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Statistical methods in process management — Capability and performance —

Part 9:
Process capability statistics for characteristics defined by geometrical specifications

Méthodes statistiques dans la gestion de processus — Aptitude et performance —

Partie 9: Méthodes statistiques pour l'aptitude des processus dont les caractéristiques sont définies par des spécifications géométriques

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Contents

Foreword

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This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in products and process management*.

This document is a second draft for approval and only editorial changes will be made before publication.

A list of all parts in the ISO 22514 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Many organizations will need to evaluate the capability and performance of their key processes when the specifications are defined by requirements other than linear size. The methods described in this document are intended to assist the organization in this respect.

During the last couple of years, it has been more common in the design and development departments in companies to not only use linear tolerances alone, but also including modifiers as well as geometrical tolerances with or without use of the maximum material requirements.

This situation has been supported by new measurement methods used in production, where it is common to use measurement equipment, where the results are given in form of point clouds instead of one single value.

It is a challenge in such cases to calculate capability and performance, but organizations and customers still require the capability indices in acceptance of produced or delivered batches of parts.

This document describes how to calculate capability or performance where functional requirements on parts are given.

As an example, the “maximum material requirement”, MMR, covers “assemble ability” and the “least material requirement”, LMR, covers, for example, “minimum wall thickness” of a part. Each requirement (MMR and LMR) combines two independent requirements into one collective requirement, which simulates the intended function of the workpiece. In some cases of both MMR and LMR, the “reciprocity requirement”, RPR, can be added.

In Annex D, a case study of process analysis, where the characteristic to be improved is perpendicularity, is introduced.

Statistical methods in process management — Capability and performance —

Part 9:
Process capability statistics for characteristics defined by geometrical specifications

1 Scope

This document describes process capability and performance measures when the specifications are given by geometrical product specifications e.g. maximum material requirements or linear size with a modifier.

The purpose of this document of the international series of standards on capability calculation is to assist the organizations to calculate the PCIs (process capability index) when geometrical product specifications are used on drawings.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1 Terms

3.1.1
feature of size
feature of linear size
geometrical feature, having one or more intrinsic characteristics, only one of which can be considered as variable parameter, that additionally is a member of a "one parameter family", and obeys the monotonic containment property for that parameter

EXAMPLE 1 A single cylindrical hole or shaft is a feature of linear size. Its linear size is its diameter.

EXAMPLE 2 Two opposite parallel plane surfaces are a feature of linear size. Its linear size is the distance between the two parallel planes.

[SOURCE: ISO 17450-1:2011, 3.3.1.5.1., modified: "may" replaced by "can", deleted Note 1 to Note 4, deleted reference to Figure 5 (ISO 17450-1:2011), deleted EXAMPLE 2, added new EXAMPLE 2]

3.1.2
local size
local linear size
local size characteristic
local linear size characteristic
size characteristic having by definition a non-unique result of evaluation along and around the feature of size

Note 1 to entry: For a given feature, an infinity of local sizes exists.

[SOURCE: ISO 14405-1:2016, 3.6, modified: “and/or” replaced by “and”, Note 2 to entry to Note 4 to entry deleted.]

3.1.3
two-point size
<local size>
distance between two opposite points on an extracted integral linear feature of size

Note 1 to entry: A two-point size taken on a cylinder can be called a “two-point diameter”. In ISO 17450-3, this is defined as a local diameter of an extracted cylinder.

Note 2 to entry: A two-point size taken on two opposite planes can be called “two-point distance”. In ISO 17450-3, this is defined as a local size of two parallel extracted surfaces.

[SOURCE: ISO 14405-1:2016, 3.6.1, modified: deleted Note 1 to entry to Note 3 to entry, added two new notes]

3.1.4
envelope requirement
combination of the two-point size applied for the least material limit of the size and either the minimum circumscribed size or the maximum inscribed size applied for the maximum material limit of the size

Note 1 to entry: The “envelope requirement” was previously referred to as the “Taylor principle”.

Note 2 to entry: According to ISO 8015, the surface of a single feature of size (e.g. cylindrical surface or a feature based on two parallel plane surfaces) cannot violate the envelope of a geometrical ideal form at a maximum material limit of size

[SOURCE: ISO 14405-1:2016, 3.8, modified: Note 2 to entry added]

3.1.5
maximum material virtual size
MMVS
size generated by the collective effect of the maximum material size, MMS, of a feature of size and the geometrical tolerance (form, orientation or location) given for the derived feature of the same feature of size

Note 1 to entry: Maximum material virtual size, MMVS, is a parameter for size used as a numerical value connected to maximum material virtual condition, MMVC.

Note 2 to entry: For external features, MMVS is the sum of MMS and the geometrical tolerance, whereas for internal features, it is the difference between MMS and the geometrical tolerance.

Note 3 to entry: The MMVS for external features of size, $l_{\text{MMVS,e}}$, is given by the following formula:

$$l_{\text{MMVS,e}} = l_{\text{MMS}} + \delta$$

and the MMVS for internal features of size, $l_{\text{MMVS,i}}$, is given by the following one:

$$l_{\text{MMVS,i}} = l_{\text{MMS}} - \delta$$

where

l_{MMS} is the maximum material size;

δ is the geometrical tolerance.

3.1.6
least material virtual size
LMVS
size generated by the collective effect of the least material size, LMS, of a feature of size and the geometrical tolerance (form, orientation or location) given for the derived feature of the same feature of size

Note 1 to entry: Least material virtual size, LMVS, is a parameter for size used as a numerical value connected to least material virtual condition, LMVC.

Note 2 to entry: For external features, LMVS is the difference between LMS and the geometrical tolerance, whereas for internal features, it is the sum of LMS and the geometrical tolerance.

Note 3 to entry: The LMVS for external features of size, $l_{LMVS,e}$, is given by the following formula:

$$l_{LMVS,e} = l_{LMS} - \delta$$

and the LMVS for internal features of size, $l_{LMVS,i}$, is given by the following one:

$$l_{LMVS,i} = l_{LMS} + \delta$$

where

l_{LMS} is the least material size;

δ is the geometrical tolerance.

3.1.7
maximum material requirement
MMR
requirement for a feature of linear size, defining a geometrical feature of the same type and of perfect form, with a given value for the intrinsic characteristic (dimension) equal to the maximum material virtual size, which limits the non-ideal feature on the outside of the material

Note 1 to entry: Maximum material requirement, MMR, is used to control the assembly ability of a workpiece.

[SOURCE: ISO 2692:2021, 3.12, modified Note 2 to entry deleted.]

3.1.8
least material requirement
LMR
requirement for a feature of linear size, defining a geometrical feature of the same type and of perfect form, with a given value for the intrinsic characteristic (dimension) equal to the last material virtual size, which limits the non-ideal feature on the inside of the material

Note 1 to entry: Least material requirements, LMR is used, for example, to control the minimum wall thickness between two symmetrical or coaxially located similar features of size.

[SOURCE: ISO 2692:2021, 3.13, modified Note 2 to entry deleted.]

3.1.9
reciprocity requirement
RPR
additional requirement for a feature of linear size indicated in addition to the maximum material requirement, MMR, or the least material requirement, LMR to indicate that the size tolerance is increased by the difference between the geometrical tolerance and the actual geometrical deviation

[SOURCE: ISO 2692:2021, 3.14]

3.2 Abbreviated terms

ASME	American Society of Mechanical Engineers
LMC	least material conditions
LMS	least material size
LMR	least material requirement
LMVC	least material virtual condition
LMVS	least material virtual size
MMC	maximum material condition
MMR	maximum material requirement
MMS	maximum material size
MMVS	maximum material virtual size
PCI	process capability index
RPR	reciprocity requirement

3.3 Symbols

In addition to the symbols listed below, some symbols are defined where they are used within the text.

C_{p}	process capability index
C_{pk}	minimum process capability index
C_{pk_L}	lower process capability index
C_{pk_U}	upper process capability index
D	Diameter
Δ	geometrical tolerance
δ_{A}	measured geometrical tolerance
l_{LMS}	least material size
$l_{\mathrm{LMVS,e}}$	LMVS for external features of size
$l_{\mathrm{LMVS,i}}$	LMVS for internal features of size
l_{MMS}	maximum material size

l_{MMVS} maximum material virtual size

$l_{MMVS,e}$ MMVS for external features of size

$l_{MMVS,i}$ MMVS for internal features of size

L_{SL} lower specification limit

N total sample size

n subgroup sample size

μ location of the process; population mean value

P_p P_{po} process performance index

P_{pk} P_{pok} minimum process performance index

P_{pk_L} lower process performance index

P_{pk_U} upper process performance index

θ Scale parameter of the Rayleigh distribution

s standard deviation, sample statistic

$\bar{s}$ average sample standard deviation

σ standard deviation, population

U_{SL} upper specification limit

$\bar{X}$ average from sample

$X_{99,865\,\%}$ upper 99,865 % quantile

$X_{0,135\,\%}$ lower 0,135 % quantile

4 Statistical measures used in the calculation of process capability or performance

4.1 General

The statistical analysis described in this document is designed to determine capability or performance indices when the characteristic of interest is a feature of linear size, and this size has a geometrical modifier added to the specification or a geometrical tolerance with or without maximum material condition.

4.2 Independency principle

4.2.1 General

A GPS specification for a feature or relation between features can be fulfilled independent of other specifications except when it is stated by special indication e.g. Ⓜ modifiers according to ISO 2692, CZ according to ISO 1101 or Ⓔ modifiers according to ISO 14405-1 as part of the specification. Each requirement (Ⓔ, MMR and LMR) combines two independent requirements into one collective requirement, which more accurately simulates the intended function of the workpiece. In some cases of both MMR and LMR, the “reciprocity requirement”, RPR, can be added.

If those special indications are used as requirements, they need to be considered as a collective requirement and the capability indices can be calculated as one common value.

4.2.2 Maximum Material ISO versus ASME

In this standard the ISO definitions as defined in ISO 8015 are used. Geometrical product specifications in ASME are defined in Y 14,5 that often differs from the definitions in ISO. Tolerancing in ISO geometrical features are individual and independent of each other. In ASME tolerancing of the mating behaviour of the part in the assembly group used.

4.2.3 Measurement procedure

The measurement procedure is especially important when measuring properties with modifiers or geometric tolerances. The tolerance applies to the entire surface of the workpiece in three dimensions with an infinite number of points, therefore a sufficient number of measuring points defined in the procedure can be measured on every workpiece. You also have to consider the distribution of these points. More information can be found in Annex C.

4.3 Location

It is a precondition, that the size of the characteristic of interest can have only one value assigned and a characterisations of process location can be the mean, μ, or the median, $X_{50\ \%}$. If the variation of the characteristic can be described by a symmetric distribution the mean is the most natural selection, with non-symmetric distributions the median is the preferred selection.

4.4 Dispersion

It is important to differentiate between a standard deviation that measures only variation based on e.g. 50 samples and the standard deviation which measures variation from more than 100 samples. Methods for calculating standard deviations representing these two cases are given in Annex A. Very often, when data are gathered over a long period of time, the standard deviation is larger due to the effects of fluctuations in the process. It is important that the use and calculation of the standard deviation in the formulae only make sense if the data are normally distributed.

In case of a characteristics with modifiers added or characteristics defined with geometrical tolerances the actual distribution in most cases cannot be described by a normal distribution therefore, the capability calculation formula based on reference limits can be used instead. The formulae for the distribution models can be found in Annex B.

4.5 Reference limits

The lower and upper reference limits are respectively defined as the 0,135 % and the 99,865 % quantiles of the distribution that describes the output of the process characteristic. They are described as $X_{0,135\ \%}$ and $X_{99,865\ \%}$.

4.6 Reference interval

The reference interval is the interval between the upper and the lower reference limits. The reference interval includes 99,73 % of the individual values in the population from a process.

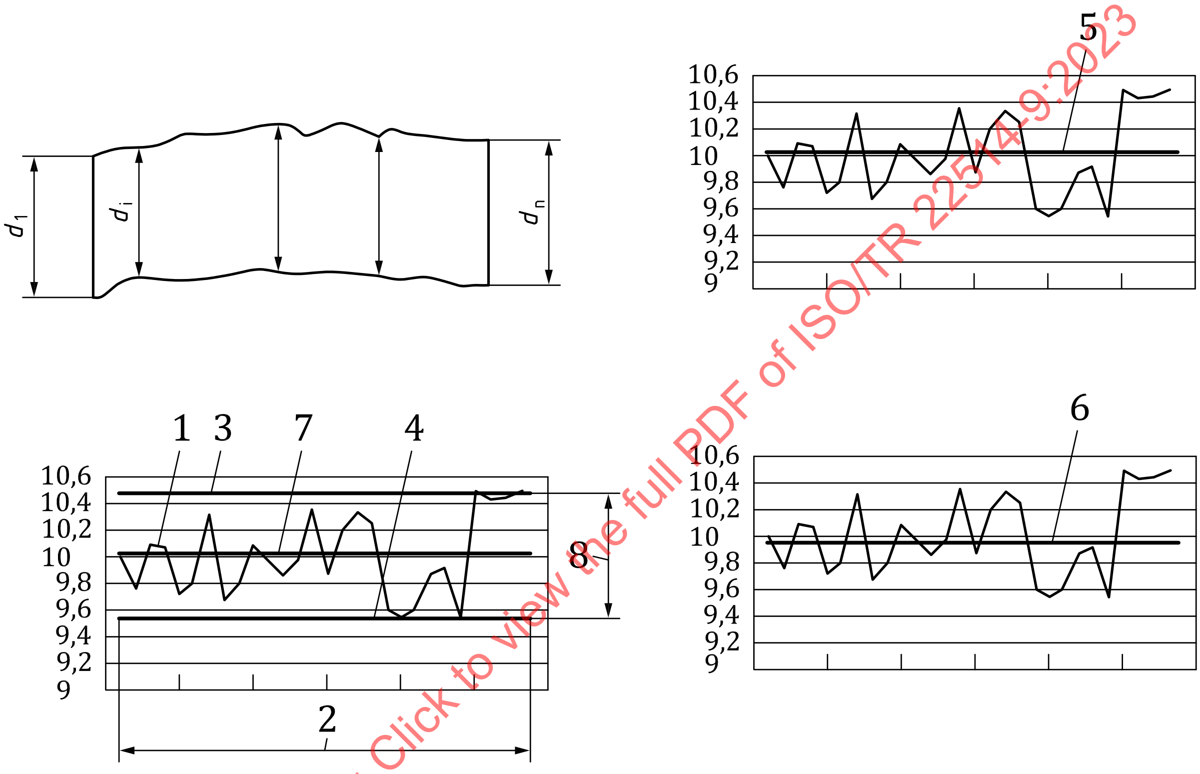
5 Geometrical product specifications

5.1 General

Produced workpieces exhibit deviations from the ideal geometric form shown on a drawing. The real value of the dimension of a feature of size is dependent on the form deviations and on the specific type of size applied.

The definition of an indication of a size tolerance by direct indication (plus and minus tolerancing), or indication by the limit values of the upper and the lower deviation limits, e.g. 25,65 ± 0,05 has not been defined before the first version of ISO 14405-1 was published, and therefore resulted in an ambiguous requirement when used on features of size of imperfect form.

The type of size to be applied to a feature of size depends on the function of the workpiece in the product. The type of size can be indicated on the drawing by a specification modifier for controlling the feature definition and evaluation method to be used. If no modifier has been added to the tolerance, the two-point size is the default requirement (see Figure 1.). In this case, there can be a lot of different values because a number of measurements has to be taken on the workpiece.



Key

1	set of values of local sizes
2	positions along the axis
3	maximum size (= 10,497 88)
4	minimum size (= 9,542 81)
d_i, d_1, d_n	different values of local size
5	average size (= 10,011 69)
6	median size (= 9,969 86)
7	mid-range size (= 10,020 345)
8	size range (= 0,955 07)

Figure 1 — Different results for two-point requirement (adapted from ISO 14405-1:2016, Figure 8)

The calculation of capability in case of two-point size can be based on the average and the reference interval of the minimum two-point value found on the workpieces and the average and the reference interval of the maximum measured values on the workpieces. The two distribution models will very often be extreme value distributions.

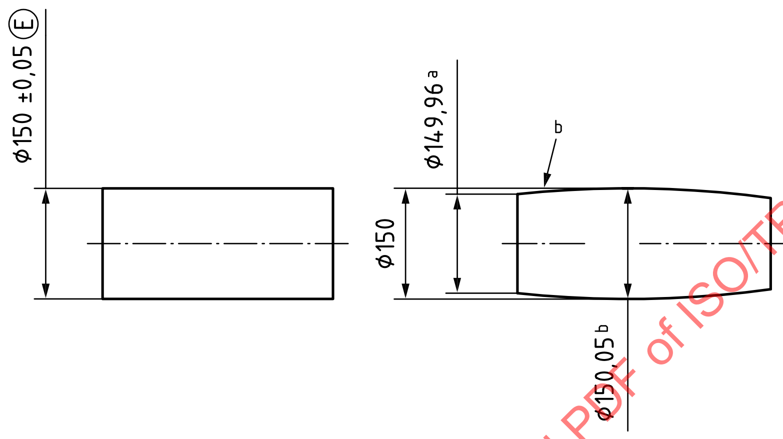
5.2 Linear size with modifiers

Geometrical product specifications with different modifiers such as (GG) or (GX) are very often used in modern drawings to specify the function of the workpiece. The combination of such requirement when the calculation of capability indices is required will often be a subject to discussions between the customers and the production because of the interpretation of the specification cannot result in only one value. ISO 14405-1 defines the different modifiers.

5.3 ISO tolerance classes with modifier

If a fit has been toleranced, the ISO tolerance class code system (in accordance with ISO 286-1) can be indicated on the drawing. An ISO tolerance class code without a modifier is a standard linear size defined as a two-point size. In case of a fit, a modifier (GX) or (GN) can be added to the tolerance. There will then be two different requirements to the characteristic: The two-point size and the envelope size.

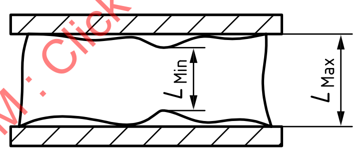
Dimensions in millimetres



a Minimum requirement value (two point).

b Envelope requirement – maximum value.

Figure 2 — ISO tolerance on a shaft with a modifier



Key

L_{Min} minimum size

L_{Max} maximum size (two point)

Figure 3 — Size with envelope modifier

For such tolerances as shown in Figure 2 and Figure 3, values from two different measurements series can be collected: The two-point size and the envelope size. In principle it is the same in case of internal diameters. The only differences are that the two-point size have a bigger value than the maximum inscribed value.

Based on the series of two measurements (the envelope size and the minimum two-point size), two different minimum indices C_{pk_U}, P_{pk_U} (envelope size) and C_{pk_L}, P_{pk_L} (two-point size) can be calculated using the following formulae:

a) for the calculation of index for envelope size, see Formula (1):

$$C_{pk_U} = \frac{U_{SL} - X_{50\,\%}}{X_{99,865\,\%} - X_{50\,\%}} \quad \text{or} \quad P_{pk_U} = \frac{U_{SL} - X_{50\,\%}}{X_{99,865\,\%} - X_{50\,\%}} \qquad (1)$$

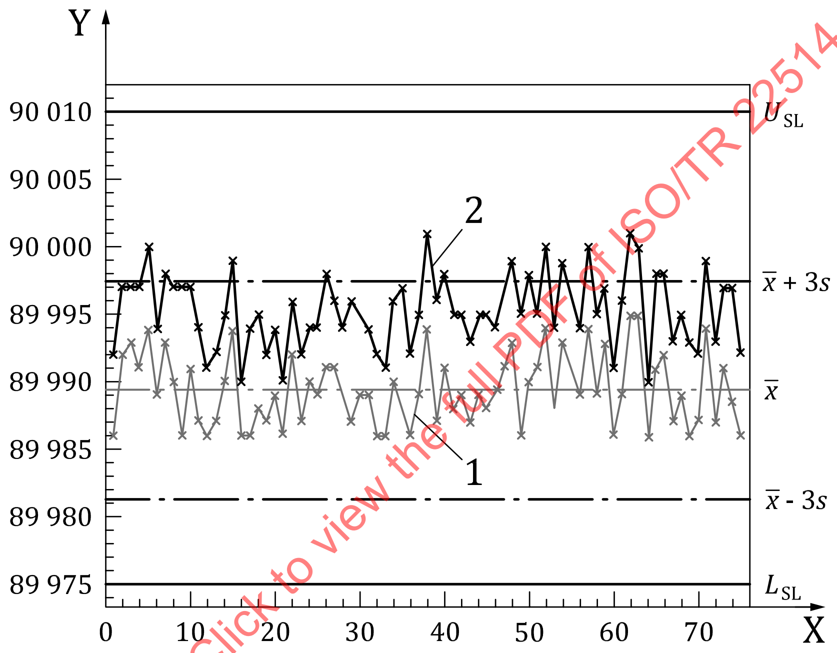
b) for the calculation of index for the minimum two-point size, see Formula (2):

$$C_{\text{pk}_L} = \frac{X_{50\,\%} - L_{\text{SL}}}{X_{50\,\%} - X_{0,135\,\%}} \quad \text{or} \quad P_{\text{pk}_L} = \frac{X_{50\,\%} - L_{\text{SL}}}{X_{50\,\%} - X_{0,135\,\%}} \tag{2}$$

The minimum capability index C_{pk}, P_{pk} is the one with the lowest value.

The corresponding P_{p} or C_{p} values can only be used for production monitoring purposes. The method for the actual calculation of these indices can be found either in ISO 22514-2 in case of time dependent production models or in ISO 22514-4.

Example from diameter measurements of a shaft diameter (Figure 4) with two different measurement series (two-point diameter and minimum subscribed diameter).



Key

X value no.
Y diameter
1 two-point requirement
2 envelope requirement

U_{SL} upper specification limit
$\bar{x}+3s$ two point measurements
$\bar{x}$ two point measurements
$\bar{x}-3s$ two point measurements
L_{SL} lower specification limit

Figure 4 — Example from measurements of a shaft

6 Capability calculation on features defined by geometrical tolerances

6.1 General

Geometrical tolerances as defined in ISO 1101 are very often used in industry to set requirements for critical and important characteristics. Geometrical tolerances define a tolerance zone where the toleranced surface or derived element (e.g. centreline or midplane) can be applied. That means that all measured points on the surface or the derived element are inside the tolerance zone.

Capability indices for geometrical tolerances are usually based on skewed distributions. The skewed distributions require a minimum of five parts in the subsamples in case of x-bar & R control charts.

In case of logarithmic distribution, the X-bar chart can be calculated based on the logarithm of the measured values.

Commonly found distribution models when geometrical tolerances are measured:

— Folded normal distribution (FD);

— Logarithmic normal distribution (LD), (This distribution can only be used in special cases because of the risk on a long “distribution tale”);

— Rayleigh distribution (RD)(often used when the specifications are coaxiality, run-out or position tolerances) ;

— Weibull distribution (WD).

6.2 Form, orientation, location and run out tolerances

6.2.1 General

Annex A contains guidance on the choice of distribution model when capability indices are calculated on characteristic defined by geometrical tolerances.

The calculation of the indices will be done by calculating the average of the maximal form failure found on the workpieces. The distribution model will very often be a folded normal distribution.

6.2.2 Example flatness

The deviation in flatness can be calculated based on the minimum zone plane. From this plane the maximum deviation in both directions is found and superposed to a maximum flatness deviation. This value found on all the workpieces are basis for the calculation of the capability or performance indices. When an Shewhart control chart has been calculated stability can be documented and the capability index calculated.

In all other cases the performance index can be used. The distribution model is also in this case often a folded normal distribution.

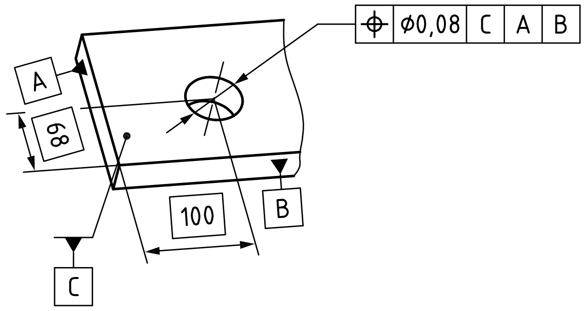
6.2.3 Example roundness

The deviation in roundness can be calculated based on the minimum zone circle. From this circle the maximum deviation in both directions (outer and inner circle) are found and added together to the maximum roundness deviation. This value found on all the workpieces are basis for the calculation of the capability or performance indices depending on documented stability.

6.3 Location tolerances (Figure 5)

The calculation of the indices can be done by calculating the average of the maximal deviation from the “true” value of the location. If the requirement is a position tolerance defined with a tolerance zone in form of a cylinder (Figure 6 shows one of a number of circles) the calculation method given in ISO 22514-6 can be used. The tolerances element is centreline in the cylinder. Therefore minimum 3 intersections can be measured and the minimum index of the three calculated indices is the resulting capability or performance index.

Dimensions in millimetres



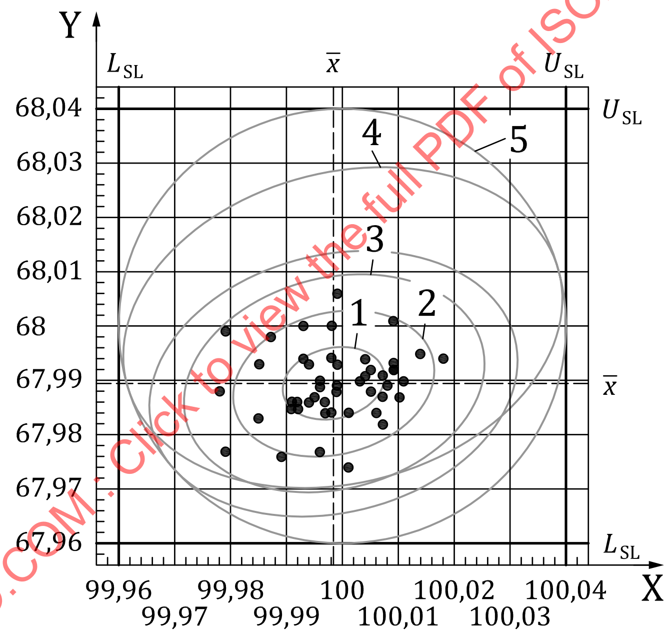
Key

A datum A

B datum B

C datum C

Figure 5 — Position tolerance



Key

X value, in mm

Y value, in mm

$\bar{x}$ average

L_{SL} lower specification limit

U_{SL} upper specification limit

1 68,27 % area

2 95,45 % area

3 99,73 % area

4 expanded variation area

5 geometrical tolerance zone

Figure 6 — Measured values

Based on the measured values (see Figure 6) can P_{po} and P_{pok} be calculated if no stability is given (see ISO 22514-6).

In case of position tolerances, there is a three-dimensional tolerance wherefore the deviation of the toleranced centreline can be found using a number of intersections. A bigger influence on the uncertainty of the result will be found, if only few intersections are used as basis for the calculations (see Annex C).

The measurements are normally done on a 3-D coordinate measurement machine in three or more sections on the cylinder surface using a tactile scanning sensor. Based on the collected points several centres are calculated. The centre with the biggest deviation from the true position are used for the capability calculation.

Geometric properties are comprehensive properties and do not have information about position and direction. Therefore, process capability information related to geometric characteristics is not useful for process improvement. To utilize the geometric characteristics for process improvement, it is necessary to decompose the geometric characteristics (see Figure 7).

Dimensions in millimetres

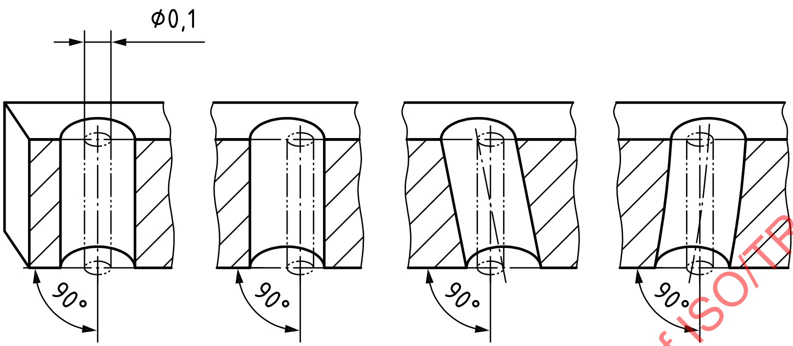


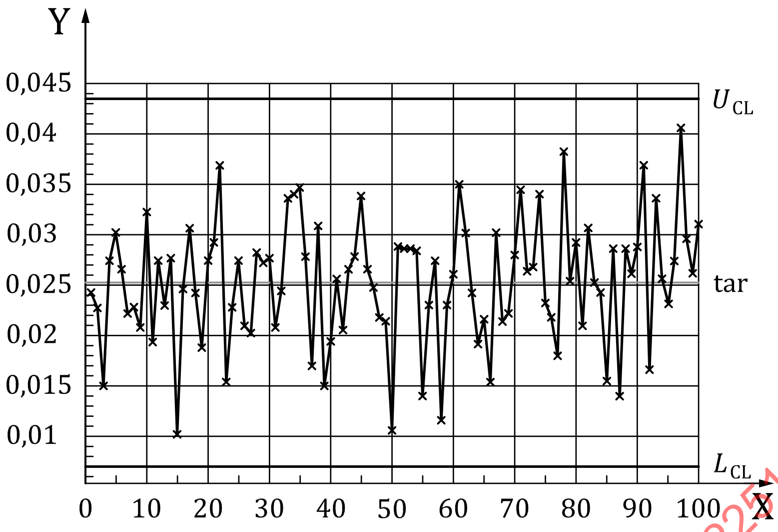
Figure 7 — Three different components of deviation from the true position

6.4 Example on capability calculation in case of known inverse distribution function

The calculation of the deviation from run out can be calculated by the difference of the minimum and maximum distance between the circumference line of the rotation surface and the datum axis in any cross-section perpendicular to the datum axis. The distribution model from run out will very often be a Rayleigh distribution.

Example (from ISO/TR 11462-3 test example 10)

A dataset collected from run of measurement of 500 shafts can be used for the calculation of the capability indices (only 100 measurement of the 500 is shown on the graph in Figure 8).



Key

X sample number

Y measured value (mm)

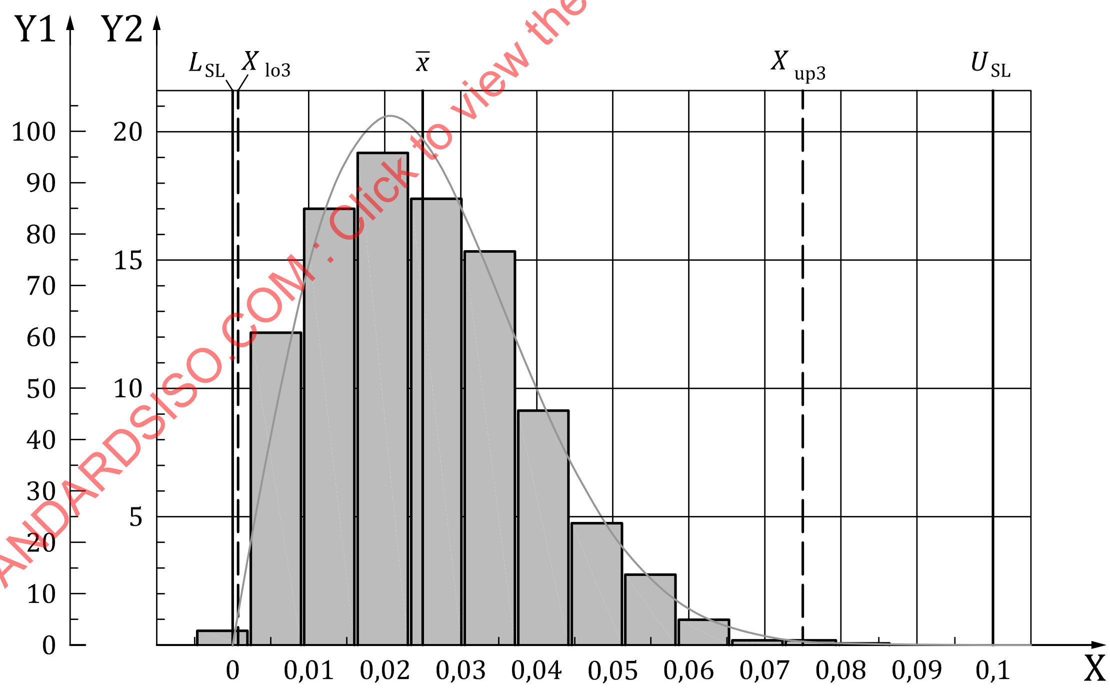
tar target value

U_{CL} upper control line

L_{CL} lower control line

Figure 8 — Shewhart control chart (averages)

$\bar{X}$ – chart (see Figure 8) calculated from the measured values shows a process in control.



Key

X measured value, in mm

Y1 absolute frequency

Y2 relative frequency, in %

L_{SL} lower specification limit

U_{SL} upper specification

X_{lo3} 0,135 %

$\bar{x}$ average

X_{up3} 99,865 %

Figure 9 — Histogram on the dataset

Based on the values the following calculations can be done.

The average is calculated to $\bar{X} =$ 0,252 7 mm and the $X_{50\ \%}$ = 0,024 5 mm

The histogram (see Figure 9) shows a Rayleigh distribution using the estimated scale-parameter σ = 0,020 17 based on the Method of Moments ($\mu_{\text{Rayleigh}} = \sigma \times \sqrt{\frac{\pi}{2}} = \hat{\mu} = \bar{x}$).

The use of the inverse distribution function of the Rayleigh distribution $F^{-1}(p|\sigma) = \sigma \times \sqrt{-2 \times \ln(1-p)}$ leads to the following estimators of distribution percentiles and of the process dispersion according to method d1 of ISO 22514-2:

$$\hat{X}_{0,135\ \%} = F^{-1}\left(0,001\ 35\ |\ 0,020\ 165\ 73\right) \approx 0,001\ 05$$

$$\hat{X}_{99,865\ \%} = F^{-1}\left(0,998\ 65\ |\ 0,020\ 165\ 73\right) \approx 0,073\ 31$$

The specification is one limited U_{SL} = 0,1 mm (and a theoretical lower limit) wherefore only the capability index C_{pk_U} can be calculated:

$$C_{\text{pk}_U} = (U_{\text{SL}} - X_{50\ \%})/(\hat{X}_{99,865\ \%} - X_{50\ \%}) = (0,1 - 0,024\ 5)/(0,073\ 31 - 0,024\ 2) = 1,55.$$

7 Maximum material requirement

7.1 Information about maximum material requirement

Three types of requirements are defined in (ISO 2692): MMR, LMR and RPR:

They are used to cover frequently occurring workpiece functional cases in design and tolerancing, where there is a mutual dependence of size and geometry, for fulfilling the function assembly of parts.

The "maximum material requirement", MMR, covers "assembleability (mountability)" and the "least material requirement", LMR, covers "minimum wall thickness" of a part. Each requirement (MMR and LMR) combines two independent requirements into one collective requirement, which more accurately simulates the intended function of the workpiece. In some cases, the "reciprocity requirement", RPR, can be added.

The assembly of parts depends on the combined effect of the size (of one or more extracted features of size), and the geometrical deviation of the feature(s).

The minimum assembly clearance occurs when each of the mating features of size is at its maximum material size (e.g. the largest bolt size and the smallest hole size) and when the geometrical deviations (e.g. the form, orientation and location deviations) of the features of size and their derived features (median line or median surface) are also at their maximum. Assembly clearance increases to a maximum when the sizes of the assembled features of size are furthest from their maximum material sizes (e.g. the smallest shaft size and the largest hole size) and when the geometrical deviations (e.g. the form, orientation and location deviations) of the features of size and their derived features are zero. It therefore follows that if the sizes of one mating part do not reach their maximum material size, the indicated geometrical tolerance of the features of size and their derived features can be increased without endangering the assembly to the other part.

This assembly function is controlled by the maximum material requirement. This collective requirement is indicated on drawings by the symbol M in a circle (see Figure 10)

Dimensions in millimetres

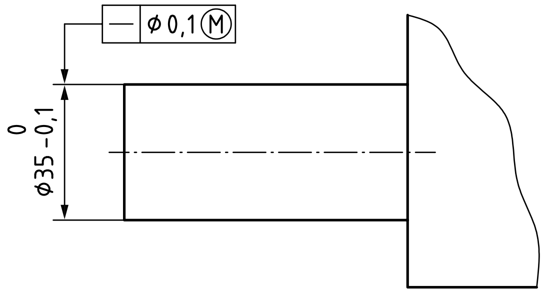


Figure 10 — Example on drawing indication (adapted from ISO 2692:2021, Figure A.2 a) “Drawing indication”)

The least material requirement is designed to control the minimum wall thickness preventing breakout or the maximum width of a series of slots, etc. The least material requirement is also characterized by a collective requirement for the size of a feature of size, the geometrical deviation of the feature of size (form deviations) and the location of its derived feature. LMR is indicated on drawings by the symbol L in a circle.

The reciprocity requirement is an additional requirement, which can be used together with the maximum material requirement and the least material requirement. It can be used in cases where it is permitted — considering the function of the toleranced feature(s) — to enlarge the size tolerance when the geometrical deviation on the actual workpiece does not take full advantage of the maximum material virtual condition or the least material virtual condition.

7.2 Calculation of results

When maximum material requirement, MMR, or least material requirement, LMR, is used, the size specification and geometrical specification are transformed into one collective requirements specification. The collective specification concerns only the integral feature, which relates to the surface(s) of the feature(s) of size(s).

The sizes used for the calculation of the result are generated by the collective effect of the maximum material size called MMVS, of a feature of size and the geometrical tolerance given for the derived feature of the same feature of size. The result is a clearance, which have a minimum of zero.

For external features of size:

— $l_{\text{MMVS,e}} = l_{\text{MMS}} + \delta$

For internal features of size:

— $l_{\text{MMVS,i}} = l_{\text{MMS}} - \delta$

where δ is the geometrical tolerance, l_{MMVS} is the virtual size.

The MMC requirement is that the clearance:

— $l_{\text{MMVS,e}} - l_{\text{MMS}} \geq 0$

For every combined characteristic, the minimum assembling clearance can be calculated.

EXAMPLE 1

Dimensions in millimetres

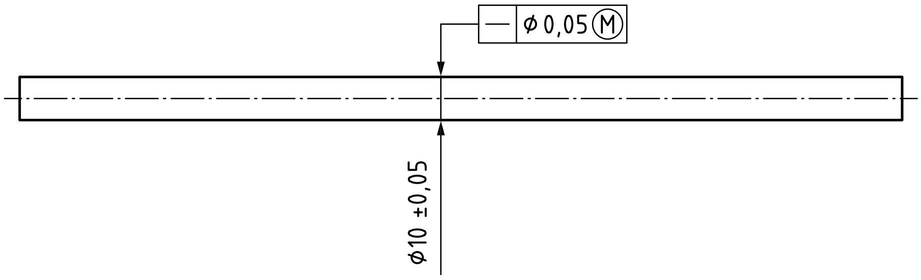


Figure 11 — Example on a MMC requirement on a part

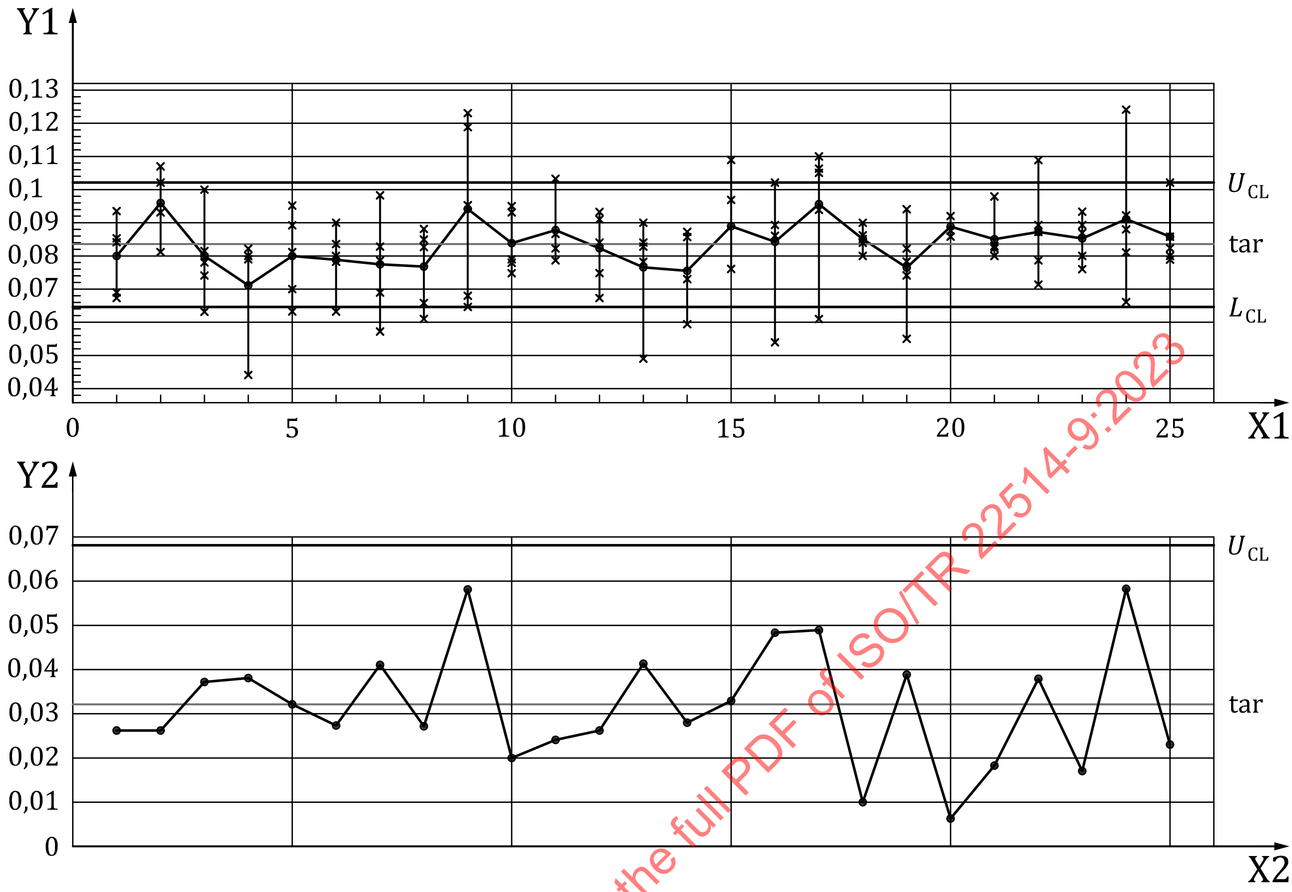
An initial process study has been done in the production on the above part (Figure 11) with the following result: A Shewhart control chart on the assembling clearance that has been used in connection to the data collection (Figure 12).

Basic information

— $\bar{X}$ −R control chart

— Sample size $n = 5$

— Total number of data $N = 125$



Key

X1, X2	number of samples	U_{CL}	upper control limit
Y1	average of samples	L_{CL}	lower control limit
Y2	range of samples	tar	target value

Figure 12 — Shewhart control chart

Results:

— Straightness; $\bar{X}$ = 0,015 9 mm;

— Diameter; $\bar{X}$ = 10,000 52 mm.

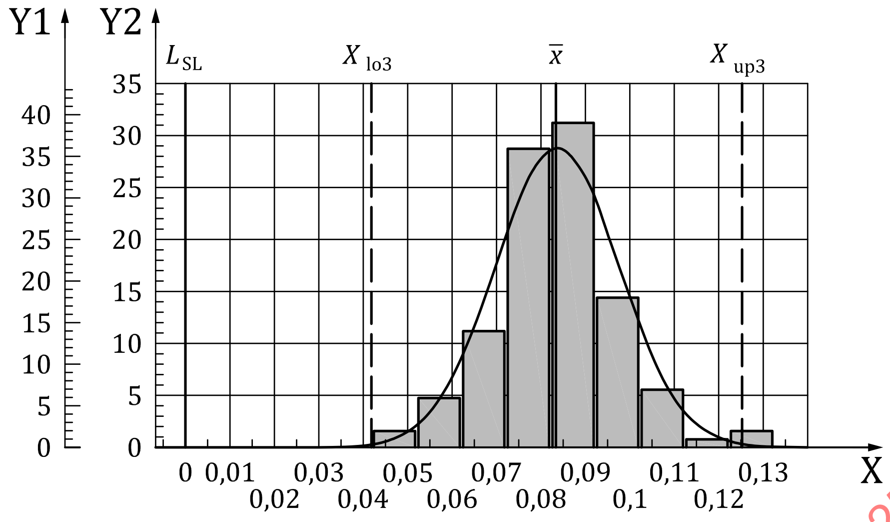
The process is in statistical control and the process capability can be calculated (Figure 12).

— $l_{MMVS,e} = l_{MMS} + \delta$ = 10,05 mm + 0,05 mm = 10,1 mm

The actual assembly clearance for a part can be calculated from:

— $l_{MMVS,e} - (d + \delta)$ = 10,1 − (estimated diameter + estimated straightness).

— Average of clearance = 0,083 58 mm.



Key

X clearance, in mm

Y1 relative frequency, in %

Y2 absolute frequency

L_{SL} lower specification limit

X_{lo3} lower 0,135 % value

X_{up3} upper 99,865 % value

$\bar{x}$ average

Figure 13 — Histogram — Clearance

Based on the distribution model of the minimum assembly clearance (see Figure 13), the capability indices can be calculated as defined in ISO 22514-4.

$$C_{pk} = \frac{\bar{X} - L_{SL}}{\bar{X} - X_{0,135\,\%}} = 2,00 \text{ (based on the assumption of normal distribution)}$$

Also, the capability index for the diameter can be calculated if necessary.

EXAMPLE 2

— Position of a hole in a plate;

— Tolerance on diameter: ⌀ 10,15 mm and 10 mm;

— Tolerance position: ⌀ 0,2 mm;

— Maximum material for the diameter: ⌀ 3,5 mm;

— Position tolerance: ⌀ 0,15 mm;

— Maximum material virtual condition:

$l_{MMVS,i} = l_{MMS} - \delta$

— Calculation on one actual measured part:

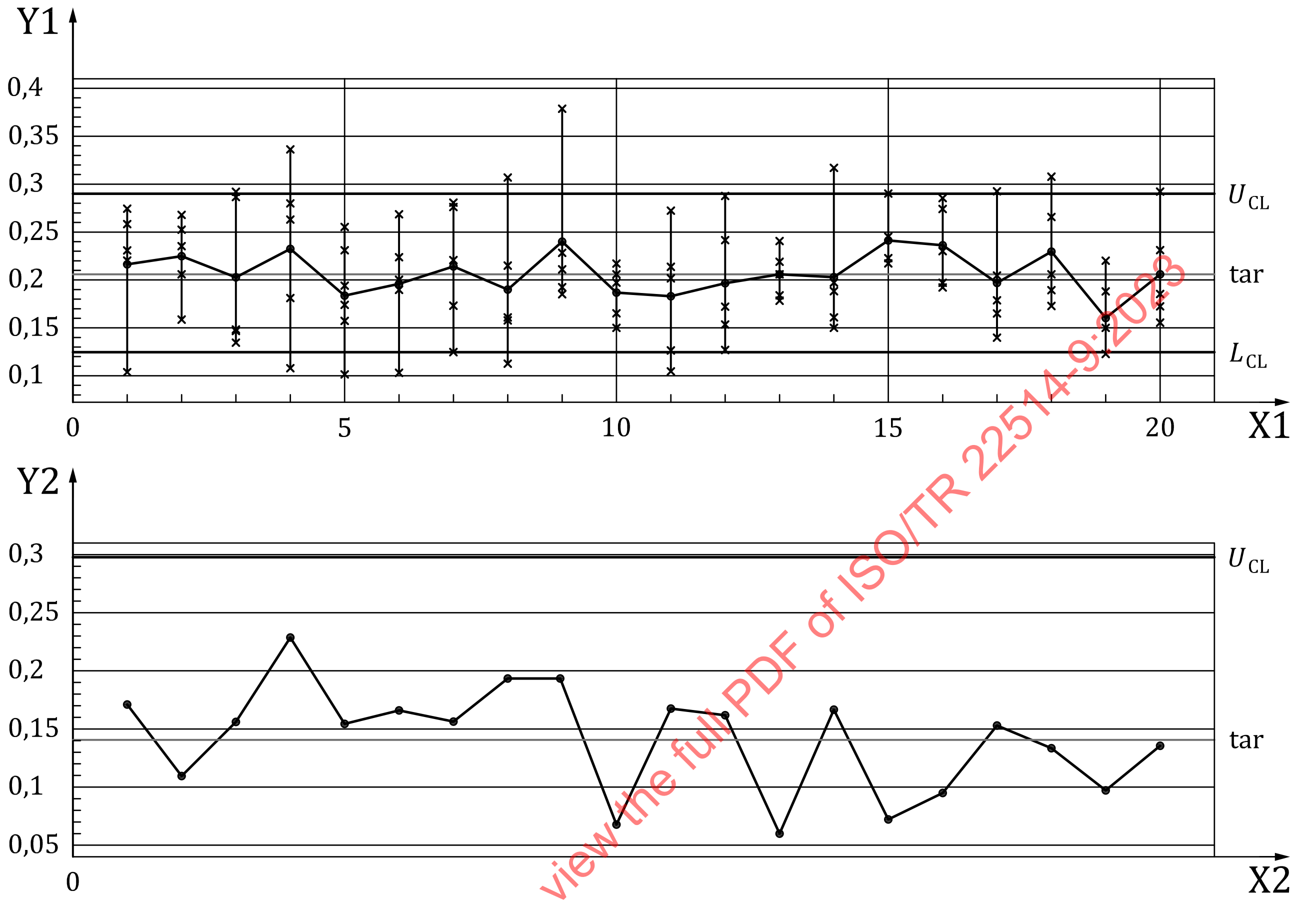
— Measured values:

Diameter (maximum inscribed): ⌀ 3,605 mm and the position deviation 0,034 mm;

— Calculated clearance:

$(d + \delta_A) - l_{MMVS,i} = (3,605 - 0,034) - 3,35 = 3,571 - 3,35 = 0,221$ mm.

A Shewhart control chart is constructed on the calculated clearance. It shows a process in statistical control (see Figure 14).



Key

X1, X2	sample number	U_{CL}	upper control limit
Y1	average of sample	L_{CL}	lower control limit
Y2	range of samples	tar	target value

Figure 14 — Control chart — Clearance

— Result of the calculation on the total sample *n*;

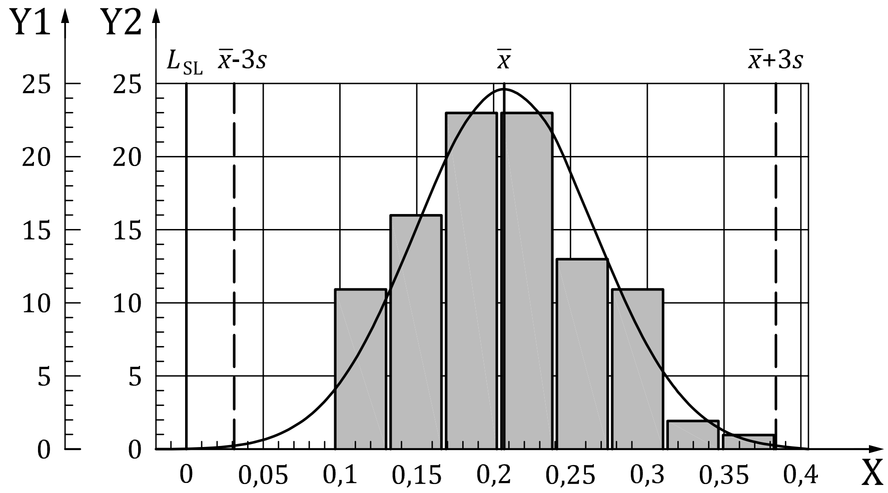
— Calculated actual average for the hole, $\overline{X}$ = 3,606 mm;

— Calculated actual average for the position deviation, $\overline{X}$ = 0,048 6 mm;

— Calculated average clearance = $(\overline{X}_{diam} - \overline{X}_{pos}) - l_{MMVS,i}$;

— (3,606 − 0,048 6) mm − 3,35 mm = 0,207 4 mm;

— Minimum value for the calculated clearance = 0.



Key

X clearance values

Y1 absolute frequency

Y2 relative frequency, in %

L_{SL} lower specification limit

$\bar{x} - 3s$ lower 3 standard limit

$\bar{x} + 3s$ upper 3 standard limit

$\bar{x}$ average

Figure 15 — Histogram normal distribution — Clearance

Distribution model (see Figure 15): Normal.

Calculation: The capability index C_p and C_{pk} (P_p and P_{pk} for processes not in control) for the hole has been calculated based on measures of the maximum inscribed diameter using the methodology defined in ISO 22514-4. If the process has been demonstrated to be in statistical control the capability [C] can be calculated. If not, then the performance [P] can be reported.

The capability of the maximum material requirement is the capability of the clearance and only the lower capability index C_{pk_L} or the performance index P_{pk_L} can be calculated.

7.3 Least material requirement LMR

Least material requirements, LMR, are used in pairs, e.g. to control the minimum wall thickness between two symmetrical or coaxially located similar features of size. The least material requirement is the local size of the toleranced feature equal to or larger than the least material size.

The calculation of the capability index follows the same methodology as described for the maximum material requirement. Instead of calculation of the clearance the least material virtual value LMVC can be calculated and the compared with the actual material thickness.

Also, here only a minimum capability can be calculated.

Dimensions in millimetres

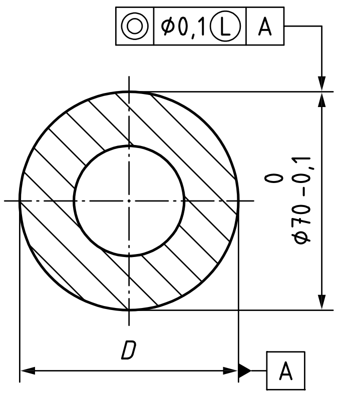


Figure 16 — Example on LMR (adapted from ISO 2692:2021, Figure A.5 "drawing indication")

Dimensions in millimetres

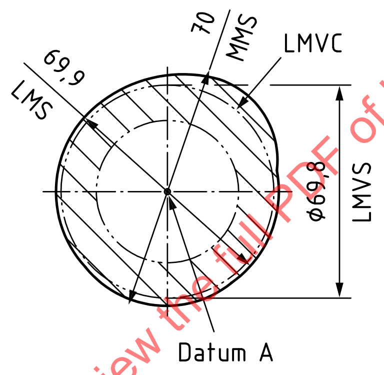


Figure 17 — Interpretation of the specification in Figure 16

The LMVS for external features of size, $l_{MMVS,e}$, is l_{MMVS}, e = $l_{LMS} - \delta$.

In example Figure 16 the result $l_{MMVS,e} = l_{LMS} - \delta$ = Ø69,9 mm − 0,1 mm = Ø69,8 mm.

The capability index in this case can be calculated based on the actual measured values as shown on Figure 17. The value is the minimum distance to the diameter LMVS.

7.4 Methodology

When the capability on the maximum material requirement or the minimum material requirement can be calculated, the following step can be necessary:

a) Define the process and its operating conditions. If there is a change in those conditions during the production, it will be necessary to run a new process study for machine performance or preliminary capability.

b) Calculate the virtual size for the characteristic of interest.

c) The uncertainty of the measurement system can be estimated and judged appropriately (see ISO 22514-7).

d) In case of MMC measure the clearance for every part from the sample. In case of LMC measure the minimum material value for every part from the sample.

e) Construct a Shewhart control chart to demonstrate a process in control.

f) Preserve the process stability and maintain its statistical control.

g) Determine the present distribution.

h) Estimate the average and reference interval; and select an appropriate measure of capability.

The following are the conditions that will apply for capability:

— multi-factor, multi-level aspects of the process are allowed but the calculations can be separated for every factor;

— the duration over which the data have been gathered to be recorded;

— the frequency of sampling can be specified, together with the start and end dates of data collection;

— the process can be controlled with a control chart.

It is necessary to check the control chart from which the data has been taken for statistical control and to examine a histogram of the data. A valid test for the chosen distribution can be used for assessing the data distribution, such as the χ^2 test or any other suitable method. Additionally, a probability plot can be used to document the distribution and determine the variation.

Annex A
(informative)

Distribution identification

A.1 General

Table A.1 shows the choice of distribution model. The formulae for the distribution models can be found in Annex B.

Table A.1 — Choice of distribution model to be used together with geometrical tolerances (adapted from ISO 1101:2017, Table 2)

Tolerances	Characteristics	Symbol	Commonly found distribution
Form	Straightness	⏤	FD LD
	Flatness	⏥	FD LD
	Roundness	○	FD
	Cylindricity	⌭	FD
	Line profile	⌒	FD
	Surface profile	⌓	FD
Orientation	Parallelism	∥	FD
	Perpendicularity	⊥	FD
	Angularity	∠	FD
	Line profile	⌒	FD
	Surface profile	⌓	FD
Location	Position – one coordinate	⌖	FD
	Position – two coordinates	⌖	RD
	Concentricity (for centre points)	◎	RD
	Coaxiality (for axes)	◎	RD
	Symmetry	⌯	FD
	Line profile	⌒	FD
	Surface profile	⌓	FD
Run-out	Circular run-out radial	↗	RD
	Circular run-out axial	↗	LD
	Total run-out radial	⌰	RD
	Total run-out axial	⌰	LN

FD folded normal distribution.

LD Logarithmic Normal distribution.

RD Rayleigh Distribution.

A.2 Calculation of dispersion

The dispersion of the process can be calculated with one of the following formulae.

Table A.2 — Different methods for calculation of dispersion

Method	Formula	No
Md1	$\hat{\Delta} = X_{99,865\,\%} - X_{0,135\,\%}$; $\hat{\Delta}_U = X_{99,865\,\%} - X_{50\,\%}$; $\hat{\Delta}_L = X_{50\,\%} - X_{0,135\,\%}$	(6)
Md2	$\hat{\Delta} = 6\hat{\sigma};\ \hat{\Delta}_U = 3\hat{\sigma};\ \hat{\Delta}_L = 3\hat{\sigma}$ where $\hat{\sigma} = \sqrt{\frac{\sum s_i^2}{k}}$	(7)
Md3	$\hat{\Delta} = 6\hat{\sigma};\ \hat{\Delta}_U = 3\hat{\sigma};\ \hat{\Delta}_L = 3\hat{\sigma}$ where $\hat{\sigma} = \frac{\sum s_i}{k \cdot c_4}$	(8)
Md4	$\hat{\Delta} = 6\hat{\sigma};\ \hat{\Delta}_U = 3\hat{\sigma};\ \hat{\Delta}_L = 3\hat{\sigma}$ where $\hat{\sigma} = \frac{\sum R_i}{k \cdot d_2}$	(9)
Md5	$\hat{\Delta} = 6\hat{\sigma};\ \hat{\Delta}_U = 3\hat{\sigma};\ \hat{\Delta}_L = 3\hat{\sigma}$ where $\hat{\sigma} = s_t = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2}$	(10)

where

k is the number of subgroups of size n;

n is the number of values;

R_i is the range of the ith subgroup;

s_i is the standard deviation of the ith subgroup;

s_i^2 is the variance of the ith subgroup;

s_t the standard deviation of the whole data set.

x_i are the individual values;

$\bar{x}$ is the average.

Calculation of $X_{0,135\,\%}$ and $X_{99,865\,\%}$

The two procedures that can be used to estimate the $X_{0,135\,\%}$ and $X_{99,865\,\%}$ are:

a) Fit a distribution to the combined data set and estimate them from the fitted resulting distribution.

b) Estimate them directly from the combined data set. In order to obtain reliable estimate of $X_{0,135\,\%}$ and $X_{99,865\,\%}$ in this procedure, the size of the given data set will be large. For instance, for a combined sample sizes of 1 000, then $X_{0,135\,\%}$ and $X_{99,865\,\%}$ are taken to be the minimum and maximum value of the data set assuming that there are no outliers.

See ISO 7870-2:2023, Table 2, for the values of c_4 and d_2 coefficients.

The symbol for the calculation of an index will be M_{ld}, where “l” defines the calculation method for location and “d” defines the calculation method for the dispersion.

EXAMPLE The calculation method M_{12} is based on calculation of average and variance.

NOTE 1 The estimator $\hat{\Delta}$ for d = 1 is the most general one, it can be used under all conditions.

NOTE 2 The estimators $\hat{\Delta}$ for d = 2, 3 and 4 estimate the subgroup spread only. They can be used for process model A1 only because they neglect the differences between subgroups.

NOTE 3 The estimators $\hat{\Delta}$ for d = 2, 3, 4 and 5 assume that the data is normally distributed. Otherwise, their result is biased depending on the type of distribution.

$\hat{\Delta}$ is also called the reference interval.

Annex B
(informative)

Distributions used in case of geometrical tolerances

B.1 General

When capability is calculated by geometrical specifications as described in this standard, the form of the distribution is known or can be reasonably assumed and can be verified by tests of fit. The approach is to estimate the parameters of that distribution and to use them to derive the relevant quantiles from which the capability estimates are obtained. The proportion out-of-specification can be directly estimated.

B.2 Normal distribution

If X_1, ..., X_N is a sample from a normal distribution with mean μ and variance σ^2, the estimates of μ and σ^2 are given by Formula (B.1):

$$\hat{\mu} = \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \quad \text{and} \qquad \hat{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2 \tag{B.1}$$

The normal distribution has the probability density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{B.2}$$

The capability indices can be estimated using the formulas given in ISO 22514-4.

B.3 Rayleigh distribution

This distribution is used almost exclusively to describe position, eccentricity and run-off in two-dimensional problems. In these situations, it is typical for there to be only a one-sided specification limit, U.

The Rayleigh distribution has the probability density function shown by Formula (B.3):

$$f(x) = \frac{x}{\sigma^2} \mathrm{e}^{-x^2/\left(2\sigma^2\right)} \qquad x \geq 0 \tag{B.3}$$

The cumulative distribution function is given by Formula (B.3):

$$F(x) = 1 - \exp\left(-\frac{X^2}{2\theta^2}\right) \tag{B.4}$$

where $X > 0$ and θ is a positive parameter. If X_1, ..., X_N is a sample from a Rayleigh distribution, the parameter θ is estimated by Formula (B.5):

$$\hat{\theta} = \sqrt{\frac{\sum_{i=1}^{N} X_i^2}{2N}} \tag{B.5}$$