

Informal Description: Z is the smaller of A and B .

Evaluation Sequence:

If either of A or B is not a **near-real number**, signal **domain-error**.
 Set $A1$ to the **real-number** nearest to A .
 Set $B1$ to the **real-number** nearest to B .
 If $A1$ is **greater-than** $B1$, return $B1$.
 Otherwise, return $A1$.

Example:

$$\begin{array}{cccc} & -2 & -1 & 0 & 1 \\ -2 & -2 & -2 & -2 \\ -2 & -1 & -1 & -1 \\ -2 & -1 & 0 & 0 \\ -2 & -1 & 0 & 1 \end{array} \circ . \begin{array}{cccc} & -2 & -1 & 0 & 1 \\ -2 & -1 & 0 & 1 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{array}$$

7.2.7 Power

$$Z \leftarrow A * B$$

Informal Description: Z is A raised to the B th power.

Evaluation Sequence:

If either of A or B is not a **number**, signal **domain-error**.

If A is **zero** and B is **zero**, return **one**.

If A is **zero** and the **real-part** of B is a **positive-number**, return **zero**, otherwise signal **domain-error**.

Return A **to-the-power** B .

Examples:

- In the following, **print-precision** is 12.

```

2*32
4294967296

4*0.5
2

-8*÷3
1J1.73205080757
```

Additional Requirement:

The foregoing example shows that, when the optional complex arithmetic facility is implemented, the **implementation-algorithm** should yield the principal value of the n th (odd n) root of a negative number, not the real negative root. If it is not implemented, a domain error should be signalled.

7.2.8 Logarithm

$$Z \leftarrow A \otimes B$$

Informal Description: Z is the logarithm of B to the base A .

Evaluation Sequence:

If either of A or B are not **numbers** signal **domain-error**.
If A and B are **equal**, return **one**.
If A is **one**, signal **domain-error**.
Set $A1$ to the **natural-logarithm** of A .
Set $B1$ to the **natural-logarithm** of B .
Return $B1$ **divided-by** $A1$.

Example:

10 2 10 0.1 $\otimes$ 2 65536 1E15 1E15
0.30103 16 15 $^{-15}$

7.2.9 Residue

$$Z \leftarrow A \mid B$$

Informal Description: Z is B modulo A . Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
 If either of A or B is not a **number**, signal **domain-error**.
 If A is **zero**, return B .
 If **comparison-tolerance** is not **zero**, and B **divided-by** A is **integral-within comparison-tolerance**, return **zero**.
 Otherwise set Z to B **modulo** A .
 If Z is A , return **zero**.
 Otherwise, return Z .

Examples:

- In the following, **print-precision** is 16 and **comparison-tolerance** is $1E^{-10}$.

```

      7 ^-7 0. | 31 28 ^-30
    3 0 5
  ^-4 0 ^-2
      0.2 | 1.4 1.5 1.6
    0 0.1 0
      1 | 1E30 1E^-30 ^-1E^-30 .9999999999
    0 0 0 0
  
```

- In the following, **comparison-tolerance** is **zero**.

```

      1 | 1E30 1E^-30 ^-1E^-30 .9999999999
    0 1E^-30 0 0.9999999999
  
```

Additional Requirement:

The range of residue is those numbers that are the product of a **fraction** and A , except when A is **zero**, in which case the range is the single **number** B .

Note: The **implementation-algorithm** P **modulo** Q provides an exact modulo operation for **real-numbers** P and Q . It evaluates $R \leftarrow P - (\times P) \times \lfloor Q \times \rfloor \mid P \div Q$ exactly, and returns R if $(\times R) = \times Q$, or $R + Q$ otherwise.

The definition of “mod” in the IEEE standard for Binary Floating-Point Arithmetic (754) provides an example of this exact evaluation.

Implementations should avoid signalling **limit-error** in **residue**. If the operation B **divided-by** A causes **exponent-overflow**, return **zero**. If it causes **exponent-underflow**, and if A and B have the same signs, return B . If they have different signs, return **zero**.

7.2.10 Binomial

$$Z \leftarrow A ! B$$

Informal Description: Z is $(\text{gamma}(1+B)) \div ((\text{gamma}(1+A)) \times \text{gamma}(1+B-A))$

If A and B are **nonnegative-integers**, Z is the number of combinations of B things taken A at a time.

Evaluation Sequence:

If either of A or B is not a **number**, signal **domain-error**.

Determine if each of A , B , and $B-A$ is a **negative-integer**.

Select the appropriate case from the following table, where a **one** indicates that the corresponding value is a **negative-integer** and a **zero** indicates that it is not.

Case Rule

$A \quad B \quad B-A$

0 0 0 Return $(!B) \div (!A) \times !B-A$.

0 0 1 Return **zero**.

0 1 0 Signal **domain-error**.

0 1 1 Return $(-1 \times A) \times A ! A-B+1$.

1 0 0 Return **zero**.

1 0 1 (Case cannot arise.)

1 1 0 Return $(-1 \times B-A) \times (|B+1) ! (|A+1)$.

1 1 1 Return **zero**.

Example:

	⁻⁴	⁻³	⁻²	⁻¹	0	1	2	3	4	∘	!	⁻⁴	⁻³	⁻²	⁻¹	0	1	2	3	4
1		⁻³	3	⁻¹	0	0	0	0	0	0	0									
0		1	⁻²	1	0	0	0	0	0	0	0									
0		0	1	⁻¹	0	0	0	0	0	0	0									
0		0	0	1	0	0	0	0	0	0	0									
1		⁻¹	1	1	1	1	1	1	1	1	1									
⁻⁴	⁻³	⁻²	⁻¹	0	1	2	3	4												
10	6	3	1	0	0	1	3	6												
⁻²⁰	⁻¹⁰	⁻⁴	⁻¹	0	0	0	1	4												
35	15	5	1	0	0	0	0	1												

Note: The APL expressions in the **rule** column indicate the result required, not the algorithm to be used. For example, $64!65$ should be 65 even if $!65$ signals **limit-error**.

7.2.11 Circular Functions

$$Z \leftarrow A \circ B$$

Informal Description: Z is the result of applying a function designated by A to B .

Evaluation Sequence:

If A is not a **near-integer**, signal **domain-error**.
 If B is not a **number**, signal **domain-error**.
 Set $A1$ to the **integer-nearest-to** A .
 If $A1$ is not in the **closed-interval-between** -12 and 12 , signal **domain-error**.
 If $A1$ is -12 , return $*0J1 \times B$.
 If $A1$ is -11 , return $0J1 \times B$.
 If $A1$ is -10 , return $+B$.
 If $A1$ is -9 , return B .
 If $A1$ is -8 , return $-(-1-B \times 2) \times .5$.
 If $A1$ is -7 ,
 If B is **negative-one** or **one**, signal **domain-error**.
 Return the **inverse-hyperbolic-tangent** of B .
 If $A1$ is -6 ,
 Return Z , the principal value of the **inverse-hyperbolic-cosine** of B , where Z is a **nonnegative-number**.
 If $A1$ is -5 , return the **inverse-hyperbolic-sine** of B .
 If $A1$ is -4 ,
 If B is -1 return **zero**
 Otherwise return $(B+1) \times ((B-1) \div B+1) \times 0.5$.
 If $A1$ is -3 , return Z , the principal value in radians of the **inverse-tangent** of B , where Z is in the **open-interval-between** $-\pi/2$ and $\pi/2$.
 If $A1$ is -2 ,
 Return Z , the principal value in radians of the **inverse-cosine** of B , where Z is either **zero** or a **number** in the **open-interval-between zero** and π .
 If $A1$ is -1 ,
 Return Z , the principal value in radians of the **inverse-sine** of B , where Z is either $\pi/2$, or a **number** in the **open-interval-between** $-\pi/2$ and $\pi/2$.
 If $A1$ is 0 ,
 If B is not in the **closed-interval-between negative-one** and **one**, signal **domain-error**.
 Return $(1-B \times 2) \times 0.5$.
 If $A1$ is 1 , return the **sine** of B radians.
 If $A1$ is 2 , return the **cosine** of B radians.
 If $A1$ is 3 ,
 If B is an odd multiple of $\pi/2$, signal **domain-error**.
 Return the **tangent** of B radians.
 If $A1$ is 4 , return $(1+B \times 2) \times 0.5$.
 If $A1$ is 5 , return the **hyperbolic-sine** of B .
 If $A1$ is 6 , return the **hyperbolic-cosine** of B .

If $A1$ is 7, return the **hyperbolic-tangent** of B .
 If $A1$ is 8, return $(-1-B*2)*.5$.
 If $A1$ is 9, return the **real-part** of B .
 If $A1$ is 10, return $|B|$.
 If $A1$ is 11, return the **imaginary-part** of B .
 If $A1$ is 12, return the **arc** of B .

Examples:

	$2 \circ -1 \circ .6$
.8	$2 \circ 0$
1	$3 \circ 0 \div 4$
1	$6 \circ 0$
1	

Note: The APL expressions used for $0 \circ X$, $-4 \circ X$, and $4 \circ X$ above indicate the result desired, not the algorithm to be used.

Note: The following article describes the reasons for the choices made in defining the circular functions for complex arguments,

Penfield, Paul, "Principal Values and Branch Cuts in Complex APL", **APL81 Conference Proceedings**, ACM, San Francisco, 1981

Note: Values of A greater than 7 in magnitude are required only for an implementation which includes the optional **complex-arithmetic-facility**.

7.2.12 And/LCM

$$Z \leftarrow A \wedge B$$

Informal Description: Z is the least common multiple of A and B . For Boolean arguments it is the Boolean product of A and B . Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
 If either A or B is not a **number**, signal **domain-error**.
 If both A and B are **near-boolean**,
 Set $A1$ to the **integer-nearest-to** A .
 Set $B1$ to the **integer-nearest-to** B .
 If either $A1$ or $B1$ is **zero**, return **zero**.
 Otherwise, return **one**.
 Otherwise, set $A1$ to the **greatest-common-divisor** of A and B , using the **implementation-algorithm greatest-common-divisor**.
 If $A1$ is **zero**, return **zero**.
 Otherwise, return A **times** (B **divided-by** $A1$).

Examples:

```

0 1 ◦ . ^ 0 1
0 0
0 1
30 ^ 36
180
3 ^ 3.6
18
-29J53 ^ -1J107
-853J329

```

7.2.13 Or/GCD

$$Z \leftarrow A \vee B$$

Informal Description: Z is the greatest common divisor of A and B . For Boolean arguments, Z is the Boolean sum of A and B . Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.

If either A or B is not a **number**, signal **domain-error**.

If both A and B are **near-boolean**,

Set $A1$ to the **nearest-integer** to A .

Set $B1$ to the **nearest-integer** to B .

If either $A1$ or $B1$ is **one**, return **one**.

Otherwise, return **zero**.

Otherwise, return the **greatest-common-divisor** of A and B , using the **implementation-defined-algorithm greatest-common-divisor**.

Examples:

```

0 1 0.5 0 1
0 1
1 1
30 36
6
3 3.6
0.6
-29753 -17107
771

```

7.2.14 Nand

$$Z \leftarrow A \wedge B$$

Informal Description: Z is the Boolean complement of the Boolean product of A and B .

Evaluation Sequence:

If either A or B is not **near-Boolean**, signal **domain-error**.

Otherwise, return $\sim A \wedge B$, with **comparison-tolerance** set to **zero**.

Example:

```

0 1 0.5 0 1
1 1
1 0

```

7.2.15 Nor

$$Z \leftarrow A \vee B$$

Informal Description: Z is the Boolean complement of the Boolean sum of A and B .

Evaluation Sequence:

If either A or B is not **near-Boolean**, signal **domain-error**.

Otherwise, return $\sim A \vee B$, with **comparison-tolerance** set to **zero**.

Example:

	0	1	∅	∞	0	1
1	0					
0	0					

STANDARDSISO.COM : Click to view the full PDF of ISO/IEC 13751:2001

7.2.16 Equal

$$Z \leftarrow A = B$$

Informal Description: Z is **one** if A and B are considered equal and **zero** otherwise. A and B are equal if they are the same **character**, or if they are both **numeric** and A is tolerantly equal to B within **comparison-tolerance**. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
 If the **type** of A is not the same as the **type** of B , return **zero**.
 If both A and B are **characters**,
 If A is the same **character** as B , return **one**.
 Otherwise, return **zero**.
 If both A and B are **numbers**,
 If A is **tolerantly-equal to** B within **comparison-tolerance**, return **one**.
 Otherwise, return **zero**.

Example:

```

      1 2 3 °.= 1 2 3
1 0 0
0 1 0
0 0 1

```

– In the following, **comparison-tolerance** is $1E^{-13}$.

```

      4 = 4 + 5E-13 2E-13 -2E-13 -5E-13
0 1 1 0

      0 = -1E-20 1E-20 0
0 0 1

      3 = 'A3'
0 0

```

Note: Comparisons of **numbers** whose signs differ are not affected by **comparison-tolerance**.

For any value of **comparison-tolerance** and any two real numbers A and B , exactly one of the expressions $A < B$, $A = B$, and $A > B$ is **one**.

Equal should not signal a **limit-error**. For example, the result of **positive-number-limit** = **negative-number-limit** is **zero**. The following is a sample technique for handling exponent-overflow and exponent-underflow when scaling **comparison-tolerance**.

Set C to the larger of the magnitudes of A and B .

Set D to **comparison-tolerance** times C .

If **exponent-underflow** occurs,

 Set $A1$ to A **divided-by** C .

 Set $B1$ to B **divided-by** C .

 Set $C1$ to the magnitude of $A1$ **minus** $B1$.

If $C1$ is **greater-than comparison-tolerance**, return **zero**.
 Otherwise, return **one**.
 Set E to the magnitude of A **minus** B .
 If **exponent-overflow** occurs, return **zero**.
 If **exponent-underflow** occurs,
 Set $A1$ to A **divided-by** C .
 Set $B1$ to B **divided-by** C .
 Set $C1$ to the magnitude of $A1$ **minus** $B1$.
 If $C1$ is **greater-than comparison-tolerance**, return **zero**.
 Otherwise, return **one**.
 If E is not **greater-than** D , return **one**.
 Otherwise, return **zero**.

7.2.17 Less than

$$Z \leftarrow A < B$$

Informal Description: Z is **one** if A is tolerantly less-than B , and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
 If either of A or B is not a **near-real number**, signal **domain-error**.
 Set $A1$ to the **real-number** nearest to A .
 Set $B1$ to the **real-number** nearest to B .
 If $A1=B1$, evaluated with the current value of **comparison-tolerance**, is **one**, return **zero**.
 If $A1$ is **less-than** $B1$, return **one**.
 Otherwise, return **zero**.

Examples:

```

      1 2 3 °.< 1 2 3
0 1 1
0 0 1
0 0 0
      0 1 °.< 0 1
0 1
0 0
  
```

7.2.18 Less than or equal to

$$Z \leftarrow A \leq B$$

Informal Description: Z is **one** if A is less than or tolerantly equal to B , and **zero** otherwise.
Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of A or B is not a **near-real number**, signal **domain-error**.
Set $A1$ to the **real-number** nearest to A .
Set $B1$ to the **real-number** nearest to B .
If $A1=B1$, evaluated with the current value of **comparison-tolerance**, is **one**, return **one**.
If $A1$ is **less-than** $B1$, return **one**.
Otherwise, return **zero**.

Examples:

```

      1 2 3 0.≤1 2 3
1 1 1
0 1 1
0 0 1

      0 1 0.≤0 1
1 1
0 1

```

7.2.19 Not equal

$$Z \leftarrow A \neq B$$

Informal Description: Z is **one** if A does not equal B , and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.

Return $\sim A=B$, evaluated with the current value of **comparison-tolerance**.

Examples:

```
'A' ≠ 4 1
1
1 2 3 ∘. ≠ 1 2 3
0 1 1
1 0 1
1 1 0

0 1 ∘. ≠ 0 1
0 1
1 0
```

7.2.20 Greater than or equal to

$$Z \leftarrow A \geq B$$

Informal Description: Z is **one** if A is greater than or tolerantly equal to B , and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
 If either of A or B is not a **near-real number**, signal **domain-error**.
 Set $A1$ to the **real-number** nearest to A .
 Set $B1$ to the **real-number** nearest to B .
 If $A1=B1$, evaluated with the current value of **comparison-tolerance**, is **one**, return **one**.
 If $A1$ is **greater-than** $B1$, return **one**.
 Otherwise, return **zero**.

Examples:

```

      1 2 3 0. ≥ 1 2 3
1 0 0
1 1 0
1 1 1
      0 1 0. ≥ 0 1
1 0
1 1
    
```

7.2.21 Greater than

$$Z \leftarrow A > B$$

Informal Description: Z is **one** if A is tolerantly greater than B , and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.

If either of A or B is not a **near-real number**, signal **domain-error**.

Set $A1$ to the **real-number** nearest to A .

Set $B1$ to the **real-number** nearest to B .

If $A1=B1$, evaluated with the current value of **comparison-tolerance**, is **one**, return **zero**.

If $A1$ is **greater-than** $B1$, return **one**.

Otherwise, return **zero**.

Examples:

```

      1 2 3 °.> 1 2 3
0 0 0
1 0 0
1 1 0

      0 1 °.> 0 1
0 0
1 0

```

8 Structural Primitive Functions

8.1 Introduction

Note: The functions in this chapter are used in the evaluation sequences of many non-scalar operations. They are defined here to avoid forward references.

8.2 Monadic Structural Primitive Functions

8.2.1 Ravel

$$Z \leftarrow , B$$

Informal Description: Z is a vector containing the elements of B in row-major order.

Evaluation Sequence:

Return Z , a **vector** such that the **ravel-list** of Z is the same as the **ravel-list** of B , the **type** of Z is the same as the **type** of B , and the **shape-list** of Z is a **list** of length **one** containing the **count** of B as its only **item**.

Examples:

```
,N22
11 12 21 22
,N222
111 112 121 122 211 212 221 222
,N2221
1111 1121 1211 1221 2111 2121 2211 2221
```

Note: *Ravel* always produces a vector result. The expressions **ravel-list** of Z , **type** of Z , and **shape-list** of Z refer to attributes of an **array** object.

8.2.2 Shape

$$Z \leftarrow \rho \ B$$

Informal Description: Z is a **numeric** vector containing the shape of the array B .

Evaluation Sequence:

Return Z , an **array** such that the **type** of Z is **numeric**, the **ravel-list** of Z is the **shape-list** of B , and the **shape-list** of Z is a **list** whose only **item** contains the **number-of-items** in the **shape-list** of B .

Examples:

	ρN
	ρ, N
1	$\rho \rho N$
0	$\rho N 3$
3	$\rho \rho N 3$
1	$\rho N 3 4$
3 4	

Note: *Shape always produces a vector result. The expression **shape-list** of Z refers to an attribute of an **array** object.*

8.2.3 Index Generator

$$Z \leftarrow \iota B$$

Informal Description: Z is a **numeric vector** of B consecutive ascending **integers**, the first of which is **index-origin**. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.

If the **rank** of B is **greater-than one**, signal **rank-error**.

If the **count** of B is not **one**, signal **length-error**.

If B is not a **near-integer**, signal **domain-error**.

Set $B1$ to the **integer-nearest-to** B .

If $B1$ is not a **nonnegative-integer**, signal **domain-error**.

If $B1$ is **zero**, return an **empty numeric vector**.

Generate a **numeric vector** $Z1$ of length $B1$ such that the **ravel-list** of $Z1$ consists of the **integers** in the **closed-interval-between one** and $B1$ in ascending order. Generate Z , a **numeric array** with the **shape-list** of $Z1$ such that each **item** of the **ravel-list** of Z is (**index-origin minus one**) plus the corresponding element of $Z1$.

Return Z .

Examples:

- In the following example, **index-origin** is **zero**.

$$\begin{array}{c} \iota 4 \\ 0 \ 1 \ 2 \ 3 \end{array}$$

- In the following example, **index-origin** is **one**.

$$\begin{array}{c} \iota 4 \\ 1 \ 2 \ 3 \ 4 \end{array}$$

8.2.4 Table

$$Z \leftarrow \overline{\overline{B}}$$

Informal Description: Z is a **matrix** containing the elements of B . Z is formed by ravelling the subarrays of B along its first axis. For example, if B is a scalar, Z has shape 1 1; if B is a five-element vector, Z has shape 5 1; if B has shape 5 4 3 2, Z has shape 5 24.

Evaluation Sequence:

If B is a **scalar** set C to 1 1.

Else set C to a two-element **list** as follows:

Set the **first-item** of C to the **first-item** of the **shape-list** of B .

Set the **last-item** of C to the **product-of** the **rest-of** the **shape-list** of B .

Set Z to the C **reshape** of B .

Return Z .

Examples:

```

      0
0
      0 0
1 1
      0 0 0 0
1
2
3
4
      0 0 0 0 0 0 0 0
11 12
21 22
      0 0 0 0 0 0 0 0 0 0
111 112 121 122
211 212 221 222
      0 0 0 0 0 0 0 0 0 0 0 0
1111 1121 1211 1221
2111 2121 2211 2221

```

8.2.5 Depth

$$Z \leftarrow \equiv B$$

Informal Description: Z is a numeric scalar describing the level of nesting of B . For **simple-scalars**, this is 0. For arrays, this is one greater than the depth of the element with the greatest depth.

Note: An enclosed array is an array.

Evaluation Sequence:

If B is a **simple-scalar**, return **zero**.
Return $1 + \lceil / \rceil \equiv B$.

Examples:

	$\equiv 5$
0	$\equiv 1 \ 2 \ 3$
1	$\equiv N234$
1	$\equiv 'ABC', 1 \ 2 \ 3$
1	$\equiv <1 \ 2 \ 3$
2	$\equiv , <1 \ 2 \ 3$
2	$\equiv 'HERO', <2 \cdot 3, <2 \ 3 \rho <5 \ 8$
4	

8.2.6 Enlist

$$Z \leftarrow \epsilon B$$

Informal Description: If B is a **simple-array**, then enlist has the same effect as ravel. Otherwise, enlist has the effect of recursively raveling each element of B (in **ravel-list** order) and joining them together. Thus, Z will always be a **simple-array** of rank one, containing all items of B .

Note: *Enlist is sometimes called “super ravel”.*

Evaluation Sequence:

If B is empty, return $0\rho\epsilon 1\uparrow, B$.
 If B is a **simple-array**, return $,B$.
 Set $B1$ to the first item of the **ravel-list** of B .
 Return $(\epsilon B1), (\epsilon 1\downarrow, B)$.

Examples:

```

       $\epsilon 1\ 2\ 3, \epsilon 4\ 5\ 6$ 
1 2 3 4 5 6
       $\equiv 1\ 2\ 3, \epsilon 4\ 5\ 6$ 
2
       $\equiv \epsilon 1\ 2\ 3, \epsilon 4\ 5\ 6$ 
1
```

8.3 Dyadic Structural Primitive Functions

8.3.1 Reshape

$$Z \leftarrow A \rho B$$

Informal Description: Z is an array of shape $,A$ whose elements are taken sequentially from $,B$ repeated cyclically as required.

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
 If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
 Set $A1$ to the **integer-array-nearest-to** $,A$.
 If any **item** of the **ravel-list** of $A1$ is not a **nonnegative-counting-number**, signal **domain-error**.
 Let RA stand for the **product-of** the **ravel-list** of $A1$.
 Let CB stand for the **count** of B .

If RA is not **zero** and CB is **zero**, signal **length-error**.

Return an **array** Z such that the **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of B , the **shape-list** of Z is the same as the **ravel-list** of $A1$, and the **ravel-list** of Z is a **list** with RA **items** such that for all I in the **index-set** of Z , **item** I of the **ravel-list** of Z is **item** $1+CB \mid I-1$ of the **ravel-list** of B .

Examples:

```

      2 4ρN213
111 112 113 211
212 213 111 112
      ρ0ρ'ABCD'
0
      B ←1E7 1E7 1E7 0 1E7 1E7 1E7 ρ 42
      ρB
1E7 1E7 1E7 0 1E7 1E7 1E7

```

Note: For any X that is not **empty**, $' '\rho X$ and $(\iota 0)\rho X$ produce the same result: a **scalar** whose value is that of the **first-scalar** in X .

8.3.2 Join

$$Z \leftarrow A , B$$

$$Z \leftarrow A \overline{;} B$$

Informal Description: If A and B are scalars or vectors, Z is the vector of length $(\rho, A) + \rho, B$ whose first ρ, A elements are $, A$ and whose last ρ, B elements are $, B$.

If either A or B has rank greater than one,

For form $Z \leftarrow A , B$

Z is $A, [(\rho, A) \overline{;} \rho, B] B$,

For form $Z \leftarrow A \overline{;} B$

Z is $A, [1] B$,

as defined under **Join Along an Axis**.

Evaluation Sequence:

If A is a **scalar** and B is a **scalar**, return $(, A), , B$.

If A is a **scalar** and B is a **vector**, return $(, A), B$.

If A is a **vector** and B is a **scalar**, return $A, , B$.

If A is a **vector** and B is a **vector**,

If A is **empty** and B is **empty** and the **type** of A differs from the **type** of B , return an **empty-list** whose **type** is determined by the **implementation-algorithm typical-element-for-mixed**.

If B is **empty**, return A .

If A is **empty**, return B .

Otherwise, return a **vector** Z , such that the **shape-list** of Z is $(\rho, A) + \rho, B$, the **ravel-list** of Z is a **list** whose first ρ, A items are the **ravel-list** of A and whose last ρ, B items are the **ravel-list** of B , and the **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type mixed**.

Otherwise, when either A or B has rank greater than **one**, return $A, [1] B$ for form $A \overline{;} B$ and $A, [(\rho, A) \overline{;} \rho, B] B$ for form A, B .

Example:

$$\begin{matrix} & 1 & 1 & , & 0 \\ 0 & & & & \end{matrix}$$

Note: This subsection intentionally contains a forward reference to **Join Along an Axis**. **Join** and **Join Along an Axis** are defined separately because the description of **Join Along an Axis** requires APL operations that depend for their definitions in turn upon **Join**.

9 Operators

9.1 Introduction

Note: The forms in this chapter are referred to as **operators**.

9.2 Monadic Operators

9.2.1 Reduction

$$Z \leftarrow f / B$$

$$Z \leftarrow f / [K] B$$

$$Z \leftarrow f \neq B$$

$$Z \leftarrow f \neq [K] B$$

Informal Description: Z is the value produced by placing the dyadic function f between adjacent items along a designated axis of B and evaluating the resulting expression. The axis designated determines how the subarrays are chosen. Uses **index-origin**.

There are two conforming definitions for Reduction; the **Implementation Parameter Reduction-Style** specifies which definition a particular implementation uses.

The **Enclose-Reduction-Style** style, informally known as the APL2 style, uses a definition based on **enclose** which, for all f , preserves the identity:

$$\rho \rho Z \text{ is } 0 \uparrow 1 + \rho \rho B.$$

The **Insert-Reduction-Style** style, informally known as the Sharp/J style, does not universally preserve the above identity. Instead f is inserted between each successive **cell** along the **frame** axis (the axis of reduction). The rank of f controls the subsequent evaluation.

Note: Since reduction is used in the evaluation sequences of other operators, the choice of the **Implementation Parameter Reduction-Style** has a pervasive effect on the implementation, and its importance should not be underestimated.

Note: The functions listed in **Table 5** are **scalar** functions, that is their left and right function ranks are **zero**.

Evaluation Sequence:

If **Implementation Parameter Reduction-Style** is **Enclose-Reduction-Style**:

If **f** is not a **dyadic-function**, signal **syntax-error**.

For form **f/B**

If **B** is a **scalar**, return **B**.

Otherwise, return **f** / $[\neg 1 \uparrow \wr \rho B]$ **B** .

For form **f≠B**

If **B** is a **scalar**, return **B**.

Otherwise, return **f** / $[\wr 1]$ **B** .

For forms **f/[K]** **B** and **f≠[K]** **B**

If **K** is not a **valid-axis** for **B**, signal **axis-error**.

Otherwise, set **K1** to the **integer-nearest-to** **K**.

If **B** is a **vector**,

If the **length** of **B** is **zero**, take the action designated in **Table 5** for **f**.

If the **length** of **B** is **one**, return a **scalar** **Z** such that the **type** of **Z** is the **type** of **B** and the **ravel-list** of **Z** is the **ravel-list** of **B**.

If the **length** of **B** is **greater-than one**,

Set **B1** to the **first-scalar** in **B**.

Set **B2** to the **remainder-of** **B**.

Return $\langle B1 \text{ f } \rangle B2$.

If the **rank** of **B** is **greater-than one**, return an **array** **Z** such that the **shape-list** of **Z** is the **shape-list** of **B** with **item** **K1** omitted, and the **ravel-list** of **Z** has the property that if **Z1** is an **item** of **Z** and **B3** is the corresponding **vector-item** **along-axis** **K1** of **B**, then **Z1** is **f/B3**.

Otherwise **Implementation Parameter Reduction-Style** is **Insert-Reduction-Style**:

If **f** is not a **dyadic-function**, signal **syntax-error**.

For form **f/B**

If **B** is a **scalar**, return **B**.

Otherwise, return **f** / $[\neg 1 \uparrow \wr \rho B]$ **B** .

For form **f≠B**

If **B** is a **scalar**, return **B**.

Otherwise, return **f** / $[\wr 1]$ **B** .

For forms **f/[K]** **B** and **f≠[K]** **B**

If **K** is not a **valid-axis** for **B**, signal **axis-error**.

Otherwise, set **K1** to the **integer-nearest-to** **K**.

Set **B1** to $(\uparrow K1, (\wr \rho B) \sim K1) \wr B$.

Set **C1** to $1 \downarrow \rho B1$.

If the number of **C1**-cells in **B1** is **zero**, take the action designated in **Table 5** for **f** to choose a result item **Z0**. Return an **array** **Z** such that the **shape-list** of **Z** is **C1**, the **type** of **Z** is the **type** of **Z0**, and each item of the **ravel-list** of **Z** is the

ravel-list of Z_0 .

If the number of C_1 -cells in B_1 is **one**, return an **array** Z such that the **shape-list** of Z is C_1 , the **type** of Z is the **type** of B_1 , and the **ravel-list** of Z is the **ravel-list** of B_1 .

If the number of C_1 -cells in B_1 is **greater-than one**,

Set B_2 to an array such that the **shape-list** of B_2 is C_1 , the **type** of B_2 is the **type** of B_1 , and the **ravel-list** of B_2 is the **ravel-list** of the first C_1 -cell in B_1 .

Set B_3 to an array such that the **shape-list** of B_3 is $(\neg 1 + 1 \uparrow \rho B_1), C_1$, the **type** of B_3 is the **type** of B_1 , and the **ravel-list** of B_3 is the **ravel-list** of all the C_1 -cells of B_1 except the first.

Return B_2 **f f** B_3 .

Examples:

$+ / 1 \ 2 \ 3$
 6
 $\times / 1 \ 2$
 2
 $= / 'A'$
 A
 $= / 'AA'$
 1
 $= / 'AAA'$
 0
 $(3 \ 8 \rho 0) \equiv, / 2 \ 3 \ 4 \rho 0 \ \text{Insert-Reduction-Style}$
 1
 $(3 \ 4 \rho < 0 \ 0) \equiv, / 2 \ 3 \ 4 \rho 0 \ \text{Enclose-Reduction-Style}$
 1

Additional Requirement:

If Z is **empty**, the **type** of Z is determined by the argument function **f**.

When applied along an empty **axis** in an array, reduction produces an array whose shape is that of the argument array with the designated **axis** deleted. The elements of the array, if any, are determined by the argument function and **Table 5**. In some cases, an error is signalled.

Example:

$+ / 2 \ 0 \ \rho 5.1$
 0 0
 $\rho + / 2 \ 0 \rho 5.1$
 2

Dyadic Function		Action
Plus	+	Return zero .
Minus	−	Return zero .
Times	×	Return one .
Divide	÷	Return one .
Residue		Return zero .
Minimum	⌊	Return positive-number-limit .
Maximum	⌈	Return negative-number-limit .
Power	*	Return one .
Binomial	!	Return one .
And	^	Return one .
Or	∨	Return zero .
Less	<	Return zero .
Not greater	≤	Return one .
Equal	=	Return one .
Not less	≥	Return one .
Greater	>	Return zero .
Not equal	≠	Return zero .
all others		Signal domain-error .

Table 5: Actions for the Reduction of an Empty Vector.

9.2.2 Scan

$$Z \leftarrow \mathbf{f} \setminus B$$

$$Z \leftarrow \mathbf{f} \setminus [K] B$$

$$Z \leftarrow \mathbf{f} \setminus B$$

$$Z \leftarrow \mathbf{f} \setminus [K] B$$

Informal Description: Z is an array having the same shape as B and containing the results produced by $\mathbf{f}$ reduction over all prefixes of a designated axis of B . Uses **index-origin**.

Evaluation Sequence:

If $\mathbf{f}$ is not a **dyadic-function**, signal **syntax-error**.

For form $\mathbf{f} \setminus B$

If B is a **scalar**, return B .

Otherwise, return $\mathbf{f} \setminus [\rho \rho B]$ B evaluated with **index-origin** set to **one**.

For form $\mathbf{f} \setminus B$

If B is a **scalar**, return B .

Otherwise, return $\mathbf{f} \setminus [1]$ B evaluated with **index-origin** set to **one**.

For forms $f \setminus [K] B$ and $f \setminus [K] B$

If K is not a **valid-axis** for B , signal **axis-error**.

Otherwise, set $K1$ to the **integer-nearest-to** K .

If B is a **vector**,

If the **count** of B is **less-than two**, return B .

Otherwise return Z , a **vector** such that the **shape-list** of Z is the **shape-list** of B , and the **ravel-list** of Z is such that **item** I of the **ravel-list** of Z is $f[B \setminus I]$ for all I in the **index-set** of the **ravel-list** of B . The **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type mixed**.

If the **rank** of B is **greater-than one**, each **vector-item along-axis** $K1$ of Z is $f \setminus B1$, where $B1$ is the corresponding **vector-item along-axis** $K1$ of B .

Examples:

```

+ \ 1 1 1
1 2 3
^ \ 1 1 1 0 0 0 1 1 1
1 1 1 0 0 0 0 0 0
- \ 'A'
A
= \ 'AB'
A 0

```

Additional Requirement:

If the operator **reduction** signals an error when called by **scan**, **scan** returns the resultant error **token**.

The evaluation sequence above describes a quadratic algorithm for **scan**. If the function f is associative, **scan** may be implemented with a linear algorithm.

Note: Various error checks have been performed on K by the phrase evaluators, and **index-origin** is **one**, before this evaluation sequence is called.

9.2.3 N-wise Reduction

$$Z \leftarrow N \text{ f} / B$$

$$Z \leftarrow N \text{ f} / [K] B$$

$$Z \leftarrow N \text{ f} \nabla B$$

$$Z \leftarrow N \text{ f} \nabla [K] B$$

Informal Description: **N-wise reduction** is the dyadic invocation of a function derived by **reduction**. Z is the value produced by placing the primitive scalar dyadic function $\mathbf{f}$ between subarrays of B and evaluating the resulting expression. If N is negative the subarrays are reversed before application of $\mathbf{f}$. Each subarray has length N along a common dimension. The axis designated determines how the subarrays are chosen. Uses **index-origin**.

Evaluation Sequence:

If the **rank** of N is **greater-than one**, signal **rank-error**.

If the length of the **ravel-list** of N is **greater-than one**, signal **length-error**.

If N is not a **near-integer**, signal **domain-error**.

Set $N1$ to the **integer-nearest-to** N .

Set $M1$ to the **magnitude** of $N1$.

If $\mathbf{f}$ is not a **dyadic-function**, signal **syntax-error**.

For form $N \mathbf{f} / B$

If B is a **scalar**,

If $M1$ is **greater-than two**, signal **domain-error**.

If $M1$ is **zero**, set R to **two** and take the action designated in **Table 6** for R and $\mathbf{f}$.

Otherwise, return $(2-M1) \rho B$.

Otherwise, return $N \mathbf{f} / [\rho \rho B]$ B evaluated with **index-origin** set to **one**.

For form $N \mathbf{f} \nabla B$

If B is a **scalar**,

If $M1$ is **greater-than two**, signal **domain-error**.

If $M1$ is **zero**, set R to **two** and take the action designated in **Table 6** for R and $\mathbf{f}$.

Otherwise, return $(2-M1) \rho B$.

Otherwise, return $N \mathbf{f} / [1]$ B evaluated with **index-origin** set to **one**.

For forms $N \mathbf{f} / [K] B$ and $N \mathbf{f} \nabla [K] B$

If K is not a **valid-axis** for B , signal **axis-error**.

Otherwise, set $K1$ to the **integer-nearest-to** K .

If B is a **vector**,

If $M1$ is **greater-than one plus the length** of B , signal **domain-error**.

If $M1$ is **zero**, set R to **one plus the length** of B and take the action designated in

Table 6 for R and $\mathbf{f}$.

Set $B1$ to $M1$ **take** B .

If $N1$ is **less-than zero**,

set $B2$ to $\phi B1$

otherwise set $B2$ to $B1$.

If $M1$ equals the length of B , return $f/B2$.

Otherwise,

Set $B3$ to $f/B2$.

Set $B4$ to one drop B .

Return $B3, Nf / B4$.

If the rank of B is greater than one, return an array Z such that the shape-list of Z is the shape-list of B with item $K1$ replaced by one plus the value of item $K1$ minus $M1$, and the ravel-list of Z has the property that if $Z1$ is the vector-item along-axis $K1$ of Z and $B3$ is the corresponding vector-item along-axis $K1$ of B , then $Z1$ is $Nf/B3$.

Examples:

```

0+/1 2 3
0 0 0 0
1+/1 2 3
1 2 3
2+/1 2 3
3 5
3+/1 2 3
6
4+/1 2 3

2-/1 4 9 16 25
-3 -5 -7 -9
-2-/1 4 9 16 25
3 5 7 9
-3-/1 2 3 4 5 6 7
2 3 4 5 6

```

Dyadic Function		Action
Plus	+	Return R reshape zero .
Minus	−	Return R reshape zero .
Times	×	Return R reshape one .
Divide	÷	Return R reshape one .
Residue		Return R reshape zero .
Minimum	⌊	Return R reshape positive-number-limit .
Maximum	⌈	Return R reshape negative-number-limit .
Power	*	Return R reshape one .
Logarithm	⊗	Signal domain-error .
Circular	○	Signal domain-error .
Binomial	!	Return R reshape one .
And	^	Return R reshape one .
Or	∨	Return R reshape zero .
Nand	⋈	Signal domain-error .
Nor	⋈	Signal domain-error .
Less	<	Return R reshape zero .
Not greater	≤	Return R reshape one .
Equal	=	Return R reshape one .
Not less	≥	Return R reshape one .
Greater	>	Return R reshape zero .
Not equal	≠	Return R reshape zero .
all others		Signal domain-error .

Table 6: Actions for the N-wise Reduction of an Empty Vector.

9.2.4 Duplicate

$$Z \leftarrow f \cdot B$$

Informal Description: Z is $B \cdot B$.

Evaluation Sequence:

Return $B \cdot B$.

Example:

$$\begin{array}{ccc} \circ & \cdot & \cdot \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array}$$

Note: If f is not an ambivalent-function, a valence-error will be signalled.

9.2.5 Commute

$$Z \leftarrow A \cdot f \cdot B$$

Informal Description: Z is $B \cdot A$.

Evaluation Sequence:

Return $B \cdot A$.

Examples:

$$\begin{array}{ccc} 3 & \cdot & 4 \\ 1 & + & 2 \\ 65 & \cdot & 65 \end{array}$$

9.2.6 Each

$$Z \leftarrow \mathbf{f} \mathbin{..} B$$

$$Z \leftarrow A \mathbf{f} \mathbin{..} B$$

Informal Description: The operand function **f** is applied independently to corresponding **items** of the arguments, or (monadic case) independently to the **items** of the argument. The corresponding results are assembled in an array of the same shape as the argument(s).

Evaluation Sequence:

For form $A \mathbf{f} \mathbin{..} B$:

If the **rank** of A differs from the **rank** of B ,

If A is a **scalar** or **one-element-vector** and B is not a **scalar**, set A to $(\rho B) \rho A$.

Otherwise,

If B is a **scalar** or **one-element-vector**, set B to $(\rho A) \rho B$.

Otherwise, Signal **rank-error**.

If the **shape-list** of A differs from the **shape-list** of B , Signal **length-error**.

For both forms:

If B is not empty, Signal **domain-error**.

Create Z , an array having the same **shape-list** as B .

For each I in the **index-set** of the **ravel-list** of Z :

For form $\mathbf{f} \mathbin{..} B$,

Set X to item I of the **ravel-list** of B .

Evaluate-Monadic-Function with $\mathbf{f}X$ giving token **T**.

For form $A \mathbf{f} \mathbin{..} B$,

Set X to item I of the **ravel-list** of A .

Set Y to item I of the **ravel-list** of B .

For each pair X and Y of corresponding items from A and B , **Evaluate-Dyadic-Function** with XfY giving token **T**.

If **T** is an error, return **T**.

Set Q to the content of **T**.

Set item I of the **ravel-list** of Z as follows:

If Q is a **simple-scalar**, the **first-item** of the **ravel-list** of Q .

Otherwise, Q .

If all items of the **ravel-list** of Z are **nil**, return **nil**.

If all items of the **ravel-list** of Z are **values**, return Z .

Otherwise, signal **value-error**.

Example:

$$\rho \mathbin{..} c'AB', c'CDE'$$

2 3

9.3 Dyadic Operators

9.3.1 Outer Product

$$Z \leftarrow A \circ .f B$$

Informal Description: Z is an array of shape $(\rho A), \rho B$. The elements of Z are the result of applying the **dyadic-function** f to every possible combination of scalar arguments where the left argument is an element of A and the right an element of B .

Z is such that if I is an **index-list** that selects a single element of Z , the first ρA **items** of I are the **index-list** that would select from A the element used as the left argument to f and the last ρB **items** of I are the **index-list** that would select from B the element used as the right argument to f when producing the selected element of Z .

Evaluation Sequence:

If f is not a **dyadic-function**, signal **syntax-error**.

Return Z , an **array** such that the **shape-list** of Z is $(\rho A), \rho B$, and the **ravel-list** of Z has the following property:

Let I stand for an **item** of the **index-set** of the **ravel-list** of A .

Let J stand for an **item** of the **index-set** of the **ravel-list** of B .

Let X stand for **item** I of the **ravel-list** of A .

Let Y stand for **item** J of the **ravel-list** of B .

Let N stand for the **count** of B .

Let P stand for $J + (N \times (I - 1))$.

Set Q to $X f Y$.

If Q is a **simple-scalar**, set $Q1$ to the **first-item** of the **ravel-list** of Q .

Otherwise, set $Q1$ to Q .

Then, **item** P of the **ravel-list** of Z is $Q1$.

Set the **type** of Z to the **sufficient-type** of the **ravel-list** of Z under mixed.

Example:

```

10 20 30 ◦.+ 1 2 3
11 12 13
21 22 23
31 32 33

```

Additional Requirement:

If the **dyadic-function** f signals an error, outer product returns the resulting error **token**.

9.3.2 Inner Product

$$Z \leftarrow A \mathbf{f} \cdot \mathbf{g} B$$

Informal Description: Z is an array of shape $(\rho A)[1:0]^{-1}+1+\rho A]$, $(\rho B)[1+1:0]^{-1}+1+\rho B]$. The elements of Z are the results obtained from evaluating the expression $\mathbf{f}/X\mathbf{g}Y$ for all possible combinations of X and Y , where X is a **vector-item along-axis** ρA of A and Y a **vector-item along-axis one** of B .

Evaluation Sequence:

If either $\mathbf{f}$ or $\mathbf{g}$ is not a **dyadic-function**, signal **syntax-error**.
 If A is a **scalar** or **one-element-vector** and B is not, set $A1$ to $(1\rho B)\rho A$.
 If A and B are **scalars** or **one-element-vectors**, set $A1$ to $,A$.
 Otherwise, set $A1$ to A .
 If B is a **scalar** or **one-element-vector** and A is not, set $B1$ to $(\rho A)[\rho A]\rho B$.
 If A and B are **scalars** or **one-element-vectors**, set $B1$ to $,B$.
 Otherwise, set $B1$ to B .
 If the **last-item** in the **shape-list** of $A1$ is not the same as the **first-item** in the **shape-list** of $B1$, signal **length-error**.
 If $A1$ and $B1$ are both **vectors**, return $\mathbf{f}/A1\mathbf{g}B1$.
 Otherwise, set Z to an **array** such that the **shape-list** of Z is $(\rho A1)[1:0]^{-1}+1+\rho A1]$, $(\rho B1)[1+1:0]^{-1}+1+\rho B1]$ and the **ravel-list** of Z has the following property:
 Let I stand for an **item** of the **index-set** of the **ravel-along-axis** $(\rho A1)$ of $A1$.
 Let X stand for **vector-item** I of the **ravel-along-axis** $(\rho A1)$ of $A1$.
 Let J stand for an **item** of the **index-set** of the **ravel-along-axis one** of $B1$.
 Let Y stand for **vector-item** J of the **ravel-along-axis one** of $B1$.
 Let N stand for the **number-of-items** in the **ravel-along-axis one** of $B1$.
 Let P stand for $(N \times (I-1)) + J$.
 Set Q to $\mathbf{f}/X\mathbf{g}Y$.
 If Q is a **simple-scalar**, set $Q1$ to the **first-item** of the **ravel-list** of Q .
 Otherwise, set $Q1$ to Q .
 Then, **item** P of the **ravel-list** of Z is $Q1$.
 Set the **type** of Z to the **sufficient-type** of Z under mixed.
 Return Z .

Examples:

```

      4 2 1+.×1 0 1
5
      N22+.×0 1
12 22
      N22+.×1 0
11 21
      N22+.×2 2p0 1 1 0
12 11
22 21

```

Additional Requirement:

If **scalar-function f** or **g** signals an error, inner product returns the resulting error **token**.

Note: The evaluation sequence rule for when *A1* and *B1* are **vectors** holds if *A1* or *B1* is the empty vector. The result returned is **f** / 1 0.

The type of the result of inner product is numeric only because all permitted argument functions return numeric results.

9.3.3 Rank operator definitions

The following definitions are used in connection with the **rank** operator:

cell: The last **k** items of the **shape-list** of an **array** determine **rank-k cells** of the **array**.

frame: For an array of **rank r**, its **frame** with respect to its **cells** of **rank k** is the **r-k** leading elements of its **shape-list**.

conform: The **shape** of a result is the **frame** of the argument (relative to the **cells** to which the function applies) catenated with the **shape** of the individual results produced by applying the function to the individual **cells**. If the results do not agree in **shape** they are brought to a common **shape** as follows:

If the **ranks** differ, they are brought to a common maximum **rank** by reshaping each individual result to introduce leading unit lengths.

If the individual **shapes** differ (after being brought to a common **rank**), each is brought to a common **shape** by using **take** on each individual result, using as the argument to **take** the **shape** which is the maximum over the **shapes**.

rank-vector: This is the right argument of the rank operator, and it is used to specify the rank of the cells to which the function left argument is to be applied. For the monadic case of a function this is a single value. For the dyadic case of a function it is in general a two-element vector, the first element specifying the rank of the cells of the left argument, and the second element specifying the rank of the cells of the right argument to which the function is to be applied. If only a single value is supplied for the dyadic case of a function, it is used to specify the cells of both the left and right arguments to which the function will be applied. In general, for an arbitrary function, the rank vector is a three-element vector, with the elements referring to the monadic rank, and the two dyadic ranks, in order. No matter whether a one-, two-, or three-element vector **v** is supplied as the rank vector, the expression $\phi \circ \rho \phi \mathbf{v}$ produces a canonical rank-vector of three elements, in which the first element defines the monadic rank, the second element defines the dyadic left rank, and the third element defines the dyadic right rank.

9.3.4 Rank operator deriving monadic function

$$Z \leftarrow f \circ y \ B$$

Informal Description: The result of $f \circ y$ is a function which, when applied to B , returns Z , the result of applying the function f to the **rank-y cells** of B .

Evaluation Sequence:

If y is a **scalar**, set $y1$ to y . Otherwise set $y1$ to y .

If $y1$ is not a **vector**, signal **domain-error**.

If $y1$ has more than **three** elements, signal **length-error**.

If any element of $y1$ is not a **near-integer**, signal **domain-error**.

Set $y2$ to $\phi_3 \phi y1$.

Set $y3$ to the **first-item** in $y2$.

Set $y4$ to the **integer-nearest-to** $y3$.

If $y4$ exceeds the **rank** of B , set $y5$ to the **rank** of B , otherwise set $y5$ to $y4$.

If $y5$ is **negative**, set $y6$ to $0 \uparrow y5$ plus the **rank** of B , otherwise set $y6$ to $y5$.

Apply f to the **rank- $y6$ cells** of B .

Conform the individual result **cells**. Let their common **shape** after conforming be q , and let p be the **frame** of B with respect to f , that is, (**rank** of B) minus $y6$, and return the overall result with **shape** p, q .

Examples:

```

      ,∘2 N233
111 112 113 121 122 123 131 132 133
211 212 213 221 222 223 231 232 233
      ∅∘2 N233
111 121 131
112 122 132
113 123 133

211 221 231
212 222 232
213 223 233
      1∘0 N3
1 0 0
1 2 0
1 2 3

```

9.3.5 Rank operator deriving dyadic function

$$Z \leftarrow A \mathbf{f} \ddot{\circ} \mathbf{y} B$$

Informal Description: The result of $\mathbf{f} \ddot{\circ} \mathbf{y}$ is a function which, when applied dyadically between A and B , returns Z , the result of applying the function $\mathbf{f}$ between the **rank-l cells** of A and the **rank-r cells** of B , where $\mathbf{l}$ and $\mathbf{r}$ are given by $\mathbf{y}$.

Evaluation Sequence:

If $\mathbf{y}$ is a **scalar**, set $\mathbf{y1}$ to $\mathbf{y}$. Otherwise set $\mathbf{y1}$ to $\mathbf{y}$.
 If $\mathbf{y1}$ is not a **vector**, signal **domain-error**.
 If $\mathbf{y1}$ has more than **three** elements, signal **length-error**.
 If any element of $\mathbf{y1}$ is not a **near-integer**, signal **domain-error**.
 Set each element of $\mathbf{y2}$ to the **integer-nearest-to** each corresponding element of $\mathbf{y1}$.
 Set $\mathbf{y3}$ to $\phi 3 \rho \phi \mathbf{y2}$.
 Set $\mathbf{y4}$ to the second element of $\mathbf{y3}$.
 Set $\mathbf{y5}$ to the third element of $\mathbf{y3}$.
 If $\mathbf{y4}$ exceeds the **rank** of A , set $\mathbf{y6}$ to the **rank** of A , otherwise set $\mathbf{y6}$ to $\mathbf{y4}$.
 If $\mathbf{y5}$ exceeds the **rank** of B , set $\mathbf{y7}$ to the **rank** of B , otherwise set $\mathbf{y7}$ to $\mathbf{y5}$.
 If $\mathbf{y6}$ is **negative**, set $\mathbf{y8}$ to the maximum of **zero** and $\mathbf{y6}$ plus the **rank** of A , otherwise set $\mathbf{y8}$ to $\mathbf{y6}$.
 If $\mathbf{y7}$ is **negative**, set $\mathbf{y9}$ to the maximum of **zero** and $\mathbf{y7}$ plus the **rank** of B , otherwise set $\mathbf{y9}$ to $\mathbf{y7}$.
 Set $\mathbf{y10}$ to the **shape** of A with the last $\mathbf{y8}$ items removed.
 Set $\mathbf{y11}$ to the **shape** of B with the last $\mathbf{y9}$ items removed.
 Execute the indicated action for whichever of the following cases is true:
 Case $\mathbf{y10}$ and $\mathbf{y11}$ are empty:
 Set $A1$ to A and $B1$ to B .
 Case $\mathbf{y10}$ is empty and $\mathbf{y11}$ is nonempty:
 Set $A1$ to A **conform** to B and $B1$ to B .
 Case $\mathbf{y10}$ is nonempty and $\mathbf{y11}$ is empty:
 Set $A1$ to A and $B1$ to B **conform** to A .
 Case $\mathbf{y10}$ and $\mathbf{y11}$ are nonempty:
 If $\mathbf{y10}$ is not the same length as $\mathbf{y11}$, signal a **rank-error**.
 If $\mathbf{y10}$ does not **match** $\mathbf{y11}$, signal a **length-error**.
 Set $A1$ to A and $B1$ to B .
 Apply $\mathbf{f}$ between each **rank-y8 cell** of $A1$ and each **rank-y9 cell** of $B1$.
Conform the individual result **cells** to give the overall result.

Examples:

0 1 2 ϕ 0 1 'ABC'
 ABC
 BCA
 CAB

2 2 2 τ 1 0 N5
 0 0 1
 0 1 0
 0 1 1
 1 0 0
 1 0 1

$N_{34+} 2 N_{234}$
 122 124 126 128
 142 144 146 148
 162 164 166 168

222 224 226 228
 242 244 246 248
 262 264 266 268

$N_3, 1 N_{34}$
 1 2 3 11 12 13 14
 1 2 3 21 22 23 24
 1 2 3 31 32 33 34

$N_2, 0 2 N_{34}$
 1 11 12 13 14
 1 21 22 23 24
 1 31 32 33 34

2 11 12 13 14
 2 21 22 23 24
 2 31 32 33 34

10 Mixed Functions

10.1 Monadic Mixed Functions

10.1.1 Roll

$$Z \leftarrow ? B$$

Informal Description: Z , for B a scalar integer, is **index-origin** plus R , where R is an integer selected pseudorandomly from the set of all nonnegative integers less than B . Each integer in the set has an equal chance of being selected. Uses **index-origin**. Uses and sets **random-link**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.

If **random-link** is **nil**, signal **implicit-error**.

Let J stand for **item** I of the **ravel-list** of B .

If B is not **scalar**, return a **numeric array** Z such that the **shape-list** of Z is the **shape-list** of B and, for all I in the **index-set** of the **ravel-list** of Z , **item** I of the **ravel-list** of Z is $?J$.

If B is not a **near-integer**, signal **domain-error**.

Set $B1$ to the **integer-nearest-to** B .

If $B1$ is not a **positive-integer**, signal **domain-error**.

Using the **implementation-algorithm pseudorandom-number-generator**, whose only inputs are $B1$ and **random-link**, set $Z0$ to a **nonnegative-integer less-than** $B1$, then set **random-link** to a new value.

Return $Z0$ **plus index-origin**.

Examples:

```

      ?1 1 1
1 1 1

```

```

S ← RL
?4p 1E30
1.69584E29 2.5949E29 5.38382E29 8.38274E29
RL ← S
?4p 1E30
1.69584E29 2.5949E29 5.38382E29 8.38274E29
( ?1E20 1E20 ) ÷ 1E20
0.218959 0.678865

```

Additional Requirement:

The operation of **roll** is **atomic**: If **roll** signals an error, **random-link** shall be unchanged.

The result of $?B$, where B is an **array**, shall be reproducible.

A **conforming-implementation** shall provide documentation describing the properties of its **pseudorandom-number-generator**.

Note: One class of appropriate algorithms is Lehmer's linear congruential method, described in Knuth, D. E., **Seminumerical Algorithms**, page 9.

Roll is often considered a scalar function. However, it does not have the property that the elements of its result array can be produced in parallel.

10.1.2 Grade Up

$$Z \leftarrow \mathbb{A} B$$

Informal Description: Z is, for B a vector, a permutation of $\iota \rho B$ for which $B[Z]$ is a monotone increasing sequence. The indices of identical elements of B occur in Z in ascending order. If B is a matrix, $\mathbb{A} B$ grades the rows, that is, it grades the base value of the rows, using a base larger than the magnitude of any of the elements. Higher rank arguments are graded as if their major cells were ravelled. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.

If the **rank** of B is **zero**, signal **rank-error**.

If ρB is **zero**, return $\iota 0$.

If any **item** of the **ravel-list** of B is not a **near-real number**, signal **domain-error**.

Otherwise, set $B1$ to an array having the same shape as B , and with each element having the value of the **real-number** nearest to that of the corresponding element of B .

If the **first-item** of ρB is **one**, return a **one-element-vector** Z such that the **type** of Z is **numeric** and the **ravel-list** of Z contains **index-origin**.

Set K to the length of the first axis of $B1$.

If $B1$ is a **vector**, set $B2$ to $B1$.

Otherwise,

Set M to $1+2 \times \lceil \lceil B1 \rceil \rceil$.

Set $B2$ to a list with K items, where item I is determined as follows:

Set $B3$ to the ravel-list of subarray I along the first axis of $B1$. Set $B2[I]$ to $M \perp B3$.

Generate $Z1$, a permutation of ιK such that for I and J elements of ιK for which I is **less-than** J ,

$B2[Z1[I]]$ is not **greater-than** $B2[Z1[J]]$ and

$B2[Z1[I]]$ **equals** $B2[Z1[J]]$ implies that $Z1[I]$ is **less-than** $Z1[J]$.

Generate Z , a **numeric array** with the **shape-list** of $Z1$ such that each **item** of the **ravel-list** of Z is (**index-origin minus one**) plus the corresponding element of $Z1$.

Return Z .

Examples:

$\Delta V \leftarrow 1.1 \ 3.1 \ 1.1 \ 2.1 \ 5.1$
 1 3 4 2 5
 $\Delta \Delta V$
 1 4 2 3 5
 $V[\Delta V]$
 1.1 1.1 2.1 3.1 5.1
 B
 3 1 4
 2 7 9
 3 2 0
 3 1 4
 ΔB
 2 1 4 3

- In the following example, **index-origin** is **zero**.

ΔB
 1 0 3 2

Additional Requirement:

The **system-parameter comparison-tolerance** is not an implicit argument of **grade up**.

10.1.3 Grade Down

$$Z \leftarrow \Psi B$$

Informal Description: Grade down is like grade up except that the major cells are in descending order.

Evaluation Sequence:

Return $\nabla - B$.

Examples:

```

      V ← 1.1 3.1 1.1 2.1 5.1
      ΨV
5 2 4 1 3
      V[ΨV]
5.1 3.1 2.1 1.1 1.1
      B
3 1 4
2 7 9
3 2 0
3 1 4
      ΨB
3 1 4 2

```

– In the following example, **index-origin** is **zero**.

```

      ΨB
2 0 3 1

```

Additional Requirement:

The **system-parameter comparison-tolerance** is not an implicit argument of **grade down**.

Note: **Grade up** and **grade down** are **stable sort algorithms** because they preserve the relative order of identical elements of B .

One appropriate algorithm for grade up and down appears in Woodrum, L. J., **Internal Sorting with Minimal Comparing**, IBM System Journal, Vol. 8, No. 3, p.189, 1969.

10.1.4 Reverse

$$Z \leftarrow \phi B$$

$$Z \leftarrow \ominus B$$

$$Z \leftarrow \phi[K] B$$

$$Z \leftarrow \ominus[K] B$$

Informal Description: Z is an array whose elements are those of B taken in reverse order along a specified axis. Uses **index-origin**

Evaluation Sequence:

For form ϕB

If B is **scalar**, return B .

Otherwise, return $\phi[\rho \rho B] B$, evaluated with **index-origin** set to **one**.

For form $\ominus B$

If B is **scalar**, return B .

Otherwise, return $\phi[1] B$, evaluated with **index-origin** set to **one**.

For forms $\phi[K] B$ and $\ominus[K] B$

If K is not a **valid-axis** for B , signal **axis-error**.

Otherwise, set $K1$ to the **integer-nearest-to** K .

If B is a **vector**, return $B[(1+\rho B) \div \rho B]$, evaluated with **index-origin** set to **one**.

Otherwise, return an **array** Z , such that the **type** of Z is the **type** of B , the **shape-list** of Z is the **shape-list** of B , and the **ravel-list** of Z has the property that each **vector-item along-axis** $K1$ of Z is ϕ applied to the corresponding **vector-item along-axis** $K1$ of B .

Examples:

$$\phi N23$$

$$13 \ 12 \ 11$$

$$23 \ 22 \ 21$$

$$\phi[2] N224$$

$$121 \ 122 \ 123 \ 124$$

$$111 \ 112 \ 113 \ 114$$

$$221 \ 222 \ 223 \ 224$$

$$211 \ 212 \ 213 \ 214$$

Note: Various error checks have been performed on K by the phrase evaluators, and **index-origin** is **one**, before this evaluation sequence is called.

10.1.5 Monadic Transpose

$$Z \leftarrow \text{ } B$$

Informal Description: Z is B with the order of the axes reversed.

Evaluation Sequence:

Return $(\phi \iota \rho \rho B) \text{ } B$, evaluated with **index-origin** set to **one**.

Examples:

$\text{ } 3$
 $\rho \rho \text{ } 3$
 0
 $\text{ } N 2 3$
 $1 1 \ 2 1$
 $1 2 \ 2 2$
 $1 3 \ 2 3$
 $\text{ } N 2 3 4$
 $1 1 1 \ 2 1 1$
 $1 2 1 \ 2 2 1$
 $1 3 1 \ 2 3 1$

 $1 1 2 \ 2 1 2$
 $1 2 2 \ 2 2 2$
 $1 3 2 \ 2 3 2$

 $1 1 3 \ 2 1 3$
 $1 2 3 \ 2 2 3$
 $1 3 3 \ 2 3 3$

 $1 1 4 \ 2 1 4$
 $1 2 4 \ 2 2 4$
 $1 3 4 \ 2 3 4$

Note: This subsection contains a forward reference to **dyadic transpose**.

10.1.6 Matrix Inverse

$$Z \leftarrow \boxed{B}$$

Informal Description: Z is the result of applying a generalisation of the matrix inverse function to B . **Matrix inverse** is **matrix divide** with an appropriate identity matrix as a left argument.

Evaluation Sequence:

If the **rank** of B is **greater-than two**, signal **rank-error**.
Return $((2\rho 1\rho\rho B)\rho 1, (1\rho\rho B)\rho 0) \boxed{B}$

Note: This subsection contains a forward reference to **matrix divide**.

The following article describes the motivation for matrix inverse and an acceptable algorithm.

Jenkins, M. A., **Domino—An APL Primitive Function for Matrix Inversion—Its Implementation and Applications**. APL Quote-Quad Vol III No. 4, February 1972, pp. 4-15.

10.1.7 Execute

$$Z \leftarrow \text{execute } B$$

Informal Description: Z is the result of evaluating the **character** scalar or vector B as a line of APL.

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.

If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**. Generate a new **context** in which

mode is **execute**,

current-line is the **ravel-list** of B ,

current-function is 0 0p ' ',

current-line-number is **one**,

current-statement is the empty **list** of **tokens**, and

stack is the empty **list** of **tokens**.

Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.

Set Z to **evaluate-line**.

Remove the first **context** from the **state-indicator**.

Return Z .

Examples:

```

      ⍉ 'T←3 '
      T
3
      ⍉←⍉ 'T←3 '
3
      A←⍉ ' '
value-error

```

Note: If an error is signalled during **execute**, the user should be able to determine from information provided by the system where the error occurred in the argument of **execute** as well as where the failing execute primitive occurred in the **immediate-execution** or **defined-function** line.

10.1.8 Unique

$$Z \leftarrow \cup B$$

Informal Description: B must be a **vector** or **scalar**. Z is a vector of the same **type** as B , containing one copy of each unique **item** in B . The order of the items in Z is the order they first occur in the **ravel** of B . Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil** signal **implicit-error**.
 If the **rank** of B is **greater-than one**, signal **rank-error**.
 Set $B1$ to an **empty-list** of the **type** of B .
 Set $B2$ to the **ravel-list** of B .
 Repeat:
 If $B2$ is empty, return $B1$.
 Set T to the **vector-item one** of $B2$.
 Append T to $B1$.
 Remove from $B2$ all items **tolerantly-equal-to** T , within **comparison-tolerance**.
 (End of Repeated Block).

Examples:

```

      U2 7 1 8 2 8 1 8 2 8 4 5 9 0 4 4 9
2 7 1 8 4 5 9 0
      U 'MISSISSIPPI'
MISP
    
```

10.1.9 First

$$Z \leftarrow \uparrow B$$

Informal Description: Z is the first item of B in row-major order. If B is empty, the **typical-element** of B is returned.

Evaluation Sequence:

Return **first-thingy** of B .

Examples:

$\uparrow 'BEATA'$
 B
 $\rho\rho\uparrow 'BEATA'$
 0
 $\uparrow N432$
 111
 $\uparrow 4 \ 5\rho(\leftarrow 'OSCAR'), (\leftarrow 'BEATA')$
 $OSCAR$

10.2 Dyadic Mixed Functions

10.2.1 Join Along an Axis

$$Z \leftarrow A, [K] B$$

Informal Description: Z is formed by joining A and B along a designated axis. There are two suboperations, catenate and laminate. The catenate suboperation joins the arrays along an existing axis, the laminate suboperation along a new axis. The choice of axis and the choice of operation is determined by K : if K is a **near-integer**, the operation is catenate and the axis is K ; if K is not a **near-integer**, the operation is laminate, the new axis is $\lceil K$, and the axes greater than or equal to $\lceil K$ are renumbered. Uses **index-origin**.

Evaluation Sequence:

If K is not a **near-integer**,
 If A is a **scalar**, set $A1$ to $(\rho B)\rho A$.
 Otherwise, set $A1$ to A .
 If B is a **scalar**, set $B1$ to $(\rho A)\rho B$.
 Otherwise, set $B1$ to B .

If the **rank** of $A1$ differs from the **rank** of $B1$, signal **rank-error**.

If K is not in the **open-interval-between zero** and **(one plus the rank of $A1$)**, signal **axis-error**.

If the **shape-list** of $A1$ differs from the **shape-list** of $B1$, signal **length-error**.

Set T to $(1, \rho A1) [\Delta K, \text{irp} A1]$.

Set $A2$ to $T \rho A1$.

Set $B2$ to $T \rho B1$.

Return $A2, [\text{K}] B2$, evaluated with **index-origin** set to **one** and with **comparison-tolerance** set to **integer-tolerance**.

If K is a **near-integer**, evaluate the following with **comparison-tolerance** set to **zero**:

If both A and B are **scalars**, signal **axis-error**.

Set $K1$ to the **integer-nearest-to** K .

If $K1$ is not in the **closed-interval-between one** and the larger of the **rank** of A and the **rank** of B , signal **axis-error**.

If A is a **scalar**, set $A1$ to $((\rho B) * K1 - \text{irp} B) \rho A$.

Otherwise, set $A1$ to A .

If B is a **scalar**, set $B1$ to $((\rho A) * K1 - \text{irp} A) \rho B$.

Otherwise, set $B1$ to B .

If the **rank** of $B1$ minus the **rank** of $A1$ is **one**,

set $A2$ to $(1, \rho A1) [\Delta K1, \text{irp} A1] \rho A1$.

Otherwise, set $A2$ to $A1$.

If the **rank** of $A1$ minus the **rank** of $B1$ is **one**,

set $B2$ to $(1, \rho B1) [\Delta K1, \text{irp} B1] \rho B1$.

Otherwise, set $B2$ to $B1$.

If the **rank** of $A2$ differs from the **rank** of $B2$, signal **rank-error**.

If any **axis** of $A2$ except **axis** $K1$ differs from the corresponding **axis** of $B2$, signal **length-error**.

If $B2$ is empty, return $((\rho A2) + (\rho B2) * K1 - \text{irp} B2) \rho A2$.

If $A2$ is empty, return $((\rho A2) + (\rho B2) * K1 - \text{irp} B2) \rho B2$.

If $A2$ and $B2$ are **vectors**, return $A2, B2$.

Otherwise, return Z , an **array** such that the

shape-list of Z is $(\rho A2) + (\rho B2) * K1 - \text{irp} B2$ and the **ravel-list** of Z has the property that each **vector-item along-axis** $K1$ of Z is $A3, B3$, where $A3$ is the corresponding **vector-item along-axis** $K1$ of $A2$ and $B3$ is the corresponding **vector-item along-axis** $K1$ of $B2$. The **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type mixed**.

Examples:

$\square \leftarrow M \leftarrow 2 \quad 3\rho \quad ' \Delta \quad '$
 $\Delta \Delta \Delta$
 $\Delta \Delta \Delta$
 $\square \leftarrow H \leftarrow 3 \quad 3\rho \quad ' \circ \quad '$
 $\circ \circ \circ$
 $\circ \circ \circ$
 $\circ \circ \circ$
 $M, [1] H$
 $\Delta \Delta \Delta$
 $\Delta \Delta \Delta$
 $\circ \circ \circ$
 $\circ \circ \circ$
 $\circ \circ \circ$
 $\square \leftarrow L \leftarrow 2 \quad 4\rho \quad ' \square \quad '$
 $\square \square \square \square$
 $\square \square \square \square$
 M, L
 $\Delta \Delta \Delta \square \square \square \square$
 $\Delta \Delta \Delta \square \square \square \square$
 $M, ' + \quad '$
 $\Delta \Delta \Delta +$
 $\Delta \Delta \Delta +$
 $M, ' 34 \quad '$
 $\Delta \Delta \Delta 3$
 $\Delta \Delta \Delta 4$
 $M, [1] ' 345 \quad '$
 $\Delta \Delta \Delta$
 $\Delta \Delta \Delta$
 345
 $1 \quad 2 \quad 3, [.5] \quad 4 \quad 5 \quad 6$
 $1 \quad 2 \quad 3$
 $4 \quad 5 \quad 6$
 $1 \quad 2 \quad 3, [1.5] \quad 4 \quad 5 \quad 6$
 $1 \quad 4$
 $2 \quad 5$
 $3 \quad 6$
 $1 \quad 2 \quad 3, [1.5] \quad 4$
 $1 \quad 4$
 $2 \quad 4$
 $3 \quad 4$
 $(2 \quad 0\rho 5), ' A \quad '$
 A
 A
 $\rho 3, [.5] \quad ' \quad '$
 $2 \quad 0$

Note: Various error checks have been performed on K by the phrase evaluators, and **index-origin** is **one**, before this evaluation sequence is called.

10.2.2 Index of

$$Z \leftarrow A \text{ } \iota \text{ } B$$

Informal Description: Z is a numeric array of shape ρB . Each element of Z is the least index in A of a value **tolerantly-equal** within **comparison-tolerance** to the corresponding item of B . Uses **index-origin**. Uses **comparison-tolerance**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.

If **comparison-tolerance** is **nil**, signal **implicit-error**.

If A is not a **vector**, signal **rank-error**.

Set Z to $+/\wedge \backslash B \circ . \neq A$, evaluated with the current value of **comparison-tolerance**.

Return Z plus **index-origin**.

Examples:

Note: In the following example, **index-origin** is zero.

```

⊞←A←2 2ρ1.1 3.1 5.1 4.1
1.1 3.1
5.1 4.1
3.1 4.1 5.1⊞A
3 0
2 1
'ABC'⊞3
3

```

Note: In the following example, **index-origin** is one.

```

'123ABC'⊞'3BD'
3 5 7
'123ABC'⊞3
7

```

10.2.3 Member of

$$Z \leftarrow A \in B$$

Informal Description: Z is a Boolean array with the shape of A . An element of Z is **one** if the corresponding element of A is **tolerantly-equal** to some element of B ; otherwise, it is **zero**. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.

Return $\vee / A \circ . = , B$, evaluated with the current value of **comparison-tolerance**.

Examples:

```

      □ ← B ← 2 2ρ1.1 3.1 5.1 4.1
1.1 3.1
5.1 4.1
      3.1 5.1 7.1 ∈ B
1 1 0
      19 ∈ 'CLUB'
0
      'BE' ∈ 'BOP'
1 0
      'NADA' ∈ ι0
0 0 0 0
      (⌈ / ι0) ∈ L / ι0
0

```

Note: If B is empty, the result of $A \in B$ is $(\rho A) \rho 0$.

10.2.4 Deal

$$Z \leftarrow A \text{ ? } B$$

Informal Description: Z is **index-origin** plus R , where R is a vector of shape A obtained by making A pseudorandom selections without replacement from the set of nonnegative integers less than B . Uses **index-origin**. Uses and sets **random-link**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
 If **random-link** is **nil**, signal **implicit-error**.
 If the **rank** of A or the **rank** of B is **greater-than one**, signal **rank-error**.
 If either of A or B is neither a **scalar** nor a **one-element-vector**, signal **length-error**.
 If either of A or B is not a **near-integer**, signal **domain-error**.
 Set $A1$ to the **integer-nearest-to** A .
 Set $B1$ to the **integer-nearest-to** B .
 If either of $A1$ or $B1$ is not a **nonnegative-number**, signal **domain-error**.
 If $A1$ is **greater-than** $B1$, signal **domain-error**.
 If $A1$ is **zero**, return $\uparrow 0$.
 Using the **implementation-algorithm deal**, generate a **numeric vector** $Z0$ of **length** $A1$ whose elements are selected in a pseudorandom fashion without duplication from the **integers** in the **closed-interval-between zero** and $B1$ **minus one**, then set **random-link** to a new value.
 Return $Z0$ **plus index-origin**.

Example:

```
12?300
2 3 42 94 70 105 215 9 110 298 201 5
```

Additional Requirement:

The selection of elements in Z is pseudorandom (see the operation **roll**). The elements of Z , their order, and the new value of **random-link** are determined by an **implementation-algorithm** whose only inputs are $A1$, $B1$ and **random-link**. The operation of **deal** is **atomic**: if **deal** signals an error, **random-link** shall be unchanged. The result of $A?B$, where A and B are **arrays**, shall be reproducible.

10.2.5 Replicate

$$Z \leftarrow A / B$$

$$Z \leftarrow A \neq B$$

$$Z \leftarrow A / [K] B$$

$$Z \leftarrow A \neq [K] B$$

Informal Description: Z is an array formed by replicating the subarrays, along a specified axis of B , the number of times indicated by the corresponding element of the **near-integer vector** or **scalar** A . Uses **index-origin**.

Evaluation Sequence:

If A is a **scalar**, set $A1$ to A .

Otherwise, set $A1$ to A .

If B is a **scalar**, set $B1$ to $(\rho A1) \rho B$.

Otherwise, set $B1$ to B .

For form A/B

Return $A1 / [\rho \rho B1] B1$ evaluated with **index-origin** set to **one**.

For form $A \neq B$

Return $A1 / [1] B1$ evaluated with **index-origin** set to **one**.

For forms $A/[K] B$ and $A \neq [K] B$

If K is not a **valid-axis** for $B1$, signal **axis-error**.

Otherwise, set $K1$ to the **integer-nearest-to** K .

If the **rank** of $A1$ is **greater-than one**, signal **rank-error**.

If the **count** of $A1$ is **one**, set $A2$ to $(\rho B1) [K1] \rho A1$.

Otherwise, set $A2$ to $A1$.

If $\rho A2$ is not the same as $(\rho B1) [K1]$, signal **length-error**.

If any **item** of the **ravel-list** of $A2$ is not a **near-integer**, signal **domain-error**.

Set $A3$ to the **integer-array-nearest-to** $A2$.

If any **item** of the **ravel-list** of $A3$ is not a **nonnegative-integer**, signal **domain-error**.

If $B1$ is a **vector**,

If every **item** of $A3$ is **near-Boolean** return $B1 [(+/A3) \rho \nabla A3]$.

Otherwise, return $B1 [(, A3 \circ . \geq 1 \uparrow / 0 , A3) / , (1 \rho B1) \circ . + 0 \times 1 \uparrow / 0 , A3]$

Otherwise, return an **array** Z such that

The **shape-list** of Z is $((K1 \neq 1 \rho \rho B1) \times \rho B1) + (K1 = 1 \rho \rho B1) \times +/A3$ evaluated with **comparison-tolerance** set to **zero**,

The **ravel-list** of Z has the property that if $Z2$ and $B2$ are corresponding **vector-items along-axis** $K1$ of Z and $B1$ respectively, then $Z2$ is $A3/B2$.

The **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of B .

Examples:

$$\begin{array}{c} 1 \ 0 \ 1 \ / \ 1 \ 2 \ 3 \\ 1 \ 3 \\ 1 \ / \ 1 \ 2 \ 3 \\ 1 \ 2 \ 3 \\ 3 \ 2 \ 1 / 1 \ 2 \ 3 \\ 1 \ 1 \ 1 \ 2 \ 2 \ 3 \\ 1 \ 0 \ 1 / \ 2 \\ 2 \ 2 \\ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ / [2] \ N2714 \\ 1311 \ 1312 \ 1313 \ 1314 \\ \\ 1611 \ 1612 \ 1613 \ 1614 \\ \\ 2311 \ 2312 \ 2313 \ 2314 \\ \\ 2611 \ 2612 \ 2613 \ 2614 \\ \rho 1 / 1 \\ 1 \\ \rho \rho (, 1) / 2 \\ 1 \\ 3 \ 4 / 1 \ 2 \\ 1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 2 \end{array}$$

Note: Various error checks have been performed on K by the phrase evaluators, and **index-origin** is **one**, before this evaluation sequence is called.

Note: If the left argument is boolean, the function may be called **compress**.

Note: $1/A$ for a scalar A , returns a vector.

10.2.6 Expand

$$Z \leftarrow A \setminus B$$

$$Z \leftarrow A \setminus B$$

$$Z \leftarrow A \setminus [K] B$$

$$Z \leftarrow A \setminus [K] B$$

Informal Description: Z is, for a **near-Boolean** scalar or vector A , an array of the same type as B containing subarrays of B along a specified axis.

For vector B , Z is such that $A/A \setminus B$ is B and, if P is the **typical-element** of B , $(\sim A)/A \setminus B$ is $(+/\sim A) \rho P$; similarly, $A/[K]A \setminus [K]B$ is B , $A \nabla A \setminus B$ is B , and $(\sim A)/[K]A \setminus [K]B$ and $(\sim A) \nabla A \setminus B$ are arrays of the **typical-element** of B . Uses **index-origin**.

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.

If any **item** of the **ravel-list** of A is not a **near-Boolean**, signal **domain-error**.

Set $A1$ to the **Boolean-array-nearest-to** A .

If B is a **scalar**, set $B1$ to $(+ / A1) \rho B$.

Otherwise, set $B1$ to B .

For form $A \setminus B$

Return $A1 \setminus [\rho \rho B1] B1$ evaluated with **index-origin** set to **one**.

For form $A \setminus B$

Return $A1 \setminus [1] B1$ evaluated with **index-origin** set to **one**.

For forms $A \setminus [K] B$ and $A \setminus [K] B$

If K is not a **valid-axis** for $B1$, signal **axis-error**.

Otherwise, set $K1$ to the **integer-nearest-to** K .

If $(\rho B1)[K1]$ differs from $+ / A1$, signal **length-error**.

Return an **array** Z such that

the **type** of Z is the **type** of B ,

the **shape-list** of Z is $((K1 \neq \iota \rho \rho B1) \times \rho B1) + (K1 = \iota \rho \rho B1) \times \rho A1$, evaluated with **comparison-tolerance** set to **zero**, and

the **ravel-list** of Z has the property that $A1/[K1]Z$ is $B1$ and $(\sim A1)/[K1]Z$ is an **array** consisting entirely of the **typical-element** of B .

Note: Various error checks have been performed on K by the phrase evaluators, and **index-origin** is **one**, before this evaluation sequence is called.

Examples:

```

1 0 1 \1 3
1 0 3
1 1 1 0 1 \ 'ABCD'
ABC D
1 0 1\2
2 0 2
1 0 1\[2] 2 2 ρ 'ABCD'
A B
C D
1 0 1 1\1 2 3
1 0 2 3
1 0 1 1\3
3 0 3 3
0 1 \3 1 ρ 3.14 2E17 -47
0 3.14E0
0 2.00E17
0 -4.70E1
0 0\5
0 0
1 0 1 0\[2] N224
111 112 113 114
0 0 0 0
121 122 123 124
0 0 0 0
211 212 213 214
0 0 0 0
221 222 223 224
0 0 0 0

```

10.2.7 Rotate

$$Z \leftarrow A \phi B$$

$$Z \leftarrow A \ominus B$$

$$Z \leftarrow A \phi[K] B$$

$$Z \leftarrow A \ominus[K] B$$

Informal Description: Z is an array of the same type and shape as B in which elements have been shifted cyclically along a specified axis. The amount and direction of shift is controlled by A . Uses **index-origin**.

Evaluation Sequence:

For form $A \phi B$

If B is a **scalar** and A is either a **scalar** or a **vector of length one**,

If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.

Otherwise, return B .

Otherwise, return $A \phi[\rho B] B$ evaluated with **index-origin** set to **one**.

For form $A \ominus B$

If B is a **scalar** and A is either a **scalar** or a **vector of length one**,

If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.

Otherwise, return B .

Otherwise, return $A \phi[1] B$ evaluated with **index-origin** set to **one**.

For forms $A \phi[K] B$ and $A \ominus[K] B$

If K is not a **valid-axis** for B , signal **axis-error**.

Otherwise, set $K1$ to the **integer-nearest-to** K .

If A is a **scalar**, set $A1$ to $((K1 \neq 1) \rho B) / \rho B \rho A$, evaluated with **comparison-tolerance** set to **zero**.

Otherwise, set $A1$ to A .

If $A1$ is a **one-element-vector** and B is a **vector**, set $A2$ to $(10) \rho A1$.

Otherwise, set $A2$ to $A1$.

If the **rank** of B minus the **rank** of $A2$ is not **one**, signal **rank-error**.

If the **shape-list** of $A2$ is not the same as the **shape-list** of B with **axis** $K1$ omitted, signal **length-error**.

If any **item** of the **ravel-list** of $A2$ is not a **near-integer**, signal **domain-error**.

Set $A3$ to the **integer-array-nearest-to** $A2$.

If $A3$ is a **scalar** and B is a **vector**, return $B[1+(\rho B) | ^{-1} + A3 + 1 \rho B]$ evaluated with **comparison-tolerance** set to **zero**.

Otherwise return Z , an **array** such that the **shape-list** of Z is the same as the **shape-list** of B , the **type** of Z is the same as the **type** of B , and the **ravel-list** of Z has the property that if $Z0$ is a **vector-item along-axis** $K1$ of Z , $A0$ is the corresponding **item** of $A3$, and $B0$ is the corresponding **vector-item along-axis** $K1$ of B , then $Z0$ is $A0 \phi B0$.

Examples:

$3\phi 1\ 2\ 3\ 4\ 5$
 $4\ 5\ 1\ 2\ 3$
 $^{-1}\phi 1\ 2\ 3\ 4\ 5$
 $5\ 1\ 2\ 3\ 4$
 $^{-7}\phi 'ABCDEF'$
 $FABCDE$
 $1\ominus N33$
 $21\ 22\ 23$
 $31\ 32\ 33$
 $11\ 12\ 13$
 $1\phi[1]N33$
 $21\ 22\ 23$
 $31\ 32\ 33$
 $11\ 12\ 13$
 $1\ 2\ 3\phi N34$
 $12\ 13\ 14\ 11$
 $23\ 24\ 21\ 22$
 $34\ 31\ 32\ 33$
 $N23\phi[2]N243$
 $141\ 112\ 123$
 $111\ 122\ 133$
 $121\ 132\ 143$
 $131\ 142\ 113$

 $221\ 232\ 243$
 $231\ 242\ 213$
 $241\ 212\ 223$
 $211\ 222\ 233$

Note: Various error checks have been performed on K by the phrase evaluators, and **index-origin** is **one**, before this evaluation sequence is called.

10.2.8 Base Value

$$Z \leftarrow A \perp B$$

Informal Description: Z is, for A and B numeric vectors, a number produced by regarding B as the representation of a number in the mixed radix number system specified by A . If A or B is an array of rank greater than **one** the value is like inner product: each **vector-item** along the last **axis** of A is applied to each **vector-item** along the first axis of B .

Evaluation Sequence:

Set $A0$ to $1\rho\phi1, \rho A$.
 Set $B0$ to $1\rho(\rho B), 1$.
 If $A0$ differs from $B0$,
 If A is a **scalar** or **one-element-vector**, return $(B0\rho A)\perp B$.
 If B is a **scalar** or **one-element-vector**, return $A\perp A0\rho B$.
 Otherwise, signal **length-error**.
 If $A0$ is the same as $B0$,
 If any **item** of the **ravel-list** of A is not a **number**, signal **domain-error**.
 If any **item** of the **ravel-list** of B is not a **number**, signal **domain-error**.
 Return $(\phi\phi(\phi\rho A)\rho\phi\times\backslash\phi A, 1)+.\times B$

Examples:

```

10 1 1 2 3
123
24 60 60 1 1 2 3
3723
  A ← 2 3 10 10 10 12 60 60
10 10 10
12 60 60
  B ← 3 2 1 4 2 5 3 6
1 4
2 5
3 6
  A ⊥ B
123 456
3723 14706
  .001 10 10 1 1 2 3
123
60 1 1 2 3
3723
  ' ' ⊥ 3
0
  'A' ⊥ 1 0
0

```

Note: The shape requirements of A and B are intentionally stricter in this definition than in several existing systems. Specifically, this International Standard does not require that if the last **axis** of A is **one** it be replicated to match the first **axis** of B , or that if the first **axis** of B is **one** it be replicated to match the last **axis** of A .

The first element of each left argument has no actual effect on the result of base value, but permits use of the same left argument for base value and representation. If A is a positive numeric vector and B a non-negative numeric scalar such that $B < \times / A$, then $A \perp A \top B$ is B .

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10.2.9 Representation

$$Z \leftarrow A \uplus B$$

Informal Description: Z is the representation of B in mixed radix number system A .

Evaluation Sequence:

If A or B is **empty**, return $((\rho A), \rho B) \rho 0$.

If any **item** of the **ravel-list** of A is a **character**, signal **domain-error**.

If any **item** of the **ravel-list** of B is a **character**, signal **domain-error**.

If A is **scalar**, return $A \mid B$, evaluated with **comparison-tolerance** set to **zero**.

If A is a **vector** and B is a **scalar**,

Generate two **numeric vectors** Z and C that satisfy the following constraints:

The **length** of Z is ρA .

The **length** of C is $1 + \rho A$.

$C[1 + \rho A]$ is B .

For all **scalar** indices I in $1 \rho A$:

$Z[I]$ is $A[I] \uplus C[I+1]$.

If $A[I]$ is **zero**, $C[I]$ is **zero**;

Otherwise, $C[I]$ is $(C[I+1] - Z[I]) \div A[I]$.

Return Z .

Otherwise, return $Z1$, an **array** such that the **type** of $Z1$ is **numeric**, the **shape-list** of $Z1$ is $(\rho A), \rho B$ and the **ravel-list** of $Z1$ has the following property:

Let I stand for an **item** of the **index-set** of the **ravel-along-axis one** of A .

Let $A1$ stand for **vector-item** I of the **ravel-along-axis one** of A .

Let J stand for an **item** of the **index-set** of the **ravel-list** of B .

Let $B1$ stand for **item** J of the **ravel-list** of B .

Let N stand for the **count** of B .

Let P stand for $J + (N \times (I - 1))$.

Then, **vector-item** P of the **ravel-list** of $Z1$ **along-axis one** is $A1 \uplus B1$.

Examples:

$10 \ 10 \ 10 \tau 123$
 $1 \ 2 \ 3$
 $10 \ 10 \ 10 \tau 123 \ 456$
 $1 \ 4$
 $2 \ 5$
 $3 \ 6$
 $A \leftarrow 11 \ 3 \rho 10 \ 16 \ 2$
 $'0123456789ABCDEF' [\&1+A \tau 1000 \ 1024]$
 00000001000
 $000000003E8$
 01111101000

 00000001024
 00000000400
 10000000000
 $2 \ 2 \ 2 \ \tau^{-1}$
 $1 \ 1 \ 1$
 $0 \ 2 \ 2 \ \tau^{-1}$
 $^{-1} \ 1 \ 1$
 $0 \ 1 \ \tau 3.75 \ ^{-3.75}$
 $3 \ ^{-4}$
 $0.75 \ 0.25$

Note: The shape of the result of **representation** is always $(\rho A), \rho B$.

The **system-parameter comparison-tolerance** is not an implicit argument of representation.

10.2.10 Dyadic Transpose

$$Z \leftarrow A \bowtie B$$

Informal Description: Z is an array formed by rearranging and possibly coalescing the axes of B according to the vector A . Each element of A corresponds to an axis of B by position and to an axis of Z by value. The largest value in A determines the rank of Z . All axes of Z must be present in A . If A contains no repeated elements, the shape of Z is $(\rho B) \llbracket A \rrbracket$. Repeated elements in A select diagonals from B . The length of the corresponding axis of Z is the shortest of the lengths of the designated axes of B . Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.

If A is a **scalar**, set $A1$ to $\perp$, A .

Otherwise, set $A1$ to A .

If $A1$ is not a **vector**, signal **rank-error**.

If the **length** of $A1$ is not the same as the **rank** of B , signal **length-error**.

If any **item** of the **ravel-list** of $A1$ is not a **near-integer**, signal **domain-error**.

Set $A2$ to the **integer-array-nearest-to** $A1$. Generate $A3$, a **numeric array** with the **shape-list** of $A2$ such that each **item** of the **ravel-list** of $A3$ is **(one minus index-origin) plus** the corresponding element of $A2$.

If $\wedge / A3 \in \perp \uparrow / 0$, $A3$ evaluated with **comparison-tolerance** set to **zero** is not **one**, signal **domain-error**.

If $\wedge / (\perp \uparrow / 0, A3) \in A3$ evaluated with **comparison-tolerance** set to **zero** is not **one**, signal **domain-error**.

Return an **array** Z having the following properties:

The **rank** of Z is $\uparrow / 0, A3$.

The **shape-list** of Z is such that for all I in the **index-set** of the **shape-list** of Z , **item** I of the **shape-list** of Z is $\downarrow / (A3 = I) / \rho B$, evaluated with **comparison-tolerance** set to **zero**.

The **ravel-list** of Z is such that, for all J in the **index-set** of the **ravel-list** of Z , **item** J of the **ravel-list** of Z is **item** $1 + (\rho B) \downarrow ((\rho Z) \uparrow J - 1) \llbracket A3 \rrbracket$ of the **ravel-list** of B .

The **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of B .

Examples:

1 3 2 4
 111 121 131
 112 122 132
 113 123 133
 114 124 134

211 221 231
 212 222 232
 213 223 233
 214 224 234

1 1 3 4
 11 22 33
 3 1 2 4
 111 211
 112 212
 113 213
 114 214

121 221
 122 222
 123 223
 124 224

131 231
 132 232
 133 233
 134 234

0 1 0 5

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10.2.11 Take

$$Z \leftarrow A \uparrow B$$

Informal Description: Z is an array having shape ρA . Informally, Z is a corner of B . The choice of corner is based on the signs of the elements of A . If the absolute values of elements of A are greater than corresponding elements of ρB , then Z is padded with the **typical-element** of B .

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.

Set $A1$ to ρA .

If B is a **scalar**, set $B1$ to $((\rho A1) \rho 1) \rho B$.

Otherwise, set $B1$ to B .

If the **shape-list** of $A1$ does not match the **rank** of $B1$, signal **length-error**.

If any **item** of the **ravel-list** of $A1$ is not a **near-integer**, signal **domain-error**.

Set $A2$ to the **integer-array-nearest-to** $A1$.

Return an **array** Z such that:

The **shape-list** of Z is $\rho A2$.

The **ravel-list** of Z has the property that for each **scalar** I in the **index-set** of the **ravel-list** of Z , **item** I of the **ravel-list** of Z is determined as follows:

Let J stand for $((\rho A2) \uparrow I - 1) + (A2 < 0) \times A2 + \rho B1$ evaluated with **comparison-tolerance** set to **zero**.

If each element of J is in the **open-interval-between negative-one** and the corresponding element of $\rho B1$, **item** I of the **ravel-list** of Z is $(\rho B1) [1 + (\rho B1) \uparrow J]$.

Otherwise, **item** I of the **ravel-list** of Z is the **typical-element** of B .

The **type** of Z is the **sufficient-type** of the **ravel-of** Z under the **type** of B .

Examples:

```

      2↑N5
1 2  -2↑N5
      4 5
      -2 6↑N44
31 32 33 34 0 0
41 42 43 44 0 0
      -4 -4↑99
0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 99
      2↑i0
0 0

```

10.2.12 Drop

$$Z \leftarrow A \downarrow B$$

Informal Description: Z is a corner of B with shape $0 \uparrow (\rho B) - |A$. The choice of corner is based on the signs of the elements of A .

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.

Set $A1$ to A .

If B is a **scalar**, set $B1$ to $((\rho A1) \rho 1) \rho B$.

Otherwise, set $B1$ to B .

If the **shape-list** of $A1$ does not match the **rank** of $B1$, signal **length-error**.

If any **item** of the **ravel-list** of $A1$ is not a **near-integer**, signal **domain-error**.

Set $A2$ to the **integer-array-nearest-to** $A1$.

Return $(((A2 < 0) \times 0 \uparrow A2 + \rho B1) + (A2 \geq 0) \times 0 \downarrow A2 - \rho B1) \uparrow B1$ evaluated with **comparison-tolerance** set to **zero**.

Examples:

```

      1↓N5
2 3 4 5
      2 -1↓N44
31 32 33
41 42 43
      ρ1↓5
0
      ρ0↓5
1
      ρ1 2 3↓4
0 0 0
      ' '↓5
5
      ρ0 ' '↓5
0

```

10.2.13 Matrix Divide

$$Z \leftarrow A \oslash B$$

Informal Description: Z is the least squares solution to a linear system specified by A and B .

Evaluation Sequence:

If the **rank** of A or the **rank** of B is **greater-than two**, signal **rank-error**.
 Set $A1$ to $(2 \uparrow (\rho A), 1 \downarrow \rho A)$.
 Set $B1$ to $(2 \uparrow (\rho B), 1 \downarrow \rho B)$.
 If $1 \uparrow \rho A1$ is not the same as $1 \uparrow \rho B1$, signal **length-error**.
 If $1 \uparrow \rho B1$ is **less-than** $1 \uparrow \rho A1$, signal **length-error**.
 If any **item** of the **ravel-list** of A is not a **number**, signal **domain-error**.
 If any **item** of the **ravel-list** of B is not a **number**, signal **domain-error**.
 Use the **implementation-algorithm matrix-divide** to generate an **array** Z , such that the **type** of Z is **numeric**, the **shape-list** of Z is $(1 \uparrow \rho B1), 1 \downarrow \rho A1$, and the **ravel-list** of Z has the property that for each **scalar** I in $1 \downarrow \rho A1$, $Z[:, I]$ is such that it minimises $+(A1[:, I] - B1 \cdot Z[:, I])^2$.
 If Z cannot be uniquely determined within a reasonable round-off criterion by the above constraints, signal **domain-error**.
 Return $((1 \downarrow \rho B), 1 \downarrow \rho A) \rho Z$.

Additional Requirement:

The round-off criterion discussed above is not specified by this International Standard.

Note: The following article describes an acceptable algorithm for matrix division:

Jenkins, M. A., **Domino—An APL Primitive Function for Matrix Inversion—Its Implementation and Applications**. APL Quote-Quad Vol III No. 4, February 1972, pp 4-15.

10.2.14 Indexed Reference

$$Z \leftarrow A[I]$$

Informal Description: Z is an array consisting of elements of the array A selected and structured by the position and values of the arrays in the **index-list** I . Uses **index-origin**.

Evaluation Sequence:

If the **number-of-items** in the **index-list** I does not match the **rank** of A , signal **rank-error**.

In the following,

Let IX stand for (array) **item** X of I .

Let JX stand for (array) **item** X of J .

Let LX stand for (array) **item** X of L .

Let LY stand for (array) **item** $X+1$ of L .

Generate J , an **index-list** having the same **count** as I such that for every **item** X of the **index-set** of I ,

If IX is an **elided-index-marker**, set JX to $^{-1}1 + \imath(\rho A)[X]$.

Otherwise,

If any **item** of the **ravel-list** of IX is not a **near-integer**, signal **domain-error**.

Set JX to the **integer-array-nearest-to** IX .

Set JX to JX **minus index-origin**.

If any element of JX is not an element of $^{-1}1 + \imath(\rho A)[X]$, signal **index-error**.

Set JX to $JX \times \times / X \downarrow \rho A$.

Generate L , an **index-list** with $1 + \rho \rho A$ (array) **items** such that

Item $1 + \rho \rho A$ of L is **one**.

For all X in $\imath \rho \rho A$, LX is $JX \circ . + LY$.

Let K stand for **first-item** in the **list** of arrays L .

Return $(, A)[K]$.

Examples:

```

1 2 3[2]
2
N222[2 1; ; 2]
212 222
112 122
```

Note: Since an **index-list** has at least one **item**, indexing will always signal **rank-error** when argument A is a scalar.

The vector indexing on which this subsection is based is defined in the subsection **evaluate-indexed-reference**.

10.2.15 Indexed Assignment

$$Z \leftarrow V[I] \leftarrow B$$

Informal Description: Z is B . As a side effect, indexed assignment sets elements of the array V selected by the position and values of the arrays in the **index-list** I to corresponding elements of B . Note that, at this stage, V is known to be a **variable-name**, not a **value**. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.

Set A to the **current-content** of V .

If the **number-of-items** in the **index-list** I does not match the **rank** of A , signal **rank-error**.

In the following,

Let IX stand for (array) **item** X of I .

Let JX stand for (array) **item** X of J .

Let LX stand for (array) **item** X of L .

Let LY stand for (array) **item** $X+1$ of L .

Generate J , an **index-list** having the same **count** as I such that for every **item** X of the **index-set** of I ,

If IX is an **elided-index-marker**, set JX to $^{-1+1}(\rho A)[X]$.

Otherwise,

If any **item** of the **ravel-list** of IX is not a **near-integer**, signal **domain-error**.

Set JX to the **integer-array-nearest-to** IX .

Set JX to JX **minus index-origin**.

If any element of JX is not an element of $^{-1+1}(\rho A)[X]$, signal **index-error**.

Set JX to $JX \times \times / X \rho A$.

Generate L , an **index-list** with $1+\rho \rho A$ (array) **items** such that

Item $1+\rho \rho A$ of L is **one**.

For all X in $1+\rho \rho A$, LX is $JX \circ . + LY$.

Let K stand for **first-item** in the **list** of **arrays** L .

Set $K1$ to $((1 \neq \rho K) / \rho K) \rho K$ evaluated with **comparison-tolerance** set to **zero**.

Set $B1$ to $((1 \neq \rho B) / \rho B) \rho B$ evaluated with **comparison-tolerance** set to **zero**.

If $B1$ is a **scalar**, set $B2$ to $(\rho K1) \rho B1$.

Otherwise, set $B2$ to $B1$.

If the **rank** of $K1$ is not the same as the **rank** of $B2$, signal **rank-error**.

If any **item** of the **shape-list** of $K1$ is not the same as the corresponding **item** of the **shape-list** of $B2$, signal **length-error**.

Set M to **one**.

Repeat:

If M is not **greater-than** the **count** of $K1$,

Let $K2$ stand for **item** M of the **ravel-list** of $K1$.

Set **item** $K2$ of the **ravel-list** of A to **item** M of the **ravel-list** of B .

Set M to M **plus one**.

Otherwise,

Set the **type** of A to the **sufficient-type** of the **ravel-list** of A under the **type mixed**.

Set the **current-referent** of V to a **token** whose **class** is **variable** and whose **content** is A .

Return a **token** whose **class** is **committed-value** and whose **content** is B .

(End of repeated block)

Examples:

```

      X 2 3
      □←X[3 2]←4 5
4 5
      X
1 5 4
      □←Y←N222
111 112
121 122

211 212
221 222
      □←Y[2 1;;1]←N12121
11111
11121

12111
12121
      Y
12111 112
12121 122

11111 212
11121 222
    
```

Additional Requirements:

The evaluation sequence requires that after the statement $A[1\ 1\ 1]←1\ 2\ 3$ has been evaluated the value of $A[1]$ be 3.

Indexed assignment exhibits **atomic** behaviour. If **indexed assignment** signals an error, the value of V shall be unchanged.

Note: Since an **index-list** has at least one **item**, indexing will always signal **rank-error** when argument A is a scalar.

10.2.16 Without

$$Z \leftarrow A \sim B$$

Informal Description: Z is a **vector** whose elements are the elements of A without those that are also elements of B . Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**. If the **rank** of A or B is **greater-than one**, signal **rank-error**. Set $A1$ to $,A$; set $B1$ to $,B$. Return $(\sim A1 \in B1) / A1$, evaluated with the current value of **comparison-tolerance**.

Examples:

```

1 2 3 4 5~3 4 2
1 5
1 1 2 3 3 4 5~3 4 2
1 1 5
'NAPOLEON'~'NEON'
APL
```

10.2.17 Left

$$Z \leftarrow A \rightarrow B$$

Informal Description: Z is A .

Evaluation Sequence:

Return A .

Examples:

```

0→1
0
1→0
1
N2 →'FRANCE'
1 2
```

10.2.18 Right

$Z \leftarrow A \vdash B$

Informal Description: Z is B .

Evaluation Sequence:

Return B .

Examples:

$0 \vdash 1$
 1
 $1 \vdash 0$
 0
 $N2 \vdash 'FRANCE'$
 $FRANCE$

10.2.19 Character Grade Definitions

A precedes B in C order: An operation that, for **character-vectors** A and B and character **array** C of **rank greater-than zero**, returns a **Boolean** result determined by the following evaluation sequence:

Evaluation Sequence:

Set AX to the **rank** of C .

Repeat

Set $C1$ to the **array-of-vectors along-axis** AX of C .

Generate AC , a **numeric vector** with the same **length** as A , such that for all I in the **index-set** of A , $AC[I]$ is the minimum of $V \vdash A[I]$ over all items V in $C1$.

Generate BC , a **numeric vector** with the same **length** as B , such that for all I in the **index-set** of B , $BC[I]$ is the minimum of $V \vdash B[I]$ over all items V in $C1$.

If $((AC < BC) \vdash 1) < (AC > BC) \vdash 1$, return **one**.

Set AX to $AX - 1$.

If AX **equals zero**, return **zero**.

(end of repeated sequence).

A identifies-with B in C order : An operation that returns **zero** if A **precedes** B in C order or B **precedes** A in C order and otherwise returns **one**.

10.2.20 Character Grade Down

$$Z \leftarrow A \Downarrow B$$

Informal Description: Z is a permutation of the index set of the first axis of B which specifies a stable sort of the character subarrays crossing that axis into a non-increasing sequence according to an ordering relation given in the collating array A . Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.

Set IO to the **numeric-scalar** with value **index-origin** minus one.

If any **item** of the **ravel-list** of A is not a **character**, signal **domain-error**.

If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.

If A is a **scalar** signal **rank-error**.

If A is empty, return $IO + 1 \uparrow \rho B$.

Set $B1$ to $((1 \uparrow \rho B), \times / 1 \uparrow \rho B) \rho B$.

If $1 \uparrow \rho B$ is **zero**, return 10 .

If $1 \uparrow \rho B$ is **one**, return a **one-element-vector** Z such that the **ravel-list** of Z contains **index-origin**.

Otherwise, generate Z , a permutation of $1 \uparrow \rho B$ such that for every pair (I and J) of elements of $1 \uparrow \rho B$ where I is **less-than** J either:

$B1[Z[J];]$ **precedes** $B1[Z[I];]$ in A -order or

$B1[Z[I];]$ **identifies-with** $B1[Z[J];]$ in A -order and $Z[I]$ is **less-than** $Z[J]$.

Return $Z + IO$.

10.2.21 Character Grade Up

$$Z \leftarrow A \uparrow B$$

Informal Description: Z is a permutation of the index set of the first axis of B which specifies a stable sort of the character subarrays crossing that axis into a non-decreasing sequence according to an ordering relation given in the collating array A . Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.

Set IO to the **numeric-scalar** with value **index-origin** minus one.

If any **item** of the **ravel-list** of A is not a **character**, signal **domain-error**.

If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.

If A is a **scalar** signal **rank-error**.

If A is empty, return $IO + 1 \uparrow B$.

Set $B1$ to $((1 \uparrow B), \times / 1 \uparrow B) \rho B$.

If $1 \uparrow B$ is **zero**, return 10 .

If $1 \uparrow B$ is **one**, return a **one-element-vector** Z such that the **ravel-list** of Z contains **index-origin**.

Otherwise, generate Z , a permutation of $1 \uparrow B$ such that for every pair (I and J) of elements of $1 \uparrow B$ where I is **less-than** J either:

$B1[Z[I];]$ **precedes** $B1[Z[J];]$ in A -order or

$B1[Z[I];]$ **identifies-with** $B1[Z[J];]$ in A -order and $Z[I]$ is **less-than** $Z[J]$.

Return $Z + IO$.

Note: In comparing rows of $B1$ differences at low index positions are more significant than those at higher index positions. Thus, if 'a' comes before 'b' in A -order, the rows 'aa', 'ab', 'ba', and 'bb' are in ascending A -order.

Every axis of the collating array A defines an ordering of the character set. A character's lowest index along an axis defines its position in the order described by that axis (low indices precede high indices). Characters not found in A are equal and occur after all characters in A .

The order given along A 's last axis is the most significant. The result of character grade groups together rows of B which are equal under this (major) ordering. Within each such group, the result arranges B 's rows according to the hierarchy of (minor) orderings along axes of A ending with the first (least significant) axis. Character grade is stable; its results are equal under all the A -orderings in the order that they occur in B .

Hierarchical sorting allows distinctions such as spelling, case, font, and length to carry their proper lexicographic weight. For example, the rows of W (below) are taken in order from Webster's New Collegiate Dictionary.

Examples:

(2 2p'ABBA') ∇ 'AB' [?5 2p2] $\bowtie$ A AND B ARE EQUIVALENT

1 2 3 4 5

A $\leftarrow$ 2 14p' abcd~~e~~giklmnr~~t~~ ABCDEGIKLMNRT'

<i>W</i>	<i>W[(,A)∇W;]</i>	<i>W[(,A)∇W;]</i>	<i>W[A∇W;]</i>
<i>Ab</i>	<i>aba</i>	<i>aba</i>	<i>Ab</i>
<i>AB</i>	<i>abaca</i>	<i>abaca</i>	<i>AB</i>
<i>aba</i>	<i>abecedarian</i>	<i>abecedarian</i>	<i>aba</i>
<i>ABA</i>	<i>black</i>	<i>Ab</i>	<i>ABA</i>
<i>abaca</i>	<i>black belt</i>	<i>Abelian</i>	<i>abaca</i>
<i>abecedarian</i>	<i>blackball</i>	<i>AB</i>	<i>abecedarian</i>
<i>Abelian</i>	<i>blacking</i>	<i>ABA</i>	<i>Abelian</i>
<i>black</i>	<i>Ab</i>	<i>black</i>	<i>black</i>
<i>blackball</i>	<i>Abelian</i>	<i>black belt</i>	<i>black belt</i>
<i>black belt</i>	<i>AB</i>	<i>blackball</i>	<i>Black Mass</i>
<i>blacking</i>	<i>ABA</i>	<i>blacking</i>	<i>blackball</i>
<i>Black Mass</i>	<i>Black Mass</i>	<i>Black Mass</i>	<i>blacking</i>

Note: Character grade is equivalent to the function *SRT*⁴ defined in Smith, Howard J., "Sorting – A New/Old Problem", **APL Quote-Quad** 9, 1, June 1979, pp 123-127

10.2.22 Pick

$$Z \leftarrow A \triangleright B$$

Informal Description: Z is an item selected from some level of B . A is a scalar or vector. The items of A are selectors applying to one or more successive levels of B , such that the first (or only) item of A selects an item from B , the next (if any) selects an item from this item, and so on. The length of each item in A must agree with the rank of the object to which it is to be applied. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
 If the **rank** of A is **greater-than one**, signal **rank-error**.
 If A is empty, return B .
 If the **count** of a is **one**:
 Set $A1$ to the **first-thingy** of A .
 If $A1$ is not a **simple array**, signal **domain-error**.
 If the **rank** of $A1$ is **greater-than one**, signal **rank-error**.
 Set $A2$ to the **ravel-list** of $A1$.
 If any item in $A2$ is not a **near-integer**, signal **domain-error**.
 Set $A3$ to a list with each item the **integer-nearest-to** the corresponding element of $A2$.
 If the **number-of-items** in $A3$ is not the same as the *rank* of B , signal **rank-error**.
 Set $A4$ to a list, with each item equal to the corresponding item in $A3$, minus **index-origin**.
 For every member X of the **index-set** of $A4$:
 If item X of $A4$ is **less-than zero**, or **greater-than-or-equal-to** item X of the **shape-list** of B , signal **index-error**.
 Return item $1 \uparrow (\rho B) \downarrow A4$ of the **ravel-list** of B .
 Otherwise, return $(1 \downarrow A) \triangleright (1 \uparrow A) \triangleright B$.

Examples:

```

3  $\triangleright$  'OSCAR'
C
(, 3)  $\triangleright$  10 11 12 13
12
(c 3 2)  $\triangleright$  N44
32
1 5  $\triangleright$  (c 'OSCAR'), (c 'BEATA')
R
```

10.2.23 Identical

$$Z \leftarrow A \equiv B$$

Informal Description: Returns 1 if A and B match with respect to structure and content, and 0 otherwise.

Note: *Identical* is sometimes called “match”.

Evaluation Sequence:

If A and B are both **simple-scalars**, return $A=B$.

If the **shape-list** of A is different from the **shape-list** of B , return **zero**.

If A is empty, return $(1 \uparrow, A) \equiv 1 \uparrow, B$.

Set $A1$ to the first item of the **ravel-list** of A , and $B1$ to the first item of the **ravel-list** of B .

Set $Z1$ to $A1 \equiv B1$.

If $Z1$ is **zero**, return **zero**.

Set $A2$ to the rest of the **ravel-list** of A , and $B2$ to the rest of the **ravel-list** of B .

If $A2$ is empty, return **one**.

Return $A2 \equiv B2$.

Examples:

```
'SOLAR' ≡ 'SOLAR'
1
(2 3 ρ 16) ≡ 16
0
(1,0ρ ' ') ≡ 1,0ρ 0
1
```

10.2.24 Disclose

$$Z \leftarrow \supset B$$

Informal Description: For B , an array of shape A , all of whose items have the same **rank** K , Z is an array of shape A, M where M is the maximum over the shapes of the items in B , and each **rank- K cell** of Z is the result of $M^\uparrow$ of the corresponding item in B .

Evaluation Sequence:

Set M to **max-shape-of** B .
 Set $M1$ to $(\rho B), M$.
 Set K to ρM .
 Set S to the **index-set** of the **ravel-list** of B .
 Return an **array** Z such that ρZ is $M1$, and such that for every member I of S , the I 'th **rank- K cell** is $m^\uparrow C$, where C is **item** I of the **ravel-list** of B .

Examples:

$$\begin{array}{l} T \leftarrow (\subset 'OSCAR'), (\subset 'FRED') \\ \rho T \\ 2 \\ \supset T \\ OSCAR \\ FRED \\ \rho \supset T \\ 2 \ 5 \end{array}$$

10.2.25 Disclose with Axis

$$Z \leftarrow [K] \supset B$$

Informal Description: This is the same as Disclose, except that the function specifies which axes in the result Z , are new. Uses **index-origin**.

Evaluation Sequence:

Set $Z1$ to $\supset B$.
 If **index-origin** is **nil**, signal **implicit-error**.
 If A is not a **simple vector**, signal **axis-error**.
 If any **thingy-in** A is not a **valid-axis-of** $Z1$, signal **axis-error**.
 Set $A1$ to the **integer-array-nearest-to** A .
 If any item of the **ravel-list** of $A1$ is not distinct, signal **axis-error**.
 If the **count-of** $A1$ is not the **rank** of the **first-thingy-of** B , signal **axis-error**.
 Return $((\uparrow \rho \rho Z1) \sim A), A) \Downarrow Z1$.

10.2.26 Enclose

$$Z \leftarrow \subset B$$

Informal Description: Z is a scalar array whose only item is B .

Note: If B is a **simple-scalar**, Z is B .

Evaluation Sequence:

If B is a **simple-scalar**, return B .

Return an array whose **shape-list** is empty, and whose **ravel-list** has a single **item**, the array B , and whose **type** is **mixed**.

Examples:

```

      ⊃ 'OSCAR'
OSCAR
      ρρ ⊃ 'OSCAR'
0
      2 = ⊃ 2
1

```

10.2.27 Enclose with Axis

$$Z \leftarrow \subset [K] B$$

Informal Description: Z is B with the indicated axes eliminated and replaced by a corresponding new level of nesting. K specifies the axis to be enclosed, and their order in the resulting items. Uses **index-origin**.

Evaluation Sequence:

If, for some N , $K = (-N) \uparrow \iota \rho \rho B$, return an array whose shape is $(-N) \downarrow \rho B$, and whose **ravel-list** contains the **rank- N cells** of B .

Otherwise, return $\subset [(-\rho K) \uparrow \iota \rho \rho B] (\uparrow ((\iota \rho \rho B) \sim K), K) \Downarrow B$

11 System Functions

11.1 Introduction

Note: System functions are primitive functions whose names are **distinguished-identifiers** rather than **primitives**. System function names are not permitted in the **locals-list** of a **defined-function header-line**.

The system functions related to **shared-variables** and **defined-functions** are specified in those chapters.

11.2 Definitions

Note: Some system functions take an argument which is treated as an **identifier-array**.

- **Identifier-Array:** A **character array** of **rank two** each of whose **rows** matches the **character-diagram identifier-row**.
- **Event-Report:** A representation of the most recent error.
- **Event-Message:** A **character array** giving, in an **implementation-defined** form, the **event-report** and any other information the implementation deems helpful in locating the error's source.
- **Empty-Event-Message:** An empty **array**, of an **implementation-defined** shape, indicating that the most recent **event-type** corresponds to no error.

11.3 Diagram

Identifier-Row



11.4 Niladic System Functions

Note: A **niladic system function** is an implementation-provided facility associated with a **distinguished-identifier**. The primary difference between a **niladic system function** and a **system variable** is that a **niladic system function** causes an error to be signalled if a value is assigned to it, or its name is included in the list of locals of a defined function header.

11.4.1 Time Stamp

$$Z \leftarrow \square TS$$

Informal Description: Z is a seven-element numeric vector representing the current date and time relative to an epoch and a time zone.

Evaluation Sequence:

Return the seven-element **numeric vector** result of the **implementation-algorithm time-stamp**.

Example:

$$\square TS$$

1789 7 14 11 14 45 586.4

Additional Requirements:

The first through sixth elements of the result returned by **time-stamp** are integral and represent respectively the current year, month, day, hour, minute, and second relative to the underlying epoch and time zone. The hour corresponds to the number of integral hours that have elapsed within the current day.

The seventh and last element is a quantity, not necessarily integral, representing the milliseconds that have elapsed within the current second.

The accuracy, resolution, epoch, and time zone for **time-stamp** are not specified by this International Standard.

11.4.2 Atomic Vector

$Z \leftarrow \Box AV$

Informal Description: Z is an **implementation-defined character** vector containing every element of the **required-character-set** exactly once.

Evaluation Sequence:

Return **atomic-vector**.

Note: The elements of the **atomic-vector** are often ordered so that, in **index-origin zero**, $((\lceil 2 \otimes \Box AV \rceil \rho 2) \uparrow \Box AV) \uparrow 'X'$ gives the bit representation of X . This behaviour is not required.

11.4.3 Line Counter

$Z \leftarrow \Box LC$

Informal Description: Z is a vector of statement numbers of active defined functions ordered so that the most recently called function has the lowest index.

Evaluation Sequence:

Set Y to a **numeric vector** whose **length** is the same as the **number-of-items** in the **state-indicator**, such that for every I in $\uparrow \rho Y$, $Y[I]$ is the **current-line-number** of item I of the **state-indicator**, if the **mode** of item I of the **state-indicator** is **defined-function**, and **zero** otherwise.

Set M to a **Boolean vector** whose **length** is the same as the **number-of-items** in the **state-indicator**, such that for every I in $\uparrow \rho M$, $M[I]$ is **one** if the **mode** of item I of the **state-indicator** is **defined-function**, and **zero** otherwise.

Return M/Y .

Note: The subsections that affect the value of $\Box LC$ are discussed in the subsection **defined-function-control**.

11.4.4 Event Message

$$Z \leftarrow \square EM$$

Informal Description: When an error occurs, three lines of information are usually displayed: the first line indicates what happened, the second line shows the line in which the error was found, and the third line contains one or more carets pointing to the place(s) in the second line where the error was detected. **Event-message** is commonly a character matrix corresponding to this three-line message. It is set according to the contents of **event-type**. If **event-type** is 0 0 then **event-message** is an empty three-row matrix. Otherwise, **event-message** is set to an appropriate message. In some cases its first row is blank (when **event-message** is set via $\square ES$) because there is no message for the given value of **event-type**. (See the description of **event-type** for proposed values and messages.)

event-message is local to a context.

Evaluation Sequence:

Return **event-message**.

Note: While the informal description speaks of a three line message of a particular format, this is an attempt to describe common usage, rather than prescribed behaviour. The format of **event-message** is implementation defined.

11.4.5 Event Type

$Z \leftarrow \square ET$

Informal Description: The default value for **event-type** is 0 0 (no error). When an event happens which halts execution **event-type** is set to a pair of integers specifying the class and sub-class of the event. **event-type** is local to a **context**.

Evaluation Sequence:

Return **event-type**.

Note: *Classes and subclasses of event-type:*

- 0 0 – no error
- 0 1 – unclassified event

There may be additional implementation-defined types.

11.5 Monadic System Functions

Note: *Refer to the **Defined Functions** chapter for definitions of $\square FX$ and $\square CR$.*

11.5.1 Delay

$Z \leftarrow \square DL B$

Informal Description: Z is the time, in seconds, for completion of this operation. As a side effect, $\square DL$ causes a delay in execution of at least B seconds.

Evaluation Sequence:

Set $T0$ to **current-time** seconds.

If B is not a **scalar**, signal **rank-error**.

If B is not a **near-real-number**, signal **domain-error**.

Set $T1$ to $T0$ **plus** B .

Wait until **current-time** is not **less-than** $T1$.

Return Z , a **numeric scalar** such that the **first-item** in the **ravel-list** of Z is **current-time minus** $T0$.

Note: B may be fractional or negative.

11.5.2 Name Class

$$Z \leftarrow \square NC \ B$$

Informal Description: Z is a vector of name classes giving the usage of the identifier in the corresponding row of B . 0 means an identifier is available without current referent; 1, a label; 2, a variable or a shared variable; 3, a defined function; 4, a defined operator; 5, a system variable; 6, a system function.

Evaluation Sequence:

If the **rank** of B is **greater-than two**, signal **rank-error**.
 If B is **empty**, return 0.
 Set $B1$ to $(\neg 2 \uparrow 1 \ 1, \rho B) \rho B$.
 If any row of $B1$ does not match **identifier-row**, signal **domain-error**.
 Generate Z , a **numeric vector** such that the **length** of Z is the same as the **number-of-rows** in $B1$, and the **ravel-list** of Z has the property that for every I in $1 \uparrow \rho B$, the following holds:
 Let ZI stand for **item** I of the **ravel-list** of Z .
 Let N stand for the **token** that matched the **character-diagram simple-identifier** in the **identifier-row**.
 If the **current-class** of N is **nil**, ZI is **zero**.
 If the **current-class** of N is **label**, ZI is **one**.
 If the **current-class** of N is **variable** or **shared-variable**, ZI is **two**.
 If the **current-class** of N is **defined-function** or **niladic-defined-function**, ZI is **three**.
 If the **current-class** of N is **defined-operator**, ZI is **four**.
 If the **current-class** of N is **system-variable**, ZI is **five**.
 If the **current-class** of N is **system-function**, ZI is **six**.
 Otherwise, signal **domain-error**.
 Return Z .

Example:

```

      VZ←TEST;R0;R21;R22;R31;R32
[1] R1:R21←1 □SVO 'R22'
[2] Z←□FX 1 3ρ'R31'
[3] Z←□FX 1 5ρ'R32 X'
[4] Z←□NC 6 3ρ'R0 R1 R21R22R31R32'
      ∇
      TEST
0 1 2 2 3 3

```

11.5.3 Expunge

$$Z \leftarrow \square EX B$$

Informal Description: Z is a Boolean vector such that an element of Z is **one** if the identifier in the corresponding row of B is available for use when the operation completes. Expunge changes the **current-referent** of **symbols** whose **current-class** is **variable**, **shared-variable**, **defined-function**, **defined-operator**, or **niladic-defined-function** to **nil**, making them available for redefinition. The **symbols** to be changed are named by the rows of B , an **identifier-array**.

Evaluation Sequence:

If the **rank** of B is **greater-than two**, signal **rank-error**.
 Set $B1$ to $(\neg 2 \uparrow 1 \ 1, \rho B) \rho B$.
 For each **row** J of $B1$,
 If **row** J of $B1$ does not match **identifier-row**, signal **domain-error**.
 Let N stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
 If the **current-class** of N is **shared-variable**, **retract** N .
 If the **current-class** of N is **variable**, set the **current-referent** of N to a **token** whose **class** is **nil**.
 If the **current-class** of N is **defined-function** or **niladic-defined-function**, and if the **current-referent** of N is **locally-erasable**, set the **current-referent** of N to a **token** whose **class** is **nil**.
 Return $\sim \times \square NC B$.

Additional Requirement:

The operation of Expunge is **atomic**: if Expunge signals an error, **current-referents** shall be unchanged.

Note: *Because of the definition of **locally-erasable**, a **defined-function** or **defined-operator** cannot be expunged by a **conforming-program** while it is **pendent** or **waiting**. This does not prevent a **conforming-implementation** from permitting such behaviour; it simply leaves the consequences to a **conforming-program** undefined.*

11.5.4 Name List

$$Z \leftarrow \square NL B$$

Informal Description: Z is an **identifier-array**. An identifier occurs in Z if its **name-class** is in B . Elements of B can be: 1, for labels; 2, for variables or shared variables; 3, for defined functions; 4, for defined operators; 5, for system variables; 6, for system functions.

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.

If any **item** of the **ravel-list** of B is not a **near-integer**, signal **domain-error**.

Set $B1$ to the **integer-array-nearest-to** B .

If $B1$ contains any **numbers** other than **one**, **two**, **three**, **four**, **five**, or **six**, signal **domain-error**.

Generate Z , an **identifier-array** consisting of the **names** of **symbols** whose **current-class** is **label**, **variable**, **shared-variable**, **niladic-defined-function**, **defined-function**, **defined-operator**, **system-variable** or **system-function**. The **symbol names** are left-justified in the rows of Z .

Set Z to $((\square NC Z) \in B1) \neq Z$.

Remove any all-blank trailing columns from Z .

Return Z .

Note: The **rank** of Z is always **two**. If there are no **symbols** with a **name-class** designated in B , Z is 0 0p''.
 0 0p''.

11.5.5 Query Stop

$$Z \leftarrow \square STOP B$$

Informal Description: Z is an integer vector of line numbers in the function or operator whose name is B that have stop control set.

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.

If B does not match **identifier-row**, signal **domain-error**.

Let N stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.

If the **current-class** of N is not **defined-function**, **defined-operator** or **niladic-defined-function**, signal **domain-error**.

Return the **stop-vector** of the **current-content** of N .

Additional Requirement:

Query Stop is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Query Stop is not origin sensitive.*

If no lines have stop control set, the result is $\cdot 10$.

11.5.6 Query Trace

$$Z \leftarrow \square TRACE\ B$$

Informal Description: Z is an integer vector of line numbers in the function or operator whose name is B that are being traced.

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.

If B does not match **identifier-row**, signal **domain-error**.

Let N stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.

If the **current-class** of N is not **defined-function**, **defined-operator** or **niladic-defined-function**, signal **domain-error**.

Return the **trace-vector** of the **current-content** of N .

Additional Requirement:

Query Trace is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Query Trace is not origin sensitive.*

If no lines are being traced, the result is $\perp 0$.

11.5.7 Monadic Event Simulation

$\square ES \ B$

Informal Description: An exception is created in the caller's **context**. B is either a pair of integers specifying the class and sub-class of the event, or a character vector.

If B is a pair of integers, **event-type** is set to B and **event-message** is set to an appropriate message. If the pair of integers is undefined, the message part of **event-message** is blank, but an exception is signalled and **event-type** is set. If B is 0 0 (no error), **event-type** is set to 0 0 and **event-message** is set to **empty-event-message**, but no exception is signalled.

If B is empty no action is taken. This allows for conditional signalling of an event.

Evaluation Sequence:

If the rank of B is **greater-than one**, signal a **rank-error**.
 If B is **empty**, return **nil**.
 If B is **numeric**,
 If the **rank** of B is not **one**, signal **rank-error**.
 If the **count** of **items** in B is not **two**, signal **length-error**.
 If any **item** in B is not a **near-integer**, signal **domain-error**.
 Set B_1 to the **integer-nearest-to** B .
 Set **event-type** to B_1 .
 Set **event-message** to **empty-event-message**.
 If B_1 is not 0 0,
 Set **event-message** as appropriate.
 Signal event corresponding to **event-type**.
 If B is **character**,
 Set **event-type** to 0 1.
 Set message portion of **event-message** to B .
 Signal event corresponding to **event-type**.

11.6 Dyadic System Functions

11.6.1 Name List

$Z \leftarrow A \ \square NL \ B$

Informal Description: Z is an **identifier-array** of all names having a **name-class** in B that begin with a letter in A .

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
 If any **item** of the **ravel-list** of A does not match **letter**, signal **domain-error**.
 Set $Z1$ to $\square NL B$.
 If $0 \in \rho Z1$, return $Z1$.
 Return $(Z1[; 1] \in A) \neq Z1$.

11.6.2 Set Stop

$Z \leftarrow A \square STOP B$

Informal Description: Z is a numeric vector representing the stop controls in effect for the function or operator whose name is in B before $\square STOP$ began evaluation. As a side effect, $\square STOP$ sets stop controls for the function or operator whose name is in B at lines specified by A .

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.
 If B does not match **identifier-row**, signal **domain-error**.
 Let N stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
 If the **current-class** of N is not **defined-function**, **defined-operator** or **niladic-defined-function**, signal **domain-error**.
 If the **rank** of A is **greater-than one**, signal **rank-error**.
 If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
 Set $A1$ to the **integer-array-nearest-to** A .
 If any **item** of the **ravel-list** of $A1$ is not a **positive-integer**, signal **domain-error**.
 Set Z to $\square STOP B$.
 Let M stand for the **last-line-number** of the **current-content** of N .
 Set the **stop-vector** of the **current-content** of N to $((\iota M) \in A1) / \iota M$, evaluated with **index-origin one**.
 Return Z .

Additional Requirement:

Set Stop is part of the **optional-facility Trace-and-Stop-Control**.

Note: Stop controls are an attribute of **defined-function** objects. $\square STOP$ affects the **current-referent** of B only.

11.6.3 Set Trace

$$Z \leftarrow A \square TRACE B$$

Informal Description: Z is a numeric vector representing the lines being traced in the function or operator whose name is in B before $\square TRACE$ began evaluation. As a side effect, $\square TRACE$ begins tracing the lines specified by A in the function or operator whose name is in B .

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.
 If B does not match **identifier-row**, signal **domain-error**.
 Let N stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
 If the **current-class** of N is not **defined-function**, **defined-operator**, or **niladic-defined-function**, signal **domain-error**.
 If the **rank** of A is **greater-than one**, signal **rank-error**.
 If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
 Set $A1$ to the **integer-array-nearest-to** A .
 If any **item** of the **ravel-list** of $A1$ is not a **positive-integer**, signal **domain-error**.
 Set Z to $\square TRACE B$.
 Let M stand for the **last-line-number** of the **current-content** of N .
 Set the **trace-vector** of the **current-content** of N to $((\iota M) \in A1) / \iota M$, evaluated with **index-origin one**.
 Return Z .

Additional Requirement:

Set Trace is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Trace controls are an attribute of defined-function objects. $\square TRACE$ affects the current-referent of B only.*

11.6.4 Execute Alternate

$$Z \leftarrow A \sqcup EA \ B$$

Informal Description: A and B are both character vectors representing executable expressions. If execution of B is successful the result is Z . If there is an error or an interrupt in the execution of B , execution is abandoned and A is executed. The execution of A is subject to normal error handling.

It is possible to get a **value-error** if the successful execution of B does not return a result and an explicit result is needed. A is not executed in this case.

Evaluation Sequence:

If the **rank** of either A or B is **greater-than one** signal **rank-error**.

If any **item** of the **ravel-list** of A is not a **character**, signal **domain-error**.

If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.

Generate a new **context** in which

mode is **execute**,

current-line is the **ravel-list** of B ,

current-function is 0 0p ' ',

current-line-number is **one**,

current-statement is the empty **list** of **tokens**,

stack is the empty **list** of **tokens**.

Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.

Set Z to **evaluate-line**.

Remove the first **context** from the **state-indicator**.

If Z is not a member of **error** return Z ,

Else return $\perp A$

Note: See **Conforming Programs in Compliance** regarding hazards to conforming programs if this facility is used.

11.6.5 Dyadic Event Simulation

$A \sqsubseteq_{ES} B$

Informal Description: An exception is created in the caller's **context**. A is a **character scalar** or **vector** and B is either a pair of integers specifying the class and sub-class of the event, or an empty **vector**.

Event-type is set to B and the message portion of **event-message** is set to A . If B is $0\ 0$ (no error), **event-type** is set to $0\ 0$ and **event-message** is set to **empty-event-message**, and no event is signalled.

If B is empty no action is taken. This allows for conditional signalling of an event (for example $A \sqsubseteq_{ES} \text{cond} / B$).

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
 If any **item** of A is not a **character**, signal **domain-error**.
 If the **rank** of B is not **one**, signal **rank-error**.
 If B is empty, return **nil**.
 If the **count** of B is not **two**, signal **length-error**.
 If any **item** in B is not a **near-integer**, signal **domain-error**.
 Set $B1$ to the **integer-nearest-to** B .
 Set **event-type** to $B1$.
 Set **event-message** to **empty-event-message**.
 If $B1$ is not $0\ 0$,
 Set the message portion of **event-message** to A .
 Set the rest of **event-message** as appropriate.
 Signal event corresponding to **event-type**.

11.6.6 Transfer Form

$$Z \leftarrow A \square TF B$$

Informal Description: B is a character vector giving the name of the object for which the transfer form is desired. If there is no transfer form for the item named by B (for example, B names a system function, a label, or has no value) the result is an empty character vector.

A is either **one** or **two**. If it is **one** B may name only defined functions, defined operators, and simple homogeneous arrays. If it is **two**, B may also name **mixed** arrays.

For a given A , $\square TF$ is its own inverse. For example, if ABC names a transferable object, $A \square TF A \square TF 'ABC'$ will reestablish the object named ABC and return the character list $'ABC'$.

If A is **two**, the result is an executable character vector giving the name and contents of the object named by B . (Numeric conversions are carried out to **full-print-precision**.)

Evaluation Sequence:

If the **rank** of A is **greater-than one** signal **rank-error**.
 If the **count** of A is not **one** signal **length-error**.
 If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
 Set $A1$ to the **integer-nearest-to** A .
 If $A1$ has the value **one**, set Z to **produce-canonical-representation-vector** of the object named by B .
 If $A1$ has the value **two**,
 If the **rank** of B is **greater-than one** signal **rank-error**.
 If the **type** of B is not **character** signal **domain-error**.
 Set NC to the **name-class** of B .
 If NC is **two**:
 Set $B1$ to ΦB
 Set Z to an executable string that will recreate the object named by B
 If NC is **three** or **four**:
 Set $B1$ to the **character-representation** of the object named by B
 If the **count** of the **shape** of B is not **zero**
 Set Z to an executable string that will recreate the object named by B
 Else set Z to an empty character vector.
 If NC is **negative-one**:
 If ΦB is successful, set Z to the name of the object created
 Otherwise, set Z to an empty character vector
 Otherwise signal **domain-error**
 Otherwise, signal **domain-error**.

12 System Variables

12.1 Definitions

- **System-Variable-Symbol:** A **symbol**, having as its name a **distinguished-identifier**, and as its **referent-list** a **list** of **tokens** representing the value assigned to an associated **system-parameter** in each **context** of the **state-indicator**.

Note: System parameters are values used implicitly and set as side effects by primitive operations.

- **Internal-Value-Set:** An **implementation-defined** set of values that the **tokens** in the **referent-list** of a **system-variable-symbol** can take on.

Note: When a **system-variable-symbol** is **localised** the **initial class** of its associated **system-parameter** is **nil**. If a primitive operation is invoked that requires a system parameter whose class is currently **nil**, the primitive operation signals **implicit-error**. Therefore, a **conforming-program** that **localises system-variable-symbols** should assign them values from their **internal-value-set** before calling primitive operations that require them.

The assignment operation for a system variable rejects attempts to set its associated system parameter to a value outside its **internal-value-set**.

12.2 Evaluation Sequences

12.2.1 Comparison Tolerance

$$Z \leftarrow \square CT \leftarrow B$$

$$Z \leftarrow \square CT$$

Informal Description: Z is the current value of **comparison-tolerance**.

The distance within which numbers are to be considered equal by certain primitives is a function of **comparison-tolerance**.

Evaluation Sequence:

For form $\square CT$

Return **comparison-tolerance**.

For form $\square CT \leftarrow B$

If the **rank** of B is **greater-than one**, signal **rank-error**.

If the **count** of B is not **one**, signal **length-error**.

Let $B1$ be the **first-scalar** in B .

If $B1$ is not a **nonnegative-number**, signal **domain-error**.

If $B1$ is not in the **internal-value-set** of **comparison-tolerance**, signal **limit-error**.

Set **comparison-tolerance** to $B1$.

Return a **token** whose **class** is **committed-value** and whose **content** is B .

Additional Requirement:

The **internal-value-set** for **comparison-tolerance** consists of **nonnegative-numbers** not greater than the **implementation-parameter comparison-tolerance-limit**.

The initial value of **comparison-tolerance** in a **clear-workspace** is that member of the **internal-value-set** for **comparison-tolerance** given by the **implementation-parameter initial-comparison-tolerance**.

Note: The following subsections reference **comparison-tolerance**: **ceiling**, **equal**, **floor**, **greater than or equal to**, **greater than**, **index of**, **less than or equal to**, **less than**, **member of**, **not equal**, **residue**, **and**, **or** and **unique**.

12.2.2 Random Link

$Z \leftarrow \square RL \leftarrow B$

$Z \leftarrow \square RL$

Informal Description: Z is the current value of **random-link**; **random-link** is the current seed of the pseudorandom number generator.

Evaluation Sequence:

For form $\square RL$

Return **random-link**.

For form $\square RL \leftarrow B$

If the **rank** of B is **greater-than one**, signal **rank-error**.

If the **count** of B is not **one**, signal **length-error**.

Set $B1$ to the **first-scalar** in B .

If $B1$ is not a **near-integer**, signal **domain-error**.

Set $B2$ to the **integer-nearest-to** $B1$.

If $B2$ is **less-than one**, signal **domain-error**.

If $B2$ is not in the **internal-value-set** for **random-link**, signal **limit-error**.

Set **random-link** to $B2$.

Return a **token** whose **class** is **committed-value** and whose **content** is B .

Additional Requirement:

The **internal-value-set** of **random-link** is **implementation-defined**.

The initial value of **random-link** in a **clear-workspace** is that member of the **internal-value-set** for **random-link** given by the **implementation-parameter** **initial-random-link**.

Note: The system parameter **random-link** is used and set by **roll** and **deal**. **Roll** gives a reference for a suitable algorithm.

12.2.3 Print Precision

$$Z \leftarrow \square PP \leftarrow B$$

$$Z \leftarrow \square PP$$

Informal Description: Z is the current value of **print-precision**, which controls the number of significant positions in the output form produced by **monadic format** and **numeric-output-conversion**.

Evaluation Sequence:

For form $\square PP$

Return **print-precision**.

For form $\square PP \leftarrow B$

If the **rank** of B is **greater-than one**, signal **rank-error**.

If the **count** of B is not **one**, signal **length-error**.

If B is not **near-integer**, signal **domain-error**.

Set $B1$ to the **integer-nearest-to** the **first-scalar** in B .

If $B1$ is **less-than one**, signal **domain-error**.

If $B1$ is not in the **internal-value-set** of **print-precision**, signal **limit-error**.

Set **print-precision** to $B1$.

Return a **token** whose **class** is **committed-value** and whose **content** is B .

Additional Requirements:

The **internal-value-set** of **print-precision** consists of the **integer** scalars in the **closed-interval-between one** and **print-precision-limit**.

The initial value of **print-precision** in a **clear-workspace** is that member of the **internal-value-set** for **print-precision** given by the **implementation-parameter** **initial-print-precision**.

Note: The system parameter **print-precision** is used by **monadic format** and **numeric-output-conversion**. **Print-precision-limit** should be greater than or equal to **full-print-precision**.

12.2.4 Index Origin

$Z \leftarrow \square IO \leftarrow B$

$Z \leftarrow \square IO$

Informal Description: Z is the current value of **index-origin**, which is the index associated with the first position of any axis of non-zero length.

Evaluation Sequence:

For form $\square IO$

Return **index-origin**.

For form $\square IO \leftarrow B$

If the **rank** of B is **greater-than one**, signal **rank-error**.

If the **count** of B is not **one**, signal **length-error**.

If B is not a **near-integer**, signal **domain-error**.

Set $B1$ to the **integer-nearest-to** the **first-scalar** in B .

If $B1$ is not in the **internal-value-set** of **index-origin**, signal **limit-error**.

Set **index-origin** to $B1$.

Return a **token** whose **class** is **committed-value** and whose **content** is B .

Additional Requirement:

The **internal-value-set** for **index-origin** is the scalar integer values **zero** and **one**.

The initial value of **index-origin** in a **clear-workspace** is that member of the **internal-value-set** for **index-origin** given by the **implementation-parameter** **initial-index-origin**.

Note: The following primitive operations refer to **index-origin**: **deal**, **dyadic transpose**, **grade down**, **grade up**, **index generator**, **index of**, **indexed assignment**, **indexed reference**, **roll**, and all functions that provide for axis specification when the form containing an axis is used.

12.2.5 Latent Expression

$$Z \leftarrow \square LX \leftarrow B$$

$$Z \leftarrow \square LX$$

$$Z \leftarrow \square LX[I] \leftarrow B$$

Informal Description: Z is the current value of **latent-expression**, which is a **character** vector that is executed when a workspace is activated.

Evaluation Sequence:

For form $\square LX$

Return **latent-expression**.

For form $\square LX \leftarrow B$

If the **rank** of B is **greater-than one**, signal **rank-error**.

If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.

If B is not in the **internal-value-set** of **latent-expression**, signal **limit-error**.

Set **latent-expression** to B .

Return a **token** whose **class** is **committed-value** and whose **content** is B .

For form $\square LX[I] \leftarrow B$

Set LX to **latent-expression**.

Evaluate $LX[I] \leftarrow B$.

Set **latent-expression** to LX .

Return a **token** whose **class** is **committed-value** and whose **content** is B .

Additional Requirement:

The **internal-value-set** for **latent-expression** is an **implementation-defined** subset of the set of all character vectors which includes the empty vector.

The initial value of **latent-expression** in a **clear-workspace** is that member of the **internal-value-set** for **latent-expression** given by the **implementation-parameter** **initial-latent-expression**.

13 Defined Functions

13.1 Introduction

Note: Algorithms written in APL are called *defined functions*. A defined function consists of a header line and zero or more body lines. The header line indicates the name and syntax class of the function and gives a list of names to be localised and, in the case of argument names, initialised. Each body line consists of an optional label followed by an APL line to be evaluated. Body lines are evaluated in the order in which they occur in the defined function unless a statement beginning with a right arrow is evaluated. In this chapter, the term defined function also includes defined operator.

A defined function is established in a workspace by the system function `⌞FX`, by actions in function definition mode, or by the system command `)COPY`.

In this International Standard, a **defined-function** is represented as an object with three attributes: **canonical-representation**, **trace-vector**, and **stop-vector**. A **defined-function** is called when its name occurs in a **prefix** of the **current-stack** matching one of the **patterns** in the **phrase-table**.

A **defined-function** is called by the operation **call-defined-function**. This operation prefixes the **state-indicator** with a new **context**, **localises** and **initialises** any local names in the **header-line**, and calls the evaluation sequence in the subsection **defined-function-control**.

A **defined-function** ends the evaluation if **defined-function-control** finds **current-line-number** set to a value not in the **closed-interval-between one and last-line-number**. At this point, the first item of the **state-indicator** is discarded and a **token** of class **result** is returned, via **call-defined-function**, to the **phrase-evaluator** that called it.

The returned **token** may be **constant** (for functions returning a value), **nil** (for functions that return no value), **unwind** (for functions that end through the evaluation of an escape arrow), or **clear-state-indicator** (as the result of an `)SIC` command).

If a line in a **defined-function** signals an error, **immediate-execution** is called to report the error and suspend the **defined-function**. **Immediate-execution** may return either a **branch** to indicate that evaluation of the **defined-function** should continue, or an **escape** or **clear-state-indicator** to indicate that the **defined-function** context should be removed from the **state-indicator**.

13.2 Definitions

Note: The following term defines the subset of **arrays** that can be used to create **defined-functions**.

- **Proper-Defined-Function:** An array A is a **proper-defined-function** if
 - The **type** of A is **character**.
 - The **rank** of A is **two**.
 - A has at least one **row**.
 - The first **row** of A matches the **character-diagram header-line**.
 - No **identifier** in the first row of A is the **distinguished-identifier** $\square$ or $\sqcap$ or is a **system-function-name** or **niladic-system-function-name**.
 - All **rows** after the first in A match the **character-diagram body-line**.

Note: The following definitions give properties of arrays that are **proper-defined-functions**.

- **Niladic:** A , an **array** that is a **proper-defined-function**, is **niladic** if the **form** of the first **row** of A is **niladic-function-header**.
- **Monadic:** A , an **array** that is a **proper-defined-function**, is **monadic** if the **form** of the first **row** of A is **monadic-function-header**.
- **Ambivalent:** A , an **array** that is a **proper-defined-function**, is **ambivalent** if the **form** of the first **row** of A is **ambivalent-function-header**.
- **Monadic-Monadic-Operator:** A , an **array** that is a **proper-defined-function**, is **monadic-monadic-operator** if the **form** of the first **row** of A is **monadic-monadic-operator-header**.
- **Monadic-Dyadic-Operator:** A , an **array** that is a **proper-defined-function**, is **monadic-dyadic-operator** if the **form** of the first **row** of A is **monadic-dyadic-operator-header**.
- **Ambivalent-Monadic-Operator:** A , an **array** that is a **proper-defined-function**, is **ambivalent-monadic-operator** if the **form** of the first **row** of A is **ambivalent-monadic-operator-header**.
- **Ambivalent-Dyadic-Operator:** A , an **array** that is a **proper-defined-function**, is **ambivalent-dyadic-operator** if the **form** of the first **row** of A is **ambivalent-dyadic-operator-header**.
- **Value-Returning:** A , an **array** that is a **proper-defined-function**, is **value-returning** if, when **header-line** is threaded with the first **row** of A , the diagram **result** is threaded.

Note: The following terms categorise **defined-functions** according to their presence on or absence from the **state-indicator**.

- **Suspended:** N , a **simple-identifier**, is **suspended** if it is the **function-name** of a **context** that immediately follows an **immediate-execution context**.
- **Pendent:** N , a **simple-identifier**, is **pendent** if it is the **function-name** of a **context** that does not immediately follow an **immediate-execution context**.

- **Waiting:** N , a **simple-identifier**, is **waiting** if a **token** whose **class** is **defined-function-name** and whose **content** is the **content** of N occurs in the **stack** of some **context** in the **state-indicator** of the **active-workspace**, and is not the first **defined-function-name** in that **stack**.

Note: For example, if F and G are defined function names, G is **waiting** during the evaluation of $(F\ 2)$ in the line

$(F\ 2)\ G\ 3.$

- **Editable:** N , a **simple-identifier**, is **editable** if the **current-referent** of N is **nil**, or if all the following are true:
The **current-class** of N is **defined-function**, **defined-operator** or **niladic-defined-function**.
The **number-of-items** in its **referent-list** is **one**.
 N is neither **pendent** nor **waiting**.
 N , if **suspended**, is **suspended** in only the **current-context**.

Note: This definition requires that a **conforming-implementation** be capable of editing the top function on the state indicator as long as it has not been **localised** and occurs nowhere else on the state indicator. This minimal definition is included to be certain that all **conforming-implementations** be capable of interactive program correction.

- **Globally-Erasable:** **Globally-Erasable** is defined for N , a **simple-identifier**, as follows:
If no **context** of the **state-indicator** contains, in its **current-statement**, a **defined-name** with **content** that of N , then N is **globally-erasable**.
Otherwise, let L be the last (most global) **context** in whose **local-name-list** N appears.
If there is no such **context** then N is not **globally-erasable**.
Otherwise, let C be the last (most global) **context** in whose **current-statement** a **defined-name** with **content** that of N appears.
If the index of L is greater-than or equal to that of C then N is **globally-erasable**.

Note: Informally, N is **globally-erasable** if it was **localised** before it was made **waiting** or **pendent**.

- **Locally-Erasable:** **Locally-Erasable** is defined for N , a **simple-identifier**, as follows:
If no **context** of the **state-indicator** contains, in its **current-statement**, a **defined-name** with **content** that of N , then N is **locally-erasable**.
Otherwise, let L be the first (most local) **context** in whose **local-name-list** N appears.
If there is no such **context** then N is not **locally-erasable**.
Otherwise, let C be the first (most local) **context** in whose **current-statement** a **defined-name** with **content** that of N appears.
If the index of C is greater than that of L , then N is **locally-erasable**.

Note: Informally, N is **locally-erasable** if it has been **localised** since the last time it was made **waiting** or **pendent**. This definition is used in $\square EX$ to specify that the expression $(\square EX\ 'F')\ F\ 1$ be the same as $0\ F\ 1$; it is used in $\square FX$ to specify that the expression $(\square FX\ 1\ 1\rho\ 'F')\ F\ 1$ signal a **domain-error**.

Note: The following operations provide information about **arrays** that are **proper-defined-functions**.

- **Function-Name** of A : For A a **proper-defined-function**, the **simple-identifier** that matches **function-name** when the **character-diagram header-line** is threaded with the first **row** of A .
- **Last-Line-Number** of A : For A a **proper-defined-function**, **negative-one plus the number-of-rows** in A .
- **Header-Name-List** of A : For A a **proper-defined-function**, a **list of identifiers** developed as follows:
Thread the **character-diagram header-line** with the first **row** of A .
Return a **list of tokens** consisting of **identifiers** that matched any of the **character-diagrams local-name, result-name, left-argument-name, right-argument-name, right-operand-name, or left-operand-name**.
- **Label-Name-List** of A : For A a **proper-defined-function**, a **list of simple-identifiers** developed as follows:
Set Z to the **empty-list** of **tokens**.
For each **index** I in the **closed-interval-between one and last-line-number** of A ,
 Thread the **character-diagram body-line** with **row** I **plus one** of A .
 If **labelled-line** was matched, append the **simple-identifier token** that matched the **character-diagram label-name** to Z as a new last **item**.
Return Z .
- **Local-Name-List** of A : For A a **proper-defined-function**, a **list of identifiers** consisting of the **label-name-list** followed by the **header-name-list**.
- **Identifier-Matching** D in A : For A a **proper-defined-function** and D a **character-diagram**, a **simple-identifier token** developed as follows:
Thread the **character-diagram header-line** with the first **row** of A .
Return a **token** of class **simple-identifier** whose **content** is the **list of characters** that matched D .

Note: **Identifier-Matching** is used to refer to the arguments and result of an arbitrary defined function.

- **Function-Line** I of A : For A a **proper-defined-function**, a **list of characters** developed as follows:
Thread the **character-diagram body-line** with **row** I **plus one** of A .
Return the **list of characters** that matched the **character-diagram line** in **body-line**.

Note: The following operations affect symbols.

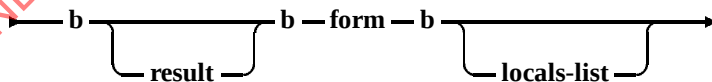
- **Localise** N :
Append a new first **item**, a **token** of class **nil**, to the **referent-list** of the **symbol-named-by** N .
- **Delocalise** N :
If the **current-class** of N is **shared-variable**, **retract** N .
Remove the first **item** from the **referent-list** of the **symbol-named-by** N .

- **Current-Canonical-Representation:** The **canonical-representation** of the **current-function** of the **current-context** in the **active-workspace**.
- **Current-Function-Line I :** **Function-Line I** of the **current-canonical-representation**.
- **Current-Last-Line-Number:** The **last-line-number** of the **current-canonical-representation**.
- **Current-Stop-Vector:** The **stop-vector** of the **current-function** of the **current-context** in the **active-workspace**.
- **Current-Trace-Vector:** The **trace-vector** of the **current-function** of the **current-context** in the **active-workspace**.
- **Current-Local-Names:** The **local-name-list** of the **current-canonical-representation**.
- **Current-Right-Argument-Name:** The **identifier-matching right-argument-name** in the **current-canonical-representation**.
- **Current-Left-Argument-Name:** The **identifier-matching left-argument-name** in the **current-canonical-representation**.
- **Current-Result-Name:** The **identifier-matching result-name** in the **current-canonical-representation**.
- **Current-Left-Operand-Name:** The **identifier-matching left-operand-name** in the **current-canonical-representation**.
- **Current-Right-Operand-Name:** The **identifier-matching right-operand-name** in the **current-canonical-representation**.

13.3 Diagrams

In these diagrams,
b stands for **permitted-blanks**.
r stands for **required-blanks**.

Header-Line



Example:

$Z \leftarrow A \ F \ B; C; \square IO$

Note: The header line indicates the name and syntactic class of the **defined-function**. It also gives a list of identifiers to be **localised** when the **defined-function** is called.

This International Standard does not provide for end-of-line comments in header lines.

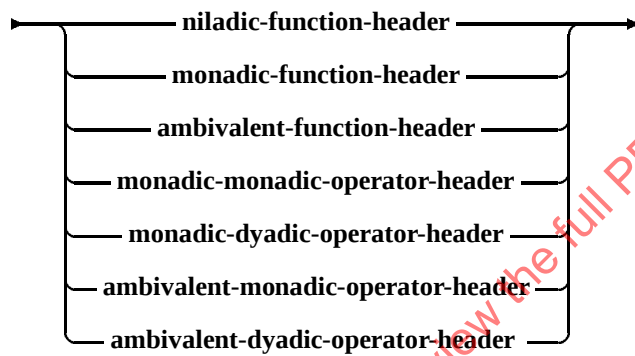
Result

► **result-name** — **b** — **assignment-arrow** ►

Result-Name

► **simple-identifier** ►

Form



Niladic-Function-Header

► **function-name** ►

Monadic-Function-Header

► **function-name** — **r** — **right-argument-name** ►

Ambivalent-Function-Header

► **left-argument-name** — **r** — **function-name** — **r** — **right-argument-name** ►

In the next four diagrams,

la stands for **left-argument-name**

ra stands for **right-argument-name**

mop stands for **monadic-operator-part**

dop stands for **dyadic-operator-part**

Monadic-Monadic-Operator-Header

► b — mop — b — ra —►

Monadic-Dyadic-Operator-Header

► b — dop — b — ra —►

Ambivalent-Monadic-Operator-Header

► la — b — mop — b — ra —►

Ambivalent-Dyadic-Operator-Header

► la — b — dop — b — ra —►

Monadic-Operator-Part

► (— b — left-operand-name — r — function-name — b —) —►

Dyadic-Operator-Part

► (— b — left-operand-name — r — function-name — r — right-operand-name — b —) —►

Right-Argument-Name

► simple-identifier —►

Left-Argument-Name

► simple-identifier —►

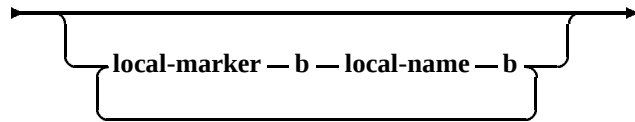
Right-Operand-Name

► simple-identifier —►

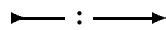
Left-Operand-Name

► simple-identifier —►

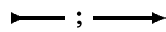
Locals-List



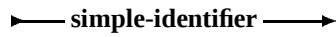
Colon



Local-Marker



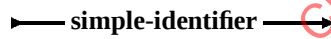
Function-Name



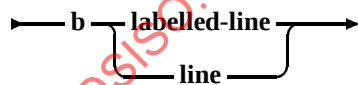
Local-Name



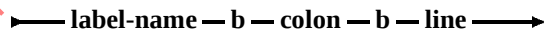
Label-Name



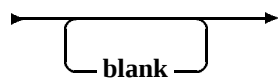
Body-Line



Labelled-Line



Permitted-Blanks



Required-Blanks



13.4 Operations

13.4.1 Call-Defined-Function

$Z \leftarrow DFN$

$Z \leftarrow DFN \ B$

$Z \leftarrow A \ DFN \ B$

$Z \leftarrow \mathbf{f} \ DFN \ B$

$Z \leftarrow A \ \mathbf{f} \ DFN \ B$

$Z \leftarrow \mathbf{f} \ DFN \ \mathbf{g} \ B$

$Z \leftarrow A \ \mathbf{f} \ DFN \ \mathbf{g} \ B$

Informal Description: Z is the value of the variable given as the result name in the header line of the defined function whose name is DFN . If there is no result name, Z is **nil**.

Evaluation Sequence:

Generate a new **context** in which
mode is **defined-function**,
current-line is the empty **list** of **characters**,
current-function is the **current-referent** of DFN ,
current-line-number is **one**,
current-statement is the empty **list** of **tokens**, and
stack is the empty **list** of **tokens**.

Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.

Localise the **current-local-names**.

For form $Z \leftarrow A \ \mathbf{f} \ DFN \ \mathbf{g} \ B$

If **current-function** is not **ambivalent-defined-dyadic-operator**, signal **valence-error**.

Set the **symbol-named-by current-left-argument-name** to A .

Set the **symbol-named-by current-left-operand-name** to $\mathbf{f}$.

Set the **symbol-named-by current-right-operand-name** to $\mathbf{g}$.

Set the **symbol-named-by current-right-argument-name** to B .

For form $Z \leftarrow \mathbf{f} \ DFN \ \mathbf{g} \ B$

If **current-function** is not **monadic-defined-dyadic-operator**, signal **valence-error**.
 Set the **symbol-named-by current-left-operand-name** to **f**.
 Set the **symbol-named-by current-right-operand-name** to **g**.
 Set the **symbol-named-by current-right-argument-name** to **B**.
 For form $Z \leftarrow A \text{ f } DFN \ B$
 If **current-function** is not **ambivalent-defined-monadic-operator**, signal **valence-error**.
 Set the **symbol-named-by current-left-argument-name** to **A**.
 Set the **symbol-named-by current-left-operand-name** to **f**.
 Set the **symbol-named-by current-right-argument-name** to **B**.
 For form $Z \leftarrow \text{f } DFN \ B$
 If **current-function** is not **monadic-defined-monadic-operator**, signal **valence-error**.
 Set the **symbol-named-by current-left-operand-name** to **f**.
 Set the **symbol-named-by current-right-argument-name** to **B**.
 For form $Z \leftarrow A \ DFN \ B$
 If **current-function** is not **ambivalent**, signal **valence-error**.
 Set the **symbol-named-by current-left-argument-name** to **A**.
 Set the **symbol-named-by current-right-argument-name** to **B**.
 For form $Z \leftarrow DFN$
 If **current-function** is not **niladic**, signal **valence-error**.
 For all indices **I** in the **closed-interval-between one** and **current-last-line-number**,
 If **row I plus one** of **canonical-representation** matches the diagram **labelled-line**,
 set the **current-referent** of the **simple-identifier** that matches **label-name** to a
token of class **label** and **content** the **numeric-scalar** with value **I**.
 Set **Z** to **defined-function-control**.
Delocalise the **current-local-names**.
 Remove the **first-item** in the **state-indicator** of the **active-workspace**.
 If **Z** is **escape**, signal **unwind**.
 Otherwise, return **Z**.

Note: Tokens of class **escape** are converted to tokens of class **unwind** so that **immediate-execution** can distinguish an escape issued in **immediate-execution** mode from one issued in a **defined-function**.

The **context** attribute **current-function** includes the function itself plus its trace and stop vectors.

13.4.2 Defined-Function-Control

Informal Description: This procedure controls the execution of a user defined function.

Evaluation Sequence:

Set *Z* to a **token** whose **class** is **nil**.
Let *I* stand for **current-line-number**.
Set *I* to **one**.
Repeat:
 If *I* is in the **current-stop-vector**, set **attention-flag** to **one**.
 If *Z* is an **error** or **attention-flag** is **one**,
 Set *N* to **immediate-execution** with *Z*.
 If *N* is **escape** or **clear-state-indicator**, return *N*.
 If *N* is **branch**, set *I* to the **content** of *N*.
 If *I* is not in the **closed-interval-between one** and **current-last-line-number**,
 If the **current-function** is not **value-returning**, return **nil**.
 If the **current-class** of the **current-result-name** is **variable** or **nil**, return the
 current-referent of the **current-result-name**.
 Otherwise, signal **value-error**.
 Set *Z* to **evaluate-line** of **current-function-line** *I*.
 If *Z* is **escape**, **unwind**, or **clear-state-indicator**, return *Z*.
 If *Z* is **branch**, set *I* to the **content** of *Z*.
 If *Z* is **value** or **nil**, set *I* to *I* plus **one**.
(End of repeated block)

Note: The **attention-flag** is reset by the user facility **enter**.

The **class** of the **token** returned by **evaluate-line** for $\rightarrow 10$ is **nil**, not **branch**.

The **value-error** signalled from **defined-function-control** is signalled in the line that invoked the defined function; the error is introduced to prevent **conforming-programs** from exiting a **defined-function** with the **current-class** of the **result-name** set to such classes as **defined-function**, **niladic-defined-function**, and **shared-variable**.

13.4.3 Function Fix

$$Z \leftarrow \square FX B$$

Informal Description: Z is, for B the **canonical-representation** of a defined function or operator having **function-name** N , a **character** vector holding N . As a side effect, the defined function that B represents is established as the current referent of N . Rows of B that are all blanks give rise to blank lines in the established function.

Evaluation Sequence:

If the **rank** of B is not **two**, signal **rank-error**.

If B is an **empty array**, signal **length-error**.

If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.

If B is not a **proper-defined-function**, signal **domain-error**.

If **identifiers** are duplicated in the **local-name-list** of B , signal **domain-error**.

Let N stand for the **simple-identifier** that matches **function-name** in the first **row** of B .

If the **current-class** of N is not **defined-operator**, **defined-function**, **niladic-defined-function**, or **nil**, signal **domain-error**.

If N is not **locally-erasable**, signal **domain-error**.

Set the **current-referent** of N to a **defined-function** or **defined-operator** for which **canonical-representation** is set from B .

stop-vector is $\uparrow 0$,

trace-vector is $\uparrow 0$.

Return a **character vector** Z such that the **length** of Z is the **number-of-items** in the **content** of N and the **ravel-list** of Z is the **content** of N .

13.4.4 Character Representation

$Z \leftarrow \square CR B$

Informal Description: Z is a **character** matrix representation of the defined function or operator named by B .

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.
 If the **length** of B is **zero**, signal **length-error**.
 If B does not match the **character-diagram identifier-row**, signal **domain-error**.
 If the **current-class** of B is not **defined-operator**, **defined-function** or **niladic-defined-function**, signal **domain-error**.
 Otherwise return a **token** whose **class** is **constant** and whose **content** is the **canonical-representation** of the **current-content** of B .

Additional Requirement:

The formal model of APL used in this International Standard assumes that the internal representation of a defined function is precisely the **character** matrix satisfying the requirements for a **proper-defined-function** that was supplied as an argument to **function fix**. Representation of defined functions in this way in an implementation is neither required nor suggested.

The preservation of numeric literals and blanks as entered is desirable in the character representation, but it is not required by this International Standard. $\square CR \square FX \square CR \square FX X$ shall be the same as $\square CR \square FX X$, if X is a **proper-defined-function**.

If **numeric-literals** are stored as **numbers** in **defined-functions**, they are to be formatted with **print-precision** set to **full-print-precision**.

13.5 Function Editing

Note: The function editing facilities described here are included to provide the APL programmer with a standard method for entering and correcting defined functions.

13.5.1 Evaluate-Function-Definition-Request

Evaluate-Function-Definition-Request Q .

Note: This subsection provides minimal facilities for the establishment and alteration of **defined-functions**.

An identifier can be edited by the function editing facilities only if it is **editable**. The header line of a **defined-function** can be altered only if the **function-name** is **locally-erasable**.

In **function-definition mode**, lines of the function being edited are temporarily associated with numbers in **decimal-rational** form by an unspecified mechanism. When a function is closed, the order of the lines given by the line number associated with each line is maintained.

The display of a function should be the same as the entries required to add the function to a **clear-workspace**.

This operation is called by **immediate-execution** when it recognises a **function-definition-line**.

Evaluation Sequence:

Thread **opening-request** with Q ; if Q does not match **opening-request**, signal **definition-error**.

Rethread **opening-request** with Q , taking the following actions:

When **creation-request** is matched,

Set G to the **list of characters** that matched **creation-request**.

Set N to the **simple-identifier** that matched **function-name** in G .

If the **current-class** of N is not **nil** and G does not match **identifier-row**, signal **definition-error**.

When **change-request** is matched,

Set N to the **simple-identifier** that matched the diagram **subject-function** in **change-request**.

Set G to the **list of characters** that matched **initial-request**.

If the **current-class** of N is **nil**, set M to a new **defined-function** for which

Canonical-Representation is $0\ 0p\ ' \ ' \ '$

Stop-Vector is $\imath 0$

Trace-Vector is $\imath 0$.

Otherwise,

If N is not **editable**, signal **definition-error**.

Set M to the **current-referent** of N .

Generate a new **context** in which

mode is **function-definition**,

current-line is G ,

current-function is M ,

current-line-number is the **number-of-rows** in the **canonical-representation** of M ,

current-statement is the empty **list of tokens**, and

stack is the empty **list of tokens**.

Append the **context** as a new first **item** to the **state-indicator**.

Repeat:

Set Z to **evaluate-editing-request**.

If Z is **command-complete**,

Remove the **first-item** in the **state-indicator** of the **active-workspace**.

Return Z .

If Z is an **error**, display Z .

Set **current-prompt** to **function-definition-prompt** of **current-line-number**.

Set **current-line** to **read-keyboard**.

(End of repeated block)

Additional Requirement:

The prompt in definition mode is a line number enclosed in brackets. Line numbers are always displayed in **decimal-rational** form. The line number of the header is always **zero**; it is not affected by **index-origin**.

13.5.2 Evaluate-Editing-Request

Evaluate-Editing-Request

Informal Description: This subsection acts on a single line of input to the editor.

Evaluation Sequence:

Let C stand for the **current-canonical-representation**.
 Let N stand for the **simple-identifier** that matches **function-name** in **function-line**.
 Let S stand for the **current-stop-vector**.
 Let T stand for the **current-trace-vector**.
 Thread **general-request** with **current-line**; if **general-request** does not match **current-line**, signal **definition-error**.
 Otherwise, rethread **general-request** with **current-line**, taking the following actions:
 When **positioning-request** is threaded,
 Set L to the **numeric-input-conversion** of the **list** of **characters** that matched **line-number** in **positioning-request**.
 If L is a **negative-number**, signal **definition-error**.
 If L is **greater-than definition-line-limit**, signal **limit-error**.
 Set **current-line-number** to L .
 When **deletion-request** is threaded,
 Set L to the **numeric-input-conversion** of the **list** of **characters** that matched **line-number** in **deletion-request**.
 If L is not a **positive-number**, signal **definition-error**.
 If L is **greater-than definition-line-limit**, signal **limit-error**.
 Set **current-line-number** to L .
 Delete from C the row associated with L , if it exists.
 Delete from S and T the item associated with L , if it exists.
 When **display-request** is threaded, **display function-display** of C .
 When **function-line** is threaded, if **function-line** does not match **permitted-blanks**,
 If **current-line-number** is **zero**,
 If **function-line** does not match **header-line**, signal **definition-error**.
 If N is not **locally-erasable**, signal **definition-error**.
 Set the **row** of C associated with **current-line-number** to the **list** of **characters** that matched **function-line**.
 If **current-line-number** is not an **integer**, or is **greater-than current-last-line-**

number, set the **items** of S and T associated with **current-line-number** to **zero**.

Set **current-line-number** to **next-definition-line** of **current-line-number**.

When **end-definition** is matched,

If C is not a **proper-defined-function**, signal **definition-error**.

If **identifiers** are duplicated in the **local-name-list** of C , signal **definition-error**.

Set the **current-referent** of N to a **defined-function** for which

Canonical-Representation is C .

Stop-Vector is S .

Trace-Vector is T .

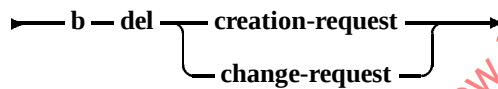
Return **command-complete**.

13.5.3 Diagrams

In these diagrams,

b stands for **permitted-blanks**.

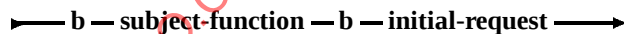
Opening-Request



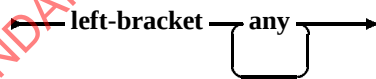
Creation-Request



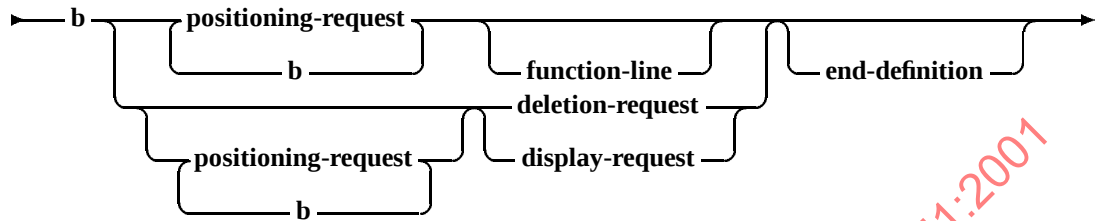
Change-Request



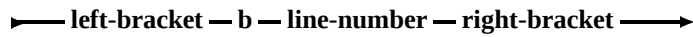
Initial-Request



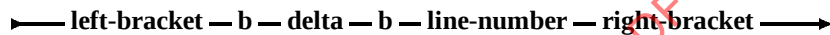
General-Request



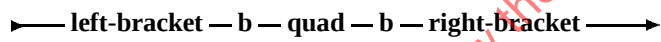
Positioning-Request



Deletion-Request



Display-Request



Function-Line



End-Definition



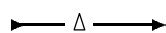
Line-Number



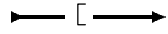
Subject-Function



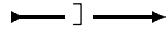
Delta



Left-Bracket



Right-Bracket



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14 Shared Variables

14.1 Informal Introduction

This section gives an informal introduction to the **Shared-Variable-Protocol**. The evaluation sequences which follow this introduction determine the required behaviour of **shared-variables** exactly.

The **Shared-Variable-Protocol** is an **optional-facility**. The **Shared-Variable-Protocol** is described in this document as an interface between cooperating **sessions**, but it may also be used to provide an interface between a **session** and an **auxiliary processor**. An **auxiliary processor** is a program, written in any programming language, that typically provides access to facilities in the underlying operating system, such as file systems.

A **shared-variable** can be shared by precisely two partners, each either a **session** or an **auxiliary processor**.

Conforming-programs that use the **Shared-Variable-Protocol** should document this fact, since the **Shared-Variable-Protocol** is not a **defined-facility** of this International Standard.

Operations

There are conceptually four operations involved in the **Shared-Variable-Protocol**: **offer**, **retract**, **set**, and **reference**.

Sharing

Offer is an operation provided to permit **sessions** to share **shared-variables**. A given **shared-variable** can be shared between at most two **sessions**. Once a **shared-variable** has been shared, assignments made to it in one **session** are visible in the other **session**.

In the evaluation sequences for the **Shared-Variable-Protocol**, sharing is accomplished by appending **tokens** of **class shared-variable** to the **shared-variable-list** which can be accessed by all **active-workspaces** in the **system**.

Any **Shared-Variable-Protocol** operation may cause a **session** to be delayed while another **session** changes a **shared-variable** or adds an item to the **shared-variable-list**.

Sharing begins when one **session** offers to share a **symbol** whose **current-class** is **nil** or **variable**. The **system** adds a new **shared-variable** to the **shared-variable-list** and returns the **shared-variable** to the offering **session**. The **shared-variable** then replaces the **current-referent** of the offered **symbol**.

There are two forms of offers. A **specific-offer** identifies a particular **session** as the partner; the **general-offer** does not.

Two offers match if both **sessions** use the same **shared-name** and:

At least one of the offers is a **specific-offer**.

Either **specific-offer** identifies the other **session**.

Consequently, two **general-offers** do not match.

Once both **sessions** have shared the **shared-variable**, the **shared-variable** is the **current-referent** of a **symbol** in both **active-workspaces**, and a change made to the **shared-variable** by one **session** will be visible to the other **session**.

Note that two **sessions** can share more than one **shared-variable**, that a given **session** can share **shared-variables** with more than one other **session**, and that each **shared-variable** can be shared by only two **sessions**.

Retracting

Two **sessions**, coupled by a **shared-variable**, decouple when either **retracts** the **shared-variable**. This leaves the **shared-variable** in the same state it would have been had the remaining **session** made a **specific-offer** to the **session** that **retracted**. If the remaining **session** **retracts**, the **shared-variable** becomes inaccessible to the **sessions**.

Degree of Coupling

All **symbols** in the **symbol-table** of an **active-workspace** have a **degree of coupling**. This is **zero** if the **current-referent** is not a **shared-variable**, **one** if the **current-referent** is a **shared-variable** that is offered to another **session**, and **two** if **current-referent** is a **shared-variable** shared with another **session**.

Naming

For any session, there are two **identifiers** associated with a given **shared-variable**. The first is the **name** of the **symbol** whose **current-referent** is the **shared-variable**; the second is the **shared-name** of the **shared-variable**. The **shared-name** is the name used in matching offers to share.

Only the **name** of the **symbol** is required for invoking any of the **Shared-Variable-Protocol** system functions other than **Shared Variable Offer**.

Setting, Referencing, and Using

Each partner can **set** and **reference** the **shared-variable**.

A **shared-variable** is **set** when the operation **shared-variable-assignment** or **shared-variable-indexed-assignment** is called. A **shared-variable** is **referenced** when the operation **shared-variable-reference** is called. A **shared-variable** is used when it is either **set** or **referenced**. Both indexed assignment and indexed reference of **shared-variables** are possible.

Synchronisation

Synchronisation of **sessions** sharing a **shared-variable** is achieved by **access control**. With each **shared-variable** is associated an **access-control-vector**, which is designated here as the *ACV*. The *ACV* is a four element Boolean vector that is accessible to a **session** through the **Shared-Variable-Access-Control-Inquiry** operation.

The order of the elements in the *ACV* is relative to the viewing **session**. When **session A**, sharing a **shared-variable** with **session B**, views the *ACV*, its elements have the following meanings:

- If *ACV*[1] is **one**, once A has set the **shared-variable** B must use it before A can set it again.
- If *ACV*[2] is **one**, once B has set the **shared-variable** A must use it before B can set it again.
- If *ACV*[3] is **one**, once A has used the **shared-variable** B must set it before A can reference it again.
- If *ACV*[4] is **one**, once B has used the **shared-variable** A must set it before B can reference it again.

If a **session** attempts to use a **shared-variable** when not permitted to by the above rules, it is delayed until the partner has performed the required intermediate operation.

The rules can be summarised in a three-state state diagram. Let A be the **session** that made the initial offer. Then the states are as follows:

- State 0: The **shared-variable** has been **referenced** by the partner of the **session** that last set it.
- State 1: The **shared-variable** was last **set** by A and has not been **referenced** by B since then.
- State 2: The **shared-variable** was last **set** by B and has not been **referenced** by A since then.

State 0 is the state entered when the **shared-variable** is initially offered. Note that only an explicit assignment is considered a **set** to the **shared-variable**. The act of transferring the **current-content** of a **symbol** to the **shared-value** of a **shared-variable** when an offer

Action attempted:	SET by A	SET by B	REF by A	REF by B		
Current state:	0	+	+	3	4	
	1	1	+	3	+	
	2	+	2	+	4	
Next state:	0	1	2	0	1	2

Figure 2: Shared Variable Access Rules.

is made is not a **set**. The state is not affected by retraction and re-sharing. **Figure 2** summarises the state transitions permitted. In the figure, the action is always permitted if there is a + in the row corresponding to the input state. The column in which the + appears determines the output state. A digit in the row indicates that the corresponding element in the *ACV* as seen by A must be zero.

For example, if the state of a **shared-variable** is 0 and the *ACV* is 0 0 1 0, **session A** can assign a new value to the variable (SET by A), but must wait until the variable is in state 2 to refer to it (REF by A).

Note that all digits lie along the diagonal in these matrices, indicating that only actions that do not change the state can be blocked by the *ACV*. That is, the *ACV* can only prevent a given session from referencing or setting a **shared-variable** twice in a row.

Other Rules

Retraction of all the **shared-variables** in a **session** takes place when a new **workspace** is loaded, the **active-workspace** is cleared, or the **session** ends. A **shared-variable** is automatically **retracted** when the **shared-variable** is erased, expunged, or replaced by a)*COPY* command, or, if it is local, when the function to which it is local terminates execution. The state of sharing is not saved in a library workspace.

The shared variable mechanism must appear to the user to be consistent with one in which there exists a single copy of each **shared-variable**. Specifically, this means:

- When a **shared-variable** is initially offered, its value, if any, becomes the shared-value of the **shared-variable**.
- When an offered **shared-variable** is matched, it retains its value if it has one. If it does not have a value, it takes on the value (if any) in the matcher's workspace.
- When a **shared-variable** is retracted, its value in the retractor's workspace is the value last set by either partner before the retraction.

Note that, since there is conceptually a single copy across a period of sharing, no use is implied in the above rules for making the value of a **shared-variable** available.

It must appear to the APL user that **sessions** communicating via **shared-variables** are

independently scheduled. Any synchronisation between the **sessions** must appear to be by means of the delays caused by accessing **shared-variables**. The procedures used to describe the **Shared-Variable-Protocol** are written as though synchronisation conflicts do not occur when two **sessions** are accessing the **shared-variable-list** or the same **shared-variable**.

A **limit-error** may occur during the retraction of an active **shared-variable** if there is not enough room in a **workspace** for the **shared-value** of the **shared-variable**. When the expected system action is to ignore the variable (as in the case of) *CLEAR* or delocalisation), a **limit-error** is not signalled.

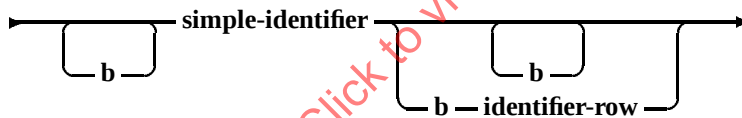
14.2 Definitions

- **Identifier-Pair-Array**: A **character array** of **rank two** in which each **row** matches the **character-diagram identifier-pair-row**.

14.3 Diagrams

In the following, **b** stands for **blank**.

Identifier-Pair-Row



14.4 Operations

14.4.1 Primary-Name

Primary-Name in A: For *A*, a **character vector** that matches **identifier-pair-row**, the first **simple-identifier** matched in **identifier-pair-row**.

14.4.2 Surrogate-Name

Surrogate-Name in A: For *A*, a **character vector** that matches **identifier-pair-row**, the last **simple-identifier** matched in **identifier-pair-row**.

14.4.3 Degree-of-Coupling

Degree-of-Coupling of N : For **symbol** N , a **number** defined as follows:

If the **current-class** of N is not **shared-variable**, **zero**.

Otherwise, using the **current-referent** of N , the sum of **session-A-active** and **session-B-active**.

14.4.4 Access-Control-Vector

Access-Control-Vector of N : An operation that, for N a **shared-variable**, is defined as follows:

Using N ,

Let $ACVA$ stand for **session-A-ACV** and $ACVB$ stand for **session-B-ACV**.

If **this-session** is **session-A**, return $ACVA \vee ACVB$.

If **this-session** is **session-B**, return $(ACVA \vee ACVB) [2 \ 1 \ 4 \ 3]$.

14.4.5 Offer

Offer S to P with value V : An operation defined as follows:

Let X stand for the **item** in the **shared-variable-list** with the smallest **index** such that **shared-name** is S .

session-A is **this-session** or, if P is not **general-offer**, **general-offer**.

session-A-active is **zero**.

session-B is P .

session-B-active is **one**.

or

shared-name is S .

session-A is P .

session-A-active is **one**.

session-B is **this-session** or, if P is not **general-offer**, **general-offer**.

session-B-active is **zero**.

If such a **shared-variable** exists,

If **shared-value** is **nil**, set **shared-value** to V .

If **session-A-active** is **zero**,

Set **session-B-event** to **one**.

Set **session-A-active** to **one**.

Set **session-A** to **this-session**.

If **session-B-active** is **zero**,

Set **session-A-event** to **one**.

Set **session-B-active** to **one**.

Set **session-B** to **this-session**.

Return X .

Otherwise, create a new **shared-variable**, Y , as follows:

shared-name is S .

session-A is **this-session**.

session-A-active is **one**.

session-A-ACV is 0 0 0 0.

session-B is P .

session-B-active is **zero**.

session-B-ACV is 0 0 0 0.

shared-value is V .

state is **zero**.

session-A-event is **zero**.

session-B-event is **one**.

Append Y to the **shared-variable-list** as a new last item.

Return Y .

14.4.6 Retract

Retract N : An operation that, for N a **shared-variable**, is defined as follows:

Using N ,

If **session-A** is **this-session**, set **session-A-active** to **zero**.

If **session-B** is **this-session**, set **session-B-active** to **zero**.

Signal-event.

Clear-event.

14.4.7 Shared-Variable-Reset

Shared-Variable-Reset

For each item in the **shared-variable-list**,

If **session-A** is **this-session**, set **session-A-active** to **zero**.

If **session-B** is **this-session**, set **session-B-active** to **zero**.

For each **symbol** N of the **symbol-table** of the **active-workspace**,

For each **token** T in the **referent-list** of N ,

If T is a **shared-variable**, set T to a **token** whose **class** is **nil**.

Note: In the model, a **shared-variable** is never reused. It is effectively discarded once **session-A-active** and **session-B-active** are both **zero**.

14.4.8 Report-State

Report-State for N : An operation that, for N a **shared-variable**, is defined as follows:

Using N ,

If **state** is **zero**, return 0 0 1 1.

Otherwise,

If **state** is **one** and **session-A** is **this-session**,

or if **state** is **two** and **session-B** is **this-session**,

Return 1 0 1 0.
Otherwise return 0 1 0 1.

14.4.9 Signal-Event

Signal-Event for N : An operation that, for N a **shared-variable**, is defined as follows:

Using N ,

If **session-A** is **this-session**, set **session-B-event** to **one**.

If **session-B** is **this-session**, set **session-A-event** to **one**.

14.4.10 Clear-Event

Clear-Event for N : An operation that, for N a **shared-variable**, is defined as follows:

Using N ,

If **session-A** is **this-session**, set **session-A-event** to **zero**.

If **session-B** is **this-session**, set **session-B-event** to **zero**.

14.5 Shared Variable Forms

14.5.1 Shared Variable Reference

$Z \leftarrow SHV$

Informal Description: Return the value of a shared variable.

Evaluation Sequence:

Using the **current-content** of SHV ,

Let ACV stand for **access-control-vector**.

Let ASV stand for **report-state**.

Set A to $ACV \wedge ASV$.

If $A[2]$ or $A[3]$ is **one**, **signal-event**.

Wait until at least one of the following is true:

$ACV[3]$ is **zero**.

State is **two** and **session-A** is **this-session**.

State is **one** and **session-B** is **this-session**.

If either of the following is true, set **State** to **zero**:

State is **two** and **session-A** is **this-session**.

State is **one** and **session-B** is **this-session**.

Clear-event.

Return **Shared-Value**.

Note: This operation is all that is required for **shared-variable-indexed-reference**, since the **phrase-evaluator** calls **indexed reference** with a value, not a name.

14.5.2 Shared Variable Assignment

$$Z \leftarrow SHV \leftarrow B$$

Informal Description: Z is B . As a side effect, the value B is assigned to the shared variable SHV .

Evaluation Sequence:

Using the **current-content** of SHV ,
Let ACV stand for **access-control-vector**.
Let ASV stand for **report-state**.
Set A to $ACV \wedge ASV$.
If $A[1]$ or $A[4]$ is **one**, **signal-event**.
Wait until at least one of the following is true:
 $ACV[1]$ is **zero**.
 State is not **one** and **session-A** is **this-session**.
 State is not **two** and **session-B** is **this-session**.
Set **shared-value** to B .
If **session-A** is **this-session**, set **state** to **one**.
If **session-B** is **this-session**, set **state** to **two**.
Clear-event.
Return a **token** whose **class** is **committed-value** and whose **content** is B .

14.5.3 Shared Variable Indexed Assignment

$$Z \leftarrow SHV[I] \leftarrow B$$

Informal Description: Z is B . As a side effect, elements of the array SHV selected by the position and values of the arrays in the **index-list** I are replaced by elements of the array B .

Evaluation Sequence:

Using the **current-content** of SHV ,
 Let ACV stand for **access-control-vector**.
 Let ASV stand for **report-state**.
 Set A to $ACV \wedge ASV$.
 If $A[1]$ or $A[4]$ is **one**, **signal-event**.
 Wait until at least one of the following is true:
 $ACV[1]$ is **zero**.
 session-A is **this-session** and **state** is not **one**.
 session-B is **this-session** and **state** is not **two**.
 If the **current-class** of **shared-value** is **nil**, **signal value-error**.
 Set C to **shared-value**.
 Search the **form-table** for $Z \leftarrow V[I] \leftarrow B$.
 Call the corresponding evaluation sequence, passing C as the value of V .
 Set Z to the **token** it returns.
 If Z is an **exception**, return Z .
 Otherwise,
 Set **shared-value** to C .
 If **session-A** is **this-session**, set **state** to **one**.
 If **session-B** is **this-session**, set **state** to **two**.
 Clear-event.
 Return Z .

Note: Indexed assignment of a shared variable constitutes a **set**, not a **reference**, of the **shared-variable**.

14.6 Shared Variable System Functions

14.6.1 Shared Variable Access Control Inquiry

$$Z \leftarrow \square SVC \ B$$

Informal Description: Each row of Z is the **access-control-vector** of the shared variable named by the corresponding row of B .

Evaluation Sequence:

If the **rank** of B is **greater-than two**, signal **rank-error**.
 Set $B1$ to $(\neg 2 \uparrow 1 \ 1, \rho B) \rho B$.
 If any **item** of the **ravel-list** of $B1$ is not a **character**, signal **domain-error**.
 Set Z to $((1 \uparrow \rho B1), 4) \rho 0$.
 For every I in $1 \uparrow \rho B1$, evaluated with **index-origin** set to **one**,
 Let $B2$ stand for **row** I of $B1$.
 If $B2$ matches **identifier-row**,
 Set N to the **simple-identifier** in $B2$.
 If the **current-class** of N is **shared-variable**,
 Set **row** I of Z to **access-control-vector** of the **current-referent** of N .
 Otherwise, signal **domain-error**.
 Return $((\neg 1 \uparrow \rho B), 4) \rho Z$.

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14.6.2 Shared Variable Query

$$Z \leftarrow \square SVQ \ B$$

Informal Description: Z is one of the following:

An array representing sessions offering to share variables with this session.

An array representing the names of shared variables offered to this session by another session.

Evaluation Sequence:

If **session-identification-type** is **character**, and B is the **general-offer**,

Set Z to $0 \ 0 \rho^1 \ 1$.

Using each **item** of the **shared-variable-list**,

If

session-A is **this-session** and
session-A-active is **zero** and
session-B-active is **one**,

Set S to **session-B**.

Set Z to $((\rho Z) \uparrow 0, \rho S), [1] ((\neg 1 \uparrow \rho Z) \uparrow \rho S) \uparrow S$ evaluated with **index-origin** set to **one**.

If

session-A-active is **one** and
session-B is **this-session** and
session-B-active is **zero**,

Set S to **session-B**.

Set Z to $((\rho Z) \uparrow 0, \rho S), [1] ((\neg 1 \uparrow \rho Z) \uparrow \rho S) \uparrow S$ evaluated with **index-origin** set to **one**.

Return Z .

If **session-identification-type** is **numeric**, and B is **zero**, signal **domain-error**.

If **session-identification-type** is **numeric**, and B is $\neg 0$,

Set Z to $\neg 0$.

Using each **item** of the **shared-variable-list**,

If all of the following are true,

session-A is **this-session**.
session-A-active is **zero**.
session-B-active is **one**.

append **session-B** as a new last element to the **vector** Z .

If all of the following are true,

session-A-active is **one**.
session-B is **this-session**.
session-B-active is **zero**.

append **session-A** as a new last element to the **vector** Z .

Return $((Z \neg Z) = \neg \rho Z) / Z$.

Otherwise,

If B is not a **session-identification**, signal **domain-error**.

If B is **this-session**, signal **domain-error**.

Set Z to the **empty character array** of **rank two**.
 Using each **item** of the **shared-variable-list**,
 If all of the following are true,
 session-A is **this-session**.
 session-A-active is **zero**.
 session-B is B .
 session-B-active is **one**.
 append **shared-name** as a new last row to the **identifier-array** Z .
 If all of the following are true,
 session-A is B .
 session-A-active is **one**.
 session-B is **this-session**.
 session-B-active is **zero**.
 append **shared-name** as a new last row to the **identifier-array** Z .
 Return Z .

14.6.3 Shared Variable Degree of Coupling

$Z \leftarrow \square_{SVO} B$

Informal Description: Z is a numeric vector in which each element is the degree of coupling of the symbol named by the corresponding row of B .

Evaluation Sequence:

If the **rank** of B is **greater than two**, signal **rank-error**.
 Set $B1$ to $(\neg 2 \uparrow 1 \ 1, \rho B) \rho B$.
 If any **item** of the **ravel-list** of $B1$ is not a **character**, signal **domain-error**.
 Set Z to $(1 \uparrow \rho B1) \rho 0$.
 For every I in $1 \uparrow \rho B1$,
 Let $B2$ stand for **row** I of $B1$.
 If $B2$ matches **identifier-row**,
 Set N to the **primary-name** in $B2$.
 Set **item** I of the **ravel-list** of Z to **degree-of-coupling** of the **symbol-named-by** N .
 Otherwise, signal **domain-error**.
 Return Z .

Note: **Degree-of-coupling** is a property of **symbols**, not **shared-variables**.