
**Guidelines for implementation of
statistical process control (SPC) —**

**Part 3:
Reference data sets for SPC software
validation**

*Lignes directrices pour la mise en œuvre de la maîtrise statistique des
processus (MSP) —*

*Partie 3: Jeux de données de référence pour la validation de logiciels
pour MSP*



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Contents

	Page
Foreword.....	iv
Introduction.....	v
1 Scope.....	1
2 Normative references.....	1
3 Terms and definitions, and symbols and abbreviated terms.....	1
3.1 Terms and definitions.....	1
3.2 Symbols and abbreviated terms.....	2
4 Overview of the test examples.....	3
5 Reference data sets description and evaluation.....	4
5.1 Test data set 1.....	4
5.1.1 Test data set 1 information.....	4
5.1.2 Test data set 1 results.....	6
5.2 Test data set 2.....	13
5.2.1 Test data set 2 information.....	13
5.2.2 Test data set 2 results.....	14
5.3 Test data set 3.....	21
5.3.1 Test data set 3 information.....	21
5.3.2 Test data set 3 results.....	23
5.4 Test data set 4.....	32
5.4.1 Test data set 4 information.....	32
5.4.2 Test data set 4 results.....	34
5.5 Test data set 5.....	43
5.5.1 Test data set 5 information.....	43
5.5.2 Test data set 5 results.....	45
5.6 Test data set 6.....	53
5.6.1 Test data set 6 information.....	53
5.6.2 Test data set 6 results.....	55
5.7 Test data set 7.....	63
5.7.1 Test data set 7 information.....	63
5.7.2 Test data set 7 results.....	65
5.8 Test data set 8.....	73
5.8.1 Test data set 8 information.....	73
5.8.2 Test data set 8 results.....	75
5.9 Test data set 9.....	82
5.9.1 Test data set 9 information.....	82
5.9.2 Test data set 9 results.....	85
5.10 Test data set 10.....	92
5.10.1 Test data set 10 information.....	92
5.10.2 Test data set 10 results.....	95
5.11 Test data set 11.....	102
5.11.1 Test data set 11 information.....	102
5.11.2 Test data set 11 results.....	103
Annex A (informative) Test data values.....	105
Bibliography.....	129

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

A list of all parts in the ISO 11462 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The test examples given in this document were developed for the assessment of statistical process control (SPC) systems. They allow SPC software developers to evaluate their systems. Thus, the end user of those systems can be sure that the data sets are evaluated correctly with a high level of reliability. In order to cover the widest possible spectrum, suitable data sets were prepared individually for various constellations. The evaluation results of those data sets are documented and commented on the following pages.

The results were verified multiple times using different computer programs. This turns the data sets and the results into references for validation of the software. The data sets are listed in Annex A. An electronic version is available at <https://standards.iso.org/iso/tr/11462/-3/ed-1/en/>.

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Guidelines for implementation of statistical process control (SPC) —

Part 3: Reference data sets for SPC software validation

1 Scope

This document describes examples for software validation for SPC software implementing the standards of the ISO 7870 series on control charts and the ISO 22514 series on capability and performance. In detail ISO 7870-2, ISO 22514-2 and ISO 22514-8 are covered.

It provides data sets and test results for testing the implementation of the evaluation methods described in these standards. This includes the detection of out of control situations as well as the calculation of sample statistics and process capability indices.

The test examples cover the following situations:

- a) General:
 - different sample and subgroup sizes, accuracy of calculation for large/small numbers;
- b) ISO 22514 series:
 - calculation of sample statistics for location and dispersion;
 - different distribution models;
- c) ISO 7870-2:
 - calculation of control limits;
 - visualization of data (histogram, control charts);
 - detection of out of control situations.

2 Normative references

There are no normative references in this document.

3 Terms and definitions, and symbols and abbreviated terms

3.1 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

3.2 Symbols and abbreviated terms

Symbols used in this document are identical to the symbols used in ISO 22514-2 and ISO 7870-2:

C_p	process capability index
C_{pk}	minimum process capability index
C_{pkU}	upper process capability index
C_{pkL}	lower process capability index
U_{CL}	upper control limit
L_{CL}	lower control limit
m	number of subgroups
n	sample size of each subgroup
P_m	machine performance index
P_{mk}	minimum machine performance index
P_{mkU}	upper machine performance index
P_{mkL}	lower machine performance index
P_p	process performance index
P_{pk}	minimum process performance index
P_{pkU}	upper process performance index
P_{pkL}	lower process performance index
T	Centreline (target value) for the respective characteristic in the control charts
U	upper specification limit
L	lower specification limit
U_{tr}	upper specification limit, transformed values
L_{tr}	lower specification limit, transformed values

Abbreviated terms:

SPC	statistical process control
-----	-----------------------------

4 Overview of the test examples

See [Table 1](#) for an overview of the test examples.

Table 1 — Overview of test examples

Test data set number	Subclause	Distribution model	Resulting distribution	Decimal points	Total sample size	Subgroup sample size	Description of data set
1	5.1	A1	normal	4	125	5	Data follows a normal distribution with no outliers
2	5.2	A2	Weibull	3	600	3	Correct calculations for a sample following a Weibull distribution
3	5.3	B	non-normal	2	1 000	5	Normal distribution with time dependent shift of mean
4	5.4	C1	normal	2	1 000	5	Location: random normally distributed; dispersion: constant
5	5.5	C2	non-normal	2	1 000	5	Location: random non-normally distributed; dispersion: constant; resulting distribution: non-normal
6	5.6	C3	non-normal	2	600	6	Capability indices for trend production (tool wear)
7	5.7	C4	non-normal	2	500	5	Capability indices for fix tooling (tool change)
8	5.8	D	non-normal	3	500	5	Systematic and random changes in location and dispersion - non-normal
9	5.9	A2	Rayleigh	3	500	5	Correct calculations for a sample following a Rayleigh distribution
10	5.10	A2	Weibull	0	200	5	Correct calculations for a sample following a Weibull distribution
11	5.11	C2	normal	1	30	(10)	ISO 22514-8 capability of multi-state production processes

NOTE

- The decision which distribution model fits the data and its use is up to the statistical expert. Within this document distribution models for each example are therefore assumed. The procedure to select a statistical distribution is not part of this document.
- According to ISO 22514-2 the calculation method $d = 5$ for the dispersion is not suited for non-normal distributions. It is calculated and given in this document even for non-normal distributions for the purpose of validating the implementation of the calculation method of $d = 5$.
- The resulting distributions for test 2, 9 and 10 are pre-selected models and with no means the "best natural" resulting distribution models for the same reason no goodness of fit statistics are given, knowing they will lead to reject H_0 for the pre-selected distribution models.
- Two more decimal places for sample statistics and control limits than digits of the input values. Resulting values are rounded (ISO/IEC/IEEE 60559, rounded-to-the-nearest).
- Capability index are always given with 2 digits (rounded).

- Control limits were calculated using all values in the test data set.
- All control limits were calculated using tabulated correction factors in ISO 7870-2. Using the exact quantiles of the corresponding distribution functions instead can lead to a deviation in the results <0,1 %.
- The centrelines for the control charts were chosen according to the respective statistics given by ISO 7870-2:2013, 6.1 to 6.3
- The histogram plot is not part of the validation, since the procedure is not specified in ISO 7870 series nor in ISO 22514 series. Histograms are for visualization purposes only.
- No units are given, no unit conversion has to be done.
- Time-dependent distribution models A-D are specified in ISO 22514-2.
- Internally a precision of 15 digits is used, when already calculated values are used again in further calculations.
- If the data are not normal distributed an individual chart and a MR chart cannot be used.

5 Reference data sets description and evaluation

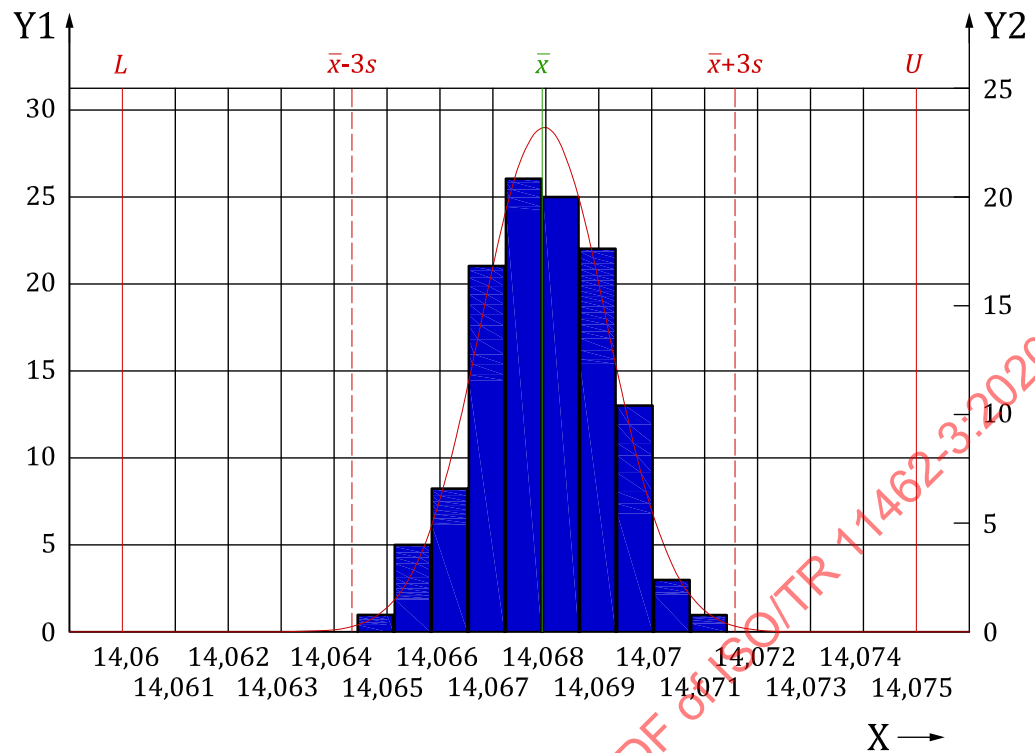
5.1 Test data set 1

5.1.1 Test data set 1 information

This set of test data is taken from a process following a normal distribution and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 1 is given in [Table 2](#). [Figure 1](#) shows a histogram and [Figure 2](#) a probability plot of the test data set 1 with the purpose of data visualization.

Table 2 — Description of test data set 1

Description of data input			
Distribution model	A1	Resulting distribution	Normal
Data set	Annex A, Table A.1	decimal points	4
Total sample size	125	<i>U</i>	14,075
Size of subgroups	5	<i>L</i>	14,060

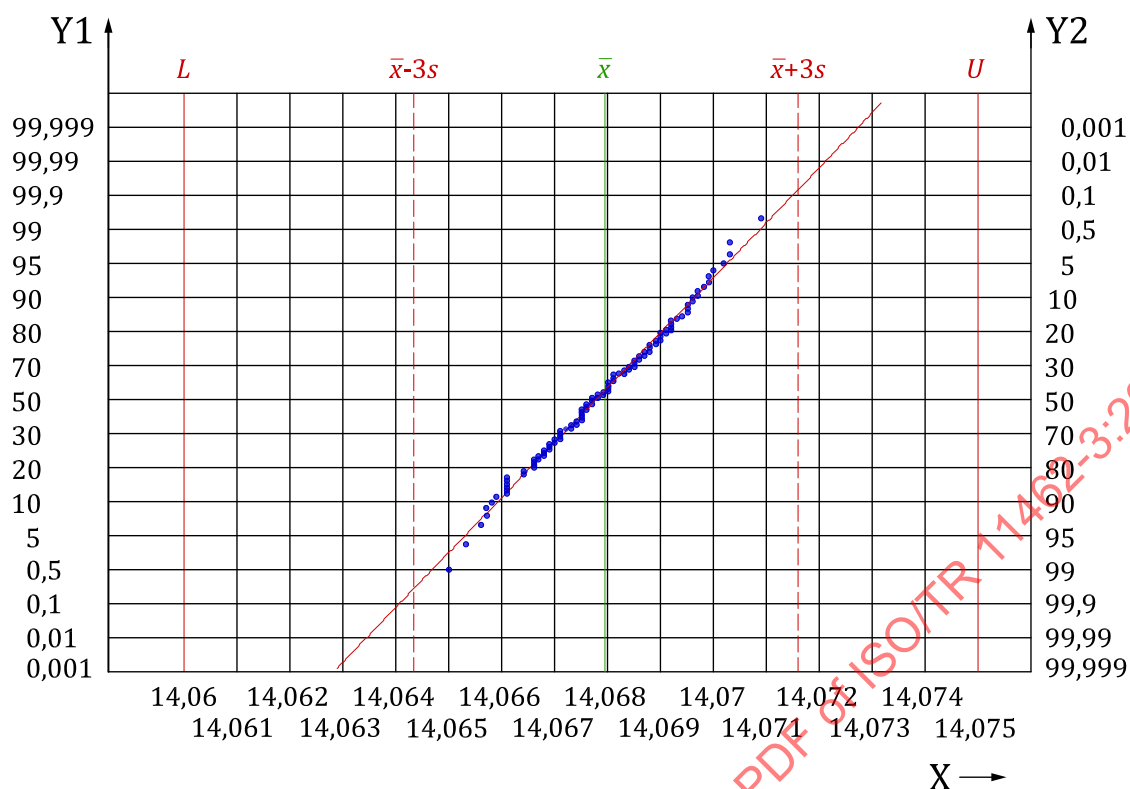
**Key**

X value

Y1 absolute frequency

Y2 relative frequency in %

Figure 1 — Histogram of test data set 1

**Key**

X value

Y1 probability in %

Y2 1-probability in %

Figure 2 — Probability plot of test data set 1

NOTE The width of the class intervals is 0,000 7.

The class interval with the highest frequency of values is from 14,067 95 to 14,068 02.

The density plot is based on the assumption of normality and estimated parameters $l = 1$, $d = 5$.

5.1.2 Test data set 1 results**5.1.2.1 List of sample statistics**

Table 3 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 3 — List of sample statistics for test data set 1

Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	14,067 958	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	14,068 000	ISO 22514-2:2017, Formula (12)
$\bar{\bar{x}}$ ($l = 3$)	14,067 958	ISO 22514-2:2017, Formula (13)
$\bar{\bar{\bar{x}}}$ ($l = 4$)	14,067 828	ISO 22514-2:2017, Formula (14)

Table 3 (continued)

Statistic	Value	Reference
Dispersion		
$\hat{\Delta} (d = 1)$	0,007 240 (normal distribution)	ISO 22514-2:2017: Formula (15)
$\hat{\sigma} (d = 2)$	0,001 187	ISO 22514-2:2017, Formula (16)
$\hat{\sigma} (d = 3)$	0,001 200	ISO 22514-2:2017, Formula (17)
$\hat{\sigma} (d = 4)$	0,001 180	ISO 22514-2:2017, Formula (18)
$\hat{\sigma} (d = 5)$	0,001 207	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,001 128	ISO 7870-2:2013, 3.2
R	0,005 900	ISO 7870-2:2013, 3.2
$\bar{R}$	0,002 744	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,001 379	ISO 7870-2:2013, 3.2

5.1.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 3.

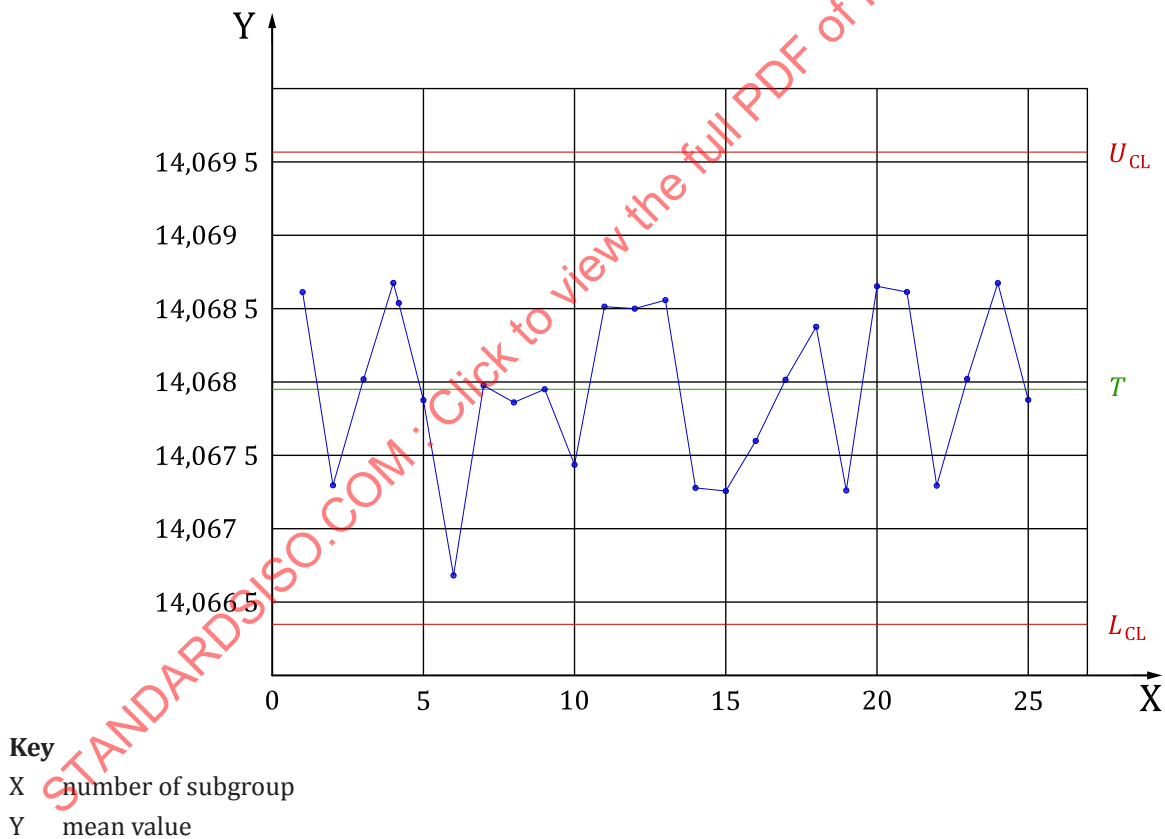


Figure 3 — Mean control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = 14,066\ 349$$

$$U_{CL} = 14,069\ 568$$

5.1.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 4](#).

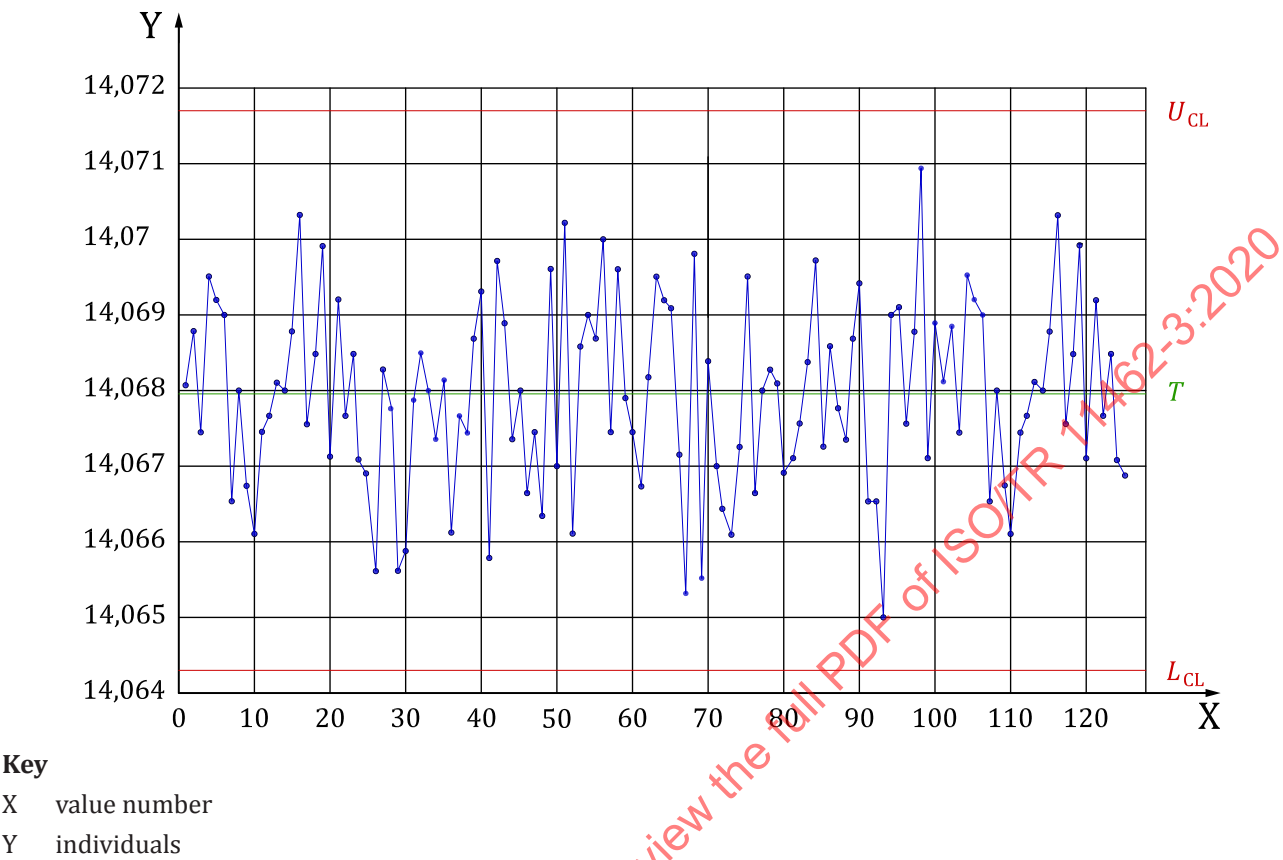


Figure 4 — Individuals control chart for test data set 1

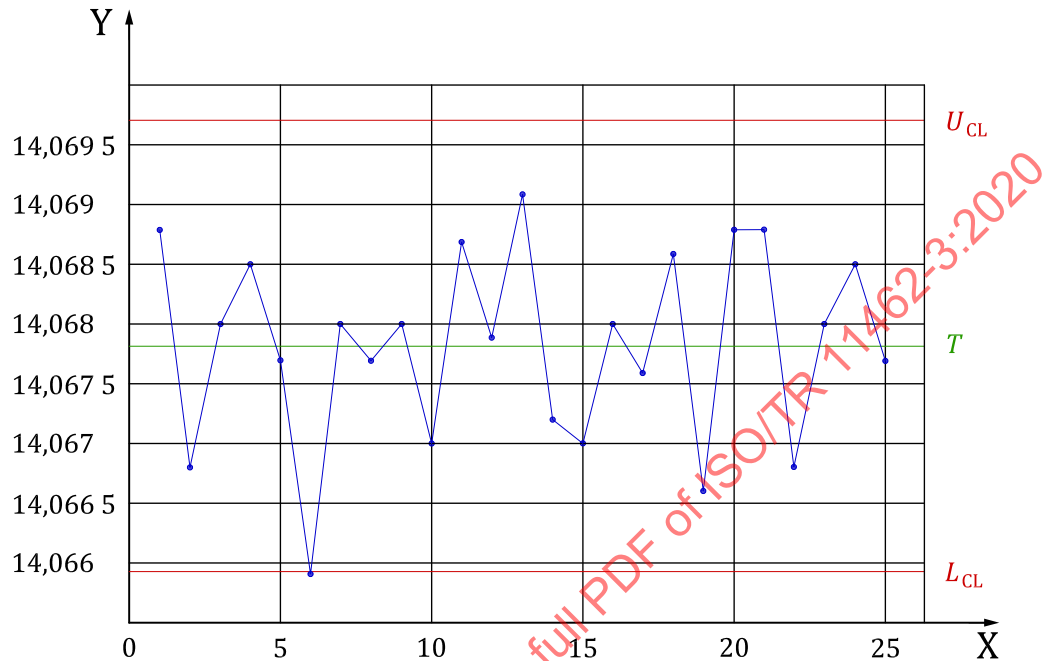
The control limits are (see ISO 7870-2:2013, Table 3):

$L_{CL} = 14,064\ 29$

$U_{CL} = 14,071\ 63$

5.1.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 5](#).



Key

X subgroup number
Y median value

Figure 5 — Median control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = 14,065\ 93$$

$$U_{CL} = 14,069\ 72$$

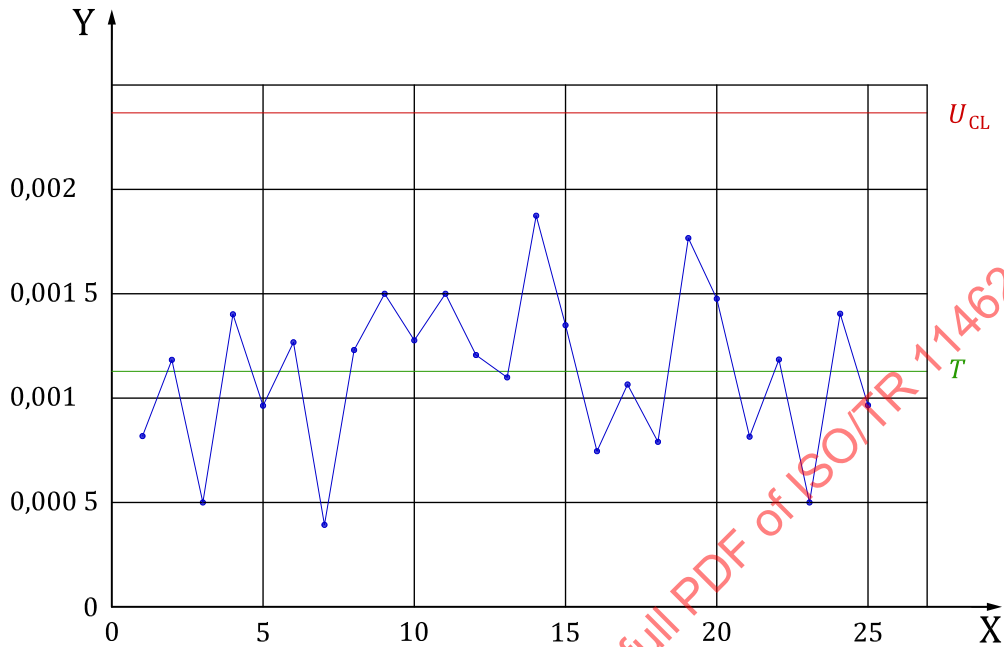
Out of control situations are given in [Table 4](#).

Table 4 — Results of pattern tests for the median control chart and test data set 1

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
6	Violation of L_{CL}		

5.1.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 6.



Key
X subgroup number
Y empirical standard deviation s value

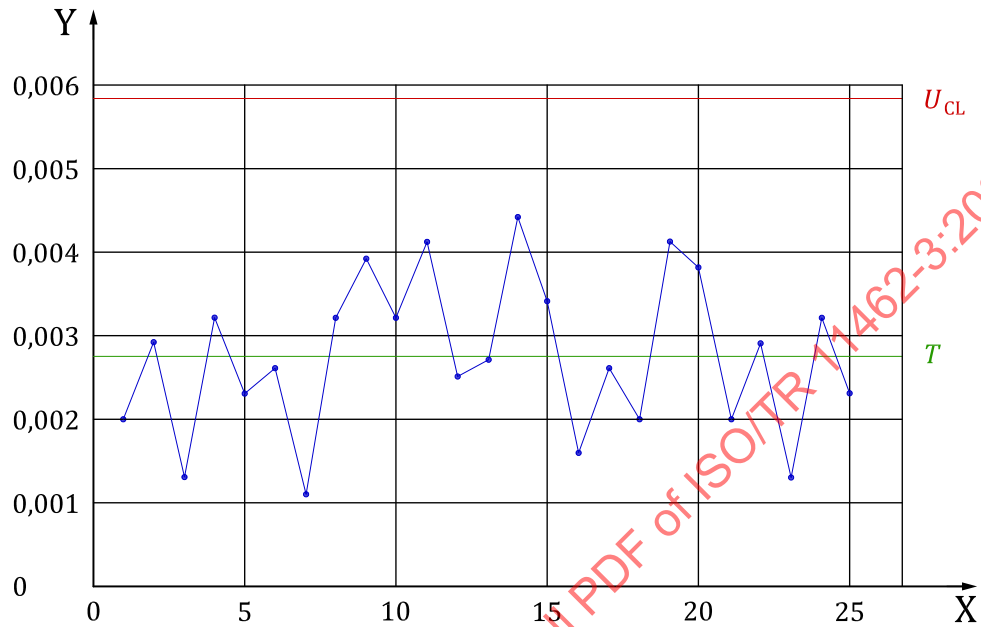
Figure 6 — s control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$
$$U_{CL} = 0,002\ 356$$

5.1.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 7](#).



Key

X subgroup number

Y range R value

Figure 7 — R control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,005\,801$$

5.1.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 8](#).

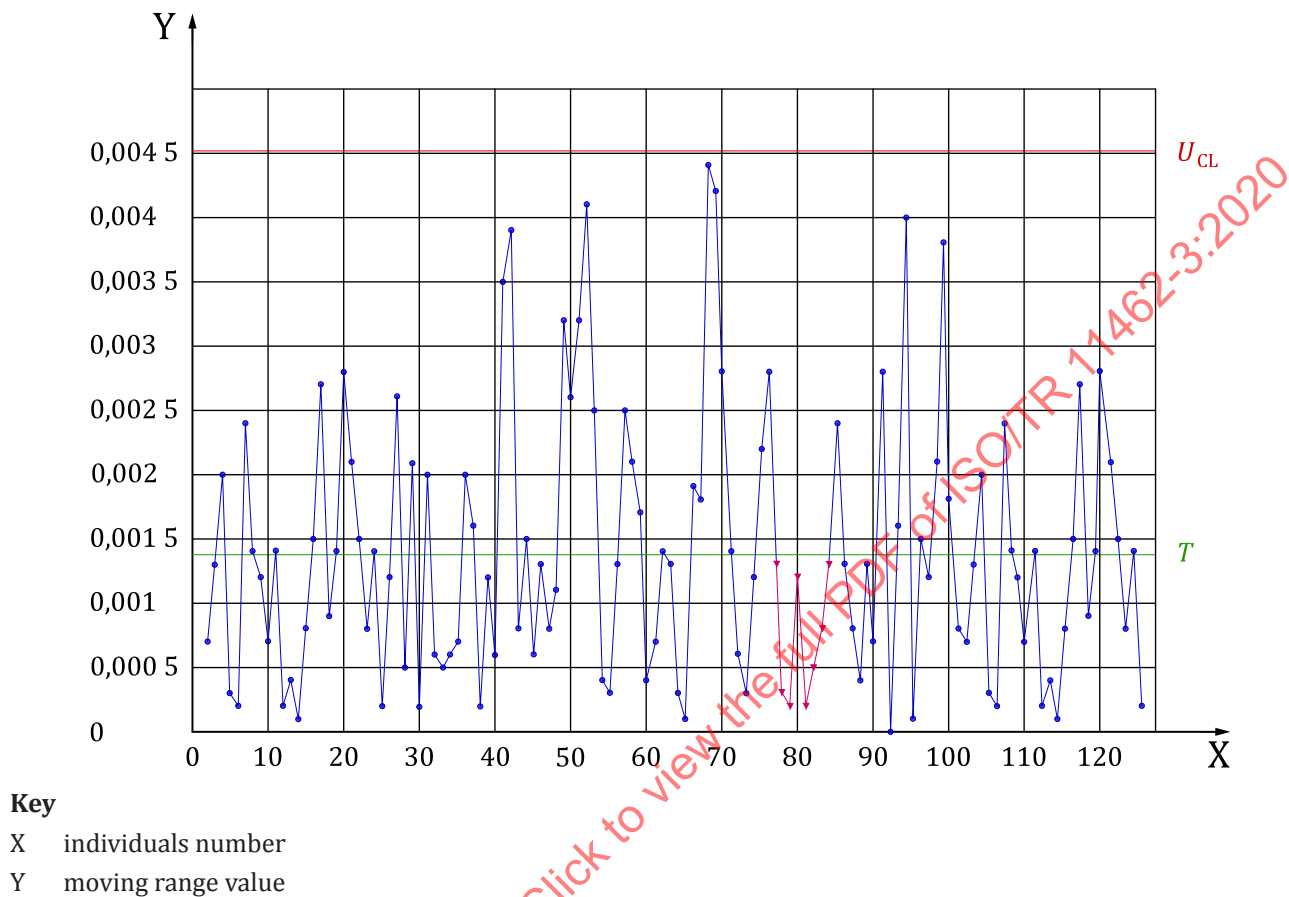


Figure 8 — Moving range control chart for test data set 1

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 0,004\ 51$$

Out of control situations are given in [Table 5](#).

Table 5 — Results of pattern tests for the moving range control chart and test data set 1

Out of control situations			
Value	Result/violation	Value	Result/violation
77 to 84	run below centreline		

5.1.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	2,07	C_{pk}	1,95
C_{pkL}	2,20	C_{pkU}	1,95

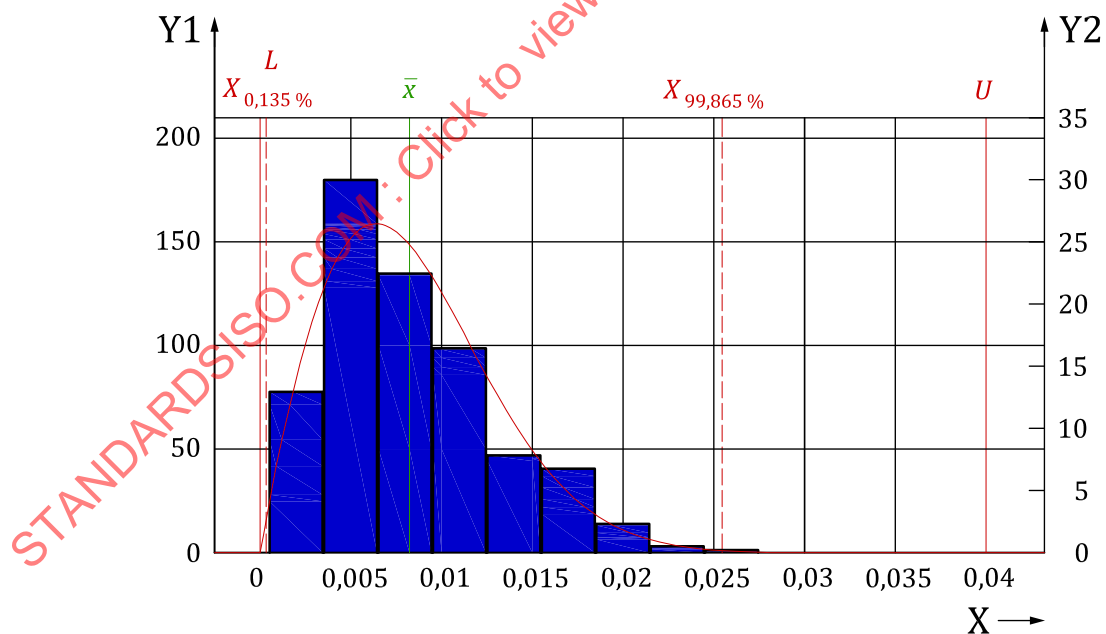
5.2 Test data set 2

5.2.1 Test data set 2 information

This set of test data is taken from a process following a non-normal distribution and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 2 is given in [Table 6](#). [Figure 9](#) shows a histogram and [Figure 10](#) a probability plot of the test data set 2 with the purpose of data visualization.

Table 6 — Description of test data set 2

Description of data input			
Distribution model	A2	Resulting distribution	Weibull
Data set	Annex A, Table A.1	decimal points	3
Total sample size	600	U	0,04
Size of subgroups	3	L	0



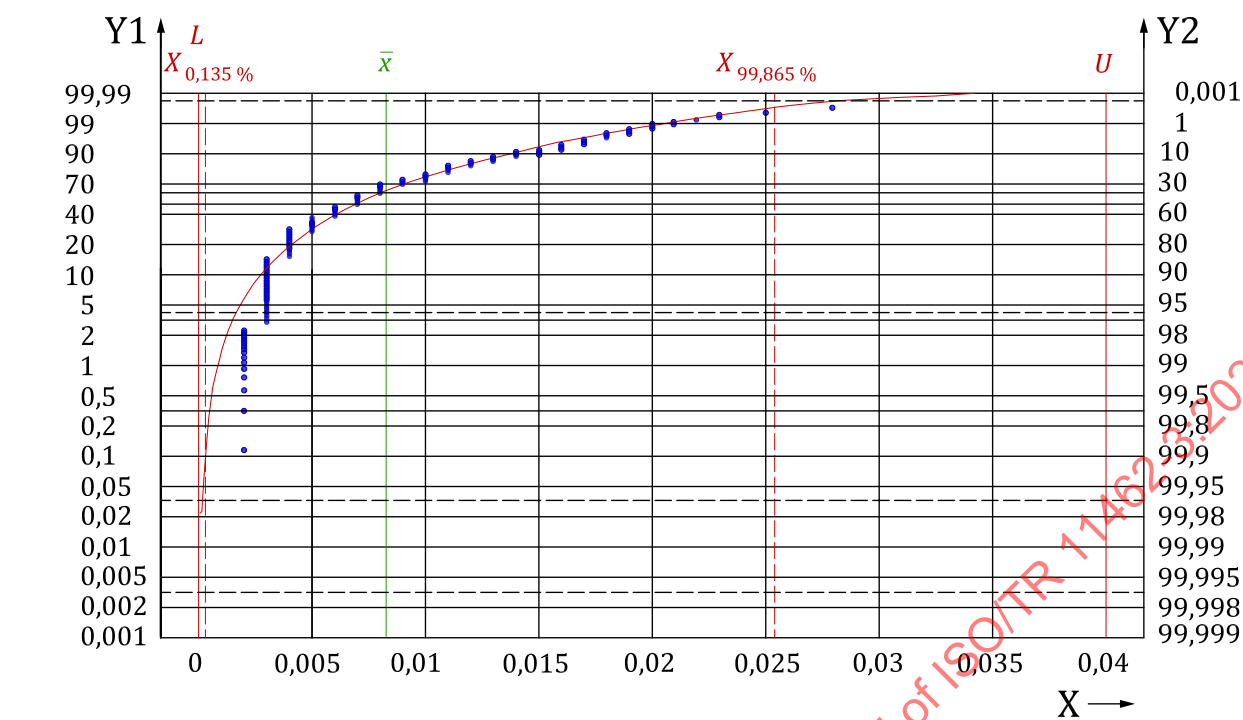
Key

X value

Y1 absolute frequency

Y2 relative frequency in %

Figure 9 — Histogram of test data set 2



Key
X value
Y1 probability in %
Y2 1-probability in %

Figure 10 — Probability plot of test data set 2

NOTE The width of the class intervals is 0,003.
The class interval with the highest frequency of values is from 0,003 5 to 0,006 5.

The density plot is based on the assumption of a two parametric Weibull distribution with estimated parameters scale $a = 0,009\ 39$ and shape $b = 1,897\ 82$. Both estimators have been calculated using the method of Maximum Likelihood.

The 0,135 %- and the 99,865 %-percentiles are calculated using the inverse distribution function of the Weibull distribution

$$F^{-1}(p|a,b) = a \times [-\ln(1-p)]^{\left(\frac{1}{b}\right)}$$

$X_{0,135\ \%} = 0,000\ 29$
 $X_{99,865\ \%} = 0,025\ 39$

5.2.2 Test data set 2 results

5.2.2.1 List of sample statistics

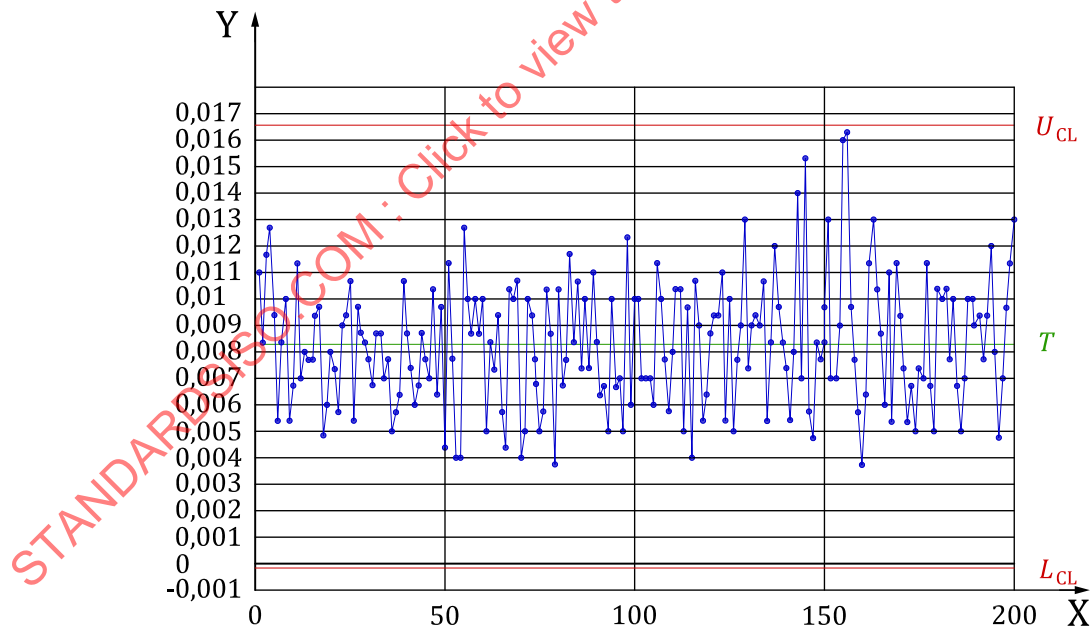
Table 7 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 7 — List of sample statistics for test data set 2

Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	0,008 29	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	0,007 00	ISO 22514-2:2017, Formula (12)
$\bar{\bar{x}}$ ($l = 3$)	0,008 29	ISO 22514-2:2017, Formula (13)
$\tilde{\tilde{x}}$ ($l = 4$)	0,007 51	ISO 22514-2:2017, Formula (14)
Dispersion		
$\hat{\Delta}$ ($d = 1$)	0,025 11 (Weibull using parameters in 5.2.1)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	0,004 88	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	0,004 87	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	0,004 81	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	0,004 67	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,004 32	ISO 7870-2:2013, 3.2
R	0,026 00	ISO 7870-2:2013, 3.2
$\bar{R}$	0,008 13	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,005 14	ISO 7870-2:2013, 3.2

5.2.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 11.



Key

- X number of subgroup
- Y mean value

Figure 11 — Mean control chart for test data set 2

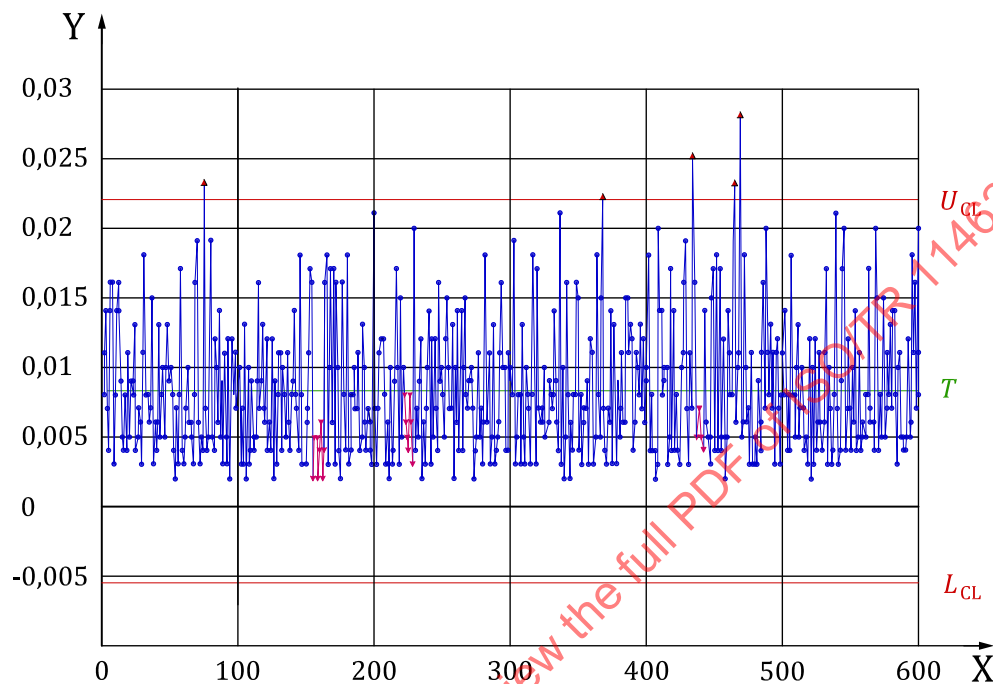
The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$L_{CL} = -0,000\ 15$ (The calculated lower control limit is negative and the dataset is one-sided against 0 limited characteristic. Therefore, the lower control limit is not shown, $L_{CL} = ---$)

$$U_{CL} = 0,016\ 72$$

5.2.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 12](#).



Key

X value number
Y individuals

Figure 12 — Individuals control chart for test data set 2

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = -0,005\ 39$$

$$U_{CL} = 0,021\ 96$$

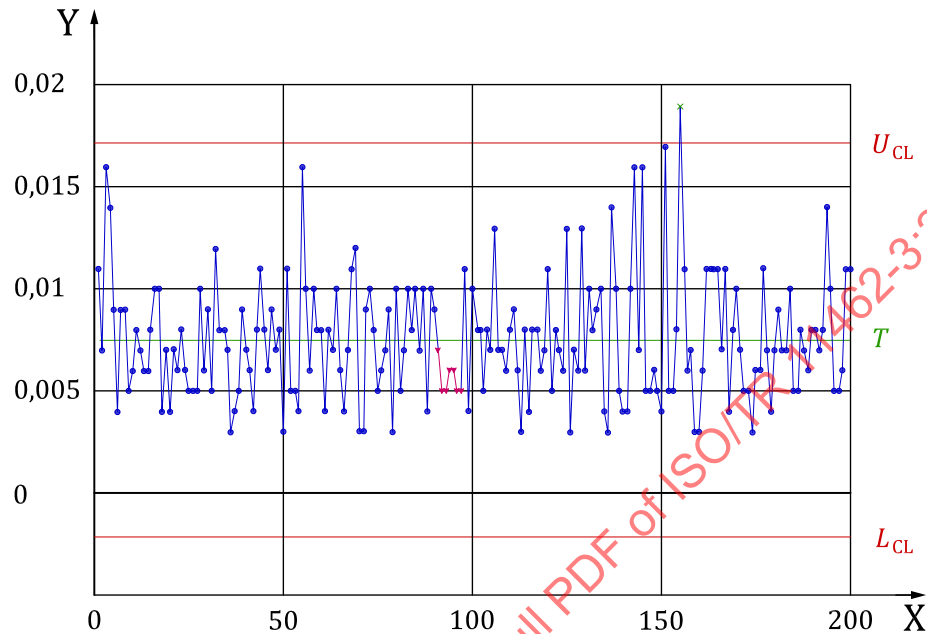
Out of control situations are given in [Table 8](#).

Table 8 — Results of pattern tests for the individuals control chart and test data set 2

Out of control situations			
Value	Result/violation	Value	Result/violation
75	violation of U_{CL}	433	violation of U_{CL}
155 to 163	run below centreline	435 to 441	run below centreline
222 to 228	run below centreline	464	violation of U_{CL}
367	violation of U_{CL}	468	violation of U_{CL}

5.2.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 13](#).



Key

X subgroup number
Y median value

Figure 13 — Median control chart for test data set 2

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = -0,002\ 15$$

$$U_{CL} = 0,017\ 17$$

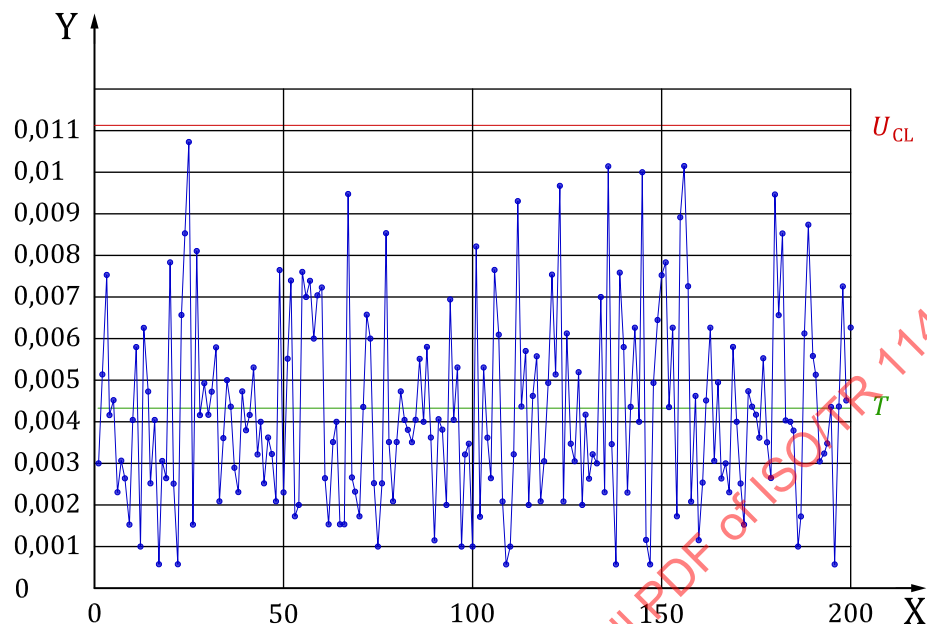
Out of control situations are given in [Table 9](#).

Table 9 — Results of pattern tests for the median control chart and test data set 2

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
91 to 97	run below centreline	155	violation of U_{CL}

5.2.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 14](#).

**Key**

X subgroup number

Y empirical standard deviation s value

Figure 14 — s control chart for test data set 2

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,011\ 083$$

5.2.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 15](#).

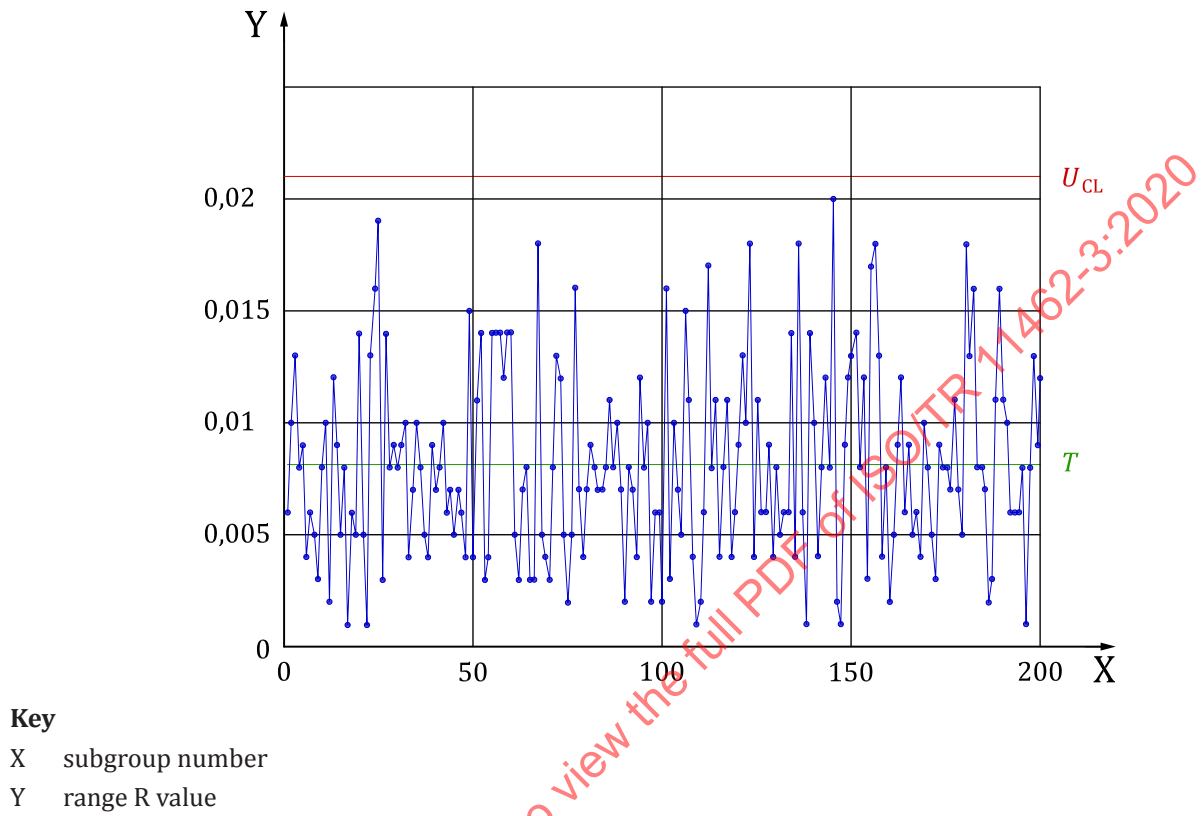


Figure 15 — R control chart for test data set 2

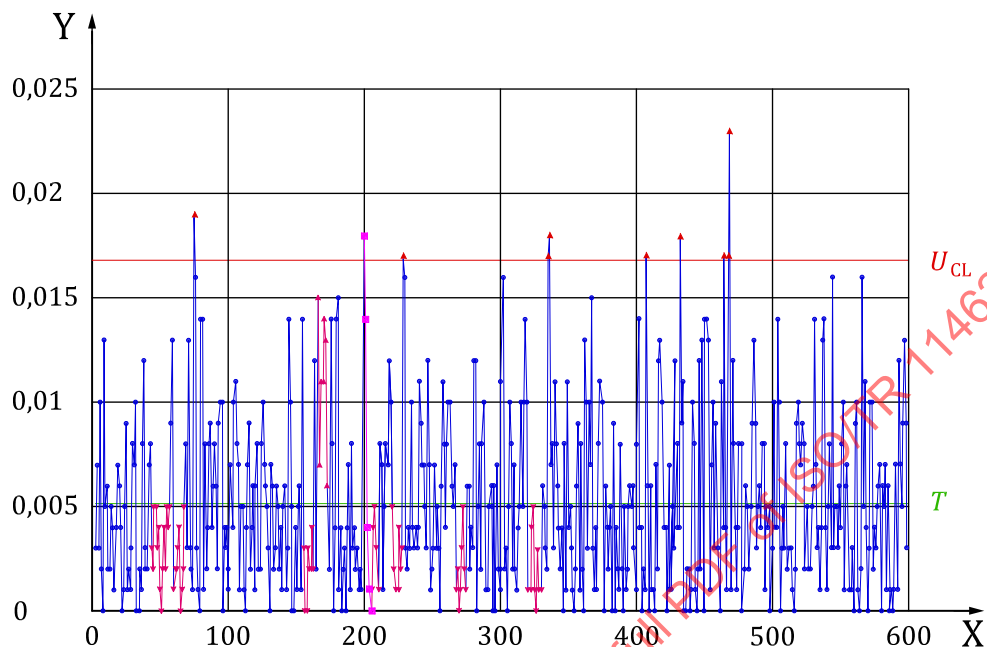
The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,020\ 94$$

5.2.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 16](#).



Key

X individuals number
Y moving range value

Figure 16 — Moving range control chart for test data set 2

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 0,016\ 80$$

Out of control situations are given in [Table 10](#).

Table 10 — Results of pattern tests for the moving range control chart and test data set 2

Out of control situations			
Value	Result/violation	Value	Result/violation
44 to 57	run below centreline	268 to 275	run below centreline
60 to 68	run below centreline	321 to 331	run below centreline
75	violation of U_{CL}	336	violation of U_{CL}
156 to 163	run below centreline	337	violation of U_{CL}
166 to 174	run below centreline	408	violation of U_{CL}
199 to 206	trend decreasing	433	violation of U_{CL}
199	violation of U_{CL}	465	violation of U_{CL}
201 to 211	run below centreline	468	violation of U_{CL}
221 to 228	run below centreline	469	violation of U_{CL}
229	violation of U_{CL}		

5.2.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	1,43	C_{pk}	0,59
C_{pkL}	0,59	C_{pkU}	2,26

Because of non-normal distributed data (Weibull) $M_{3,5}$ cannot be used – instead $M_{3,1}$ is used.

Capability indices (calculation method $M_{3,1}$)			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	(1,59) theoretical limit	C_{pk}	1,83
C_{pkL}	---	C_{pkU}	1,83

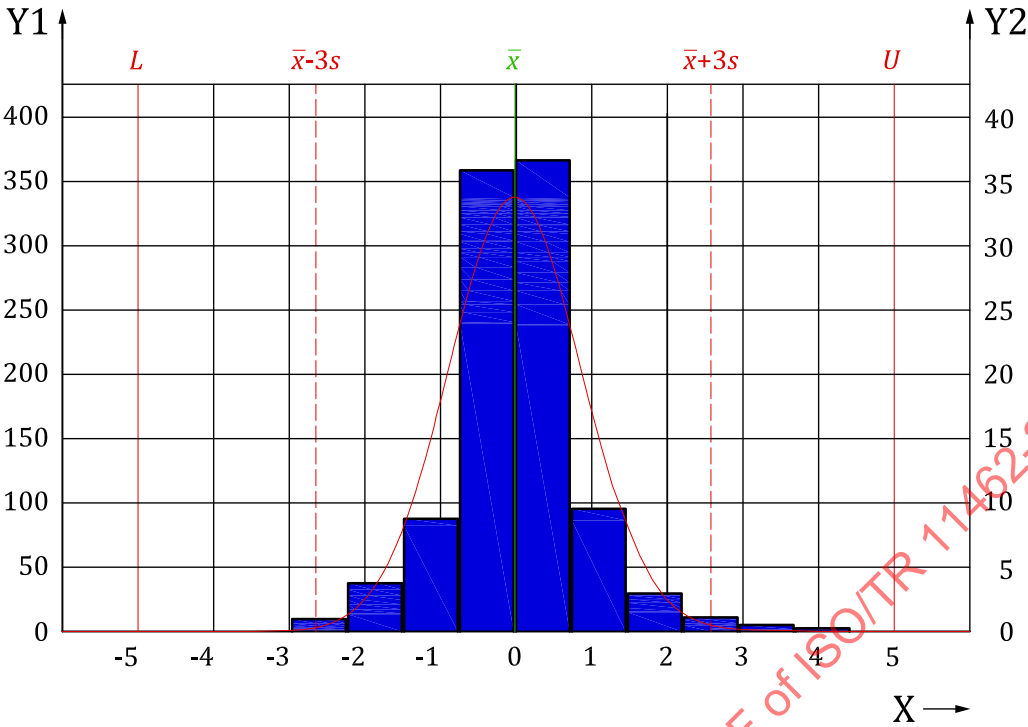
5.3 Test data set 3

5.3.1 Test data set 3 information

This set of test data is taken from a process following a non- normal distribution with a varying dispersion parameter σ and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 3 is given in [Table 11](#). [Figure 17](#) shows a histogram and [Figure 18](#) a probability plot of the data set with the purpose of data visualization.

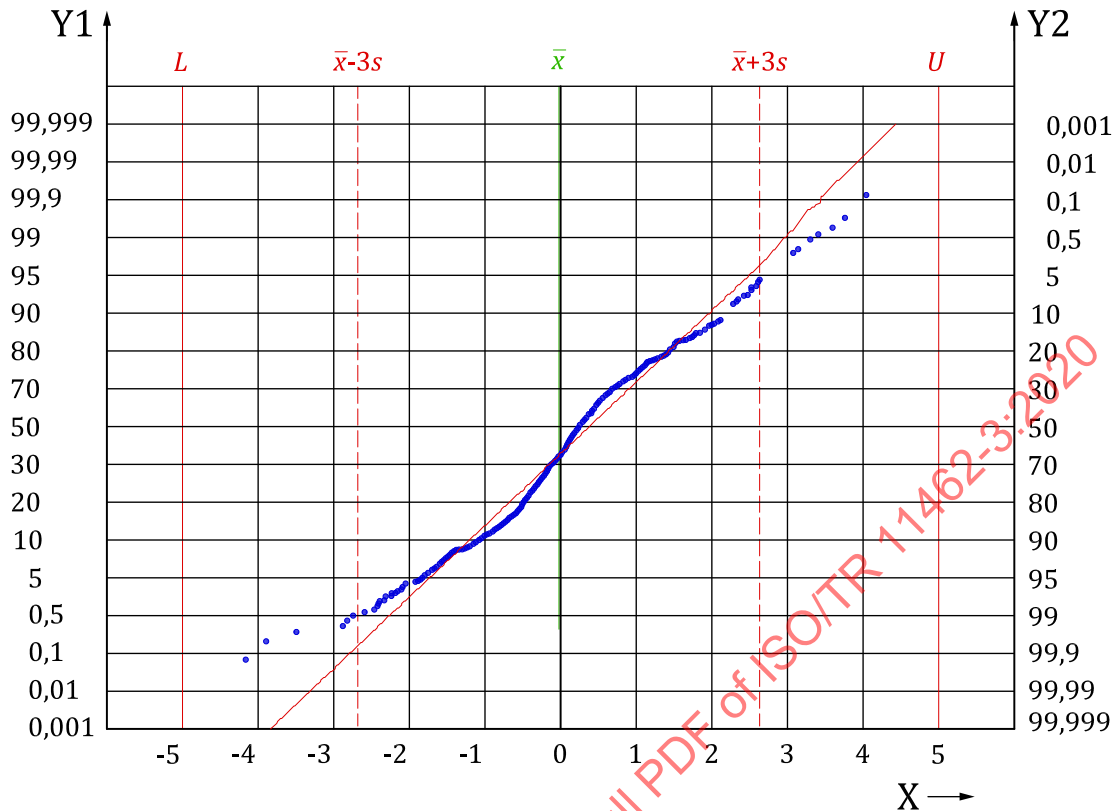
Table 11 — Description of test data set 3

Description of data input			
Distribution model	B	Resulting distribution	non-normal, unimodal
Data set	Annex A, Table A.1	decimal points	2
Total sample size	1 000	U	5
Size of subgroups	5	L	-5



Key
Y1 absolute frequency
Y2 relative frequency in %

Figure 17 — Histogram of test data set 3

**Key**

X value

Y1 probability in %

Y2 1-probability in %

Figure 18 — Probability plot of test data set 3

NOTE The width of the class intervals is 0,74.

The class interval with the highest frequency of values is from $-0,005$ to $0,735$.

The density plot is based on the assumption of normality and estimated parameters $l = 1$, $d = 5$.

5.3.2 Test data set 3 results**5.3.2.1 List of sample statistics**

Table 12 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 12 — List of sample statistics for test data set 3

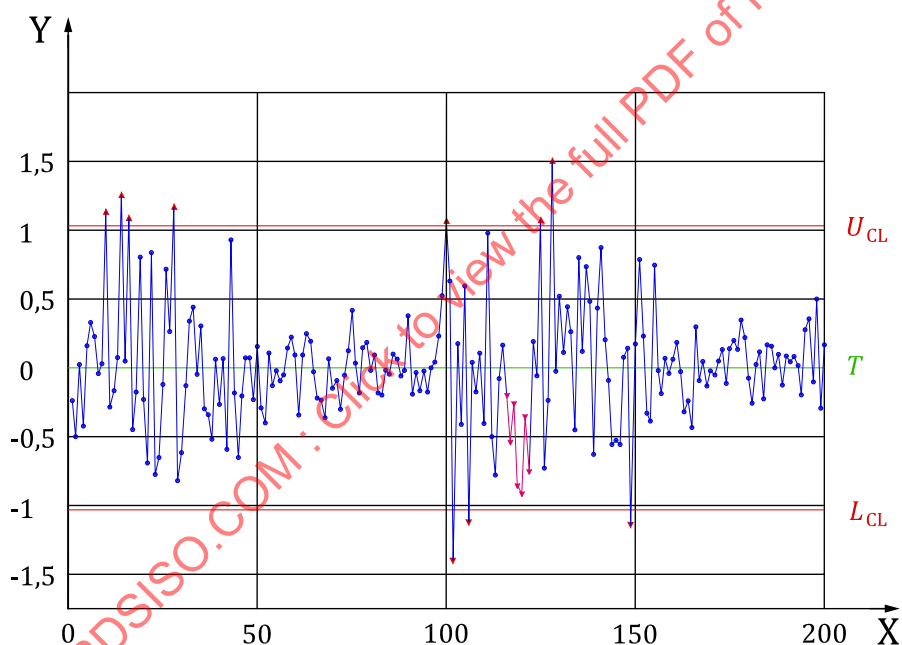
Statistic	Value	Reference
Location		
$\bar{X}$ ($l = 1$)	$-0,002\ 5$	ISO 22514-2:2017, Formula (11)
$\tilde{X}$ ($l = 2$)	$0,000\ 0$	ISO 22514-2:2017, Formula (12)
$\bar{\bar{X}}$ ($l = 3$)	$-0,002\ 5$	ISO 22514-2:2017, Formula (13)
$\tilde{\tilde{X}}$ ($l = 4$)	$0,000\ 6$	ISO 22514-2:2017, Formula (14)

Table 12 (continued)

Statistic	Value	Reference
Dispersion		
$\hat{\Delta} (d = 1)$	5,292 3 (normal distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma} (d = 2)$	0,856 1	ISO 22514-2:2017, Formula (16)
$\hat{\sigma} (d = 3)$	0,759 9	ISO 22514-2:2017, Formula (17)
$\hat{\sigma} (d = 4)$	0,767 5	ISO 22514-2:2017, Formula (18)
$\hat{\sigma} (d = 5)$	0,882 1	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,714 3	ISO 7870-2:2013, 3.2
R	8,150 0	ISO 7870-2:2013, 3.2
$\bar{R}$	1,785 2	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,859 0	ISO 7870-2:2013, 3.2

5.3.2.2 $\bar{x}$ - chart

The $\bar{x}$ - control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 19.



Key

X number of subgroup
Y mean value

Figure 19 — Mean control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$)

$$L_{CL} = -1,021\ 8$$

$$U_{CL} = 1,016\ 8$$

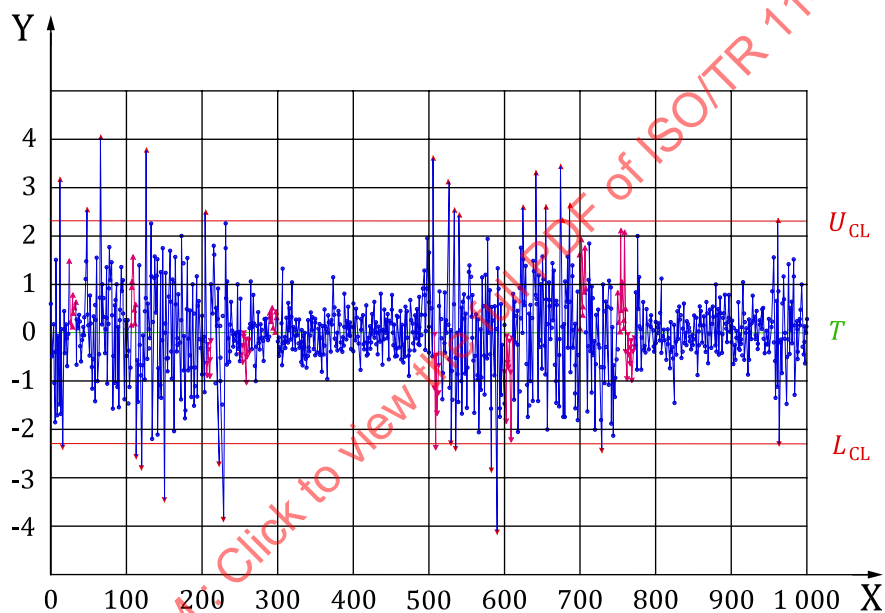
Out of control situations are given in Table 13.

Table 13 — Results of pattern tests for the mean control chart and test data set 3

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
10	violation of U_{CL}	106	violation of L_{CL}
14	violation of U_{CL}	116 to 122	run below centreline
16	violation of U_{CL}	125	violation of U_{CL}
28	violation of U_{CL}	128	violation of U_{CL}
100	violation of U_{CL}	149	violation of L_{CL}
102	violation of L_{CL}		

5.3.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 20](#).

**Key**

X value number
Y individuals

Figure 20 — Individuals control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = -2,287\ 4$$

$$U_{CL} = 2,282\ 5$$

Out of control situations are given in [Table 14](#).

Table 14 — Results of pattern tests for the individuals control chart and test data set 3

Out of control situations			
Value	Result/violation	Value	Result/violation
12	violation of U_{CL}	523	violation of U_{CL}
16	violation of L_{CL}	528	violation of L_{CL}
24 to 33	run above centreline	532	violation of U_{CL}
48	violation of U_{CL}	534	violation of L_{CL}
66	violation of U_{CL}	538	violation of U_{CL}
106 to 112	run above centreline	581	violation of L_{CL}
113	violation of L_{CL}	588	violation of L_{CL}
120	violation of L_{CL}	599 to 608	run below centreline
126	violation of U_{CL}	622	violation of U_{CL}
150	violation of L_{CL}	639	violation of U_{CL}
204	violation of U_{CL}	652	violation of U_{CL}
205 to 211	run below centreline	672	violation of U_{CL}
222	violation of L_{CL}	673	violation of U_{CL}
228	violation of L_{CL}	684	violation of U_{CL}
251 to 261	run below centreline	697 to 705	run above centreline
288 to 298	run above centreline	726	violation of L_{CL}
504	violation of U_{CL}	747 to 770	run above centreline
505 to 511	run below centreline	958	violation of U_{CL}
507	violation of L_{CL}	960	violation of U_{CL}

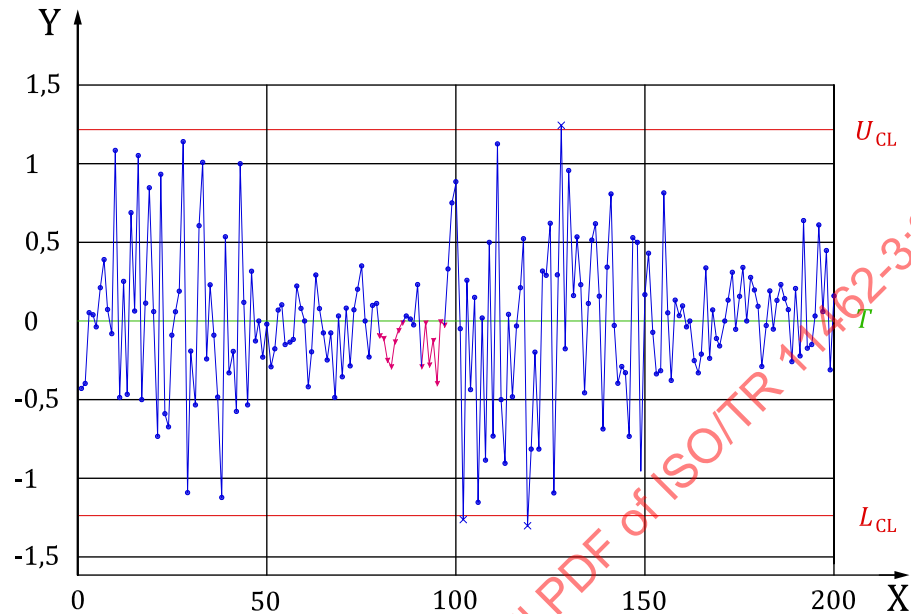
Because of the variation in dispersion another set of control limits can be calculated e.g. based on a Johnson transformation.

$$L_{CL} = -2,964\ 6$$

$$U_{CL} = 2,959\ 7$$

5.3.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 21](#).



Key

X subgroup number
Y median value

Figure 21 — Median control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = -1,233\ 0$$

$$U_{CL} = 1,234\ 2$$

Out of control situations are given in [Table 15](#).

Table 15 — Results of pattern tests for the median control chart and test data set 3

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
80 to 86	run below centreline	119	violation of L_{CL}
91 to 97	run below centreline	128	violation of U_{CL}
102	violation of L_{CL}		

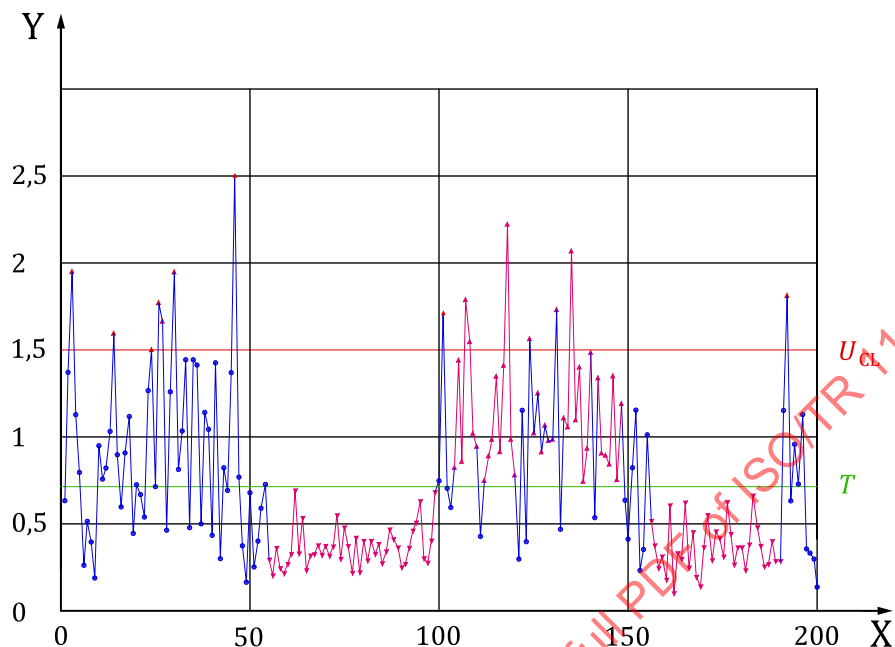
Because of the variation in dispersion another set of control limits can be calculated based on a Johnson Transformation.

$$L_{CL} = -1,377\ 4$$

$$U_{CL} = 1,372\ 4$$

5.3.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 22](#).



Key

X subgroup number

Y empirical standard deviation s value

Figure 22 — s control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 1,492\ 1$$

Out of control situations are given in [Table 16](#).

Table 16 — Results of pattern tests for the s control chart and test data set 3

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
3	violation of U_{CL}	108	violation of U_{CL}
14	violation of U_{CL}	112 to 120	run above centreline
24	violation of U_{CL}	118	violation of U_{CL}
26	violation of U_{CL}	124	violation of U_{CL}
27	violation of U_{CL}	124 to 131	run above centreline
30	violation of U_{CL}	131	violation of U_{CL}
46	violation of U_{CL}	133 to 140	run above centreline
55 to 99	run below centreline	135	violation of U_{CL}
101	violation of U_{CL}	142 to 148	run above centreline
104 to 110	run above centreline	156 to 190	run below centreline

Table 16 (continued)

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
107	violation of U_{CL}	192	violation of U_{CL}

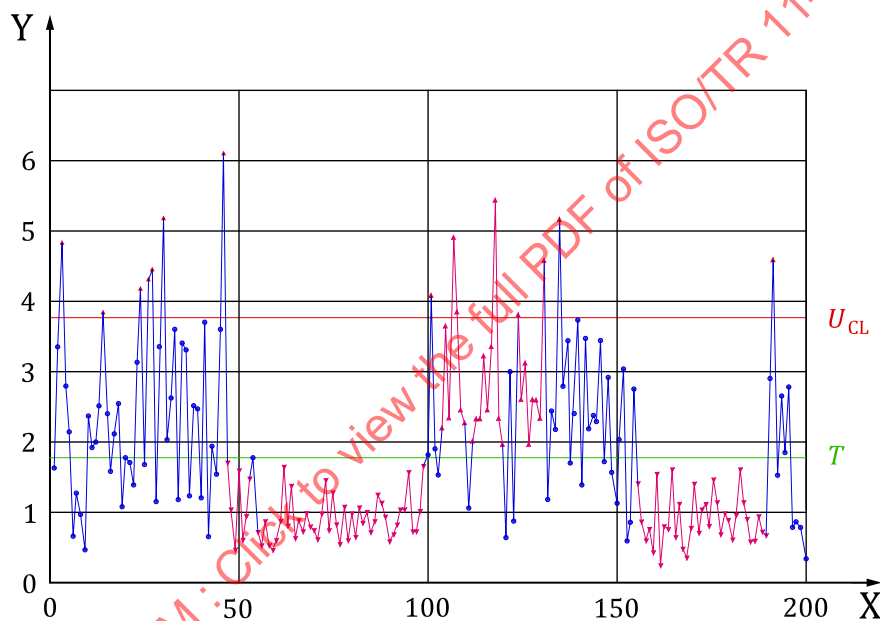
Because of the variation in dispersion another set of control limits can be calculated (Johnson transformation)

$$L_{CL} = 0,139\ 22$$

$$U_{CL} = 1,806\ 07$$

5.3.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in Figure 23.



Key

X subgroup number

Y range R value

Figure 23 — R control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 3,773\ 9$$

Out of control situations are given in Table 17.

Table 17 — Results of pattern tests for the R control chart and test data set 3

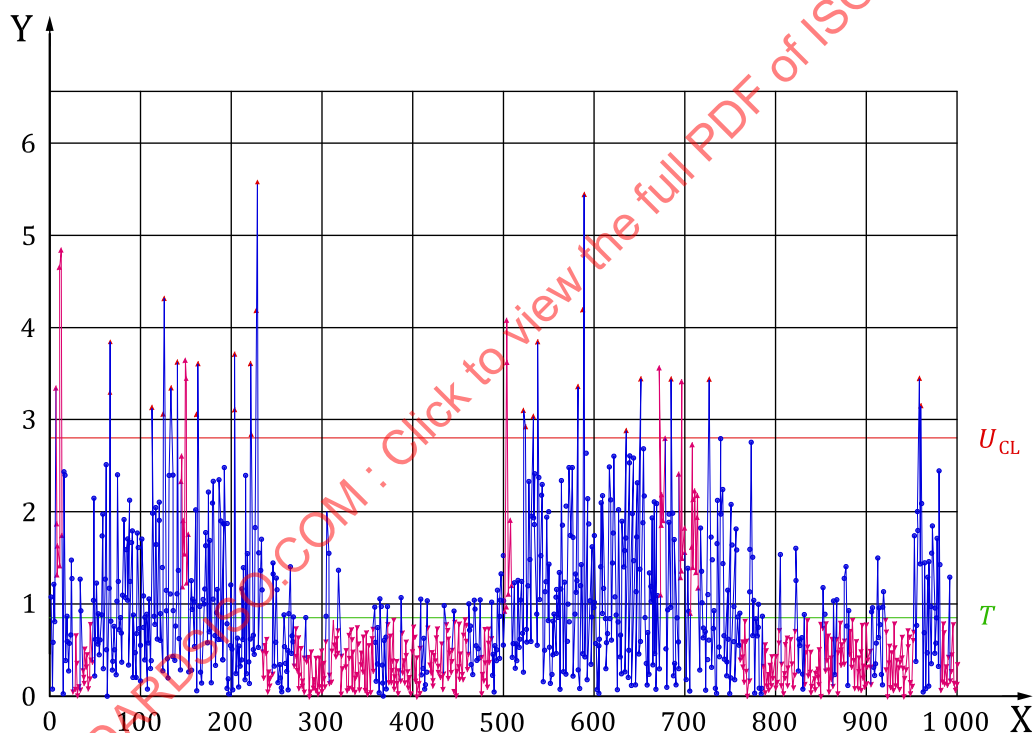
Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
3	violation of U_{CL}	107	violation of U_{CL}
14	violation of U_{CL}	108	violation of U_{CL}

Table 17 (continued)

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
24	violation of U_{CL}	112 to 120	run above centreline
26	violation of U_{CL}	118	violation of U_{CL}
27	violation of U_{CL}	124 to 131	run above centreline
29	violation of U_{CL}	124	violation of U_{CL}
46	violation of U_{CL}	131	violation of U_{CL}
47 to 53	run below centreline	135	violation of U_{CL}
55 to 99	run below centreline	156 to 190	run below centreline
101	violation of U_{CL}	192	violation of U_{CL}
104 to 110	run above centreline		

5.3.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in Figure 24.



Key

X individuals number
Y moving range value

Figure 24 — Moving range control chart for test data set 3

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 2,806\ 3$$

Out of control situations are given in Table 18.

Table 18 — Results of pattern tests for the moving range control chart and test data set 3

Out of control situations			
Value	Result/violation	Value	Result/violation
7 to 14	run above centreline	446 to 461	run below centreline
7	violation of U_{CL}	475 to 486	run below centreline
12	violation of U_{CL}	502 to 509	run above centreline
13	violation of U_{CL}	504	violation of U_{CL}
26 to 33	run below centreline	505	violation of U_{CL}
36 to 47	run below centreline	523	violation of U_{CL}
66	violation of U_{CL}	524	violation of U_{CL}
67	violation of U_{CL}	533	violation of U_{CL}
113	violation of U_{CL}	538	violation of U_{CL}
126	violation of U_{CL}	582	violation of U_{CL}
127	violation of U_{CL}	588	violation of U_{CL}
134	violation of U_{CL}	589	violation of U_{CL}
141	violation of U_{CL}	636	violation of U_{CL}
145 to 153	run above centreline	652	violation of U_{CL}
150	violation of U_{CL}	672 to 678	run above centreline
151	violation of U_{CL}	672	violation of U_{CL}
163	violation of U_{CL}	685	violation of U_{CL}
164	violation of U_{CL}	694 to 700	run above centreline
205	violation of U_{CL}	697	violation of U_{CL}
205	violation of U_{CL}	706 to 715	run above centreline
222	violation of U_{CL}	727	violation of U_{CL}
223	violation of U_{CL}	761 to 770	run below centreline
228	violation of U_{CL}	786 to 804	run below centreline
229	violation of U_{CL}	806 to 821	run below centreline
235 to 245	run below centreline	832 to 851	run below centreline
270 to 281	run below centreline	854 to 863	run below centreline
283 to 304	run below centreline	867 to 875	run below centreline
308 to 318	run below centreline	881 to 905	run below centreline
320 to 357	run below centreline	920 to 952	run below centreline
373 to 386	run below centreline	958	violation of U_{CL}
388 to 408	run below centreline	960	violation of U_{CL}
417 to 434	run below centreline	982 to 990	run below centreline
436 to 444	run below centreline	992 to 1 000	run below centreline

Because of the variation in dispersion another set of control limits can be calculated.

5.3.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used			
P_p	1,89	P_{pk}	1,89
P_{pkL}	1,89	P_{pkU}	1,89

Because of non-normal distributed data (dispersion not stable) $M_{3,5}$ cannot be used – instead $M_{3,1}$ and a Pearson or Johnson transformation is used.

Capability indices (calculation method $M_{3,1}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used. Calculation based on:			
Pearson transformation			
P_p	1,42	P_{pk}	1,39
P_{pkL}	1,89	P_{pkU}	1,46
Johnson Transformation			
P_p	1,02	P_{pk}	1,02
P_{pkL}	1,03	P_{pkU}	1,02

5.4 Test data set 4

5.4.1 Test data set 4 information

This set of test data is taken from a process following a normal distribution (having changes in the values of parameters μ and σ over time) and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 4 is given in [Table 19](#). [Figure 25](#) shows a histogram and [Figure 26](#) a probability plot of the data set with the purpose of data visualization.

Table 19 — Description of test data set 4

Description of data input			
Distribution model	C1	Resulting distribution	normal
Data set	Annex A, Table A.1	decimal points	2
Total sample size	1 000	U	10
Size of subgroups	5	L	-10

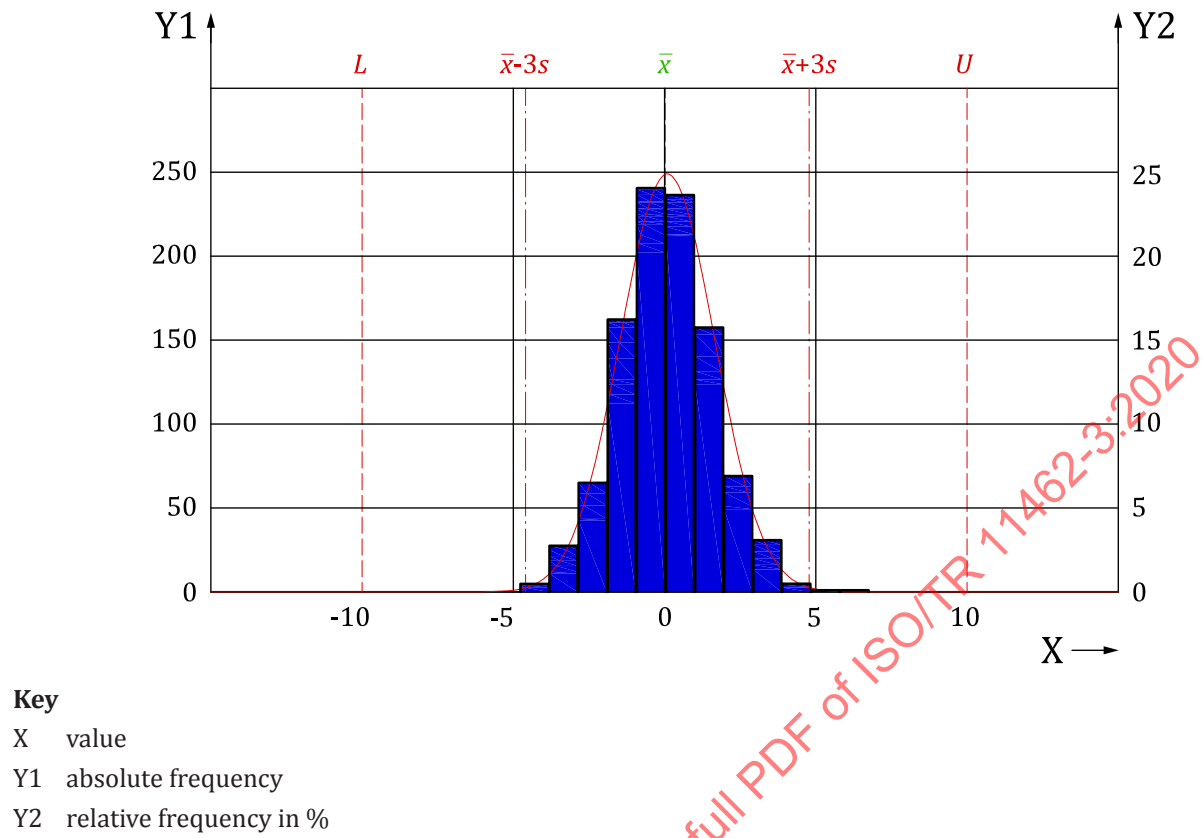
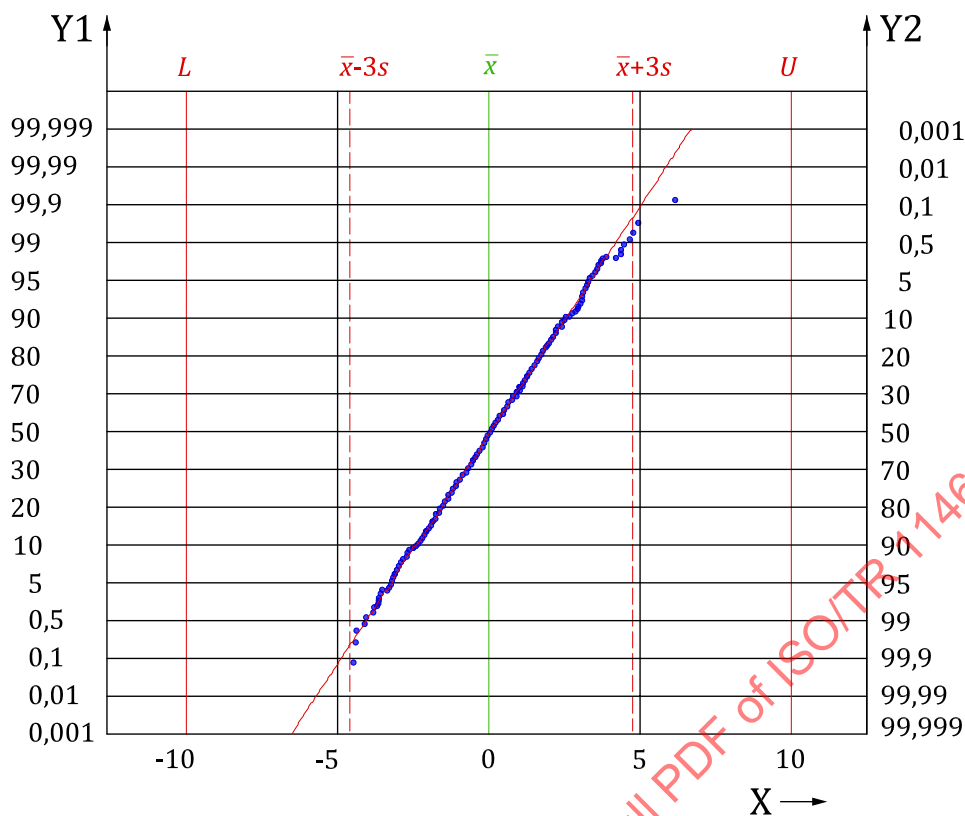


Figure 25 — Histogram of test data set 4



Key
X value
Y1 probability in %
Y2 1-probability in %

Figure 26 — Probability plot of test data set 4

NOTE The width of the class intervals is 0,97.
The class interval with the highest frequency of values is from -0,91 to 0,06.
The density plot is based on the assumption of normality and estimated parameters $l = 1$, $d = 5$.

5.4.2 Test data set 4 results

5.4.2.1 List of sample statistics

Table 20 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 20 — List of sample statistics for test data set 4

Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	0,088 5	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	0,065 0	ISO 22514-2:2017, Formula (12)
$\bar{\bar{x}}$ ($l = 3$)	0,088 5	ISO 22514-2:2017, Formula (13)
$\tilde{\tilde{x}}$ ($l = 4$)	0,114 3	ISO 22514-2:2017, Formula (14)
Dispersion		
$\hat{\Delta}$ ($d = 1$)	9,349 0 (normal distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	1,157 9	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	1,158 3	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	1,165 0	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	1,558 2	ISO 22514-2:2017, Formula (19)
$\bar{s}$	1,088 8	ISO 7870-2:2013, 3.2
R	10,640 0	ISO 7870-2:2013, 3.2
$\bar{R}$	2,709 7	ISO 7870-2:2013, 3.2
$\bar{R}_m$	1,344 9	ISO 7870-2:2013, 3.2

5.4.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 27.

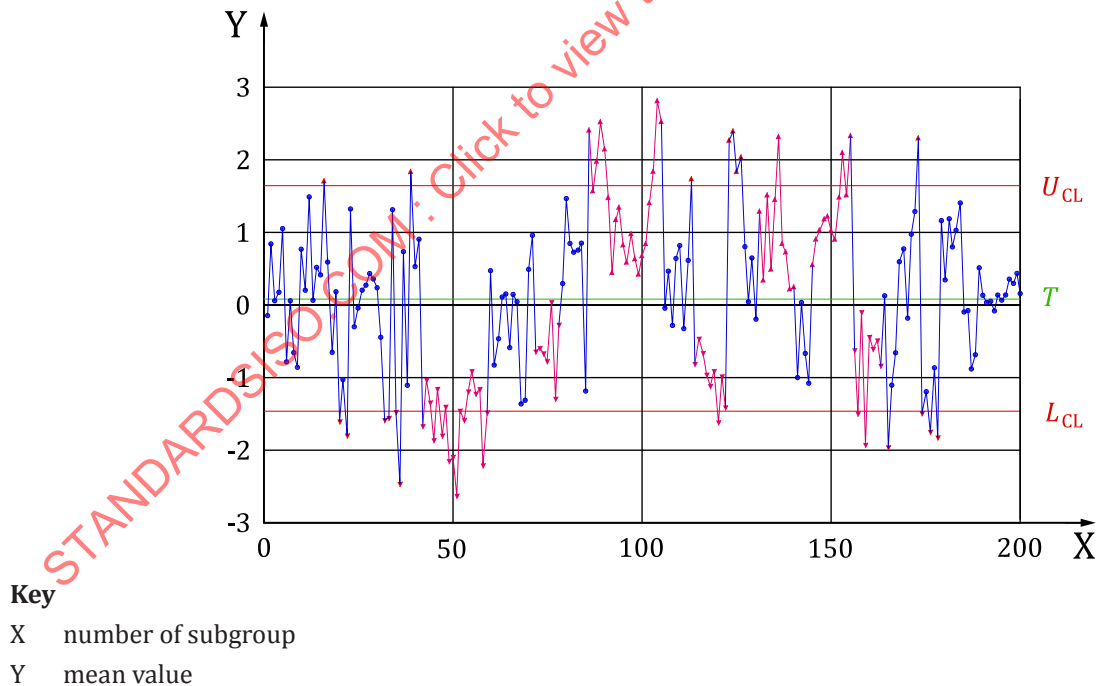


Figure 27 — Mean control chart for test data set 4

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = -1,465\ 2$$

$$U_{CL} = 1,642\ 2$$

Out of control situations are given in [Table 21](#).

Table 21 — Results of pattern tests for the mean control chart and test data set 4

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
16	violation of U_{CL}	103 to 105	violation of U_{CL}
20	violation of L_{CL}	113	violation of U_{CL}
22	violation of L_{CL}	114 to 122	run below centreline
32	violation of L_{CL}	120	violation of L_{CL}
33	violation of L_{CL}	123 to 126	violation of U_{CL}
35	violation of L_{CL}	131 to 140	run above centreline
36	violation of L_{CL}	136	violation of U_{CL}
39	violation of L_{CL}	145 to 155	run above centreline
42 to 59	run below centreline	153	violation of U_{CL}
42	violation of L_{CL}	155	violation of U_{CL}
45	violation of L_{CL}	156 to 163	run below centreline
47	violation of L_{CL}	157	violation of L_{CL}
49 to 53	violation of L_{CL}	159	violation of L_{CL}
58	violation of L_{CL}	165	violation of L_{CL}
59	violation of L_{CL}	173	violation of U_{CL}
72 to 78	run below centreline	174	violation of L_{CL}
86 to 105	run above centreline	176	violation of L_{CL}
86	violation of U_{CL}	178	violation of L_{CL}
88 to 90	violation of U_{CL}		

Because of the variation in location (not stable) another set of control limits can be calculated.

The calculation can be done using s_{tot} or extended control limits.

In this example extended control limits are proposed (extension = 3).

$$L_{CL} = -2,965\ 2$$

$$U_{CL} = 3,142\ 2$$

5.4.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 28](#).

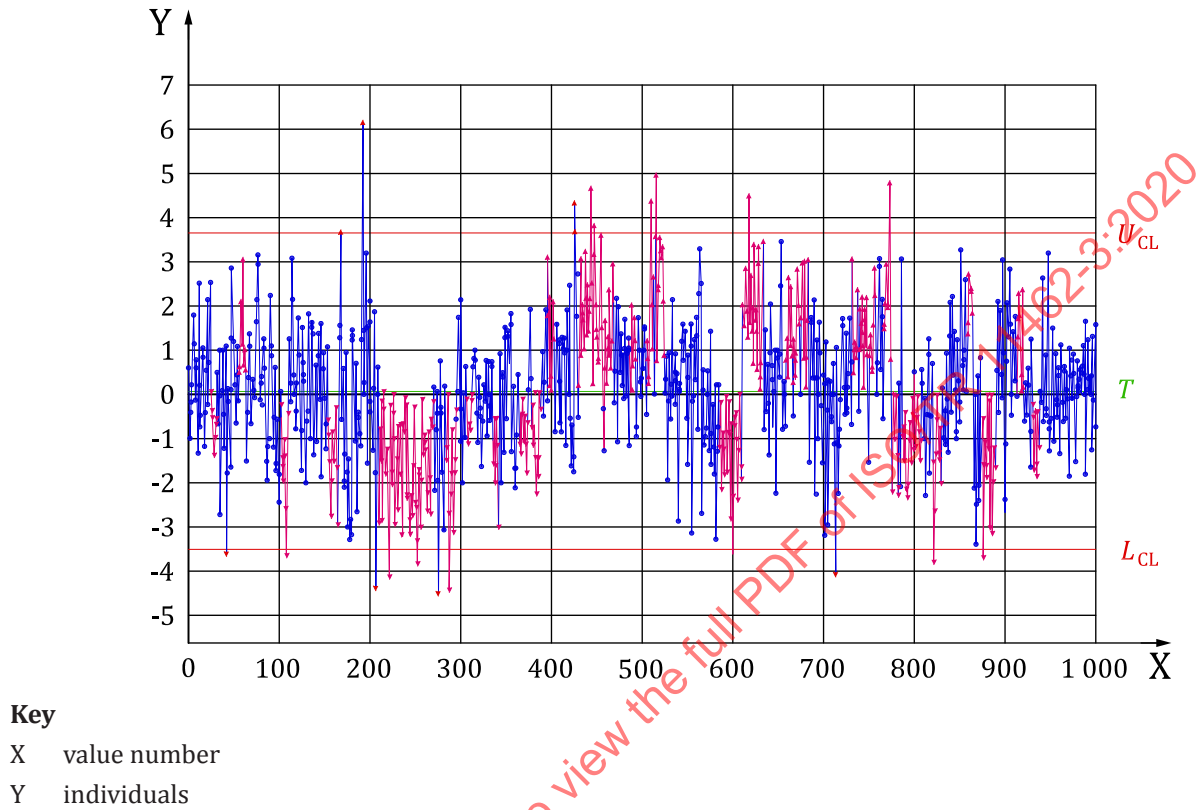


Figure 28 — Individuals control chart for test data set 4

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = -3,488\ 9$$

$$U_{CL} = 3,665\ 8$$

Out of control situations are given in [Table 22](#).

Table 22 — Results of pattern tests for individuals control chart and test data set 4

Out of control situations			
Value	Result/violation	Value	Result/violation
25 to 31	run below centreline	506 to 513	run above centreline
42	violation of L_{CL}	510	violation of U_{CL}
56 to 63	run above centreline	515 to 525	run above centreline
102 to 110	run below centreline	516	violation of U_{CL}
108	violation of L_{CL}	586 to 610	run below centreline
154 to 165	run below centreline	600	violation of L_{CL}
168	violation of U_{CL}	611 to 634	run above centreline
192	violation of U_{CL}	618	violation of U_{CL}
207	violation of L_{CL}	659 to 665	run above centreline

Table 22 (continued)

Out of control situations			
Value	Result/violation	Value	Result/violation
210 to 270	run below centreline	667 to 674	run above centreline
222	violation of L_{CL}	676 to 684	run above centreline
246	violation of L_{CL}	714	violation of L_{CL}
253	violation of L_{CL}	732 to 738	run above centreline
276	violation of L_{CL}	740 to 749	run above centreline
284 to 295	run below centreline	751 to 758	run above centreline
288	violation of L_{CL}	767 to 775	run above centreline
307 to 313	run below centreline	774	violation of U_{CL}
336 to 342	run below centreline	776 to 783	run below centreline
365 to 374	run below centreline	787 to 799	run below centreline
378 to 389	run below centreline	801 to 808	run below centreline
396 to 404	run above centreline	821 to 830	run below centreline
426	violation of U_{CL}	822	violation of L_{CL}
428	violation of U_{CL}	859 to 865	run above centreline
431 to 456	run above centreline	875 to 890	run below centreline
444	violation of U_{CL}	876	violation of L_{CL}
448	violation of U_{CL}	914 to 921	run above centreline
459 to 469	run above centreline	931 to 938	run below centreline
487 to 493	run above centreline		

Because of the variation in location (not stable) another set of control limits can be calculated.

The calculation can be done using s_{tot} or extended control limits.

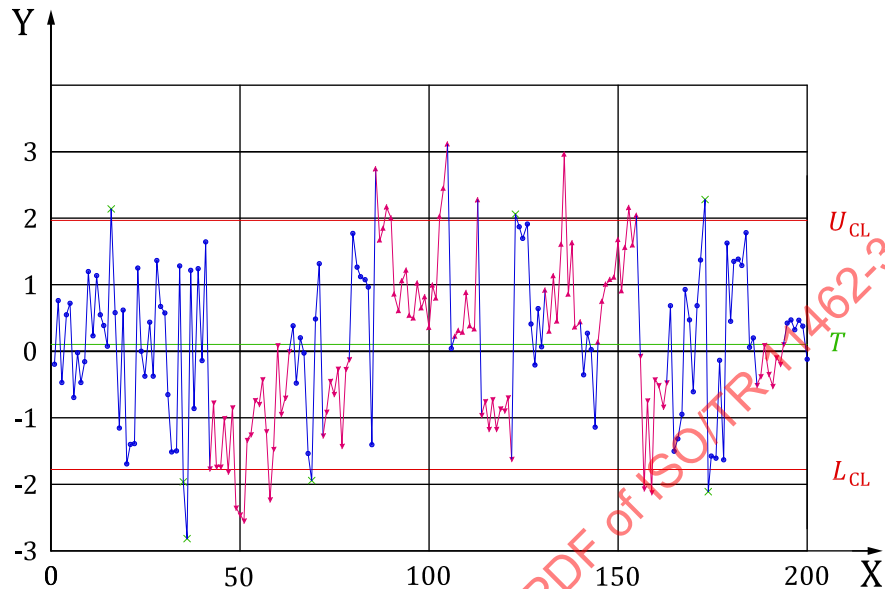
In this example extended control limits are proposed (extension = 3).

$$L_{CL} = -5,417\ 6$$

$$U_{CL} = 5,594\ 5$$

5.4.2.4 $\tilde{x}$ - chart

The median control chart calculated according to ISO 7870-2:2013, 6.3 is shown in [Figure 29](#).



Key

X subgroup number
Y median value

Figure 29 — Median control chart for test data set 4

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = -1,758\ 1$$

$$U_{CL} = 1,986\ 7$$

Out of control situations are given in [Table 23](#).

Table 23 — Results of pattern tests for median control chart and test data set 4

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
16	violation of U_{CL}	107 to 113	run above centreline
35	violation of L_{CL}	113	violation of U_{CL}
36	violation of L_{CL}	114 to 122	run below centreline
42 to 63	run below centreline	123	violation of U_{CL}
42	violation of L_{CL}	131 to 140	run above centreline
47	violation of L_{CL}	136	violation of U_{CL}
49 to 51	violation of L_{CL}	145 to 155	run above centreline
58	violation of L_{CL}	153	violation of U_{CL}
69	violation of L_{CL}	155	violation of U_{CL}
72 to 79	run below centreline	156 to 163	run below centreline
86 to 105	run above centreline	157	violation of L_{CL}

Table 23 (continued)

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
86	violation of U_{CL}	159	violation of L_{CL}
89	violation of U_{CL}	173	violation of U_{CL}
90	violation of U_{CL}	174	violation of L_{CL}
103 to 105	violation of U_{CL}	187 to 194	run below centreline

Because of the variation in location (not stable) another set of control limits can be calculated

The calculation can be done using s_{tot} or extended control limits.

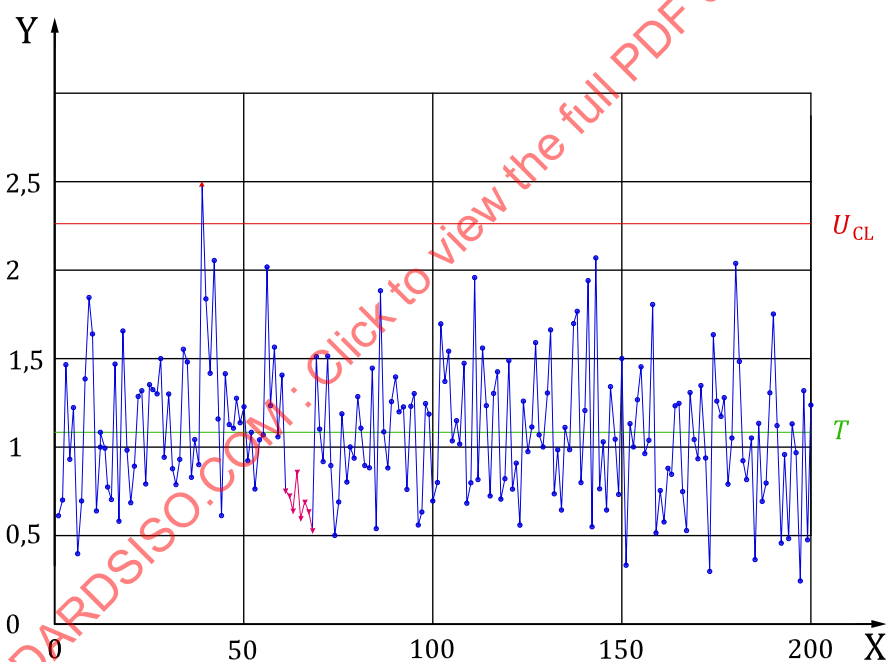
In this example extended control limits are proposed (extension = 3).

$$L_{CL} = -3,271\ 0$$

$$U_{CL} = 3,447\ 9$$

5.4.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 30.



Key

X subgroup number

Y empirical standard deviation s value

Figure 30 — s control chart for test data set 4

The control limits are (see ISO 7870-2:2013, Table 1)

$$L_{CL} = 0$$

$$U_{CL} = 2,274\ 5$$

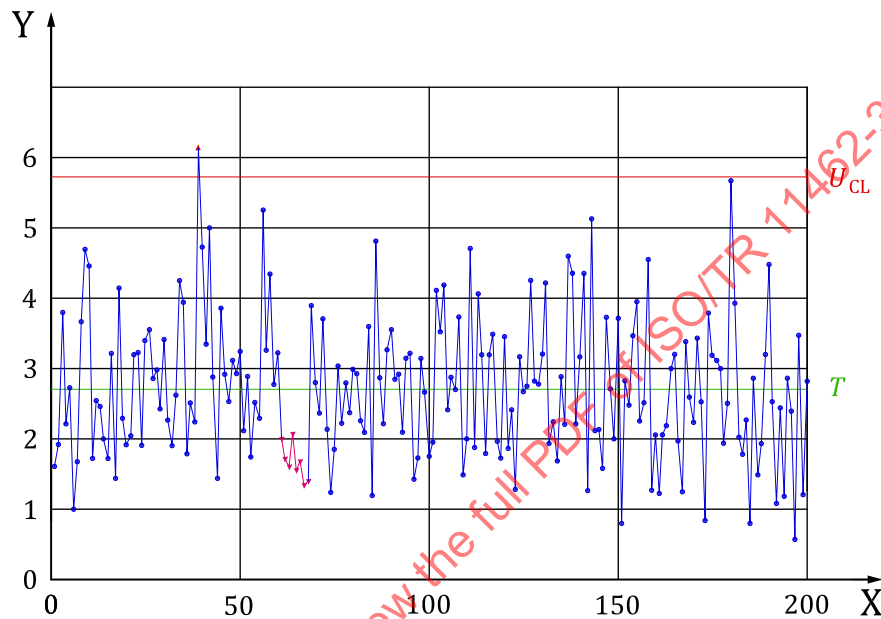
Out of control situations are given in Table 24.

Table 24 — Results of pattern tests for the s control chart and test data set 4

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
39	violation of U_{CL}	61 to 68	run below centreline

5.4.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 31](#).



Key

X subgroup number
Y range R value

Figure 31 — R control chart for test data set 4

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 5,728\ 3$$

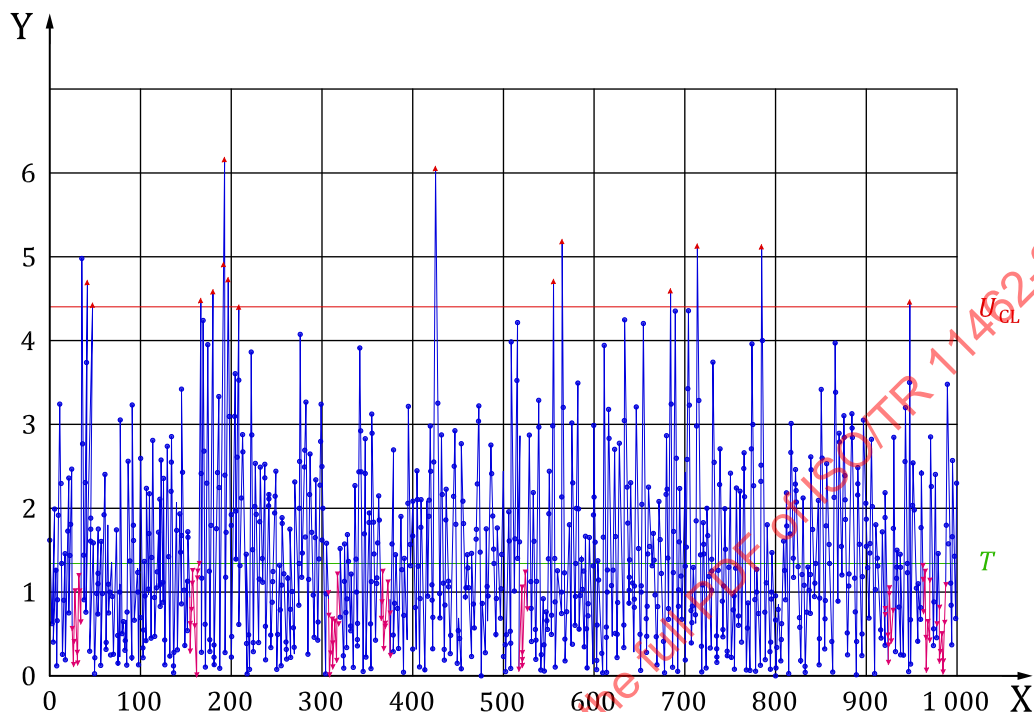
Out of control situations are given in [Table 25](#).

Table 25 — Results of pattern tests for R control chart and test data set 4

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
39	violation of U_{CL}	61 to 68	run below centreline

5.4.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 32](#).



Key

X individuals number
Y moving range value

Figure 32 — Moving range control chart for test data set 4

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 4,393\ 7$$

Out of control situations are given in [Table 26](#).

Table 26 — Results of pattern tests for moving range control chart and test data set 4

Out of control situations			
Value	Result/violation	Value	Result/violation
26 to 35	run below centreline	426	violation of U_{CL}
42	violation of U_{CL}	519 to 527	run below centreline
48	violation of U_{CL}	555	violation of U_{CL}
155 to 165	run below centreline	566	violation of U_{CL}
166	violation of U_{CL}	685	violation of U_{CL}
181	violation of U_{CL}	715	violation of U_{CL}
192	violation of U_{CL}	786	violation of U_{CL}
193	violation of U_{CL}	922 to 929	run below centreline

Table 26 (continued)

Out of control situations			
Value	Result/violation	Value	Result/violation
197	violation of U_{CL}	949	violation of U_{CL}
208	violation of U_{CL}	963 to 971	run below centreline
307 to 319	run below centreline	980 to 988	run below centreline
366 to 376	run below centreline		

5.4.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used			
P_p	2,14	P_{pk}	2,12
P_{pkL}	2,16	P_{pkU}	2,12

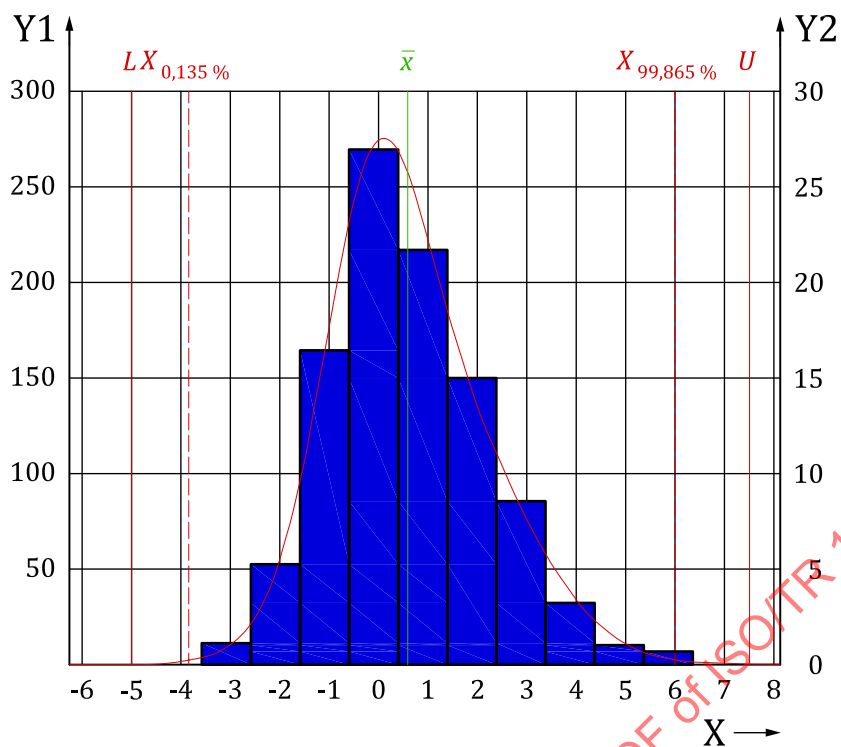
5.5 Test data set 5

5.5.1 Test data set 5 information

This set of test data is taken from a process following a non-normal distribution with a shift of parameter μ and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 5 is given in [Table 27](#). [Figure 33](#) shows a histogram and [Figure 34](#) a probability plot of the data set with the purpose of data visualization.

Table 27 — Description of test data set 5

Description of data input			
Distribution model	C2	Resulting distribution	non-normal, unimodal
Data set	Annex A, Table A.1	decimal points	2
Total sample size	1 000	U	7,5
Size of subgroups	5	L	-5

**Key**

X value

Y1 absolute frequency

Y2 relative frequency in %

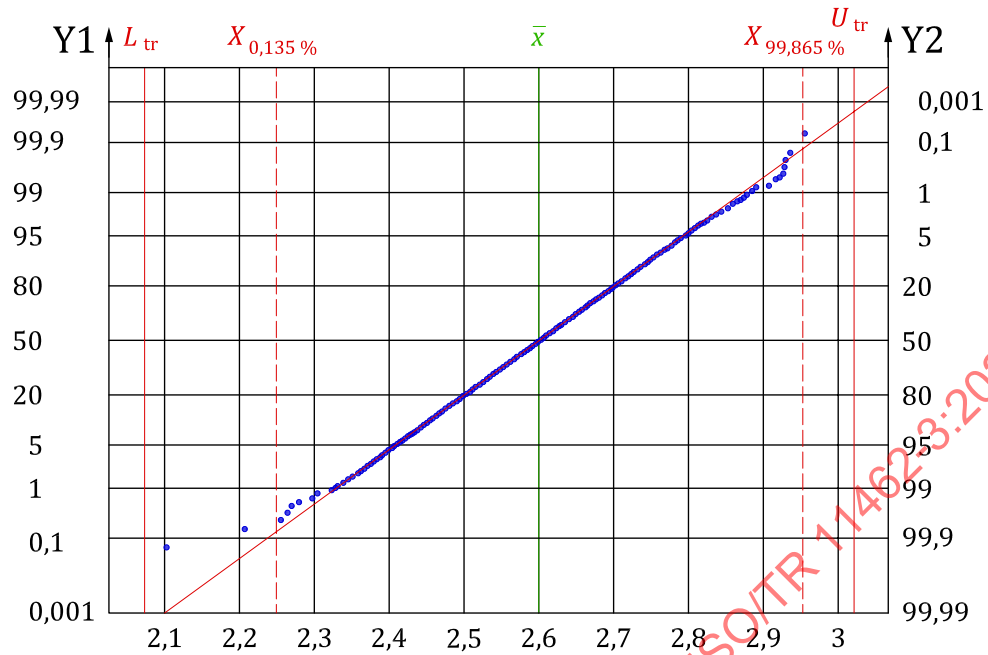
Figure 33 — Histogram of test data set 5

NOTE The width of the class intervals is 1.

The class interval with the highest frequency of values is from -0,585 to 0,415.

The probability plot is based on the assumption of a logarithmic normal distribution.

The transformation used here is $f(x) = \ln(12,969 \cdot 3 + x)$.

**Key**

X transformed value

Y1 probability in %

Y2 1-probability in %

Figure 34 — Probability plot of test data set 5**5.5.2 Test data set 5 results****5.5.2.1 List of sample statistics**

Table 28 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 28 — List of sample statistics for test data set 5

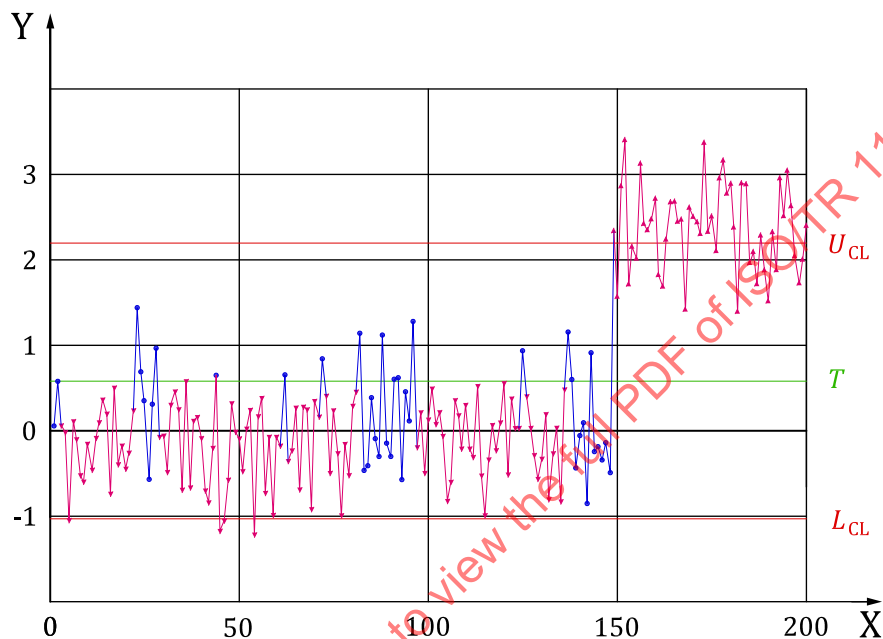
Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	0,585 1	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	0,420 0	ISO 22514-2:2017, Formula (12)
$\bar{\bar{x}}$ ($l = 3$)	0,585 1	ISO 22514-2:2017, Formula (13)
$\bar{\tilde{x}}$ ($l = 4$)	0,603 9	ISO 22514-2:2017, Formula (14)
Dispersion		
$\hat{\Delta}$ ($d = 1$)	9,572 0 (normal distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	1,204 7	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	1,198 2	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	1,200 4	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	1,595 4	ISO 22514-2:2017, Formula (19)
$\bar{s}$	1,126 3	ISO 7870-2:2013, 3.2

Table 28 (continued)

Statistic	Value	Reference
R	10,980 0	ISO 7870-2:2013, 3.2
$\bar{R}$	2,792 1	ISO 7870-2:2013, 3.2
$\bar{R}_m$	1,310 4	ISO 7870-2:2013, 3.2

5.5.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 35.



Key

X number of subgroup

Y mean value

Figure 35 — Mean control chart for test data set 5

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = -1,022\ 1$$

$$U_{CL} = 2,192\ 2$$

Out of control situations are given in Table 29.

Table 29 — Results of pattern tests for the mean control chart and test data set 5

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
3 to 22	run below centreline	151	violation of U_{CL}
5	violation of L_{CL}	152	violation of U_{CL}
29 to 43	run below centreline	156 to 160	violation of U_{CL}
45 to 61	run below centreline	163 to 167	violation of U_{CL}
45	violation of L_{CL}	169 to 175	violation of U_{CL}

Table 29 (continued)

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
46	violation of L_{CL}	177 to 181	violation of U_{CL}
54	violation of L_{CL}	183	violation of U_{CL}
63 to 71	run below centreline	184	violation of U_{CL}
73 to 81	run below centreline	188	violation of U_{CL}
97 to 124	run below centreline	191	violation of U_{CL}
126 to 136	run below centreline	193 to 196	violation of U_{CL}
149 to 200	run above centreline	200	violation of U_{CL}
149	violation of U_{CL}		

Because of the variation in location (not stable) another set of control limits can be calculated.

The calculation can be done using extended control limits.

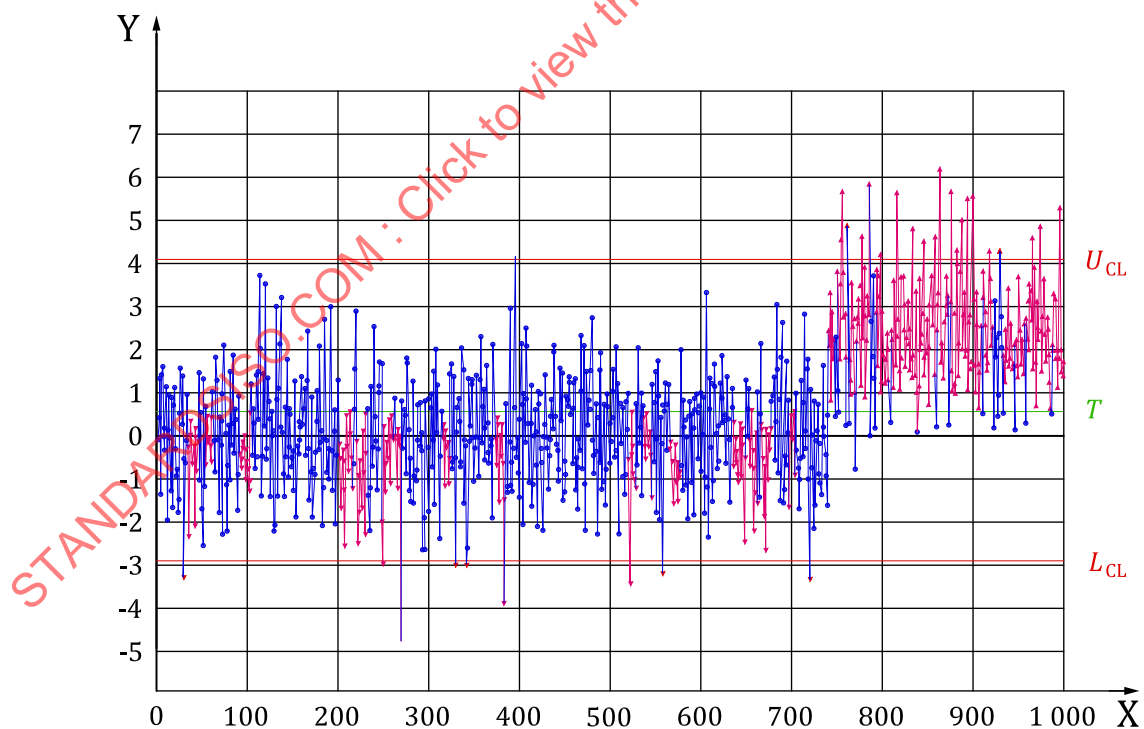
In this example extended control limits are proposed (extension = 3).

$$L_{CL} = -2,506\ 6$$

$$U_{CL} = 3,714\ 3$$

5.5.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in Figure 36.



Key

X value number

Y individuals

Figure 36 — Individuals control chart for test data set 5

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = -2,900\ 6$$

$$U_{CL} = 4,070\ 8$$

Out of control situations are given in [Table 30](#).

Table 30 — Results of pattern tests for individuals control chart and test data set 5

Out of control situations			
Value	Result/violation	Value	Result/violation
30	violation of L_{CL}	764 to 770	run above centreline
35 to 46	run below centreline	772 to 786	run above centreline
55 to 61	run below centreline	778	violation of U_{CL}
92 to 104	run below centreline	786	violation of U_{CL}
201 to 216	run below centreline	793 to 808	run above centreline
221 to 232	run below centreline	798	violation of U_{CL}
249 to 262	run below centreline	810 to 837	run above centreline
250	violation of L_{CL}	816	violation of U_{CL}
264 to 270	run below centreline	834	violation of U_{CL}
270	violation of L_{CL}	839 to 859	run above centreline
316 to 323	run below centreline	846	violation of U_{CL}
330	violation of L_{CL}	858	violation of U_{CL}
342	violation of L_{CL}	861 to 873	run above centreline
372 to 384	run below centreline	864	violation of U_{CL}
384	violation of L_{CL}	875 to 909	run above centreline
396	violation of U_{CL}	876	violation of U_{CL}
521 to 528	run below centreline	882	violation of U_{CL}
522	violation of L_{CL}	888	violation of U_{CL}
537 to 546	run below centreline	894	violation of U_{CL}
558	violation of L_{CL}	900	violation of U_{CL}
565 to 577	run below centreline	911 to 922	run above centreline
636 to 649	run below centreline	918	violation of U_{CL}
654 to 662	run below centreline	930	violation of U_{CL}
667 to 677	run below centreline	934 to 945	run above centreline
698 to 704	run below centreline	947 to 957	run above centreline
720	violation of L_{CL}	959 to 986	run above centreline
741 to 747	run above centreline	966	violation of U_{CL}
752 to 760	run above centreline	974	violation of U_{CL}
755	violation of U_{CL}	988 to 1 000	run above centreline
756	violation of U_{CL}	996	violation of U_{CL}
762	violation of U_{CL}		

Because of the variation in location (not stable) another set of control limits can be calculated

The calculation can be done using extended control limits.

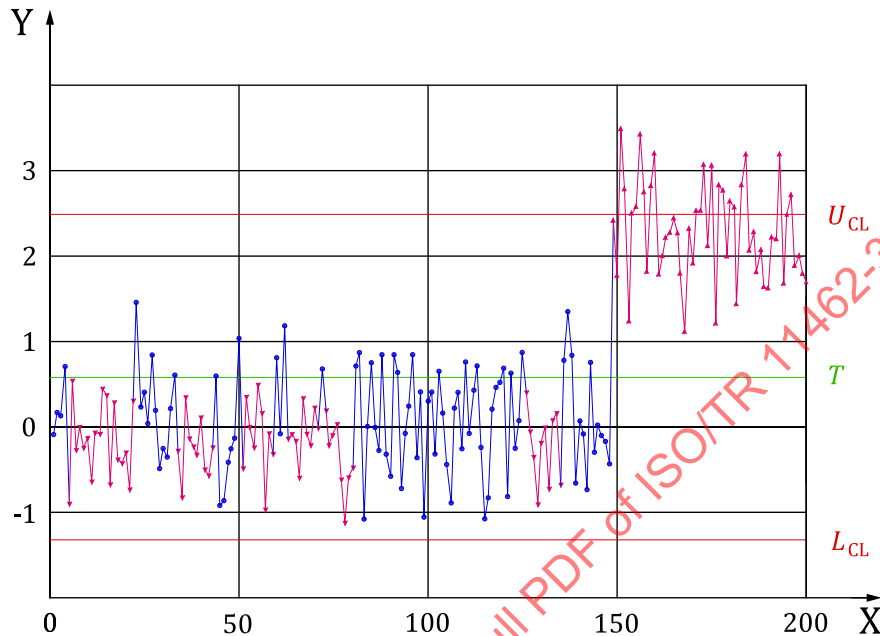
In this example extended control limits are proposed (extension = 3).

$$L_{CL} = -4,100\ 6$$

$$U_{CL} = 5,570\ 8$$

5.5.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 37](#).



Key

X subgroup number
Y median value

Figure 37 — Median control chart for test data set 5

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = 1,325\ 5$$

$$U_{CL} = 2,533\ 2$$

Out of control situations are given in [Table 31](#).

Table 31 — Results of pattern tests for median control chart and test data set 5

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
5 to 22	run below centreline	171 to 173	violation of U_{CL}
34 to 43	run below centreline	175	violation of U_{CL}
51 to 59	run below centreline	177	violation of U_{CL}
63 to 71	run below centreline	178	violation of U_{CL}
73 to 80	run below centreline	180	violation of U_{CL}
126 to 135	run below centreline	181	violation of U_{CL}
149 to 200	run above centreline	183	violation of U_{CL}
151	violation of U_{CL}	184	violation of U_{CL}
152	violation of U_{CL}	193	violation of U_{CL}
154 to 157	violation of U_{CL}	195	violation of U_{CL}

Table 31 (continued)

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
159	violation of U_{CL}	196	violation of U_{CL}
160	violation of U_{CL}		

Because of the variation in location (not stable) another set of control limits can be calculated
The calculation can be done using extended control limits.

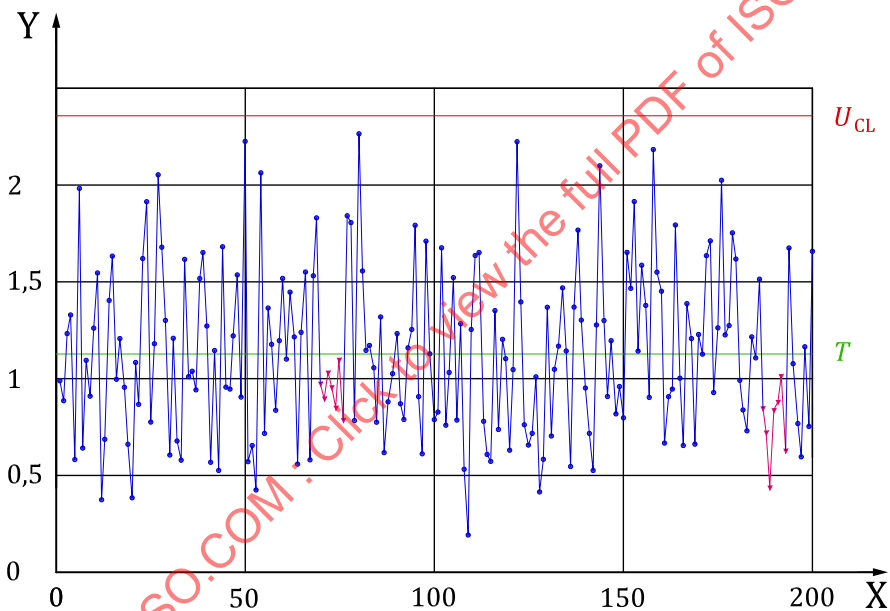
In this example extended control limits are proposed (extension = 3).

$L_{CL} = -2,825\ 5$

$U_{CL} = 4,033\ 2$

5.5.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 38](#).



Key
X subgroup number
Y empirical standard deviation s value

Figure 38 — s control chart for test data set 5

The control limits are (see ISO 7870-2:2013, Table 1):

$L_{CL} = 0$

$U_{CL} = 2,352\ 7$

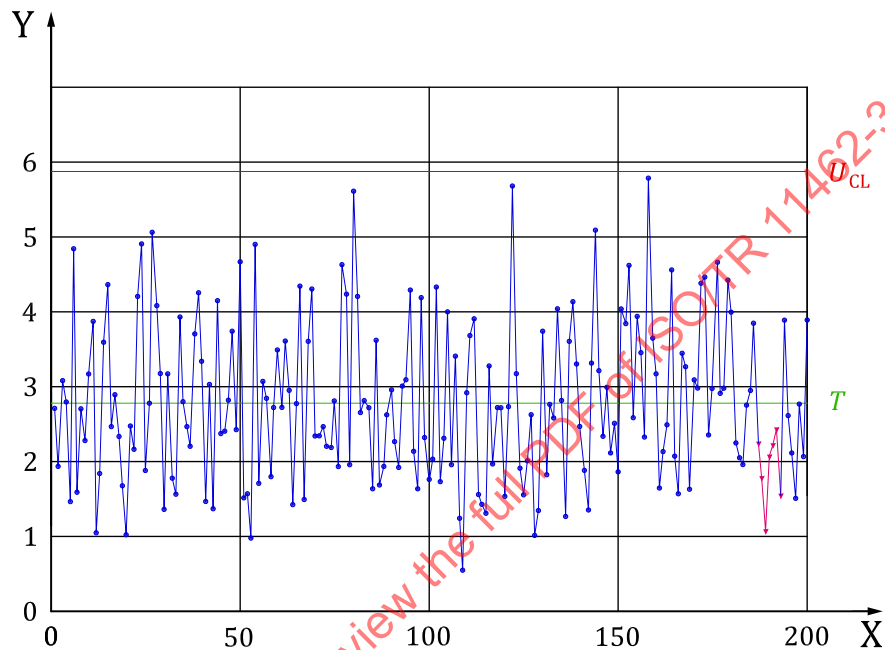
Out of control situations are given in [Table 32](#).

Table 32 — Results of pattern tests for the $\bar{s}$ control chart and test data set 5

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
70 to 76	run below centreline	187 to 193	run below centreline

5.5.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 39](#).

**Key**

X subgroup number
Y range R value

Figure 39 — R control chart for test data set 5

The control limits are (see ISO 7870-2:2013, Table 1)

$$L_{CL} = 0$$

$$U_{CL} = 5.902\ 5$$

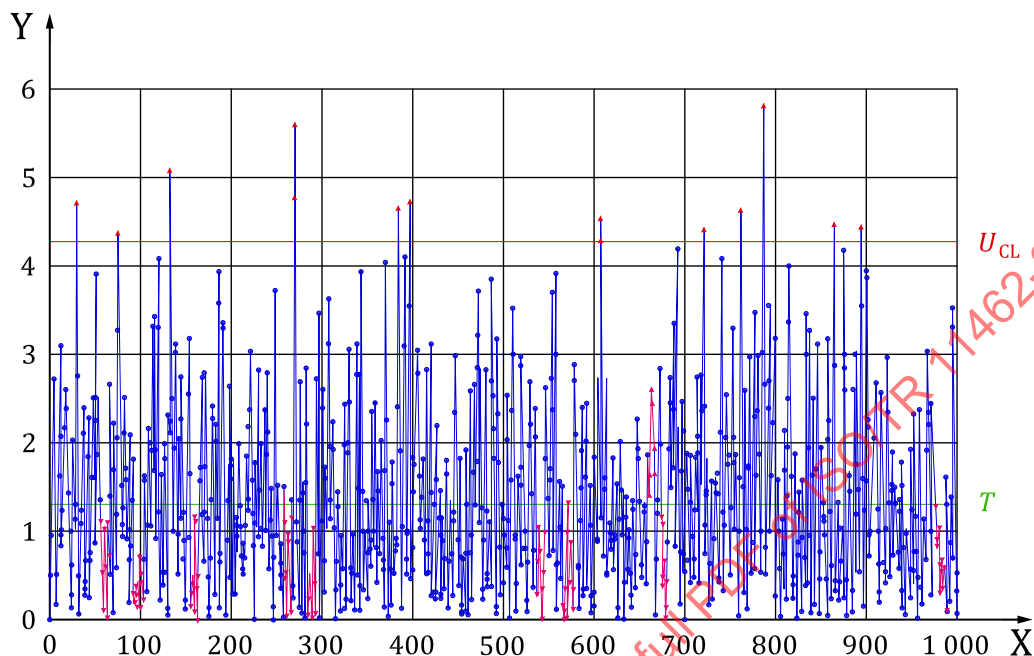
Out of control situations are given in [Table 33](#).

Table 33 — Results of pattern tests for R control chart and test data set 5

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
187 to 193	run below centreline		

5.5.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 40](#).



Key

X individuals number
Y moving range value

Figure 40 — Moving range control chart for test data set 5

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 4,281\ 1$$

Out of control situations are given in [Table 34](#).

Table 34 — Results of pattern tests for moving range control chart and test data set 5

Out of control situations			
Value	Result/violation	Value	Result/violation
30	violation of U_{CL}	566 to 577	run below centreline
56 to 65	run below centreline	606	violation of U_{CL}
75	violation of U_{CL}	607	violation of U_{CL}
93 to 103	run below centreline	661 to 667	run above centreline
132	violation of U_{CL}	674 to 681	run below centreline
156 to 164	run below centreline	721	violation of U_{CL}
259 to 266	run below centreline	762	violation of U_{CL}
270	violation of U_{CL}	763	violation of U_{CL}
271	violation of U_{CL}	787	violation of U_{CL}
285 to 293	run below centreline	865	violation of U_{CL}

Table 34 (continued)

Out of control situations			
Value	Result/violation	Value	Result/violation
385	violation of U_{CL}	895	violation of U_{CL}
397	violation of U_{CL}	976 to 987	run below centreline
537 to 546	run below centreline		

5.5.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used			
P_p	1,04	P_{pk}	0,92
P_{pkL}	1,17	P_{pkU}	0,92

In this case the general definition ($M_{3,1}$) on P_p and P_{pk} will give the following results

Capability indices (calculation method $M_{3,1}$)			
Process is not stable (extended mean, R - chart) - P_p/P_{pk} is used			
P_p	1,27	P_{pk}	1,14
P_{pkL}	1,14	P_{pkU}	1,42

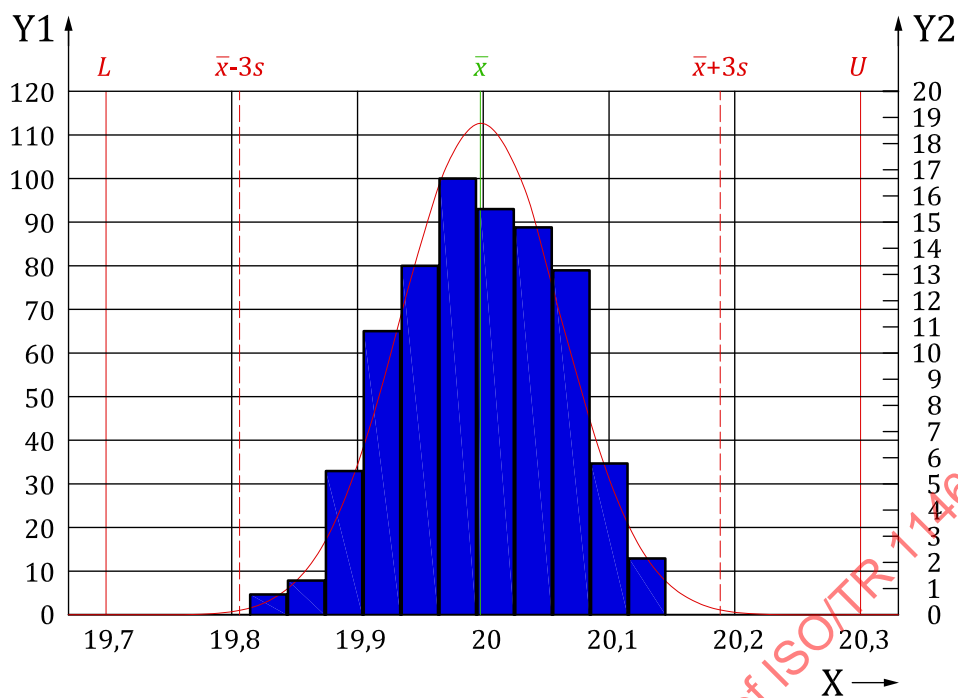
5.6 Test data set 6

5.6.1 Test data set 6 information

This set of test data is taken from a process following a non-normal distribution, showing linear decreasing trend of the parameter μ , and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 6 is given in Table 35. Figure 41 shows a histogram and Figure 42 a probability plot of the data set with the purpose of data visualization.

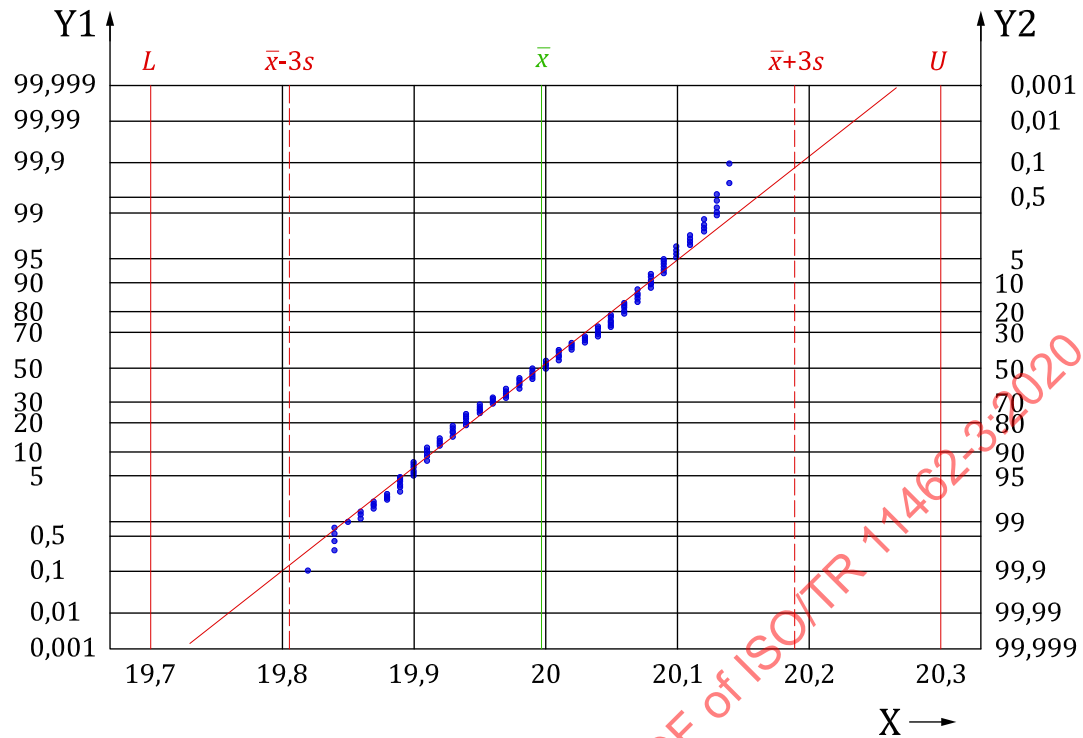
Table 35 — Description of test data set 6

Description of data input			
Distribution model	C3	Resulting distribution	non-normal, unimodal
Data set	Annex A, Table A.1	decimal points	2
Total sample size	600	U	20,3
Size of subgroups	6	L	19,7



Key
X value
Y1 absolute frequency
Y2 relative frequency in %

Figure 41 — Histogram of test data set 6

**Key**

X value

Y1 probability in %

Y2 1-probability in %

Figure 42 — Probability plot of test data set 6

NOTE The width of the class intervals is 0,03.

The class interval with the highest frequency of values is from 19,965 to 19,995.

The density plot is based on the assumption of normality and estimated parameters $l = 1$, $d = 5$.

5.6.2 Test data set 6 results**5.6.2.1 List of sample statistics**

Table 36 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 36 — List of sample statistics for test data set 6

Statistic	Value	Reference
Location		
$\bar{X}$ ($l = 1$)	19,997 3	ISO 22514-2:2017, Formula (11)
$\tilde{X}$ ($l = 2$)	20,000 0	ISO 22514-2:2017, Formula (12)
$\bar{\bar{X}}$ ($l = 3$)	19,997 3	ISO 22514-2:2017, Formula (13)
$\bar{\tilde{X}}$ ($l = 4$)	19,996 9	ISO 22514-2:2017, Formula (14)

Table 36 (continued)

Statistic	Value	Reference
Dispersion		
$\hat{\Delta}$ ($d = 1$)	0,382 4 (using normal distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	0,030 1	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	0,030 1	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	0,029 9	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	0,063 7	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,028 7	ISO 7870-2:2013, 3.2
R	0,320 0	ISO 7870-2:2013, 3.2
$\bar{R}$	0,075 8	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,033 7	ISO 7870-2:2013, 3.2

5.6.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 43](#).

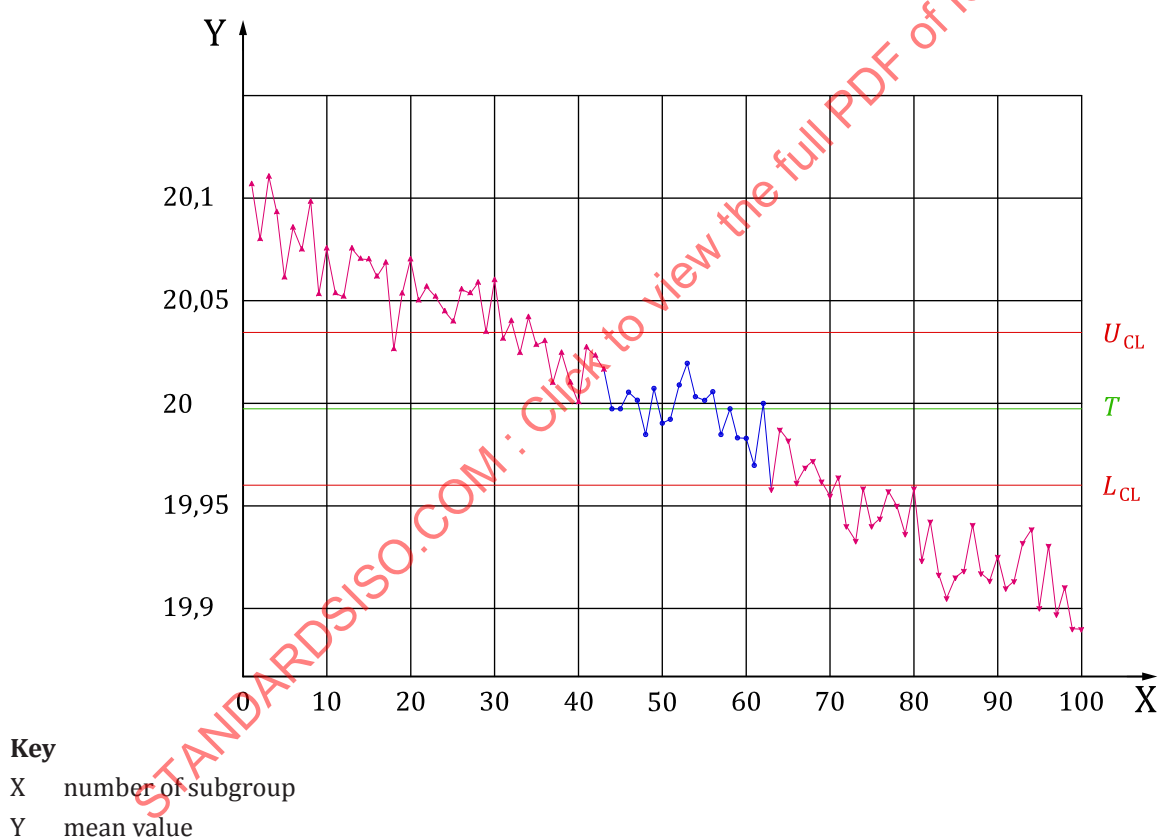


Figure 43 — Mean control chart for test data set 6

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = 19,960\ 4$$

$$U_{CL} = 20,034\ 2$$

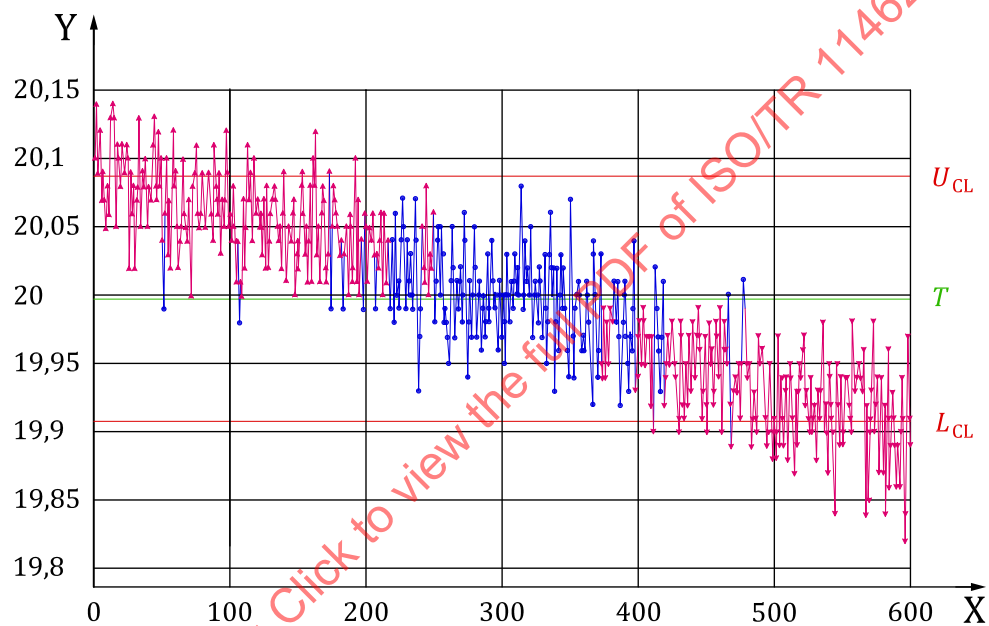
Out of control situations are given in [Table 37](#).

Table 37 — Results of pattern tests for the mean control chart and test data set 6

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
1 to 43	run above centreline	63 to 100	run below centreline
1 to 17	violation of U_{CL}	63	violation of L_{CL}
19 to 30	violation of U_{CL}	70	violation of L_{CL}
32	violation of U_{CL}	72 to 100	violation of L_{CL}
34	violation of U_{CL}		

5.6.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 44](#).

**Key**

X value number
Y individuals

Figure 44 — Individuals control chart for test data set 6

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 19,907\ 6$$

$$U_{CL} = 20,087\ 0$$

Out of control situations are given in [Table 38](#).

Table 38 — Results of pattern tests for individuals control chart and test data set 6

Out of control situations			
Value	Result/violation	Value	Result/violation
1 to 50	run above centreline	398 to 411	run below centreline
1 to 5	violation of U_{CL}	411	violation of L_{CL}
7	violation of U_{CL}	419 to 465	run below centreline

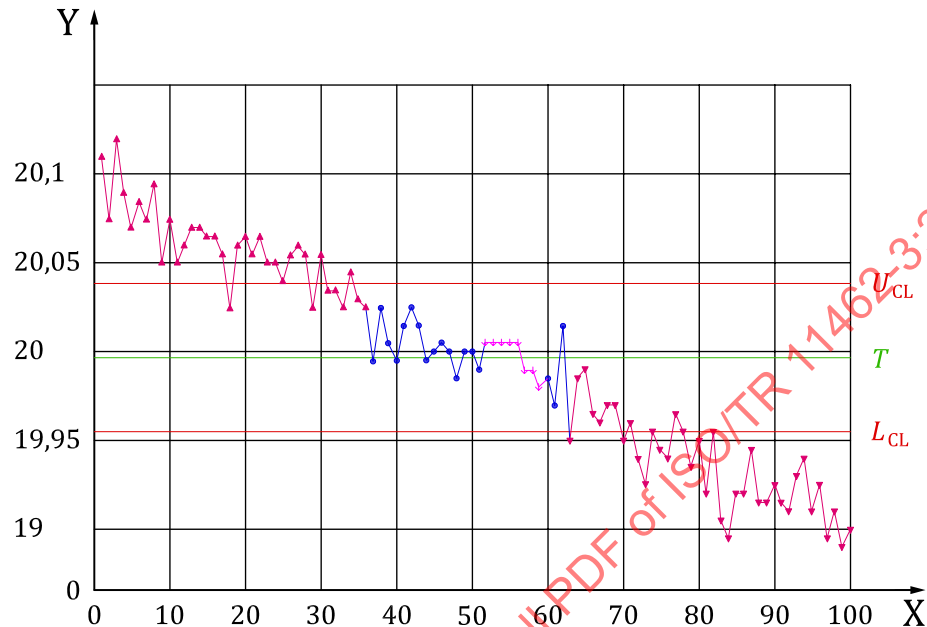
Table 38 (continued)

Out of control situations			
Value	Result/violation	Value	Result/violation
12 to 15	violation of U_{CL}	450	violation of L_{CL}
17	violation of U_{CL}	461	violation of L_{CL}
18	violation of U_{CL}	467 to 476	run below centreline
21 to 25	violation of U_{CL}	468	violation of L_{CL}
27	violation of U_{CL}	478 to 600	run below centreline
33	violation of U_{CL}	483	violation of L_{CL}
35	violation of U_{CL}	487	violation of L_{CL}
36	violation of U_{CL}	494	violation of L_{CL}
38	violation of U_{CL}	495	violation of L_{CL}
43	violation of U_{CL}	498	violation of L_{CL}
44	violation of U_{CL}	499	violation of L_{CL}
47	violation of U_{CL}	501	violation of L_{CL}
49	violation of U_{CL}	502	violation of L_{CL}
52 to 106	run above centreline	504	violation of L_{CL}
53	violation of U_{CL}	507	violation of L_{CL}
58	violation of U_{CL}	510	violation of L_{CL}
60	violation of U_{CL}	515	violation of L_{CL}
65	violation of U_{CL}	517	violation of L_{CL}
74	violation of U_{CL}	526	violation of L_{CL}
75	violation of U_{CL}	528	violation of L_{CL}
79	violation of U_{CL}	531	violation of L_{CL}
83	violation of U_{CL}	537	violation of L_{CL}
84	violation of U_{CL}	539	violation of L_{CL}
88	violation of U_{CL}	544	violation of L_{CL}
92	violation of U_{CL}	545	violation of L_{CL}
97	violation of U_{CL}	548	violation of L_{CL}
98	violation of U_{CL}	550	violation of L_{CL}
108 to 173	run above centreline	551	violation of L_{CL}
113	violation of U_{CL}	557	violation of L_{CL}
115	violation of U_{CL}	567	violation of L_{CL}
118	violation of U_{CL}	569	violation of L_{CL}
140	violation of U_{CL}	570	violation of L_{CL}
153	violation of U_{CL}	574	violation of L_{CL}
161	violation of U_{CL}	577	violation of L_{CL}
163	violation of U_{CL}	580	violation of L_{CL}
173	violation of U_{CL}	582	violation of L_{CL}
175 to 182	run above centreline	585	violation of L_{CL}
184 to 197	run above centreline	586	violation of L_{CL}
192	violation of U_{CL}	589 to 593	violation of L_{CL}
199 to 206	violation of U_{CL}	596	violation of L_{CL}
208 to 216	run above centreline	597	violation of L_{CL}
241 to 249	run above centreline	600	violation of L_{CL}
373 to 381	run below centreline		

In this case an extended normal distribution can be used.

5.6.2.4 $\bar{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 45](#).



Key

X subgroup number
Y median value

Figure 45 — Median control chart for test data set 6

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = 19,955\ 4$$

$$U_{CL} = 20,038\ 4$$

Out of control situations are given in [Table 39](#).

Table 39 — Results of pattern tests for median control chart and test data set 6

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
1 to 36	run above centreline	63 to 100	run below centreline
1 to 17	violation of U_{CL}	63	violation of L_{CL}
19 to 28	violation of U_{CL}	70	violation of L_{CL}
30	violation of U_{CL}	72 to 76	violation of L_{CL}
34	violation of U_{CL}	78 to 100	violation of L_{CL}
52 to 59	trend decreasing		

This process is a trend process. Therefore, the control limits can be expanded with 0,2.

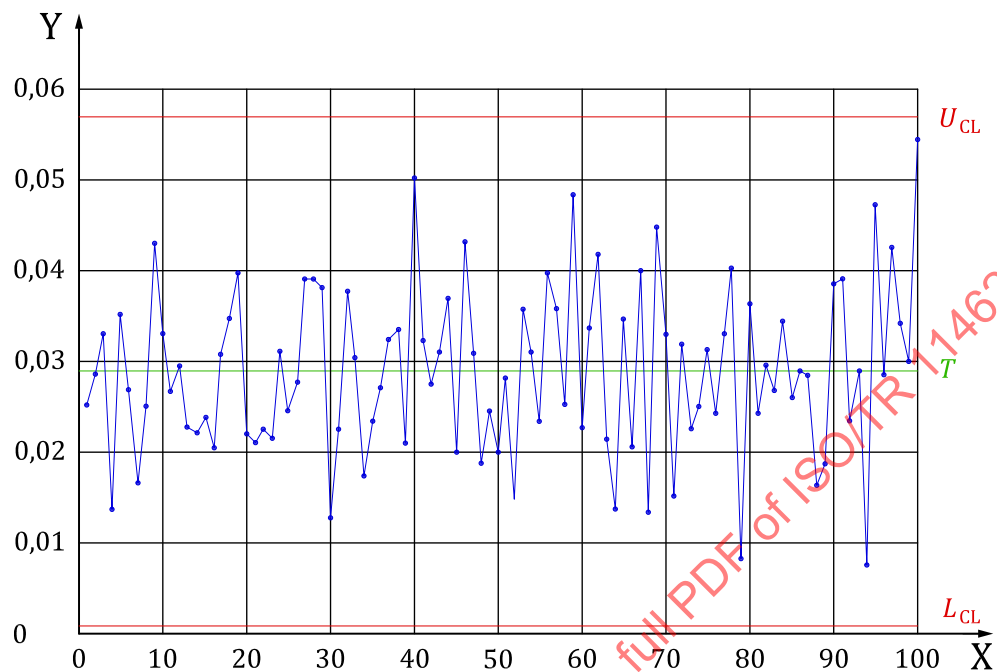
Then the control limits will be

$$L_{CL} = 19,855\ 4$$

$$U_{CL} = 20,138\ 4$$

5.6.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 46](#).



Key

- X subgroup number
Y empirical standard deviation s value

Figure 46 — s control chart for test data set 6

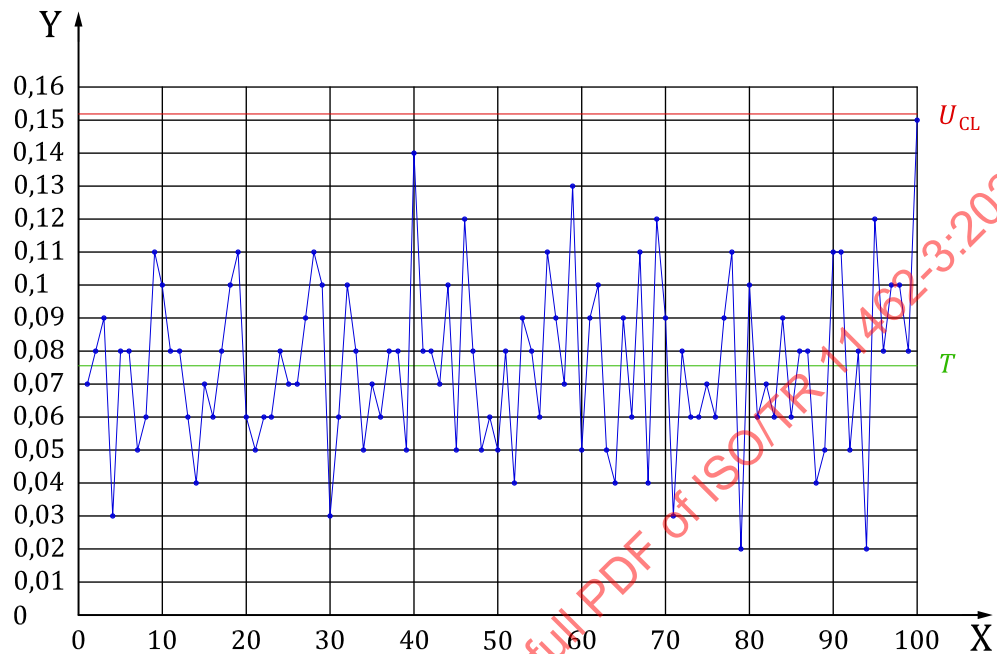
The control limits are (see ISO 7870-2:2013, Table 1)

$$L_{CL} = 0,000\ 86$$

$$U_{CL} = 0,056\ 5$$

5.6.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in Figure 47.



Key

X subgroup number

Y range R value

Figure 47 — R control chart for test data set 6

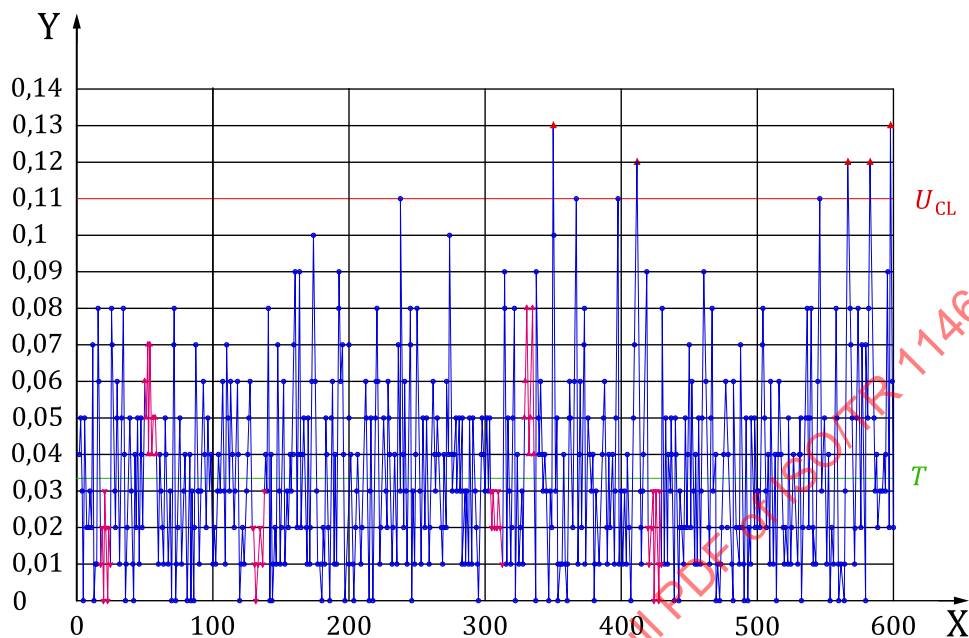
The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,1519$$

5.6.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 48](#).



Key

X individuals number
Y moving range value

Figure 48 — Moving range control chart for test data set 6

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 0,110\ 2$$

Out of control situations are given in [Table 40](#).

Table 40 — Results of pattern tests for moving range control chart and test data set 6

Out of control situations			
Value	Result/violation	Value	Result/violation
18 to 25	run below centreline	412	violation of U_{CL}
50 to 59	run above centreline	420 to 430	violation of U_{CL}
129 to 138	run below centreline	567	violation of U_{CL}
304 to 313	run below centreline	583	violation of U_{CL}
329 to 336	run above centreline	598	violation of U_{CL}
350	violation of U_{CL}		

5.6.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used			
P_p	1,57	P_{pk}	1,55
P_{pkL}	1,58	P_{pkU}	1,55

In this case an extended normal distribution can be used as calculation method ($M_{3,1}$).

Capability indices (calculation method $M_{3,1}$)			
Process is not stable (extended mean, R - chart) - P_p/P_{pk} is used			
P_p	1,83	P_{pk}	1,83
P_{pkL}	1,85	P_{pkU}	1,83

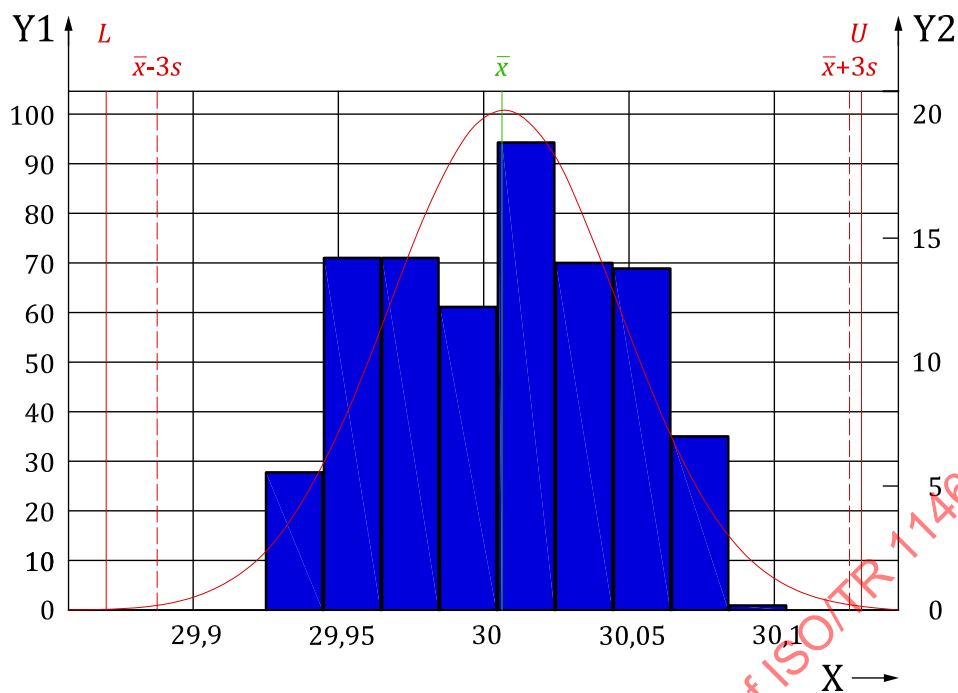
5.7 Test data set 7

5.7.1 Test data set 7 information

This set of test data is taken from a process following a non-normal distribution which has five shifts of the parameter μ over time and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 7 is given in [Table 41](#). [Figure 49](#) shows a histogram and [Figure 50](#) a probability plot of the data set with the purpose of data visualization.

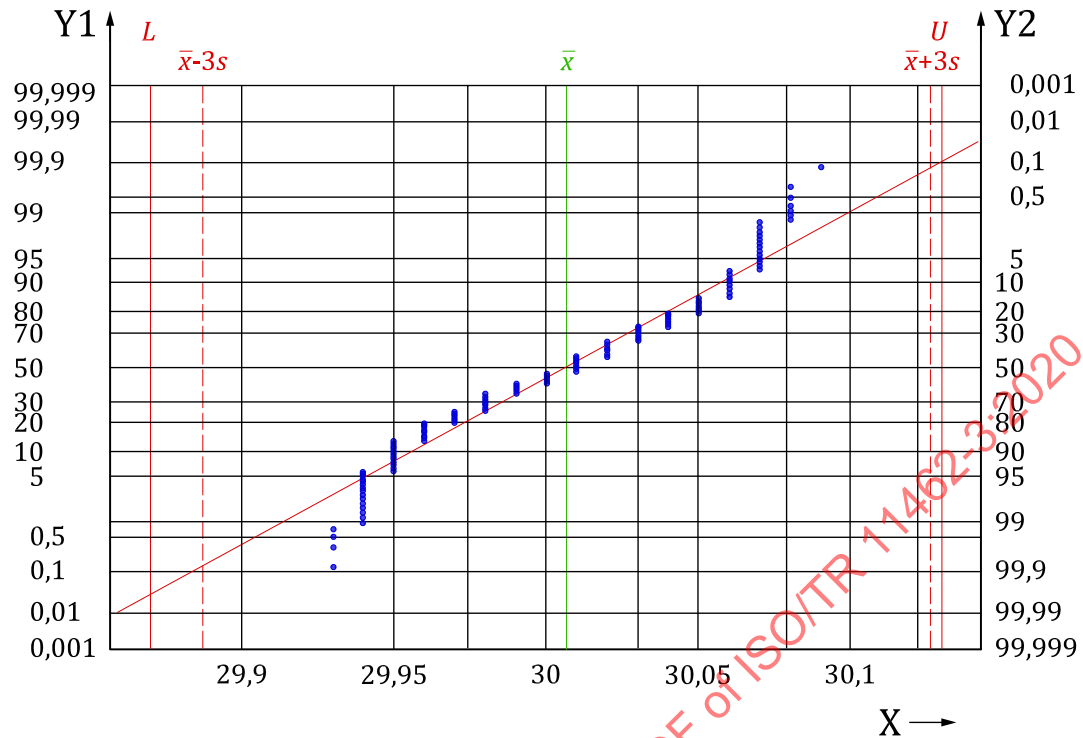
Table 41 — Description of test data set 7

Description of data input			
Distribution model	C4	Resulting distribution	non-normal, unimodal
Data set	Annex A, Table A.1	decimal points	2
Total sample size	500	U	30,13
Size of subgroups	5	L	29,87



Key
X value
Y1 absolute frequency
Y2 relative frequency in %

Figure 49 — Histogram of test data set 7

**Key**

X value

Y1 probability in %

Y2 1-probability in %

Figure 50 — Probability plot of test data set 7

NOTE The width of the class intervals is 0,02.

The class interval with the highest frequency of values is from 30,005 to 30,025.

The density plot is based on the assumption of normality and estimated parameters $l = 1$, $d = 5$.

5.7.2 Test data set 7 results**5.7.2.1 List of sample statistics**

Table 42 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 42 — List of sample statistics for test data set 7

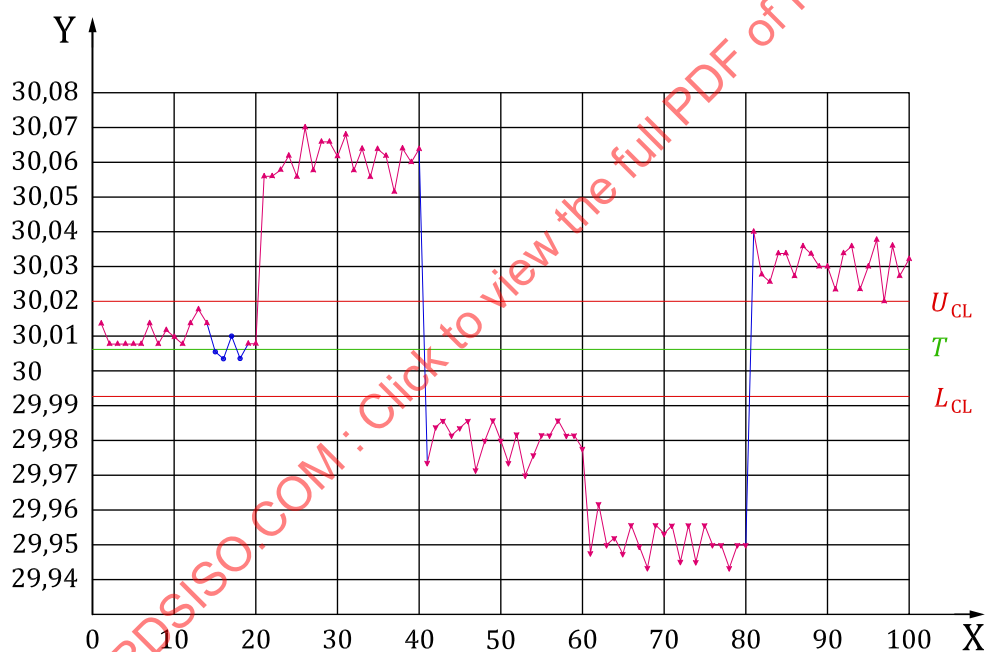
Statistic	Value	Reference
Location		
$\bar{X}$ ($l = 1$)	30,006 7	ISO 22514-2:2017, Formula (11)
$\tilde{X}$ ($l = 2$)	30,010 0	ISO 22514-2:2017, Formula (12)
$\bar{\bar{X}}$ ($l = 3$)	30,006 7	ISO 22514-2:2017, Formula (13)
$\bar{\tilde{X}}$ ($l = 4$)	30,006 3	ISO 22514-2:2017, Formula (14)

Table 42 (continued)

Statistic	Value	Reference
Dispersion		
$\hat{\Delta}$ ($d = 1$)	0,238 2 (normal distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	0,010 3	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	0,010 2	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	0,009 7	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	0,039 7	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,009 6	ISO 7870-2:2013, 3.2
R	0,160 0	ISO 7870-2:2013, 3.2
$\bar{R}$	0,022 6	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,011 7	ISO 7870-2:2013, 3.2

5.7.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 51.



Key

X number of subgroup
Y mean value

Figure 51 — Mean control chart for test data set 7

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = 29,993\ 0$$

$$U_{CL} = 30,020\ 3$$

Out of control situations are given in Table 43.

Table 43 — Results of pattern tests for the mean control chart and test data set 7

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
1 to 14	run above centreline	41 to 80	violation of L_{CL}
19 to 40	run above centreline	81 to 100	run above centreline
21 to 40	violation of U_{CL}	81 to 96	violation of U_{CL}
41 to 80	run below centreline	98 to 100	violation of U_{CL}

The mean in this dataset is not stable. Therefore, a multimodal distribution model can be used.

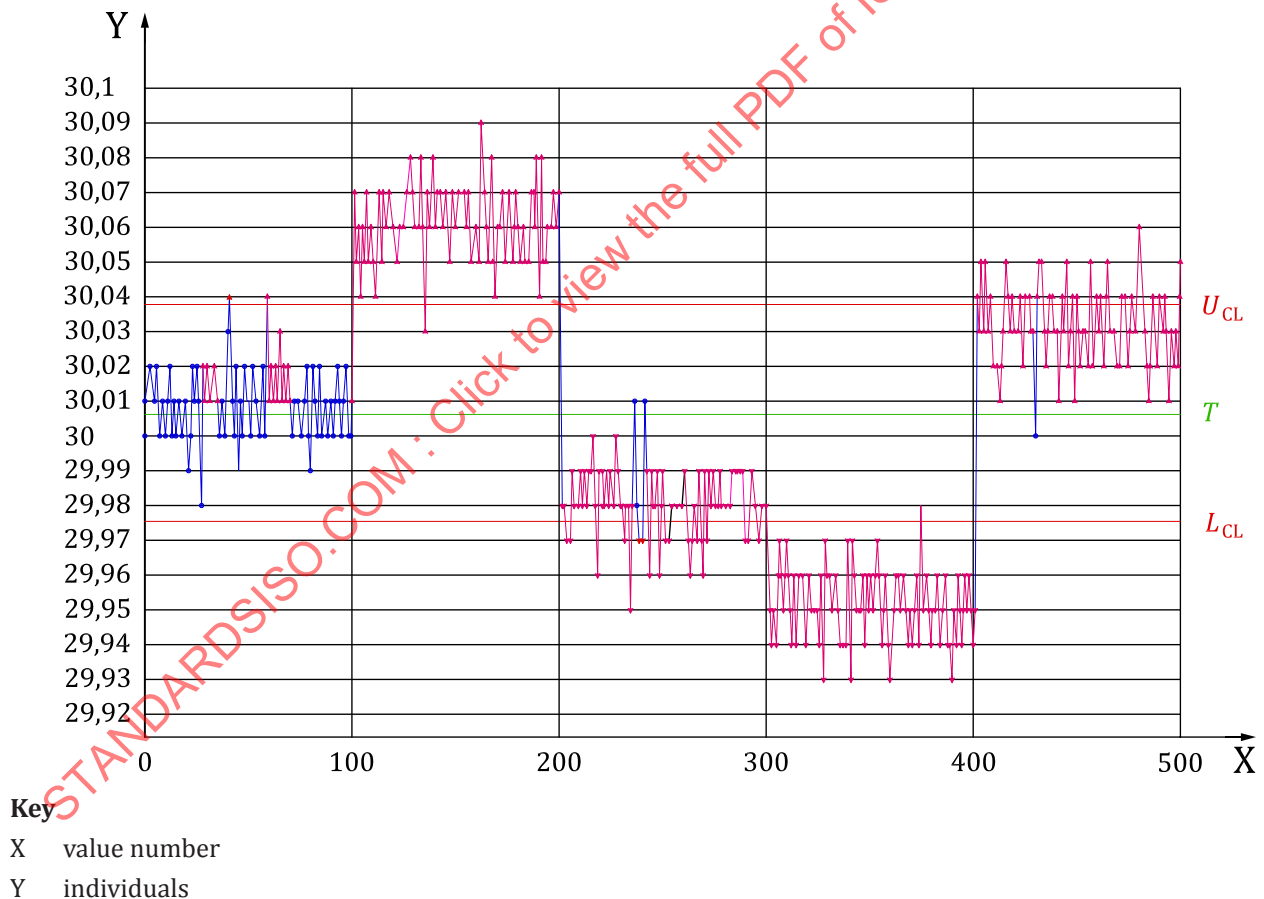
Therefore, the control chart is expanded with 0,1.

$$L_{CL} = 29,943\ 0$$

$$U_{CL} = 30,070\ 3$$

5.7.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 52](#).

**Figure 52 — Individuals control chart for test data set 7**

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 29,975\ 5$$

$$U_{CL} = 30,037\ 8$$

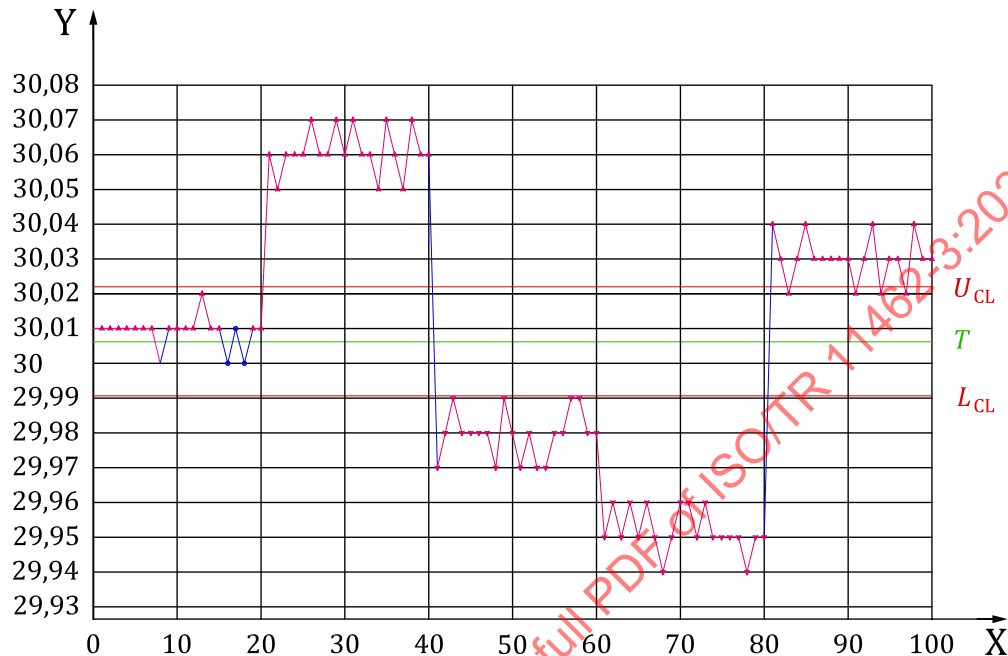
Out of control situations are given in [Table 44](#).

Table 44 — Results of pattern tests for individuals control chart and test data set 7

Out of control situations			
Value	Result/violation	Value	Result/violation
28 to 35	run above centreline	403	violation of U_{CL}
41	violation of U_{CL}	405	violation of U_{CL}
59 to 70	run above centreline	408	violation of U_{CL}
59	violation of U_{CL}	415	violation of U_{CL}
100 to 200	run above centreline	416	violation of U_{CL}
101 to 134	violation of U_{CL}	418	violation of U_{CL}
121 to 128	Trend increasing	422	violation of U_{CL}
136 to 200	violation of U_{CL}	424 to 426	violation of U_{CL}
201 to 235	run below centreline	430 to 500	run above centreline
203 to 205	violation of L_{CL}	430 to 432	violation of U_{CL}
218	violation of L_{CL}	436	violation of U_{CL}
231	violation of L_{CL}	437	violation of U_{CL}
234	violation of L_{CL}	442	violation of U_{CL}
238 to 240	violation of L_{CL}	444	violation of U_{CL}
242 to 400	run below centreline	447	violation of U_{CL}
243	violation of L_{CL}	449	violation of U_{CL}
248	violation of L_{CL}	456	violation of U_{CL}
251 to 253	violation of L_{CL}	459	violation of U_{CL}
251 to 260	trend increasing	461	violation of U_{CL}
261 to 264	violation of L_{CL}	463	violation of U_{CL}
266	violation of L_{CL}	464	violation of U_{CL}
268	violation of L_{CL}	471	violation of U_{CL}
269	violation of L_{CL}	472	violation of U_{CL}
271	violation of L_{CL}	476	violation of U_{CL}
278 to 291	trend increasing	479	violation of U_{CL}
283 to 288	trend decreasing	486	violation of U_{CL}
289 to 291	violation of L_{CL}	487	violation of U_{CL}
296	violation of L_{CL}	489	violation of U_{CL}
301 to 374	violation of L_{CL}	490	violation of U_{CL}
375 to 400	violation of L_{CL}	492	violation of U_{CL}
401 to 428	run above centreline	499	violation of U_{CL}
401	violation of U_{CL}	500	violation of U_{CL}

5.7.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 53](#).



Key

X subgroup number
Y median value

Figure 53 — Median control chart for test data set 7

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = 29,990\ 7$$

$$U_{CL} = 30,021\ 9$$

Out of control situations are given in [Table 45](#).

Table 45 — Results of pattern tests median control chart and test data set 7

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
1 to 7	run above centreline	81	violation of U_{CL}
1 to 8	trend increasing	82	violation of U_{CL}
9 to 15	run above centreline	84 to 90	violation of U_{CL}
19 to 40	run above centreline	92	violation of U_{CL}
21 to 40	violation of U_{CL}	93	violation of U_{CL}
41 to 80	run below centreline	95	violation of U_{CL}
41 to 80	violation of L_{CL}	96	violation of U_{CL}
81 to 100	run above centreline	98 to 100	violation of U_{CL}

The mean in this dataset is not stable. Therefore, a multimodal distribution model can be used.

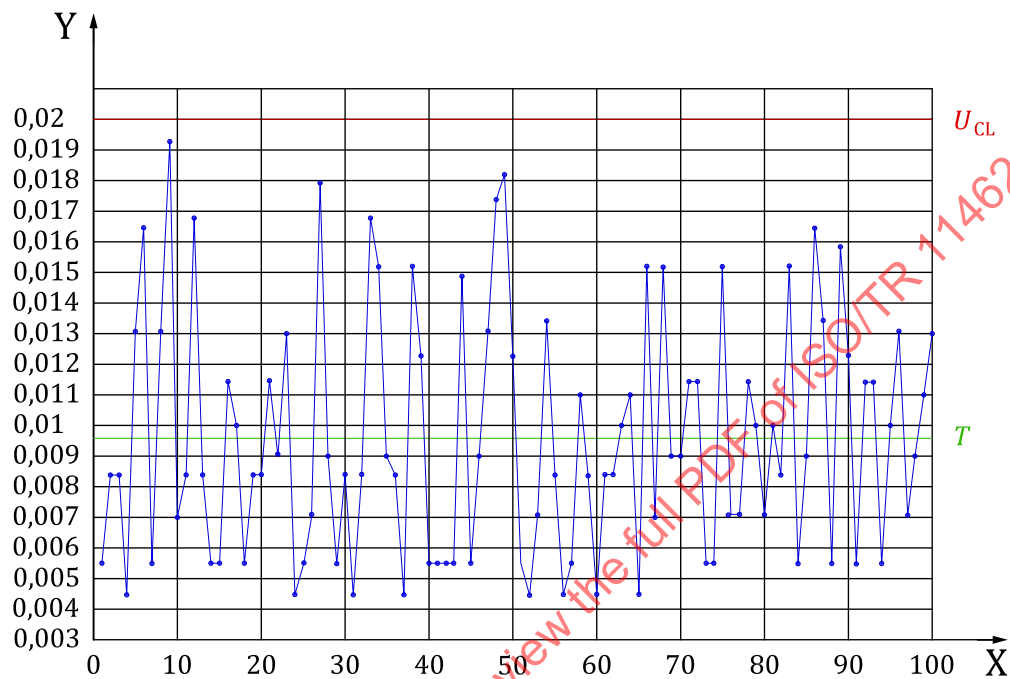
Therefore, the control chart is expanded with 0,1

$$L_{CL} = 29,940\ 7$$

$$U_{CL} = 30,071\ 9$$

5.7.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 54](#).



Key

X subgroup number

Y empirical standard deviation s value

Figure 54 — s control chart for test data set 7

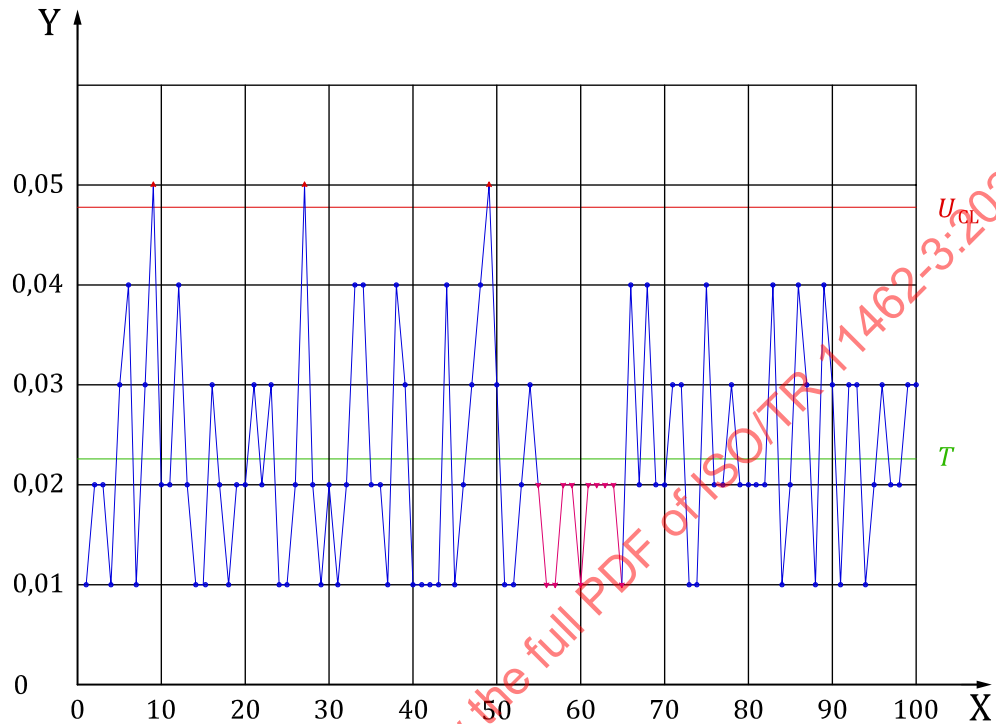
The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,020\ 0$$

5.7.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 55](#).



Key

X subgroup number
Y range R value

Figure 55 — R control chart for test data set 7

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,0478$$

Out of control situations are given in [Table 46](#).

Table 46 — Results of pattern tests R control chart and test data set 7

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
9	violation of U_{CL}	49	violation of U_{CL}
27	violation of U_{CL}	55 to 65	run below centreline

5.7.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 56](#).

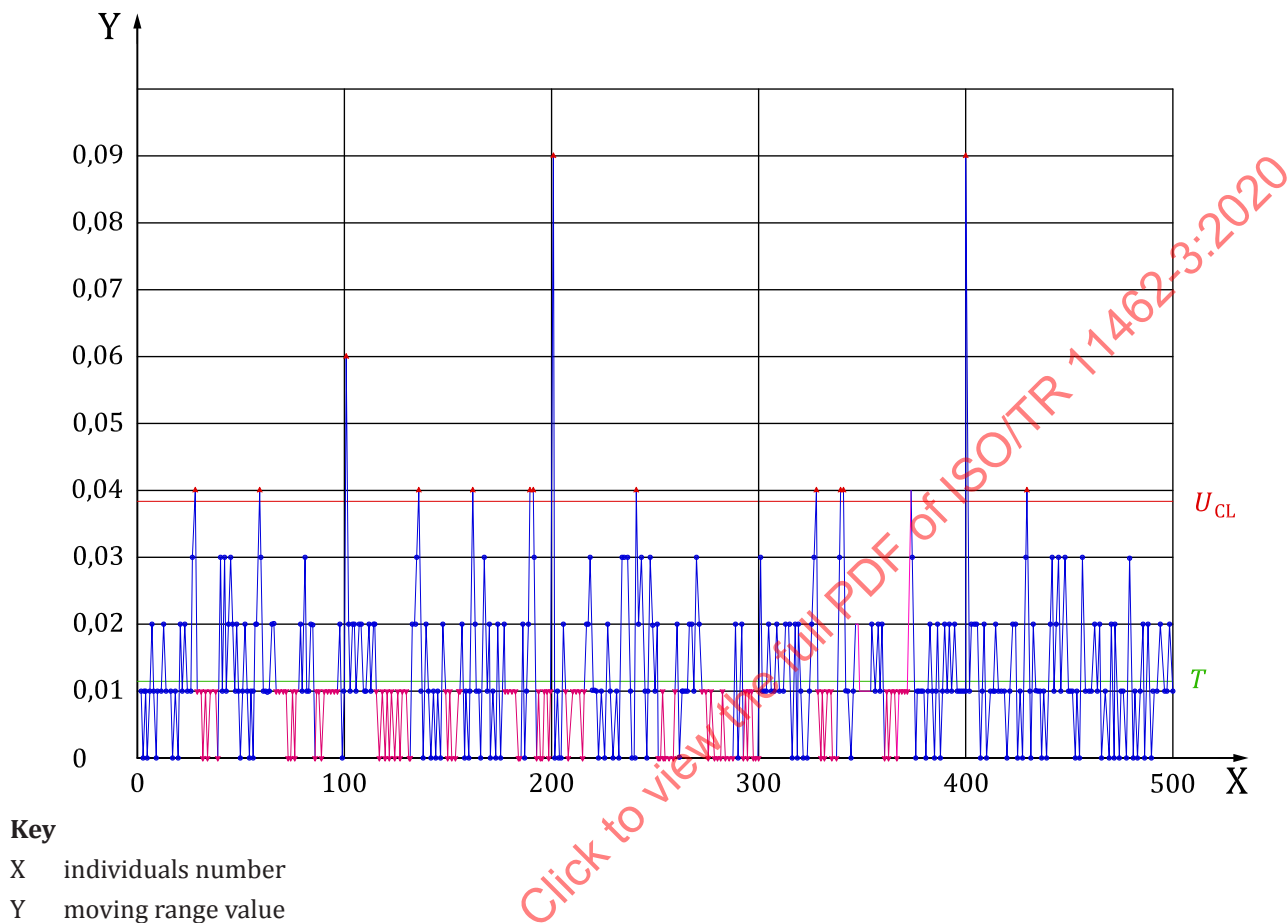


Figure 56 — Moving range control chart for test data set 7

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 0,038\ 2$$

Out of control situations are given in [Table 47](#).

Table 47 — Results of pattern tests moving range control chart and test data set 7

Out of control situations			
Value	Result/violation	Value	Result/violation
28	violation of U_{CL}	207 to 216	run below centreline
29 to 39	run below centreline	241	violation of U_{CL}
59	violation of U_{CL}	252 to 260	run below centreline
67 to 78	run below centreline	273 to 288	run below centreline
86 to 97	run below centreline	293 to 300	run below centreline
101	violation of U_{CL}	328	violation of U_{CL}

Table 47 (continued)

Out of control situations			
Value	Result/violation	Value	Result/violation
116 to 132	run below centreline	329 to 338	run below centreline
136	violation of U_{CL}	340	violation of U_{CL}
149 to 156	run below centreline	341	violation of U_{CL}
162	violation of U_{CL}	347 to 354	trend decreasing
178 to 185	run below centreline	361 to 372	run below centreline
190	violation of U_{CL}	367 to 374	trend increasing
191	violation of U_{CL}	374	violation of U_{CL}
193 to 200	run below centreline	401	violation of U_{CL}
201	violation of U_{CL}	430	violation of U_{CL}

5.7.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used			
P_p	1,09	P_{pk}	1,04
P_{pkL}	1,15	P_{pkU}	1,04

In this case an multimodal distribution can be used and the calculation based on $M_{3,1}$

Capability indices (calculation method $M_{3,1}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used			
P_p	1,54	P_{pk}	1,52
P_{pkL}	1,55	P_{pkU}	1,52

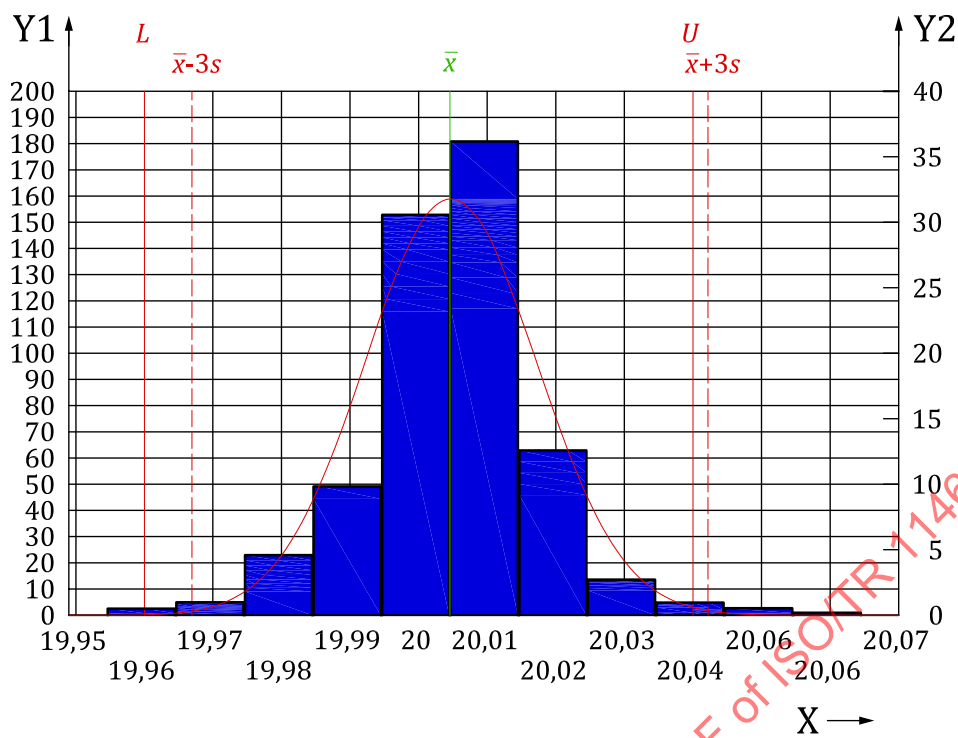
5.8 Test data set 8

5.8.1 Test data set 8 information

This set of test data is taken from a process following a non-normal distribution and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 8 is given in Table 48. Figure 57 shows a histogram and Figure 58 a probability plot of the data set with the purpose of data visualization.

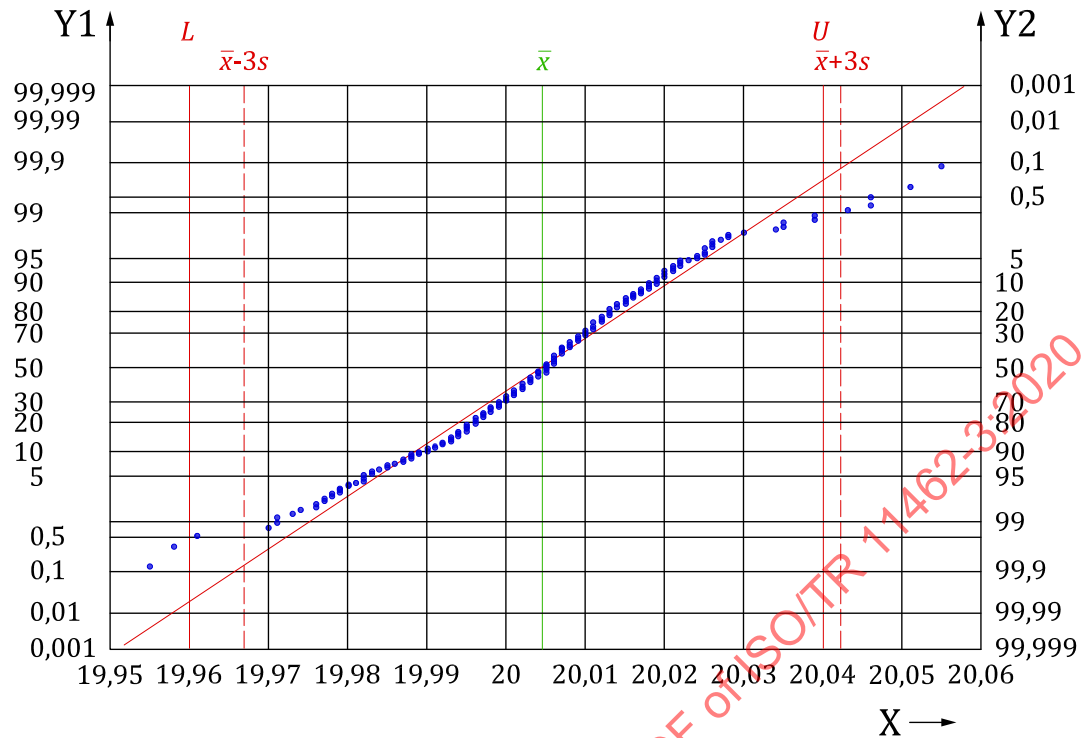
Table 48 — Description of test data set 8

Description of data input			
Distribution model	D	Resulting distribution	non-normal
Data set	Annex A, Table A.1	decimal points	2
Total sample size	500	U	20,04
Size of subgroups	5	L	19,96



Key
X value
Y1 absolute frequency
Y2 relative frequency in %

Figure 57 — Histogram of test data set 8

**Key**

X value

Y1 probability in %

Y2 1-probability in %

Figure 58 — Probability plot of test data set 8

NOTE The width of the class intervals is 0,01.

The class interval with the highest frequency of values is from 20,004 5 to 20,014 5.

The density plot is based on the assumption of normality and estimated parameters $l = 1$, $d = 5$.

5.8.2 Test data set 8 results**5.8.2.1 List of sample statistics**

Table 49 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 49 — List of sample statistics for test data set 8

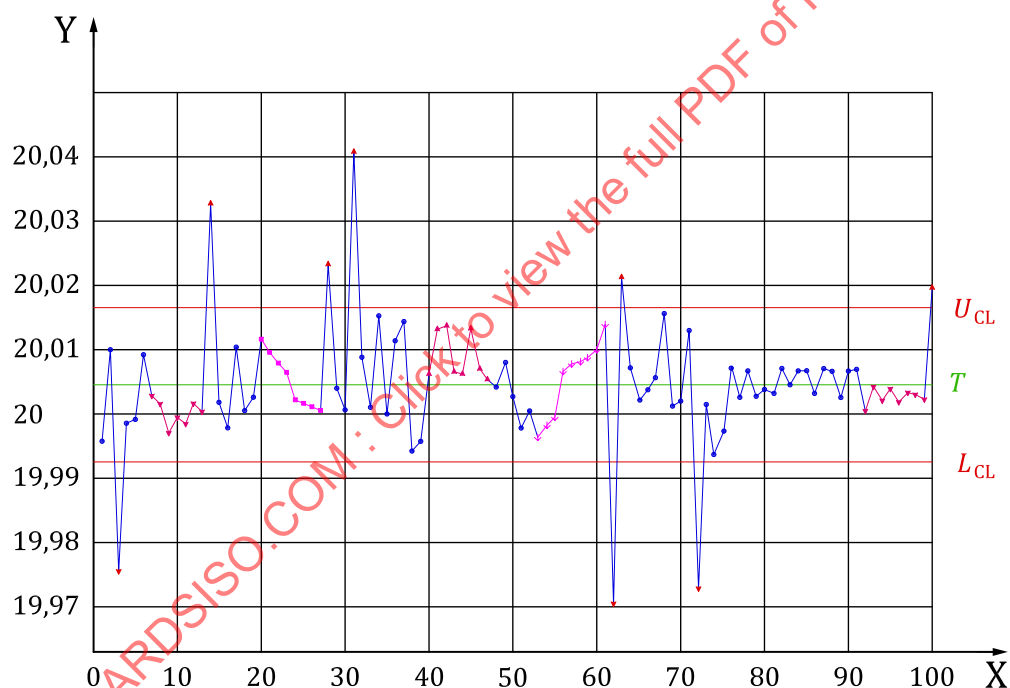
Statistic	Value	Reference
Location		
$\bar{X}$ ($l = 1$)	20,004 53	ISO 22514-2:2017, Formula (11)
$\tilde{X}$ ($l = 2$)	20,005	ISO 22514-2:2017, Formula (12)
$\bar{\bar{X}}$ ($l = 3$)	20,004 53	ISO 22514-2:2017, Formula (13)
$\bar{\tilde{X}}$ ($l = 4$)	20,005 46	ISO 22514-2:2017, Formula (14)

Table 49 (continued)

Statistic	Value	Reference
Dispersion		
$\hat{\Delta}$ ($d = 1$)	0,075 32 (normal distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	0,009 68	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	0,008 89	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	0,008 82	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	0,012 55	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,008 35	ISO 7870-2:2013, 3.2
R	0,1	ISO 7870-2:2013, 3.2
$\bar{R}$	0,020 52	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,010 85	ISO 7870-2:2013, 3.2

5.8.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 59.



Key

- X number of subgroup
Y mean value

Figure 59 — Mean control chart for test data set 8

The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = 19,992\ 61$$

$$U_{CL} = 20,016\ 45$$

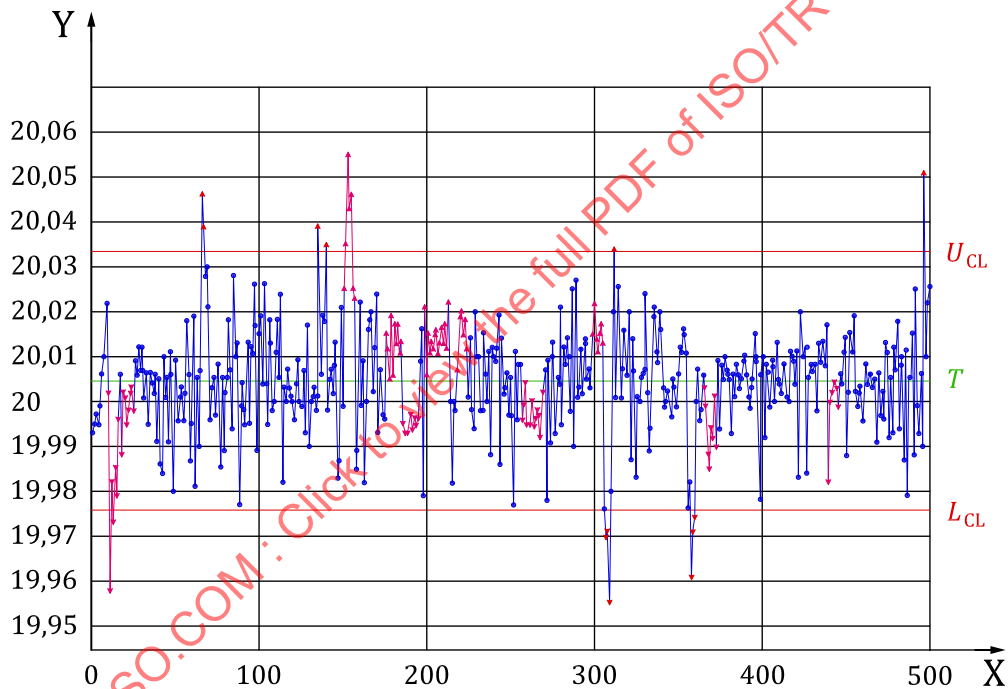
Out of control situations are given in Table 50.

Table 50 — Results of pattern tests for the mean control chart and test data set 8

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
3	violation of L_{CL}	53 to 61	trend increasing
7 to 13	run below centreline	62	violation of L_{CL}
14	violation of U_{CL}	63	violation of U_{CL}
20 to 27	trend decreasing	72	violation of L_{CL}
28	violation of U_{CL}	92 to 99	run below centreline
31	violation of U_{CL}	100	violation of U_{CL}
40 to 47	run above centreline		

5.8.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 60](#).

**Key**

X value number

Y individuals

Figure 60 — Individuals control chart for test data set 8

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 19,975\ 67$$

$$U_{CL} = 20,033\ 39$$

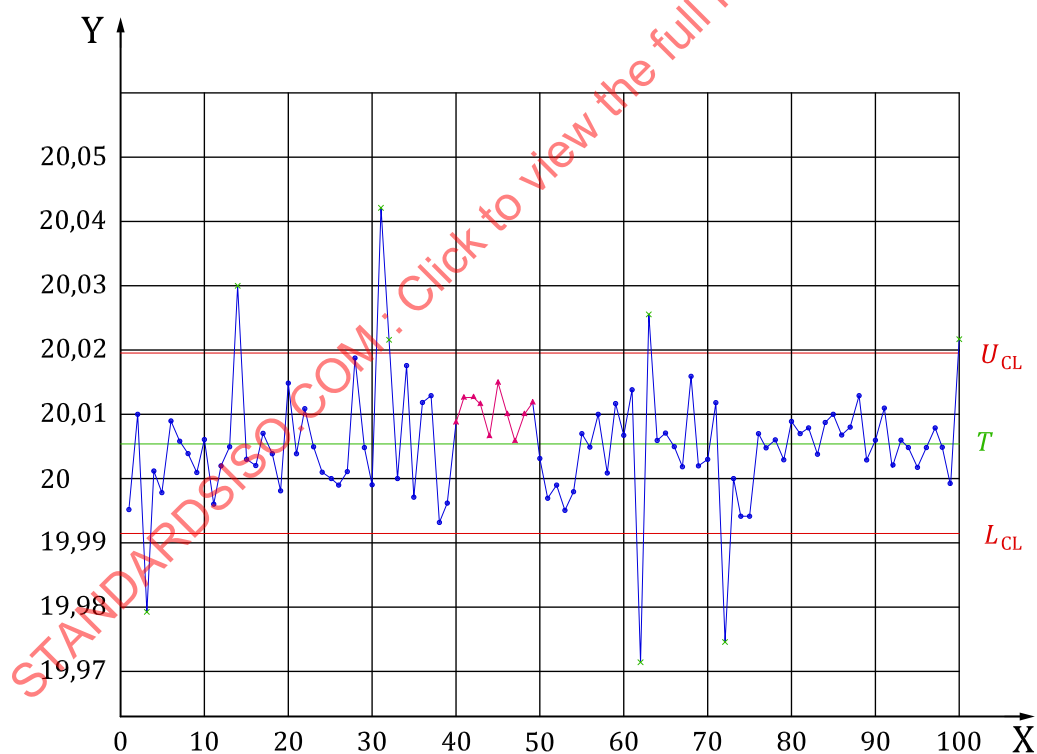
Out of control situations are given in [Table 51](#).

Table 51 — Results of pattern tests individuals control chart and test data set 8

Out of control situations			
Value	Result/violation	Value	Result/violation
10 to 16	run below centreline	186 to 195	run below centreline
11	violation of L_{CL}	199 to 213	run above centreline
13	violation of L_{CL}	218 to 224	run above centreline
18 to 25	run below centreline	257 to 270	run below centreline
66	violation of U_{CL}	299 to 305	run above centreline
67	violation of U_{CL}	307 to 309	violation of L_{CL}
136	violation of U_{CL}	312	violation of U_{CL}
140	violation of U_{CL}	358 to 360	violation of L_{CL}
151 to 157	run above centreline	366 to 373	run below centreline
152 to 155	violation of U_{CL}	440 to 446	run below centreline
176 to 185	run above centreline	497	violation of U_{CL}

5.8.2.4 $\bar{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 61](#).



Key

X subgroup number

Y median value

Figure 61 — Median control chart for test data set 8

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = 19,991\,28$$

$$U_{CL} = 20,019\ 64$$

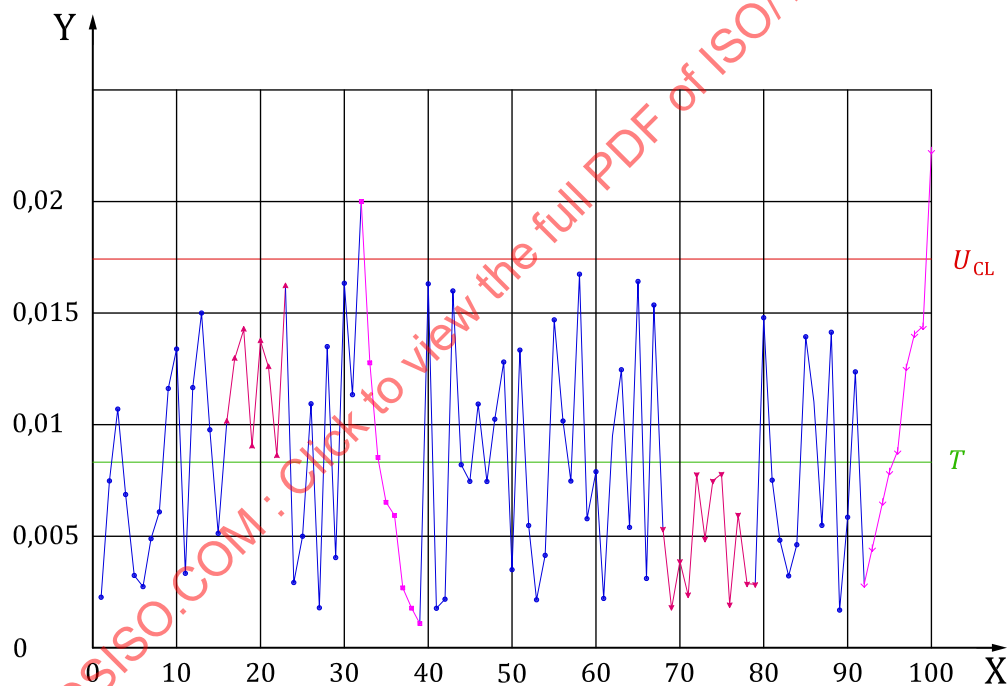
Out of control situations are given in [Table 52](#).

Table 52 — Results of pattern tests median control chart and test data set 8

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
3	violation of L_{CL}	62	violation of L_{CL}
14	violation of U_{CL}	63	violation of U_{CL}
31	violation of U_{CL}	72	violation of L_{CL}
32	violation of U_{CL}	100	violation of U_{CL}
40 to 49	run above centreline		

5.8.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 62](#).



Key

X subgroup number

Y empirical standard deviation s value

Figure 62 — s control chart for test data set 8

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,017\ 45$$

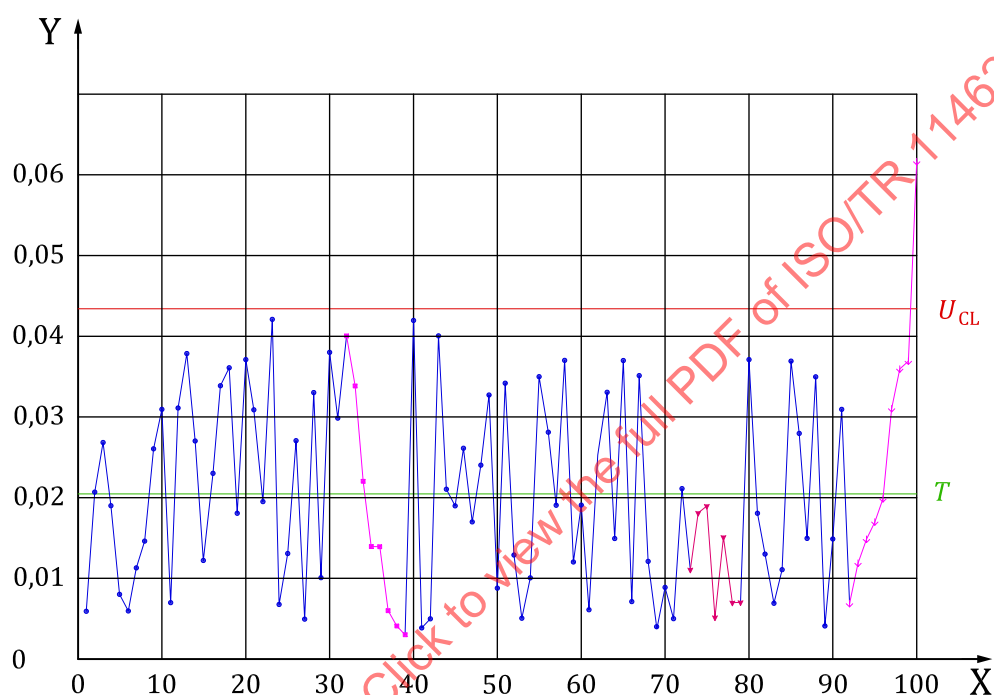
Out of control situations are given in [Table 53](#).

Table 53 — Results of pattern tests for the $\bar{x}$ control chart and test data set 8

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
16 to 23	run above centreline	68 to 79	run below centreline
32 to 39	trend decreasing	92 to 100	trend decreasing
32	violation of U_{CL}	100	violation of U_{CL}

5.8.2.6 R chart

The R control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 63](#).

**Key**

X subgroup number
Y range R value

Figure 63 — R control chart for test data set 8

The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,043\,38$$

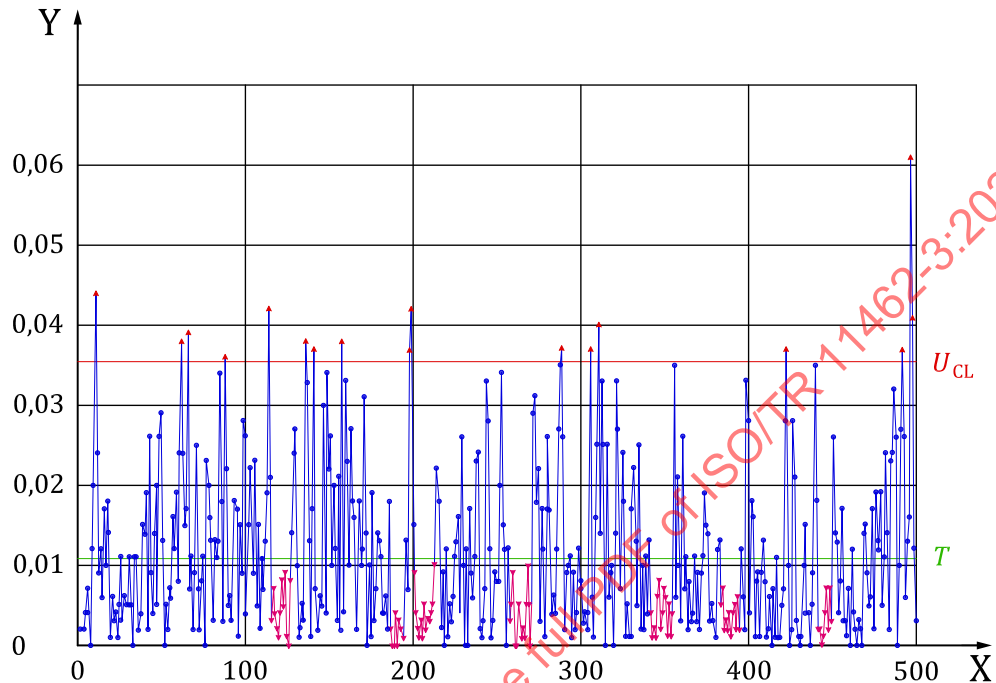
Out of control situations are given in [Table 54](#).

Table 54 — Results of pattern tests R control chart and test data set 8

Out of control situations			
Subgroup	Result/violation	Subgroup	Result/violation
32 to 39	trend decreasing	92 to 100	trend increasing
73 to 79	run below centreline	100	violation of U_{CL}

5.8.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 64](#).



Key

X individuals number

Y moving range value

Figure 64 — Moving range control chart for test data set 8

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 0,035\ 45$$

Out of control situations are given in [Table 55](#).

Table 55 — Results of pattern tests moving range control chart and test data set 8

Out of control situations			
Value	Result/violation	Value	Result/violation
11	violation of U_{CL}	201 to 213	run below centreline
62	violation of U_{CL}	258 to 271	run below centreline
66	violation of U_{CL}	289	violation of U_{CL}
88	violation of U_{CL}	306	violation of U_{CL}
114	violation of U_{CL}	311	violation of U_{CL}
116 to 127	run below centreline	342 to 355	run below centreline
136	violation of U_{CL}	384 to 395	run below centreline
141	violation of U_{CL}	423	violation of U_{CL}
158	violation of U_{CL}	442 to 450	run below centreline

Table 55 (continued)

Out of control situations			
Value	Result/violation	Value	Result/violation
187 to 195	run below centreline	492	violation of U_{CL}
198	violation of U_{CL}	497	violation of U_{CL}
199	violation of U_{CL}	498	violation of U_{CL}

5.8.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is not stable (mean, R - chart) - P_p/P_{pk} is used			
P_p	1,06	P_{pk}	0,94
P_{pkL}	1,18	P_{pkU}	0,94

In this case an extended normal distribution can be used and calculation method $M_{3,1}$.

Capability indices (calculation method $M_{3,1}$)			
Process is not stable (extended mean, R - chart) - P_p/P_{pk} is used			
P_p	1,03	P_{pk}	0,91
P_{pkL}	1,14	P_{pkU}	0,91

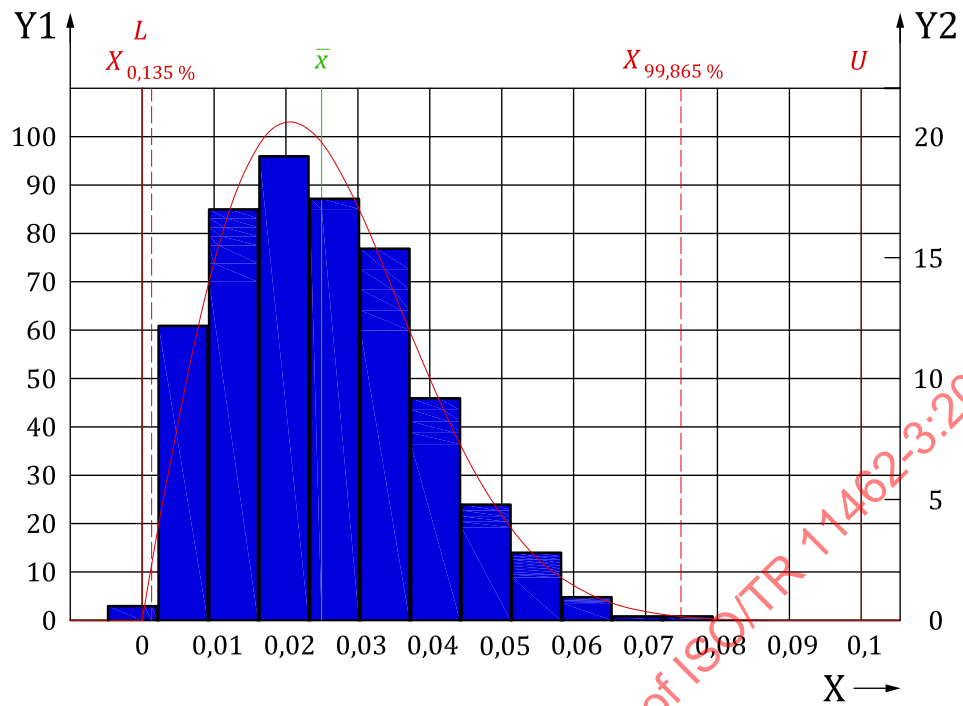
5.9 Test data set 9

5.9.1 Test data set 9 information

This set of test data is taken from a process following a non-normal distribution and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 9 is given in [Table 56](#). [Figure 65](#) shows a histogram and [Figure 66](#) a probability plot of the data set with the purpose of data visualization.

Table 56 — Description of test data set 9

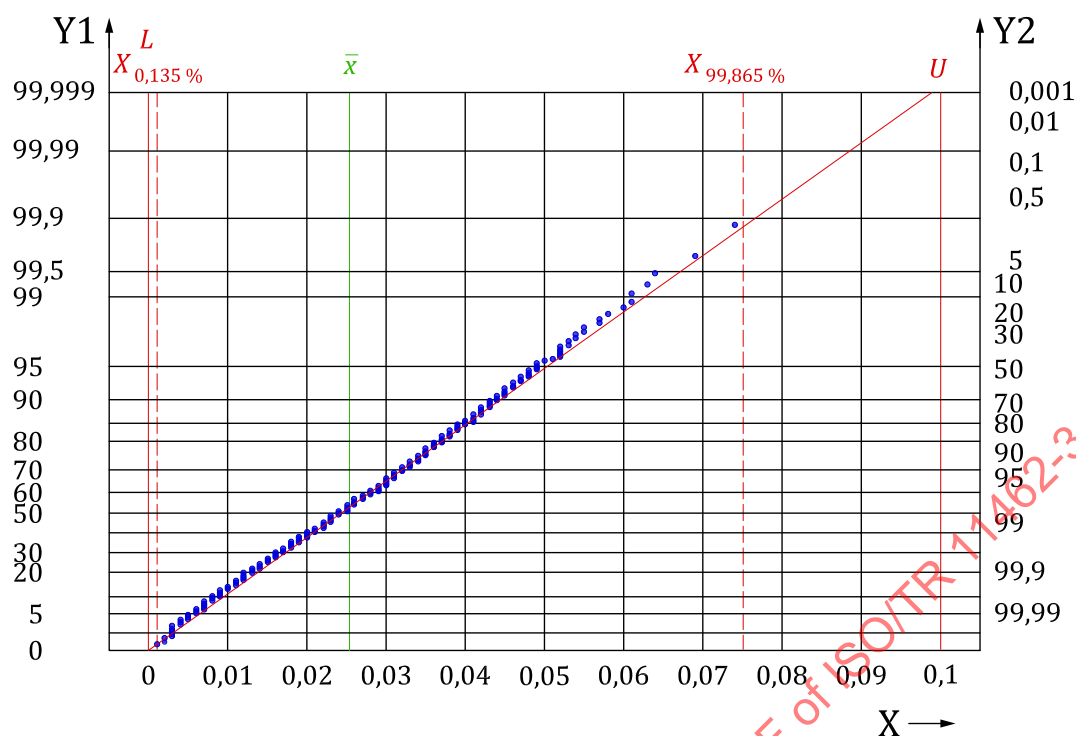
Description of data input			
Distribution model	A2	Resulting distribution	Rayleigh
Data set	Annex A, Table A.1	decimal points	3
Total sample size	500	U	0,1
Size of subgroups	5	L	0

**Key**

Y1 absolute frequency

Y2 relative frequency in %

Figure 65 — Histogram of test data set 9

**Key**

X value

Y1 probability in %

Y2 1-probability in %

Figure 66 — Probability plot of test data set 9

NOTE The width of the class intervals is 0,007.

The class interval with the highest frequency of values is from 0,016 5 to 0,023 5.

The density plot shows a Rayleigh distribution using the estimated scale-parameter $\sigma = 0,020\ 17$ based on the

Method of Moments $\left(\mu_{\text{Rayleigh}} = \sigma \times \sqrt{\frac{\pi}{2}} = \hat{\mu} = \bar{x} \right)$.

Using the inverse distribution function of the Rayleigh distribution $F^{-1}(p|\sigma) = \sigma \times \sqrt{-2 \times \ln(1-p)}$ leads to the following estimators of distribution percentiles and of the process dispersion according to method $d = 1$ of ISO 22514-2:

$$\hat{X}_{0,135\%} = F^{-1}(0,001\ 35 | 0,020\ 165\ 73) \approx 0,001\ 05$$

$$\hat{X}_{99,865\%} = F^{-1}(0,998\ 65 | 0,020\ 165\ 73) \approx 0,073\ 31$$

$$\hat{\Delta} = \hat{X}_{99,865\%} - \hat{X}_{0,135\%} = 0,073\ 308\ 2 - 0,001\ 048\ 2 = 0,072\ 26$$

5.9.2 Test data set 9 results

5.9.2.1 List of sample statistics

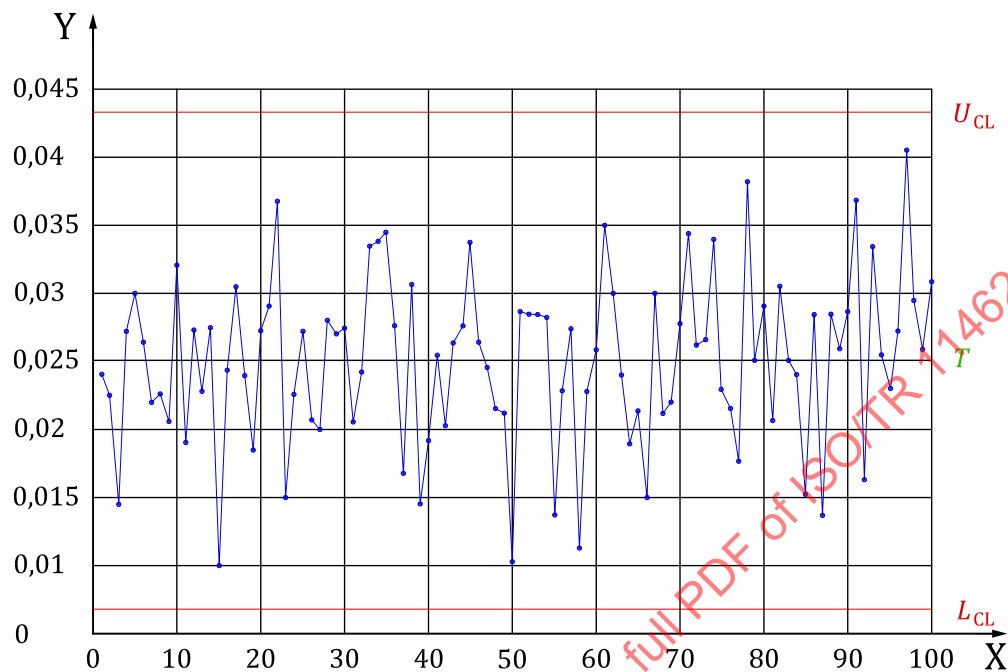
Table 57 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 57 — List of sample statistics for test data set 9

Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	0,025 27	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	0,024 00	ISO 22514-2:2017, Formula (12)
$\overline{\bar{x}}$ ($l = 3$)	0,025 27	ISO 22514-2:2017, Formula (13)
$\tilde{\tilde{x}}$ ($l = 4$)	0,024 20	ISO 22514-2:2017, Formula (14)
Dispersion		
$\hat{\Delta}$ ($d = 1$)	0,072 26 (using parameters in 5.9.1)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	0,013 46	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	0,013 51	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	0,013 30	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	0,013 52	ISO 22514-2:2017, Formula (19)
$\bar{s}$	0,012 70	ISO 7870-2:2013, 3.2
R	0,073 00	ISO 7870-2:2013, 3.2
$\bar{R}$	0,030 93	ISO 7870-2:2013, 3.2
$\bar{R}_m$	0,015 15	ISO 7870-2:2013, 3.2

5.9.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 67.

**Key**

X number of subgroup

Y mean value

Figure 67 — Mean control chart for test data set 9

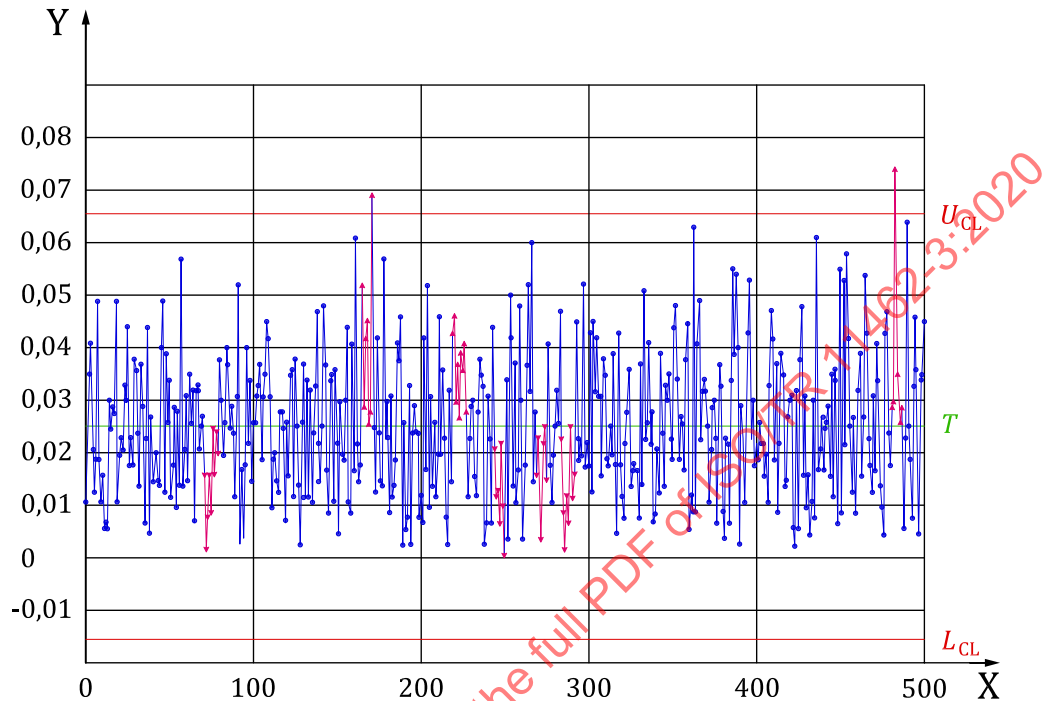
The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = 0,007\ 15$$

$$U_{CL} = 0,043\ 40$$

5.9.2.3 X chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 68](#).



Key

X value number
Y individuals

Figure 68 — Individuals control chart for test data set 9

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = -0,015\ 03$$

$$U_{CL} = 0,065\ 57$$

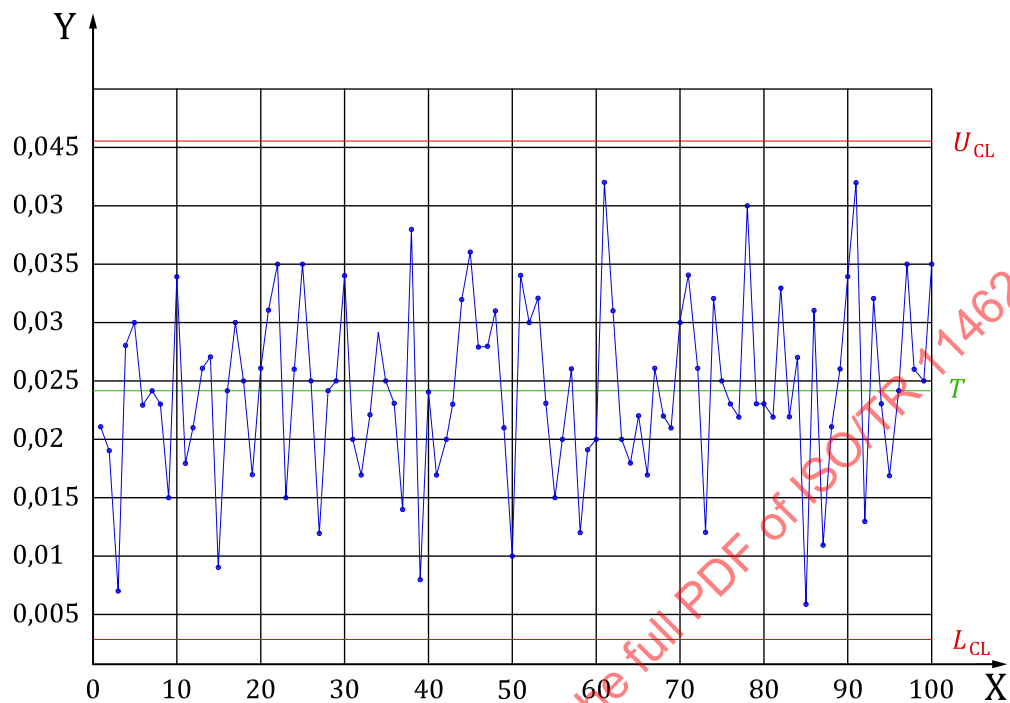
Out of control situations are given in [Table 58](#).

Table 58 — Results of pattern tests individuals control chart and test data set 9

Out of control situations			
Value	Result/violation	Value	Result/violation
171	violation of U_{CL}	483	violation of U_{CL}

5.9.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in Figure 69.

**Key**

X subgroup number
Y median value

Figure 69 — Median control chart for test data set 9

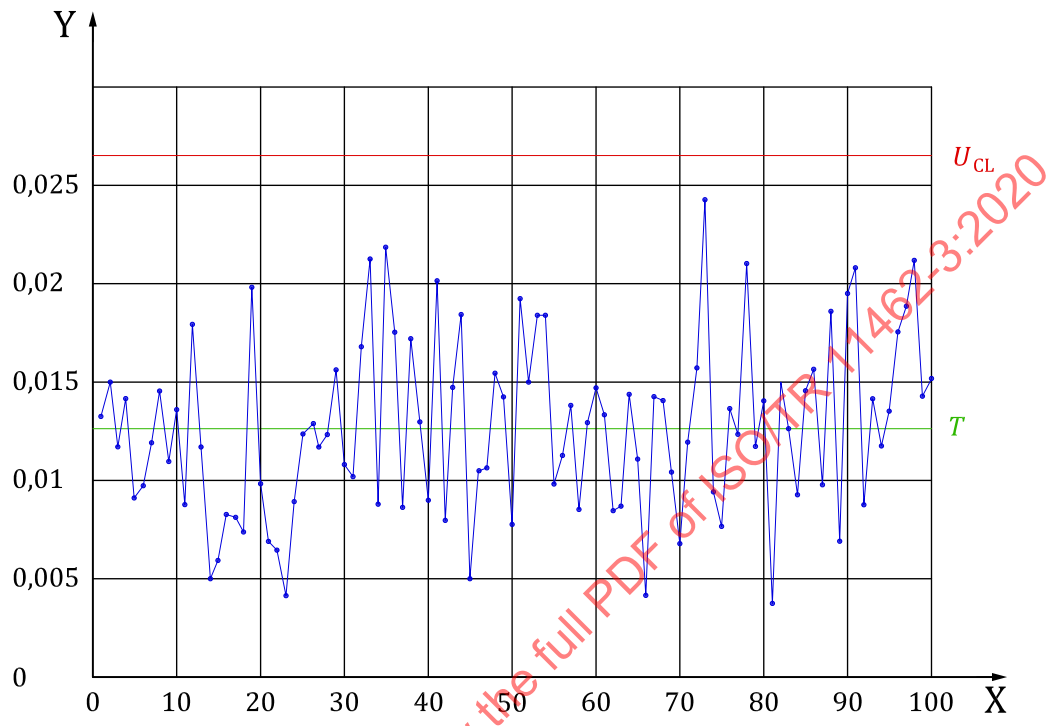
The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = 0,002\ 83$$

$$U_{CL} = 0,045\ 57$$

5.9.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 70.



Key

X subgroup number

Y empirical standard deviation s value

Figure 70 — s control chart for test data set 9

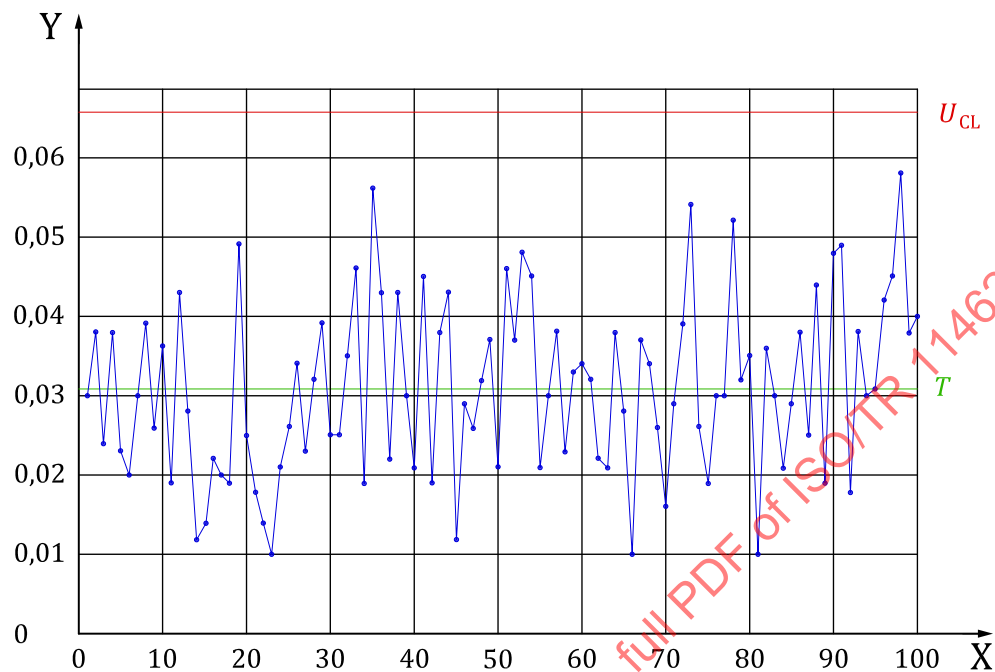
The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,026\ 53$$

5.9.2.6 *R* chart

The *R* control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 71](#).

**Key**

X subgroup number
Y range *R* value

Figure 71 — *R* control chart for test data set 9

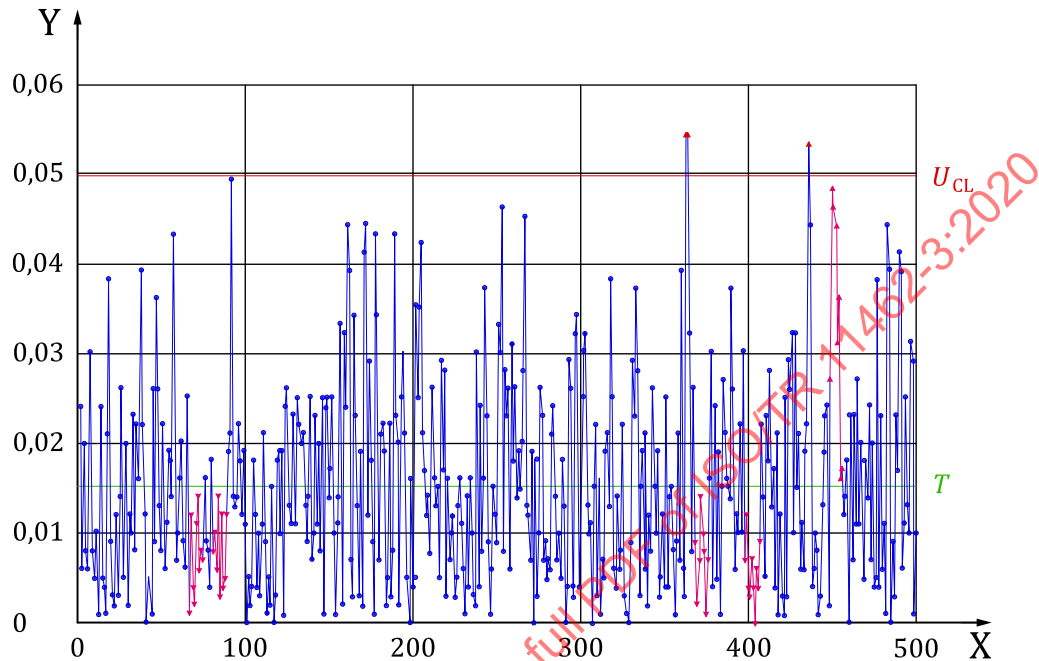
The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 0,065\ 39$$

5.9.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 72](#).



Key

X individuals number
Y moving range value

Figure 72 — Moving range control chart for test data set 9

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 0,049\ 50$$

Out of control situations are given in [Table 59](#).

Table 59 — Results of pattern moving range control chart and test data set 9

Out of control situations			
Value	Result/violation	Value	Result/violation
67 to 75	run below centreline	368 to 376	run below centreline
81 to 89	run below centreline	398 to 407	run below centreline
363	violation of U_{CL}	436	violation of U_{CL}
364	violation of U_{CL}	449 to 456	run above centreline

5.9.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	1,23	C_{pk}	0,62
C_{pkL}	0,62	C_{pkU}	1,84

In this case a Rayleigh distribution can be used and the calculation method $M_{3,1}$.

Capability indices (calculation method $M_{3,1}$) Rayleigh distribution			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	(1,34) theoretical limit – no lower specification limit	C_{pk}	1,01
C_{pkL}	Not defined – no lower specification limit	C_{pkU}	1,01

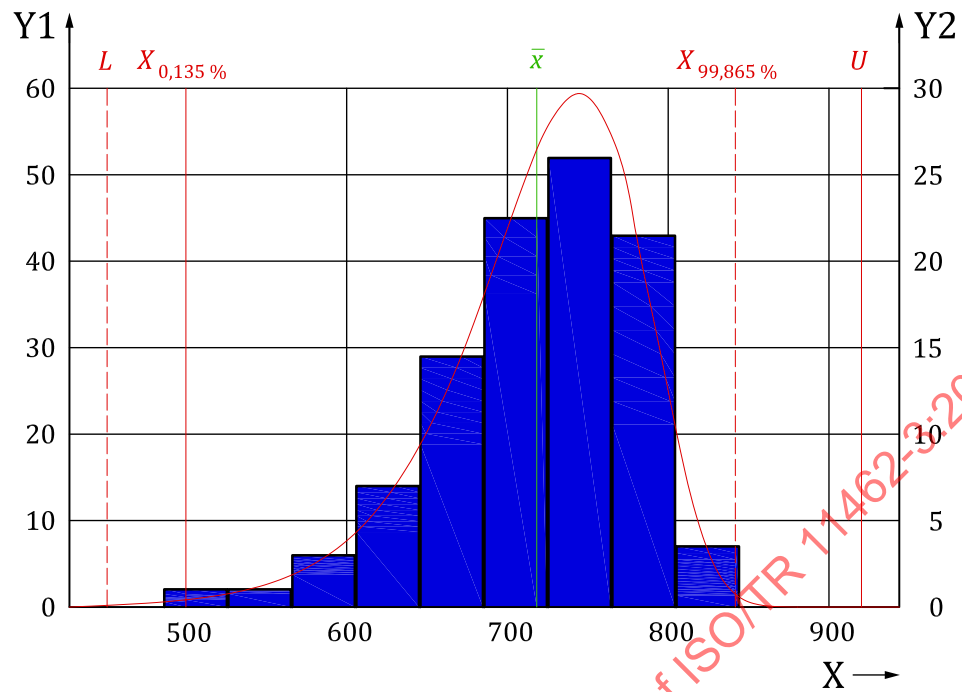
5.10 Test data set 10

5.10.1 Test data set 10 information

This set of test data is taken from a process following a non-normal distribution and is for checking the accuracy of calculation for control limits, sample statistics and process capabilities. A description of test data set 10 is given in [Table 60](#). [Figure 73](#) shows a histogram and [Figure 74](#) a probability plot of the data set with the purpose of data visualization.

Table 60 — Description of test data set 10

Description of data input			
Distribution model	A2	Resulting distribution	Weibull
Data set	Annex A, Table A.1	decimal points	0
Total sample size	200	U	920
Size of subgroups	5	L	500

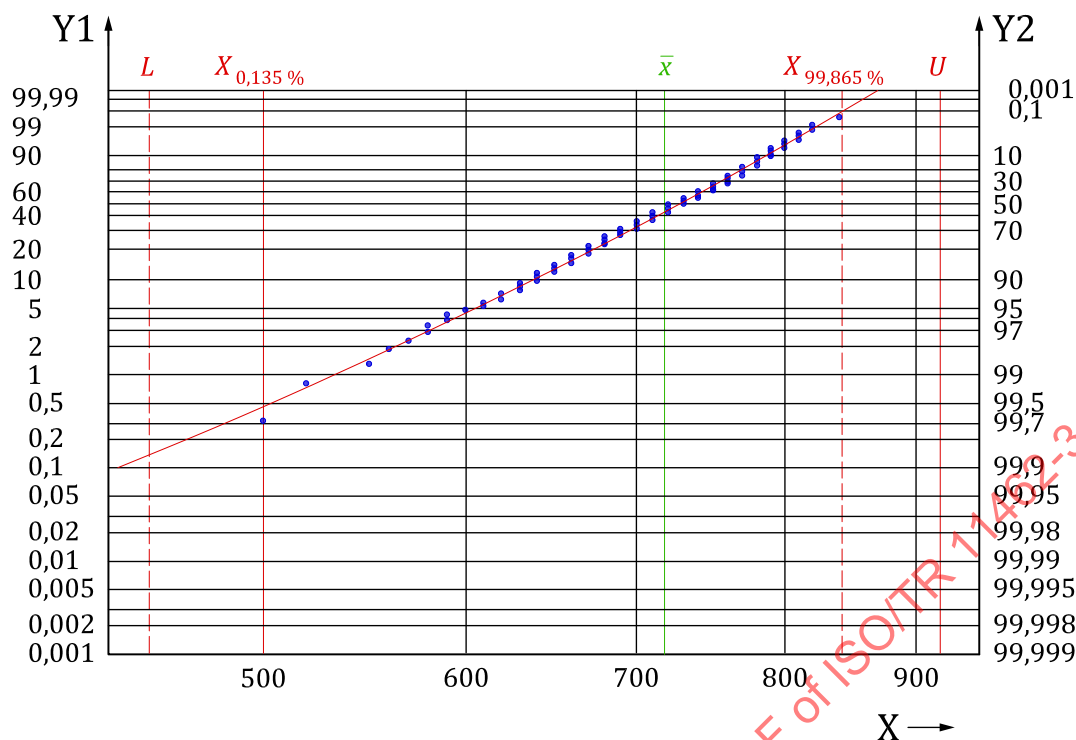
**Key**

X value

Y1 absolute frequency

Y2 relative frequency in %

Figure 73 — Histogram of test data set 10

**Key**

X value

Y1 probability in %

Y2 1-probability in %

Figure 74 — Probability plot of test data set 10

NOTE The width of the class intervals is 40.

The class interval with the highest frequency of values is from 725 to 765.

The density plot is based on the assumption of a Weibull distribution with two parameters. Using the Maximum-Likelihood-estimator to estimate both parameters leads to:

Scale $a = 744,596\ 6$

Shape $b = 14,830\ 85$

Using the inverse distribution function of the Weibull distribution

$$F^{-1}(p|a,b) = a \times [-\ln(1-p)]^{\left(\frac{1}{b}\right)}$$

leads to the following estimators for the distribution quantiles and for the process dispersion according to method $d = 1$ of ISO 22514-2:

$$\hat{X}_{0,135\ \%} = F^{-1}(0,001\ 35|744,596\ 6; 14,830\ 85) \approx 476,92$$

$$\hat{X}_{99,865\ \%} = F^{-1}(0,998\ 65|744,596\ 6; 14,830\ 85) \approx 845,70$$

$$\hat{\Delta} = \hat{X}_{99,865\ \%} - \hat{X}_{0,135\ \%} = 845,696\ 2 - 476,92\ 5 = 368,77$$

5.10.2 Test data set 10 results

5.10.2.1 List of sample statistics

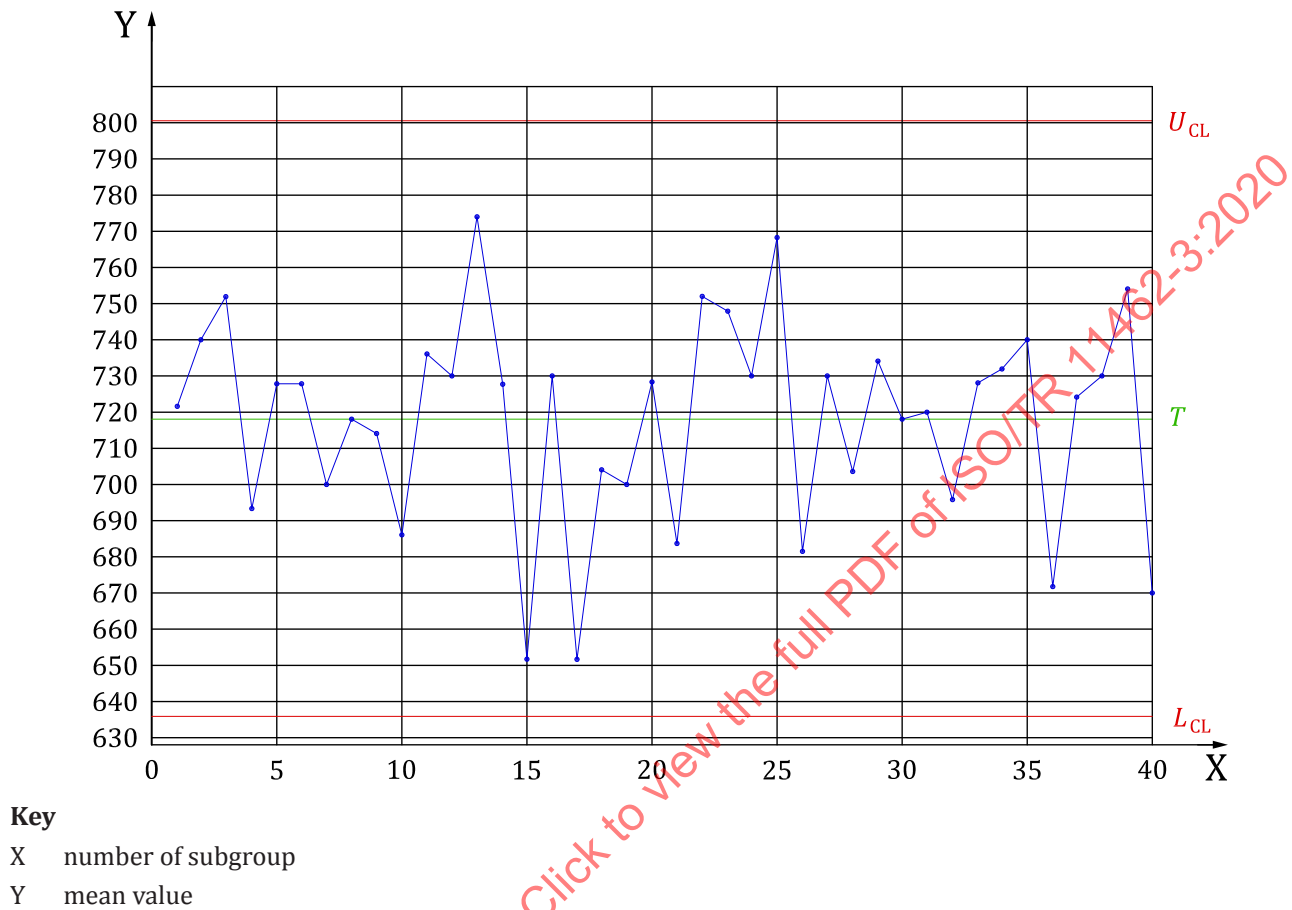
Table 61 lists all sample statistics which are necessary to calculate the target values and control limits for the control charts described in ISO 7870-2 as well as the estimators for location and dispersion given in ISO 22514-2 for the calculation of the process capability indices.

Table 61 — List of sample statistics for test data set 10

Statistic	Value	Reference
Location		
$\bar{x}$ ($l = 1$)	718,30	ISO 22514-2:2017, Formula (11)
$\tilde{x}$ ($l = 2$)	730,00	ISO 22514-2:2017, Formula (12)
$\overline{\bar{x}}$ ($l = 3$)	718,30	ISO 22514-2:2017, Formula (13)
$\tilde{\tilde{x}}$ ($l = 4$)	725,25	ISO 22514-2:2017, Formula (14)
Dispersion		
$\hat{\Delta}$ ($d = 1$)	368,77 (Weibull distribution)	ISO 22514-2:2017, Formula (15)
$\hat{\sigma}$ ($d = 2$)	60,30	ISO 22514-2:2017, Formula (16)
$\hat{\sigma}$ ($d = 3$)	61,21	ISO 22514-2:2017, Formula (17)
$\hat{\sigma}$ ($d = 4$)	62,12	ISO 22514-2:2017, Formula (18)
$\hat{\sigma}$ ($d = 5$)	61,03	ISO 22514-2:2017, Formula (19)
$\bar{s}$	57,54	ISO 7870-2:2013, 3.2
R	340,00	ISO 7870-2:2013, 3.2
$\bar{R}$	144,50	ISO 7870-2:2013, 3.2
$\bar{R}_m$	67,14	ISO 7870-2:2013, 3.2

5.10.2.2 $\bar{x}$ chart

The $\bar{x}$ control chart calculated according to ISO 7870-2:2013, 6.1, is shown in Figure 75.



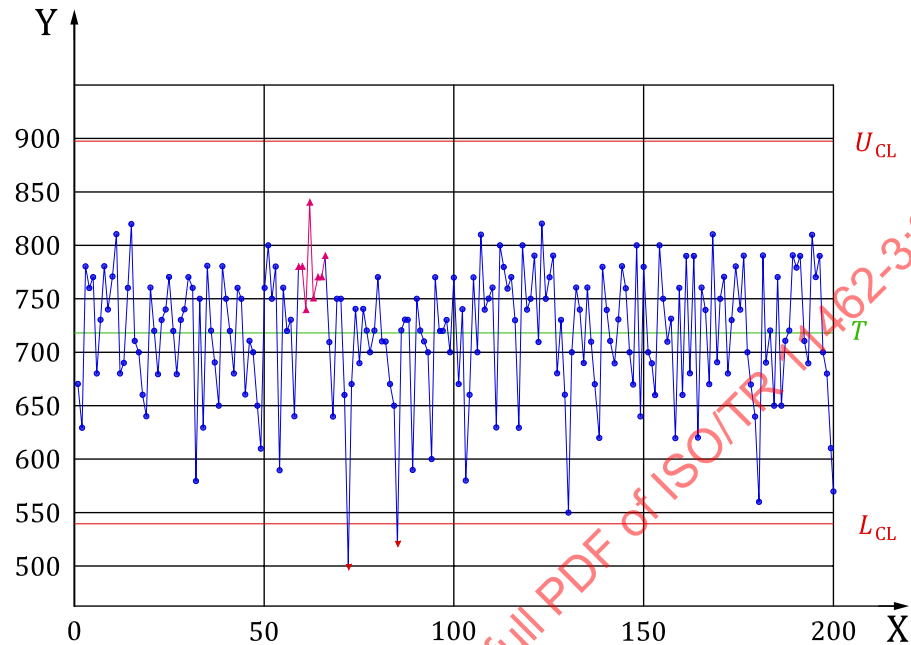
The control limits are (see ISO 7870-2:2013, Table 1, using $\bar{s}$):

$$L_{CL} = 636,19$$

$$U_{CL} = 800,41$$

5.10.2.3 $\bar{X}$ chart

The individuals control chart calculated according to ISO 7870-2:2013, 6.2, is shown in [Figure 76](#).



Key

X value number
Y individuals

Figure 76 — Individuals control chart for test data set 10

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 539,72$$

$$U_{CL} = 896,88$$

Out of control situations are given in [Table 62](#).

Table 62 — Results of pattern tests individuals control chart and test data set 10

Out of control situations			
Value	Result/violation	Value	Result/violation
59 to 66	run above centreline	85	violation of L_{CL}
72	violation of L_{CL}		

5.10.2.4 $\tilde{x}$ chart

The median control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 77](#).

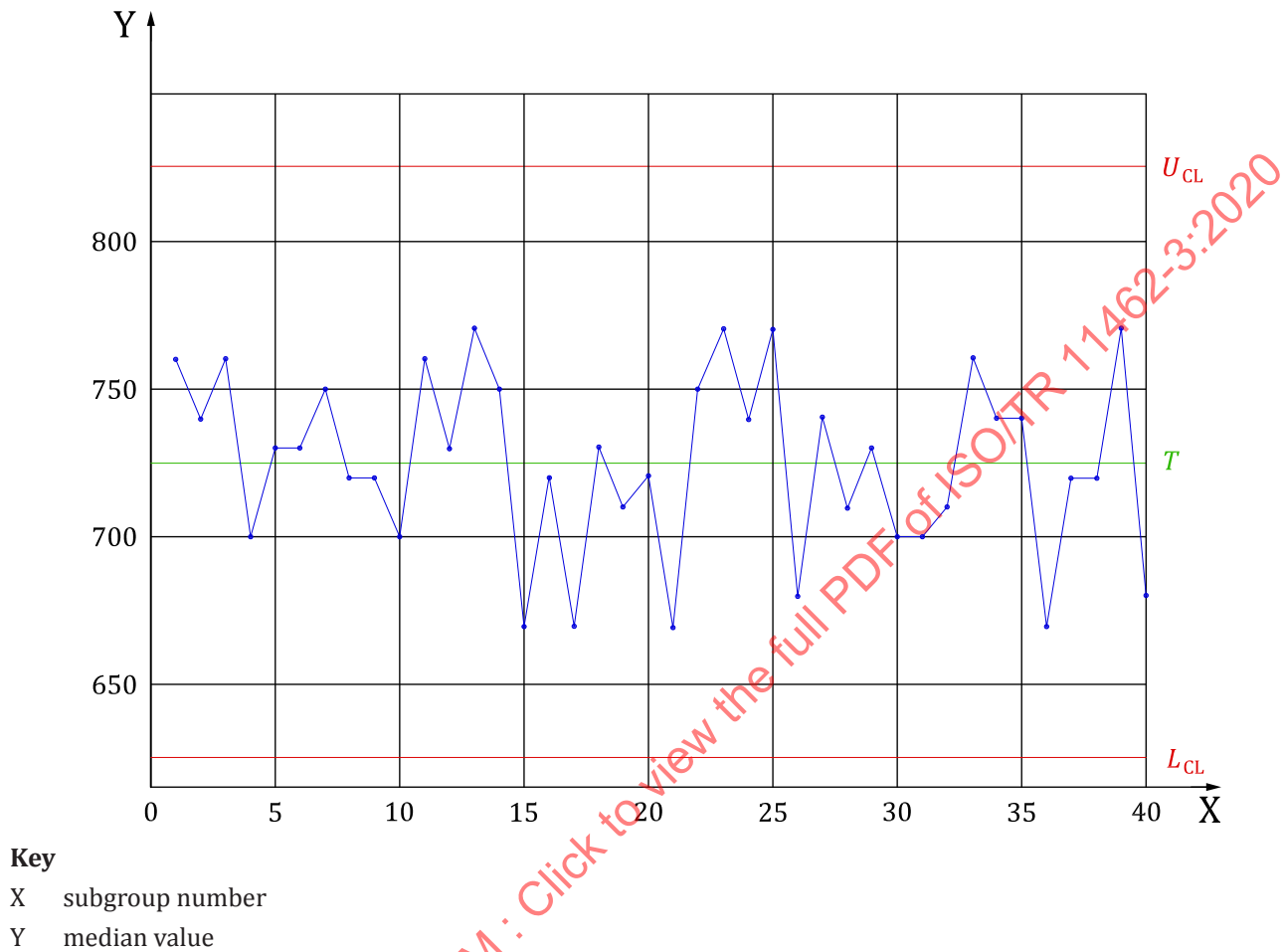


Figure 77 — Median control chart for test data set 10

The control limits are (see ISO 7870-2:2013, Table 4):

$$L_{CL} = 625,40$$

$$U_{CL} = 825,10$$

5.10.2.5 s chart

The s control chart calculated according to ISO 7870-2:2013, 6.1, is shown in [Figure 78](#).

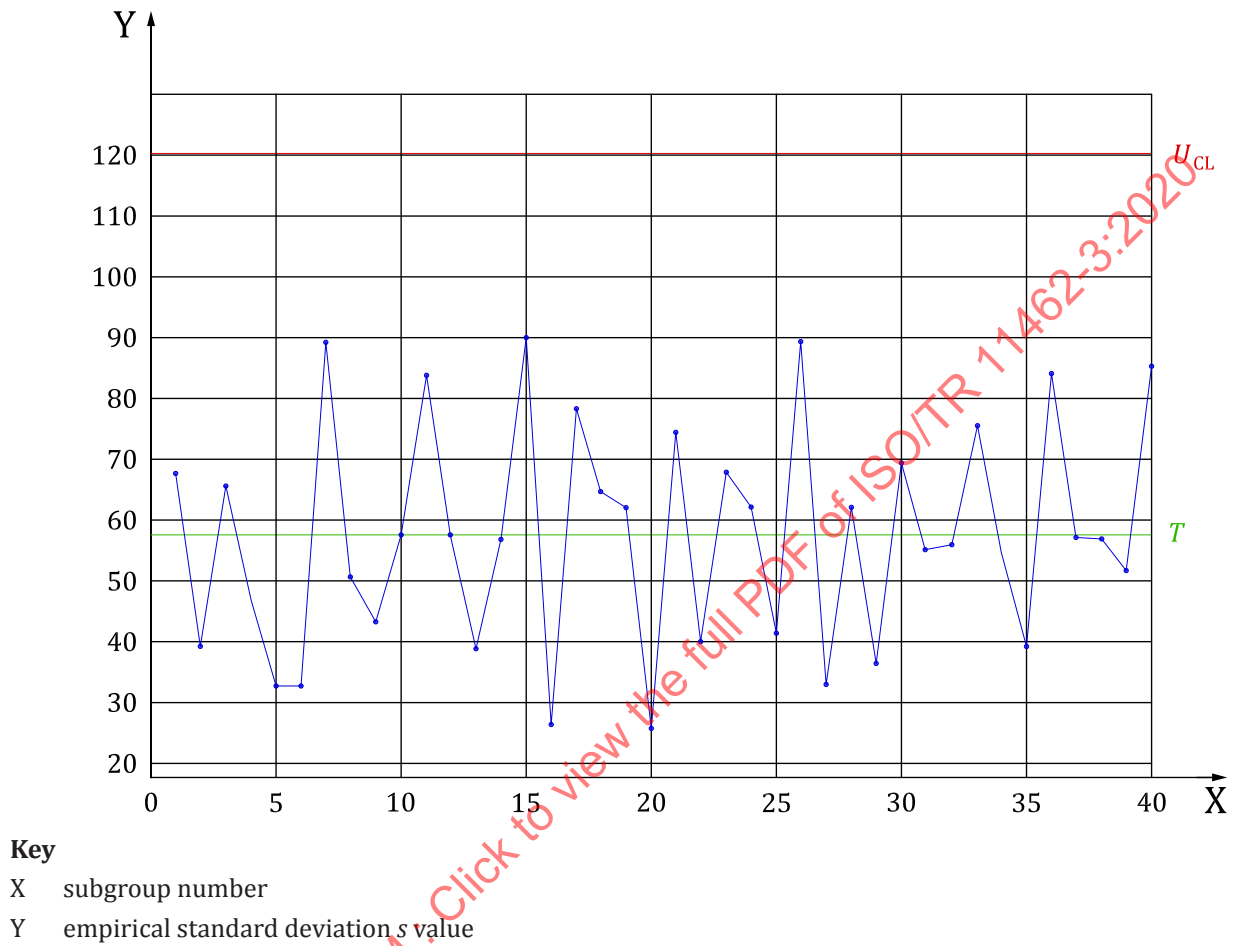


Figure 78 — s control chart for test data set 10

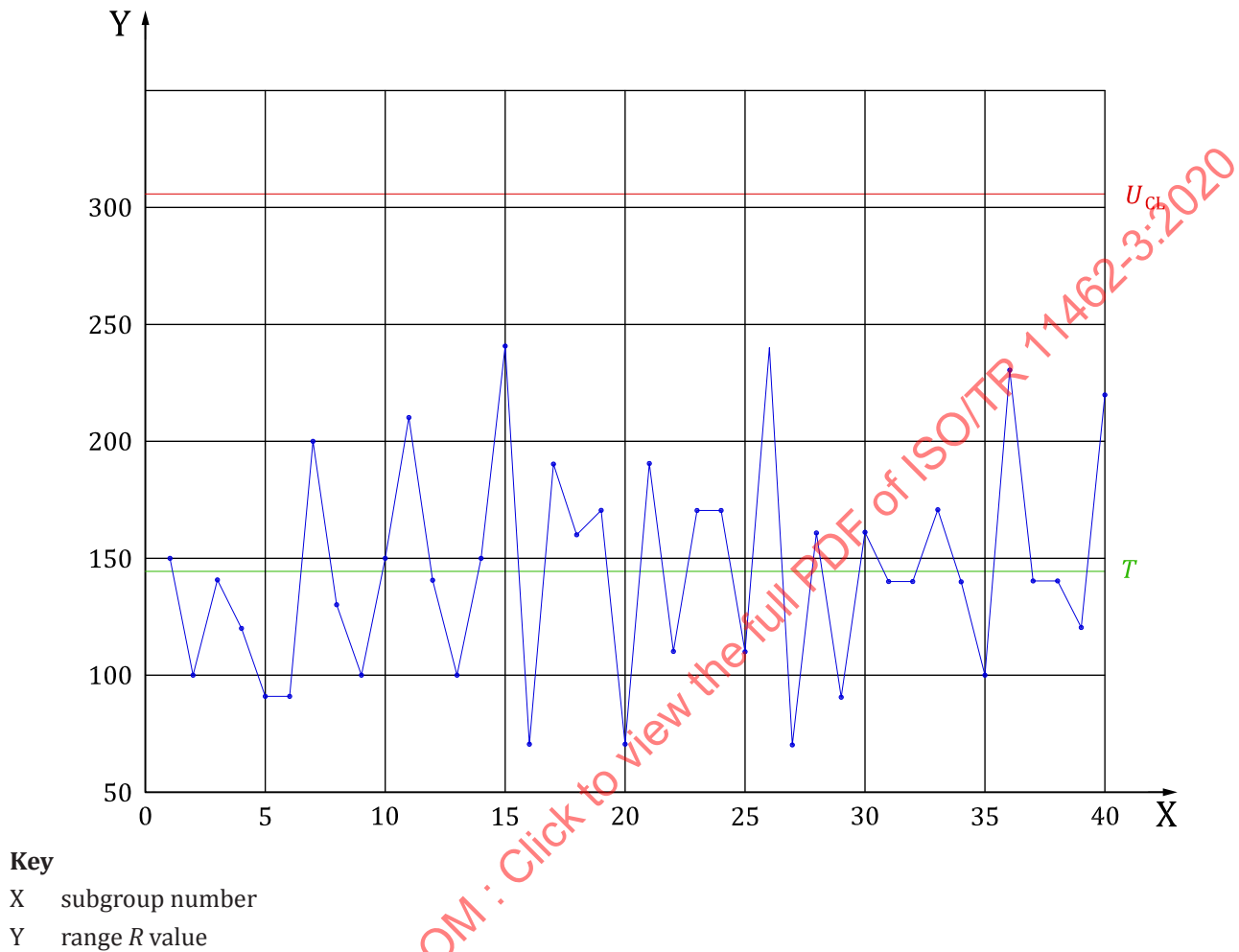
The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 120,20$$

5.10.2.6 *R* chart

The *R* control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 79](#).



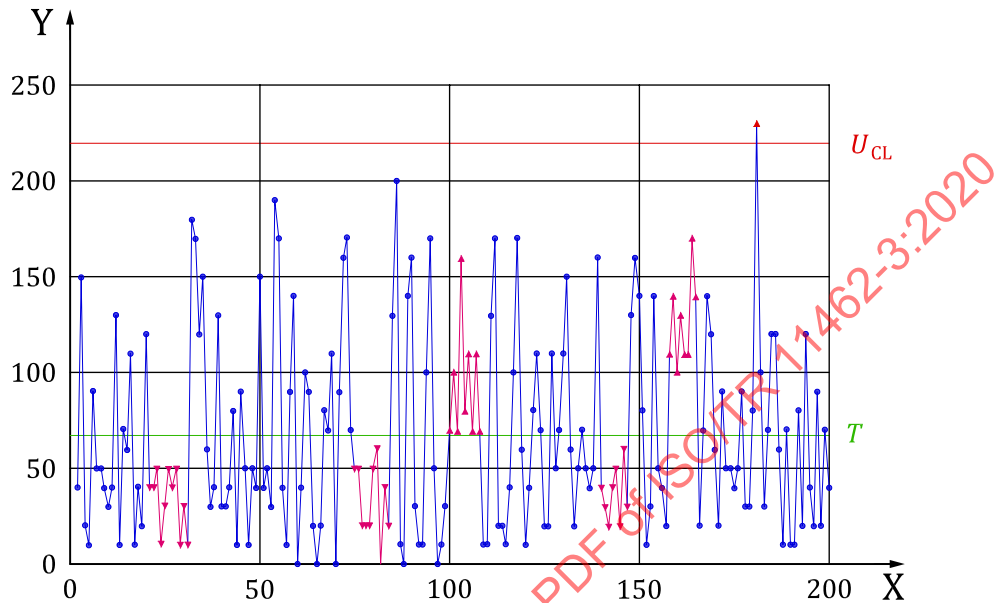
The control limits are (see ISO 7870-2:2013, Table 1):

$$L_{CL} = 0$$

$$U_{CL} = 305,47$$

5.10.2.7 Moving range chart

The moving range control chart calculated according to ISO 7870-2:2013, 6.3, is shown in [Figure 80](#).



Key

X individuals number
Y moving range value

Figure 80 — Moving range control chart for test data set 10

The control limits are (see ISO 7870-2:2013, Table 3):

$$L_{CL} = 0$$

$$U_{CL} = 219,33$$

Out of control situations are given in [Table 63](#).

Table 63 — Results of pattern tests moving range control chart and test data set 10

Out of control situations			
Value	Result/violation	Value	Result/violation
21 to 31	run below centreline	140 to 147	run below centreline
75 to 84	run below centreline	158 to 165	run above centreline
100 to 108	run above centreline	181	violation of U_{CL}

5.10.2.8 Process capability

The process capability is calculated according to ISO 22514-2:2017, Clause 6, with calculation method $l = 3$ for the location estimator and calculation method $d = 5$ for the dispersion estimator.

Capability indices (calculation method $M_{3,5}$)			
Process is stable (mean, R - chart) - C_p/C_{pk} is used			
C_p	1,15	C_p	1,10

Capability indices (calculation method $M_{3,5}$)			
C_{pkL}	1,19	C_{pkU}	1,10

In this case a Weibull distribution can be used, and the calculation method $M_{3,1}$. But the result is the same as when using the **method $M_{3,5}$** .

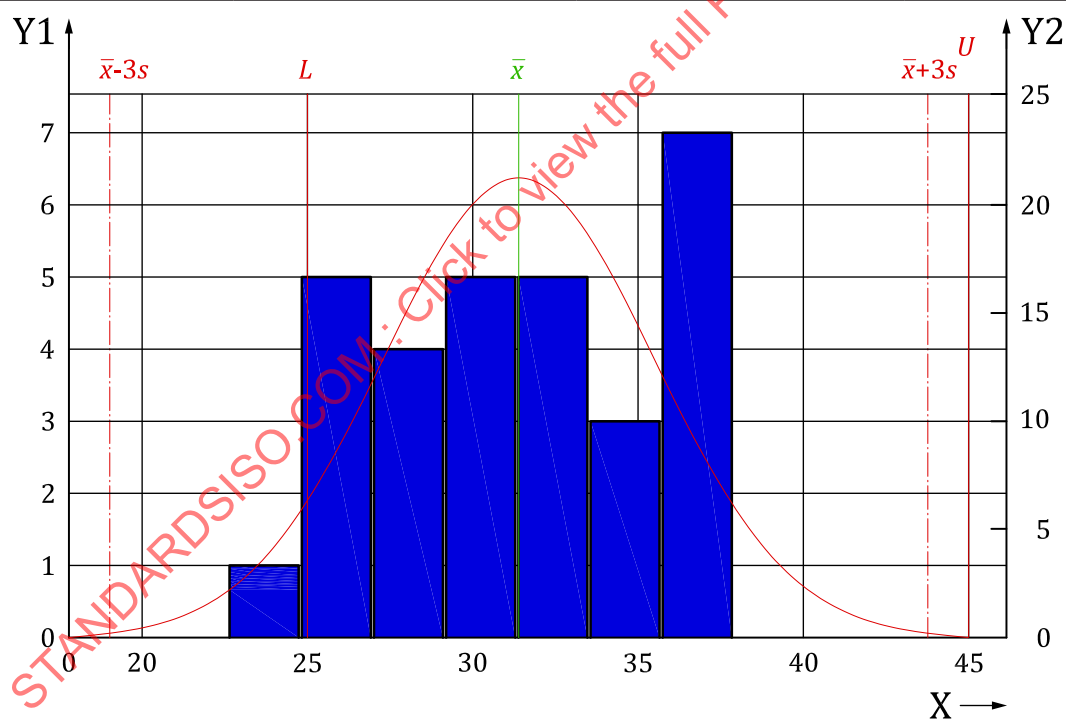
5.11 Test data set 11

5.11.1 Test data set 11 information

This set of test data is taken from a multi-stage process following a normal distribution in each state and is for checking the accuracy of calculation for process capabilities. A description of test data set 11 is given in [Table 64](#). [Figure 81](#) shows a histogram and [Figure 82](#) a probability plot of the data set with the purpose of data visualization.

Table 64 — Description of test data set 11

Description of data input			
Distribution model	C2	Resulting distribution	normal
Data set	Annex A, Table A.2	decimal points	1
Total sample size	10 per state (30 in total)	U	45
Size of subgroups	—	L	25



Key

X value

Y1 absolute frequency

Y2 relative frequency in %

Figure 81 — Histogram of test data set 11