

ISO

INTERNATIONAL ORGANIZATION FOR STANDARDIZATION

ISO RECOMMENDATION R 17

GUIDE TO THE USE OF PREFERRED NUMBERS AND OF SERIES OF PREFERRED NUMBERS

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BRIEF HISTORY

The ISO Recommendation R 17 was drawn up by the Technical Committee ISO/TC 19, *Preferred Numbers*, the Secretariat of which is held by the Association Française de Normalisation (AFNOR).

At the time when the Committee drew up the ISO Recommendation R 3, *Preferred Numbers - Series of Preferred Numbers*, it appeared necessary to provide the ISO Member Bodies with a guide for the use they would be making of preferred numbers. The work was undertaken in 1949.

Following the Committee's first meeting, held in Paris in June 1949, the Secretariat of the Committee drew up two successive draft proposals, the second of which, distributed in April 1951, was adopted by the Committee, by correspondence, as a Draft ISO Recommendation.

The Draft ISO Recommendation was submitted on 31 October 1951 to all the ISO Member Bodies and approved without reserve by 16 Member Bodies.

Nevertheless, in consideration of comments submitted by certain Member Bodies, the Committee decided, at its second plenary meeting (New York, June 1952) and its third plenary meeting (The Hague, May 1953), to make some improvements in the text of the Draft and to add to it a new section No. 5, giving examples of application, and a section No. 6, relating to the rounding-off of preferred numbers.

The text of the second Draft ISO Recommendation (finally revised at the Committee's fourth meeting, held in Paris in July 1954) was submitted to all the ISO Member Bodies in November 1954 and approved by the following 26 (out of a total of 34) Member Bodies:

Austria	India	Spain
Australia	Ireland	Sweden
Canada	Italy	Switzerland
Chile	Japan	Union of South Africa
Denmark	Mexico	United Kingdom
Finland	Netherlands	U.S.A.
France	New Zealand	U.S.S.R.
Germany	Poland	Yugoslavia
Hungary	Portugal	

The Secretariat took into consideration certain purely editorial comments submitted by two Member Bodies, and the Draft, amended accordingly, was submitted on 31 August 1955 to the ISO Council, which decided by correspondence to accept it as an ISO RECOMMENDATION, subject to certain final editorial amendments.

FOREWORD

Preferred numbers were first utilized in France at the end of the nineteenth century. From 1877 to 1879, Captain Charles RENARD, an officer in the engineer corps, made a rational study of the elements necessary in the construction of lighter-than-air aircraft. He computed the specifications for cotton rope according to a grading system, such that this element could be produced in advance without prejudice to the installations where such rope was subsequently to be utilized. Recognizing the advantage to be derived from the geometrical progression, he adopted, as a basis, a rope having a mass of a grammes per metre, and as a grading system, a rule that would yield a tenth multiple of the value a after every fifth step of the series, i.e.:

$$a \times q^5 = 10a \quad \text{or} \quad q = \sqrt[5]{10}$$

whence the following numerical series:

$$a \quad a \sqrt[5]{10} \quad a \left(\sqrt[5]{10}\right)^2 \quad a \left(\sqrt[5]{10}\right)^3 \quad a \left(\sqrt[5]{10}\right)^4 \quad 10a$$

the values of which, to 5 significant figures, are:

$$a \quad 1.5849a \quad 2.5119a \quad 3.9811a \quad 6.3096a \quad 10a$$

RENARD's theory was to substitute for the above values, more rounded but more practical values, and he adopted as a power of 10, positive, nil or negative. He thus obtained the following series:

$$10 \quad 16 \quad 25 \quad 40 \quad 63 \quad 100$$

which may be continued in both directions.

From this series, designated by the symbol R 5, the R 10, R 20, R 40 series were formed, each adopted ratio being the square root of the preceding one:

$$\sqrt[10]{10} \quad \sqrt[20]{10} \quad \sqrt[40]{10}$$

The first standardization drafts were drawn up on these bases in Germany by the Normenausschuss der Deutschen Industrie on 13 April 1920, and in France by the Commission permanente de standardisation in document X of 19 December 1921. These two documents offering few differences, the commission of standardization in the Netherlands proposed their unification. An agreement was reached in 1931 and in June 1932, the International Federation of the National Standardizing Associations organized an international meeting in Milan, where the ISA Technical Committee 32, *Preferred Numbers*, was set up and its Secretariat assigned to France.

On 19 September 1934, the ISA Technical Committee 32 held a meeting in Stockholm; sixteen nations were represented: Austria, Belgium, Czechoslovakia, Denmark, Finland, France, Germany, Hungary, Italy, Netherlands, Norway, Poland, Spain, Sweden, Switzerland, U.S.S.R.

With the exception of the Spanish, Hungarian and Italian delegations which, although favourable, had not thought fit to give their final agreement, all the other delegations accepted the draft which was presented. Furthermore, Japan communicated by letter its approval of the draft as already discussed in Milan. As a consequence of this, the international recommendation was laid down in ISA Bulletin 11 (December 1935).

After the Second World War, the work was resumed by ISO. The Technical Committee ISO/TC 19, *Preferred Numbers*, was set up and France again held the Secretariat. This Committee at its first meeting, which took place in Paris in July 1949, recommended the adoption by ISO of the series of preferred numbers defined by the table of ISA Bulletin 11, i.e. R 5, R 10, R 20, R 40. This meeting was attended by representatives of the 19 following nations: Austria, Belgium, Czechoslovakia, Denmark, Finland, France, Hungary, India, Israel, Italy, Netherlands, Norway, Poland, Portugal, Sweden, Switzerland, United Kingdom, U.S.A., U.S.S.R.

During the subsequent meetings in New York in 1952 and in The Hague in 1953, which were attended also by Germany, the series R 80 was added and slight alterations were made. The draft thus amended has become ISO Recommendation R 3.

GUIDE TO THE USE OF PREFERRED NUMBERS AND OF SERIES OF PREFERRED NUMBERS

1. GEOMETRICAL PROGRESSIONS AND PREFERRED NUMBERS

- 1.1 Standard series of numbers.** In all the fields where a scale of numbers is necessary, standardization consists primarily of grading the characteristics according to one or several series of numbers covering all the requirements with a minimum of terms.

These series should present certain essential characteristics; they should:

- (a) be simple and easily remembered,
- (b) be unlimited, as well towards the lower as towards the higher numbers,
- (c) include all the decimal multiples and sub-multiples of any term,
- (d) provide a rational grading system.

- 1.2 Characteristics of geometrical progressions which include the number 1.** The characteristics of these progressions, with a ratio q , are mentioned below.

- 1.2.1** The product or quotient of any two terms q^b and q^c of such a progression is always a term of that progression:

$$q^b \times q^c = q^{b+c}$$

- 1.2.2** The integral positive or negative power c of any term q^b of such a progression is always a term of that progression:

$$(q^b)^c = q^{bc}$$

- 1.2.3** The fractional positive or negative power $1/c$ of a term q^b of such a progression is still a term of that progression, provided that b/c be an integer:

$$(q^b)^{1/c} = q^{b/c}$$

- 1.2.4** The sum or difference of two terms of such a progression is not generally equal to a term of that progression. However, there exists one geometrical progression such that one of its terms is equal to the sum of the two preceding terms. Its ratio

$$\frac{1 + \sqrt{5}}{2}$$

approximates 1.6 (it is the *Golden Section* of the Ancients).

1.3 Geometrical progressions which include the number 1 and the ratio of which is a root of 10. The progressions chosen to compute the preferred numbers have a ratio equal to $\sqrt[r]{10}$, r being equal to 5, to 10, to 20, or to 40. The results are given hereunder.

1.3.1 The number 10 and its positive and negative powers are terms of all the progressions.

1.3.2 Any term whatever of the range $10^d \dots 10^{d+1}$, d being positive or negative, may be obtained by multiplying by 10^d the corresponding term of the range 1 ... 10.

1.3.3 The terms of these progressions comply in particular with the property given in clause 1.1, letter (c).

1.4 Rounded off geometrical progressions. The preferred numbers are the rounded off values of the progressions defined under clause 1.3.

1.4.1 The maximum roundings off are :

$$+ 1.26\% \quad \text{and} \quad - 1.01\%$$

The preferred numbers included in the range 1 ... 10 are given in the table of paragraph 1 of ISO Recommendation R 3, *Preferred Numbers - Series of Preferred Numbers*.

1.4.2 Due to the rounding off, the products, quotients and powers of preferred numbers may be considered as preferred numbers only if the modes of calculation referred to under section 3 are used.

1.4.3 For the R 10 series, it should be noted that $\sqrt[10]{10}$ is equal to $\sqrt[3]{2}$ at an accuracy closer than 1 in 1000 in relative value, so that :

- the cube of a number of this series is approximately equal to double the cube of the preceding number. In other words, the N^{th} term is approximately double the $(N - 3)^{\text{th}}$ term. Due to the rounding off, it is found that it is usually equal to exactly the double;
- the square of a number of this series is approximately equal to 1.6 times the square of the preceding number.

1.4.4 Just as the terms of the R 10 series are doubled in general every 3 terms, the terms of the R 20 series are doubled every 6 terms, and those of the R 40 series are doubled every 12 terms.

1.4.5 Beginning with the R 10 series, the number 3.15, which is nearly equal to π , can be found among the preferred numbers. It follows that the length of a circumference and the area of a circle, the diameter of which is a preferred number, may also be expressed by preferred numbers. This applies in particular to peripheral speeds, cutting speeds, cylindrical areas and volumes, spherical areas and volumes.

1.4.6 The R 40 series of preferred numbers includes the numbers 3 000, 1 500, 750, 375, which have special importance in electricity (number of revolutions per minute of asynchronous motors when running without load on alternating current at 50 hertz).

1.4.7 It follows from the features outlined above that the preferred numbers correspond faithfully to the characteristics set forth at the beginning of clause 1.1. Furthermore, they constitute a unique grading rule, acquiring thus a remarkably universal character.

2. DIRECTIVES FOR THE USE OF PREFERRED NUMBERS

2.1 Characteristics expressed by numerical values. In the preparation of a project involving numerical values of characteristics, whatever their nature, for which no particular standard exists, preferred numbers should be selected for these values. No deviation should be made, except for imperative reasons (see section 5).

At all times efforts should be made to adapt existing standards to preferred numbers.

2.2 Scale of numerical values. In selecting a scale of numerical values, that series should be chosen having the highest ratio consistent with the desiderata to be satisfied, in the order: R 5, R 10, etc. Such a scale should be carefully worked out. The considerations to be taken into account are, among others: the use that is to be made of the articles standardized, their cost price, their dependence upon other articles used in close connection with them, etc.

The best scale will be determined by taking into consideration, in particular, the two following contradictory tendencies: a scale with too wide steps involves a waste of materials and an increase in the cost of manufacture, whereas a too closely spaced scale leads to an increase in the cost of tooling and also in the value of stock inventories.

When the needs are not of the same relative importance in all the ranges under consideration, the most suitable basic series for each range should be selected so that the sequences of numerical values adopted provide a succession of series of different ratios permitting new interpolations where necessary.

2.3 Derived series. Derived series, which are obtained by taking the terms at every second, every third, every fourth, etc. step of the basic series, should be used only when none of the scales of the basic series is satisfactory.

2.4 Shifted series. A shifted series, that is, a series having the same grading as a basic series, but beginning with a term not belonging to that series, should be used only for characteristics which are functions of other characteristics, themselves scaled in a basic series.

Example: The R 80/8 (25.8 ... 165) series has the same grading as the R 10 series, but starts with a term of the R 80 series, whereas the R 10 series, from which it is shifted, would start at 25.

- 2.5 Single numerical value.** In the selection of a single numerical value, irrespective of any idea of scaling, one of the terms of the R 5, R 10, R 20, R 40 basic series should be chosen or else a term of the exceptional R 80 series, giving preference to the terms of the series of highest step ratio, choosing R 5 rather than R 10, R 10 rather than R 20, etc.

When it is not possible to provide preferred numbers for all characteristics that could be numerically expressed, preferred numbers should first be applied to the most important characteristic or characteristics, the secondary or subordinate characteristics should then be determined in the light of the principles set forth under this section.

- 2.6 Grading by means of preferred numbers.** The preferred numbers may differ from the calculated values by + 1.26 % to - 1.01 %. It follows that sizes, graded according to preferred numbers, are not exactly proportional to one another.

To obtain an exact proportionality, either the theoretical values, or the serial numbers defined under section 3, or the decimal logarithms of the theoretical values should be used.

It should be noted that when formulae are used all the terms of which are expressed in preferred numbers, the discrepancy of the result, if it is itself expressed as a preferred number, remains within the range + 1.26 % to - 1.01 %.

$$\text{Thus } \left(A + \frac{1.26\%}{-1.01\%} \right) \times \left(B + \frac{1.26\%}{-1.01\%} \right) \times \dots = C + \frac{1.26\%}{-1.01\%}$$

3. RECOMMENDATION FOR CALCULATION WITH PREFERRED NUMBERS

- 3.1 Serial numbers.** It may be noted that, for computing with preferred numbers, the terms of the arithmetical progression of the serial numbers (column 5 in table of paragraph 1 of ISO Recommendation R 3, *Preferred Numbers - Series of Preferred Numbers*) are exactly the logarithms to base $\sqrt[40]{10}$ of the terms of the geometrical progression corresponding to the preferred numbers of the R 40 series (column 4 of the same table).

The series of the serial numbers can be continued in both directions, so that if N_n is the serial number of the preferred number n , it follows:

$N_{1.00} = 0$	$N_{0.95} = -1$
$N_{1.06} = 1$	$N_{0.10} = -40$
$N_{10} = 40$	$N_{0.01} = -80$
$N_{100} = 80$	

- 3.2 Products and quotients.** The preferred number n'' which is the product or quotient of two preferred numbers n and n' is calculated by adding or subtracting the serial numbers N_n and $N_{n'}$ and finding the preferred number n'' corresponding to the new serial number thus obtained.

Example 1: $3.15 \times 1.6 = 5$
 $N_{3.15} + N_{1.6} = 20 + 8 = 28 = N_5$

Example 2: $6.3 \times 0.2 = 1.25$
 $N_{6.3} + N_{0.2} = 32 + (-28) = 4 = N_{1.25}$

Example 3: $1 : 0.06 = 17$
 $N_1 - N_{0.06} = 0 - (-49) = 49 = N_{17}$

- 3.3 Powers and roots.** The preferred number which is the integral positive or negative power of a preferred number is computed by multiplying the serial number of the preferred number by the exponent and by finding the preferred number corresponding to the serial number obtained.

The preferred number corresponding to the root or fractional positive or negative power of a preferred number is computed in the same way, provided that the product of the serial number and the fractional exponent be an integer.

Example 1: $(3.15)^2 = 10$
 $2 N_{3.15} = 2 \times 20 = 40 = N_{10}$

Example 2: $\sqrt[5]{3.15} = 3.15^{1/5} = 1.25$
 $\frac{1}{5} N_{3.15} = \frac{20}{5} = 4 \text{ (integer)} = N_{1.25}$

Example 3: $\sqrt{0.16} = 0.16^{1/2} = 0.4$
 $\frac{1}{2} N_{0.16} = \frac{-32}{2} = -16 \text{ (integer)} = N_{0.4}$

Example 4: On the other hand, $\sqrt[4]{3} = 3^{1/4}$ is not a preferred number because the product of the exponent $\frac{1}{4}$ and the serial number of 3 is not an integer.

Example 5: $0.25^{-1/3} = 1.6$
 $-\frac{1}{3} N_{0.25} = -\frac{1}{3} (-24) = +8 = N_{1.6}$

Note. — The mode of calculation with the serial numbers may introduce slight errors which are caused by the deviation between the theoretical preferred numbers and the corresponding rounded off numbers of the basic series.

- 3.4 Decimal logarithms.** The mantissae of the decimal logarithms of the theoretical values are given in column 6 of the table of paragraph 1 of ISO Recommendation R 3, *Preferred Numbers - Series of Preferred Numbers*.

Example 1: $\log_{10} 4.5 = 0.650$

Example 2: $\log_{10} 0.063 = 0.800 - 2 = \bar{2}.800$