SOCIETY OF AUTOMOTIVE ENGINEERS, Inc.
485 LEXINGTON AVENUE
NEW YORK 17, N.Y.

AEROSPACE INFORMATION REPORT

AIR 857

SPEED VARIATION OF D-C MOTORS

Issued 12-10-64
Revised

1. PURPOSE - The purpose of this report is to review the factors contributing to speed variation of d-c motors, to present typical data showing speed variation as a function of certain variables for representative motors, and to recommend a means of improving communications between motor users and manufacturers.

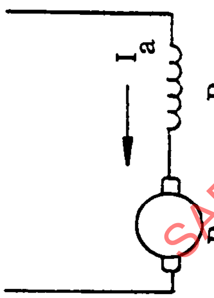
2. STATEMENT OF PROBLEM

2.1 Many applications of d-c motors require a quantitative knowledge of variations in motor speed and performance under various voltage inputs, ambient temperatures, altitudes, cooling conditions, and manufacturing tolerances. The basic specification, MIL-M-8609, is limited in that it merely requires that speed should be within 10% of the rated value after a heat run at rated voltage and sea level ambient.

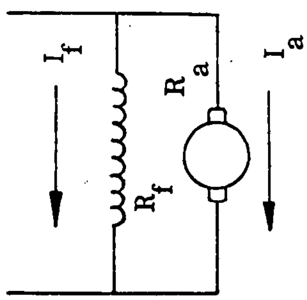
2.2 The problem facing motor users in preliminary design work is to predetermine what speeds will be encountered at other than rated torque, at other than rated voltage, at various ambient temperatures and atmospheric pressures, for a group of production motors built over a long interval of time.

2.3 Additionally, it appears that many users are in need of recommendations as to what parameters to specify, and how to specify them in order to obtain optimum designed motors meeting their requirements in size, weight, and cost.

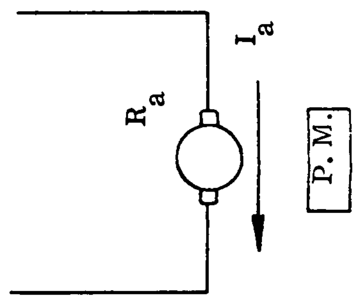
3. SCOPE - The following analysis is limited to variations in voltage, torque, temperature and altitude of the same magnitude as in MIL-M-8609.



SERIES



SHUNT



PERMANENT MAGNET

4. THEORY (GENERAL RELATIONSHIPS) - Basic equations describing the performance of a d-c motor with negligible armature reaction.

(1) $$V_t = E_c + e_b + I_a R_a + I_a R_s$$

(2) $$E_c = K_e \Phi S$$

(3) $$\Phi = \frac{a \cdot I_f}{b + I_f}$$ Froelich's equation for the machine saturation curve.

a & b	-	magnetic circuit constants
I_f	-	field excitation current
Φ	-	magnetic flux per pole
E_c	-	counter voltage developed by armature
K_e & K_t	-	winding constants
S	-	armature speed
V_t	-	line voltage at motor terminals
e_b	-	brush contact voltage drop
I_a	-	armature current
R_a	-	armature resistance
R_s	-	series field resistance

Combining and solving for speed yields the expression

$$(4) \quad S = \frac{V_t - e_b - I_a (R_a + R_s)}{K_e} \cdot \frac{b + I_f}{a \cdot I_f}$$

For a series motor ($I_a = I_f$)

$$(5) \quad S = \frac{V_t - e_b - I_a (R_a + R_s)}{K_e} \cdot \frac{b + I_a}{a \cdot I_a}$$

For a shunt motor ($R_s = o$)

$$(6) \quad S = \frac{V_t - e_b - I_a R_a}{K_e} \cdot \frac{b + I_f}{a \cdot I_f}$$

For a permanent magnet motor $\Phi = K_M$ a constant

$$(7) \quad S = \frac{V_t - e_b - I_a R_a}{K_c} \cdot \frac{1}{K_M}$$

5. <u>SPEED VARIATION DUE TO TORQUE VARIATIONS</u> - A d-c motor adjusts itself to an increase or decrease in load torque by drawing increased or decreased armature current in accordance with the equation

$$(8) \quad T = K_t \, \Phi \, I_a$$

Initially, then, changes in torque affect motor speed because of the corresponding change in armature current in accordance with equations (4); and (5), (6), or (7). The series motor is more drastically affected because I_a governs speed inversely, whereas in a shunt or permanent magnet motor I_a appears only as a subtrahend, which in most cases is a small percentage of the net numerator of equation (6) or (7).

6. SPEED VARIATION DUE TO TEMPERATURE VARIATIONS - Due to changes in heating occasioned by the change in motor losses accompanying load variations, a further effect on speed will result as winding resistances change. Generally the decrease in shunt field current due to shunt field resistance increase has more effect on speed than a proportionate change in armature or series field resistance. The speed of a shunt motor may, in general, be expected to increase as internal temperatures increase. In a series motor where $I_f = I_a$ and in a permanent magnet motor where the field is practically a constant, the speed will decrease as R_a and R_s increase.

When a shunt motor is operated well beyond the knee of the saturation curve, the expression: $\frac{a \cdot I_f}{b + I_f}$, equation (3) becomes more nearly a constant with variations in I_f caused by variations in temperature. The reciprocal of this expression in equation (6) also becomes nearly a constant. In this case the shunt motor speed will decrease as temperature increases as in the permanent magnet motor.

Another factor contributing to speed variation with temperature is the effect of increased bearing grease viscosity at low temperatures. The added torque required to overcome such losses results in a reduction of motor speed at low temperatures. This factor is generally more noticeable on miniature and/or high speed motors than on large, lower speed units. Also, it is more noticeable at the beginning of a run than it is after grease viscosity has decreased due to normal heating of the motor.

In the case of extreme temperature motors, another factor affecting speed is reduction of the permeability of steel as the Curie temperature is approached. This effects a reduction in Φ equation (3) as temperature is increased and appears as a speed increase in equations (5) and (6). Over the temperature range of MIL-M-8609 this factor is generally negligible.

7. SPEED VARIATIONS DUE TO ALTITUDE VARIATIONS - Altitude variations may act in three ways to affect motor speed:

(a) Increase in motor temperature in the case of fan-cooled motors, due to reduction in air density which also causes a small reduction in fan power requirements.

(b) Increase in temperature due to change in heat transfer parameters as altitude is increased.

(c) Change in brush contact drop and in brush friction.

Effects of items (a) and (b) on speed have been discussed previously in Sections 5 and 6.

A discussion of the effects of altitude, and the associated changes in humidity, on brush performance are beyond the scope of this report and are generally negligible on 28 volt motors.

8. SPEED VARIATIONS DUE TO INPUT VOLTAGE VARIATIONS - Equation (7) shows that the speed of a permanent magnet motor is zero until the line voltage V_t is greater than the sum of e_b and $I_a\ R_a$, the brush voltage drop and the voltage drop due to armature resistance. From this point on, the speed-voltage characteristic is practically a straight line up to rated voltage. In some cases this characteristic may extend beyond 50% overvoltage. The deviation from a straight line characteristic is caused by the $I_a\ R_a$ portion of equation (7). The armature current will increase gradually as voltage and speed increase with a constant torque load and more rapidly as overspeed conditions are reached. As $I_a\ R_a$ increases, the speed decreases.

Equation (5) shows that the speed of a series motor is zero until V_t is greater than the sum of e_b and $I_a\ (R_a + R_s)$. The armature current I_a of a series motor will also increase as voltage and speed increase with a constant torque load. Since I_a appears in two portions of equation (5) the deviation from a straight line will generally be greater than for a permanent magnet motor. An increase in $I_a\ (R_a + R_s)$ causes a decrease in speed, and since $\frac{b+I_a}{a.I_a}$ is equal to $\frac{1}{\Phi}$ (from equation 3), an increase in I_a will cause Φ to increase $\frac{1}{\Phi}$ to decrease, and the value of S, the motor speed to decrease. The change in Φ due to the increase in I_a will vary depending on what portion of the field saturation curve the motor is operating.

Similar reasoning can be applied to equation (6) for the shunt motor except I_f, the shunt field current, will increase in direct proportion to V_t rather than gradually as for I_a. Thus the factor $\frac{b+I_f}{a.I_f}$ will decrease more rapidly and the speed voltage characteristic can be expected to deviate more from a straight line than for a series motor.

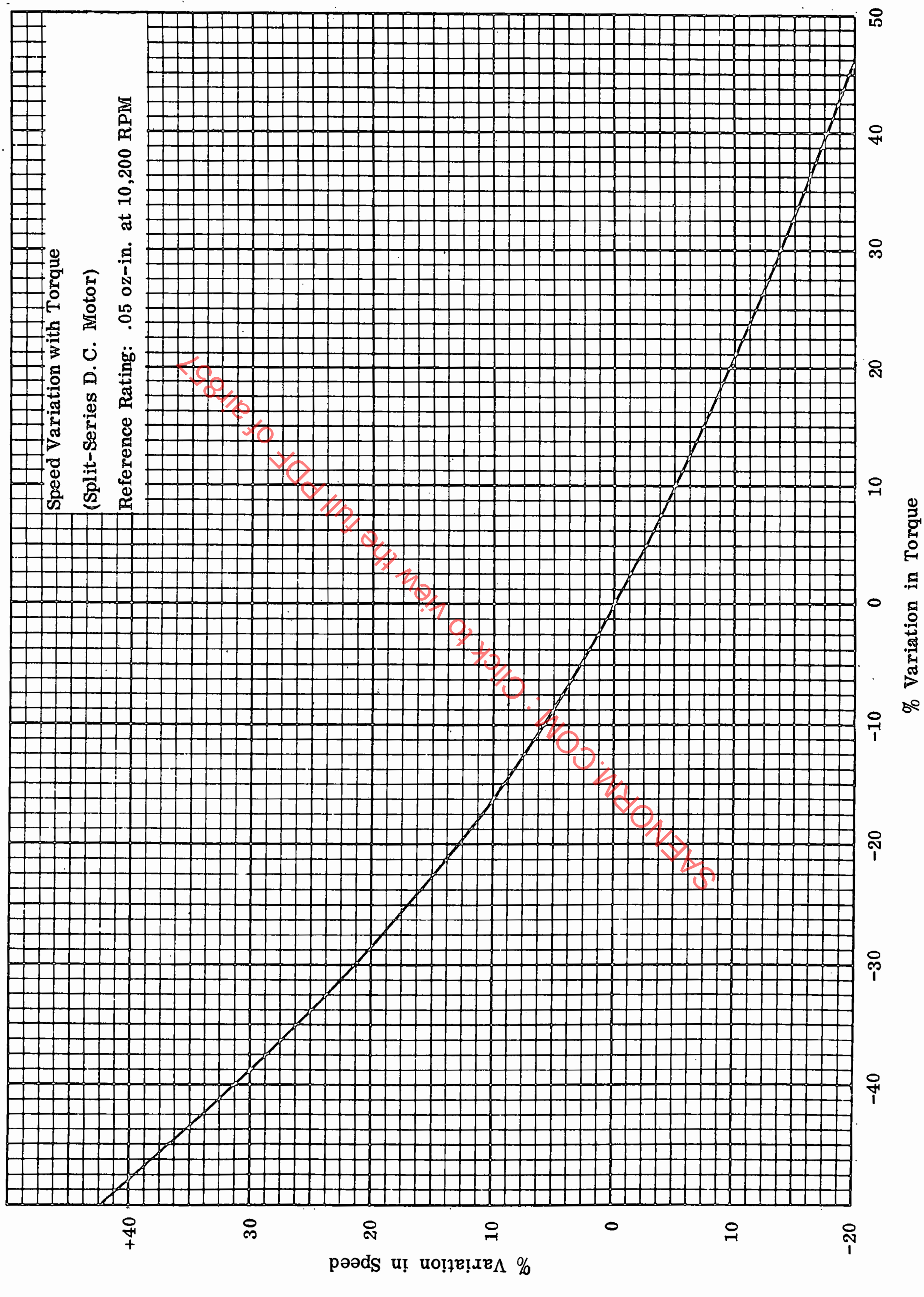
9. RECOMMENDATIONS - It is recommended that motor users accurately predetermine the percent variation which the driven device can tolerate, for each of the above governing factors, and include this information in the Detail Requirements Form, Fig. 1 of MIL-M-8609A. Only those factors of importance to the particular application should be included, in order to provide as much design latitude as possible.

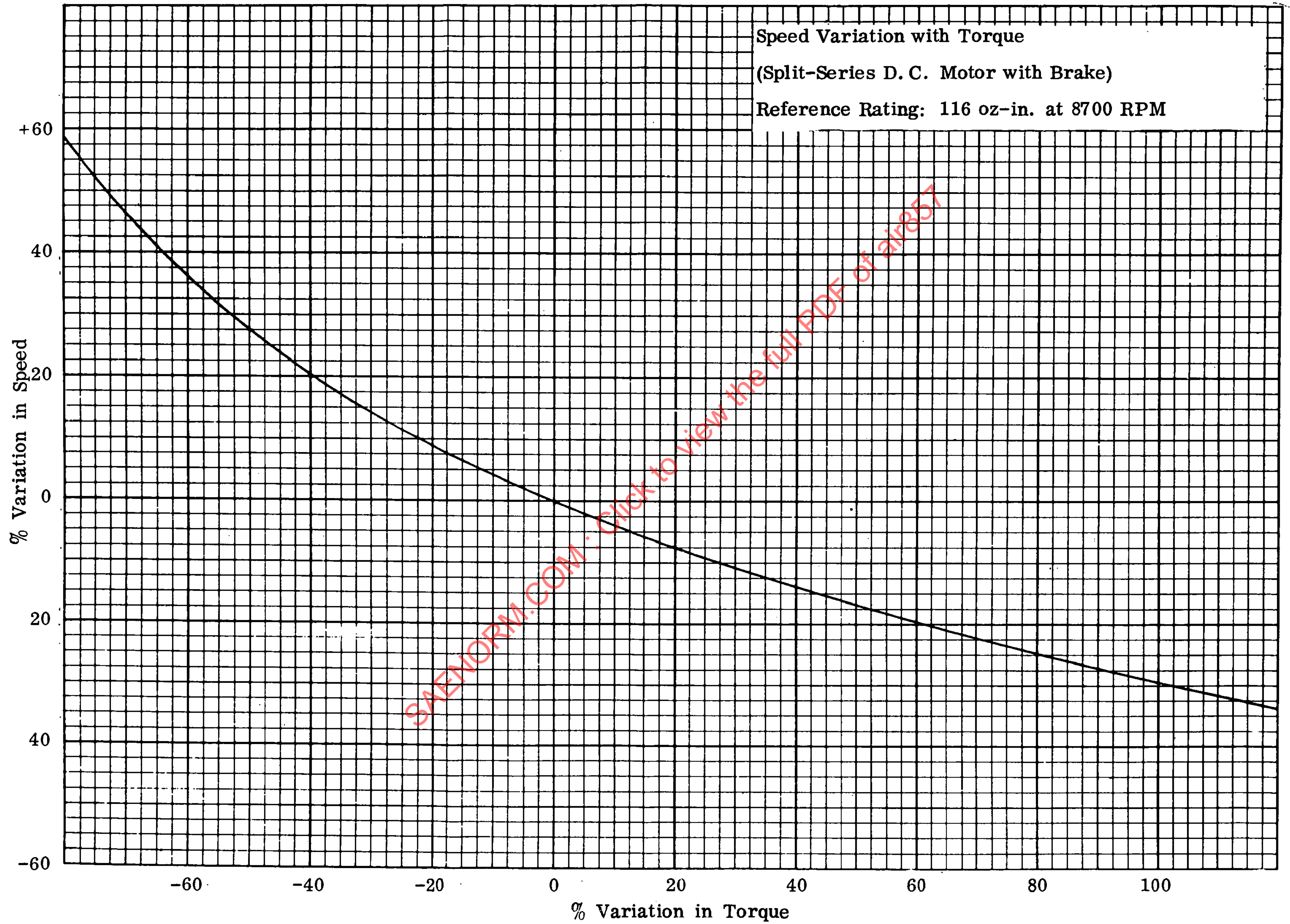
If speed variation data as a function of the other variables is desired for information, it should be obtained from test data taken on prototypes during qualification tests. Test data other than those normally taken during qualification tests may not be readily available and may require separate negotiation.

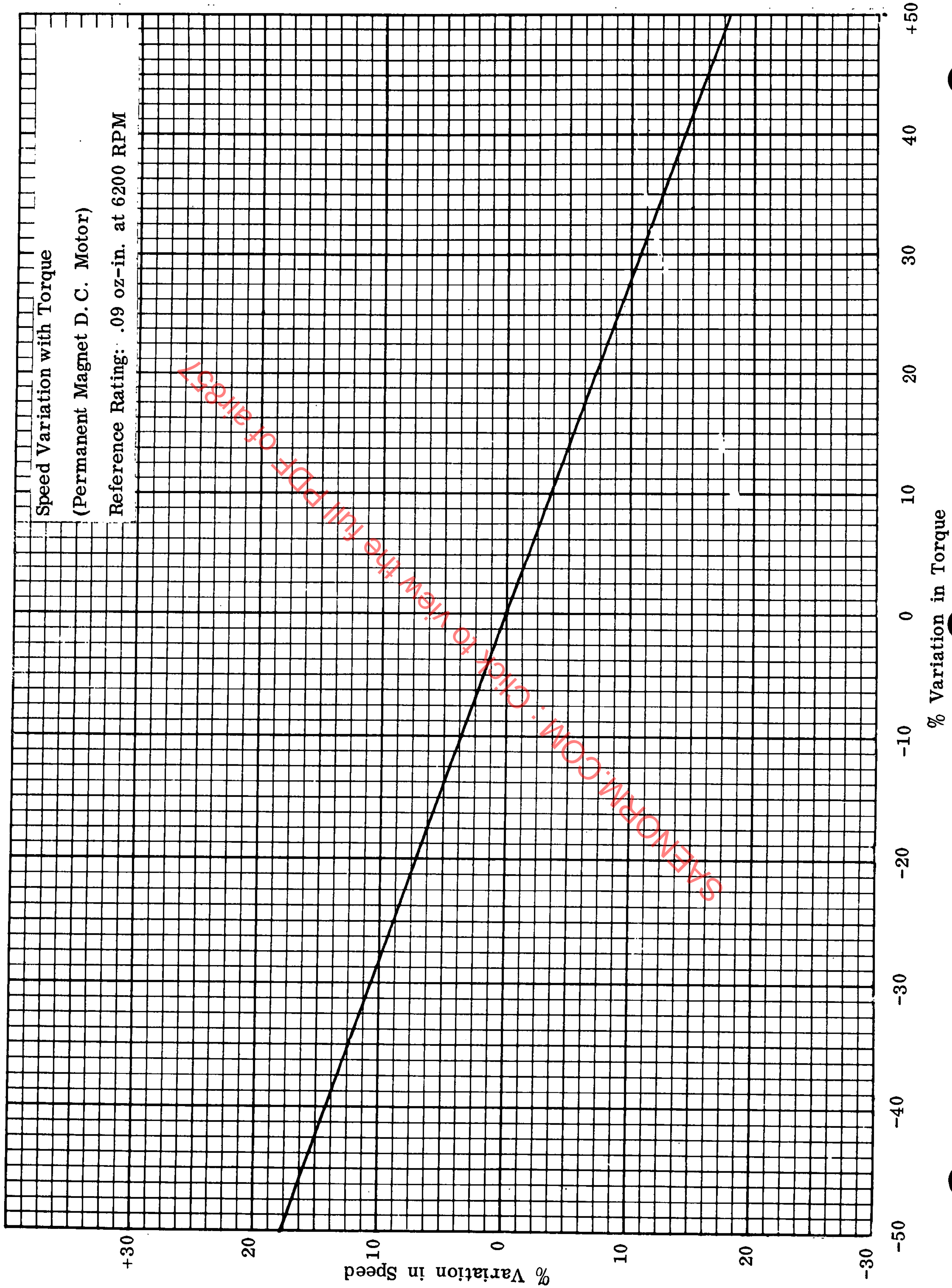
It is noted that paragraph 4.5.2.2 of MIL-M-8609A allows a $\pm$ 10% variation in speed due to manufacturing tolerances, and an additional 5% variation due to the effects of environmental tests on a motor (which should approximate the effects of normal usage and wear encountered in service). For a quantitative picture of the variation in speed due to the other factors previously discussed, it is recommended that the typical data presented herewith be studied.

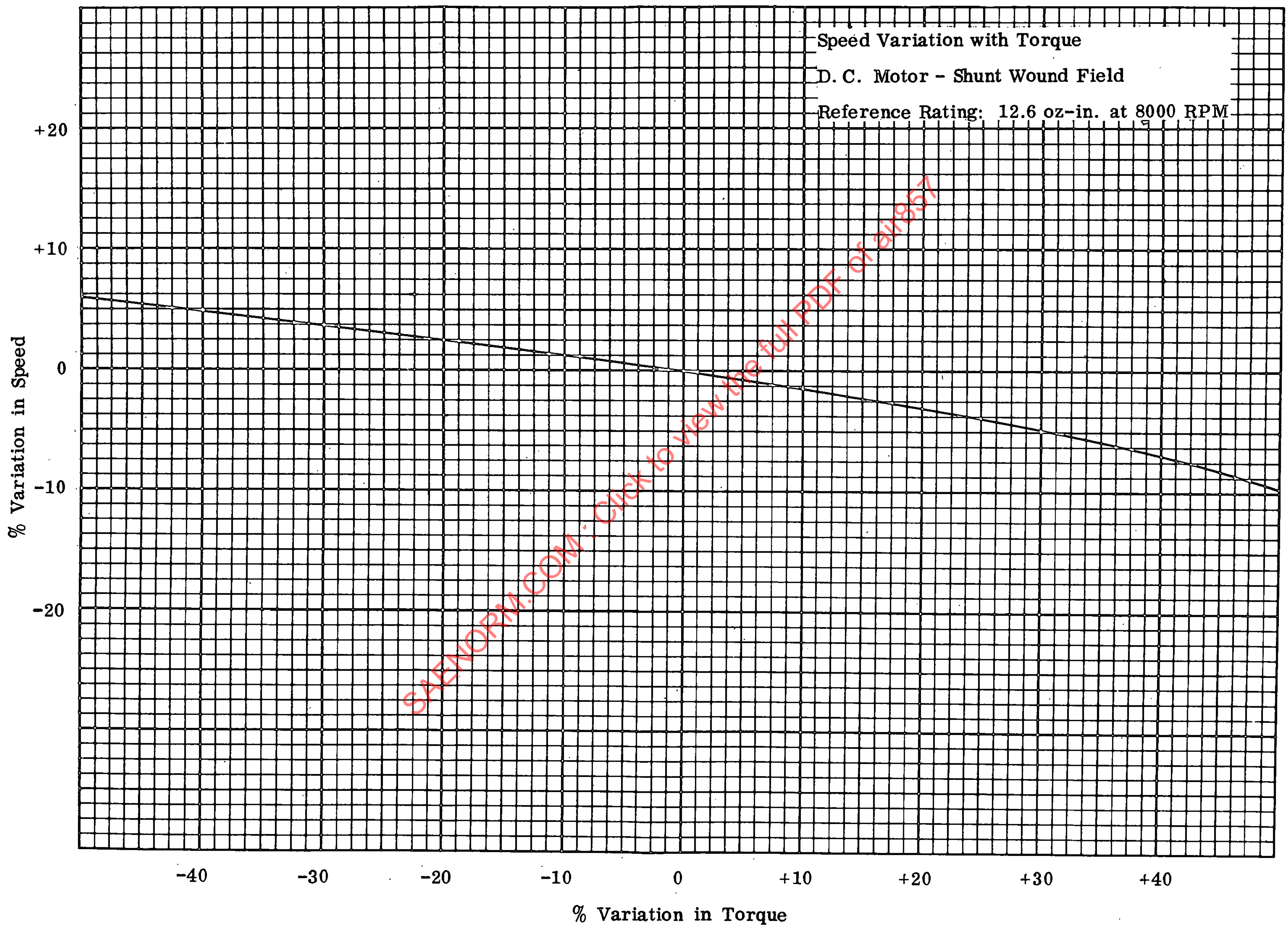
In order to obtain reasonable agreement in results obtained from motors of various sizes and designs, a "reference rating" was established for each motor by noting the value of torque at maximum power output, and then rating the unit at a torque load equal to half this value.

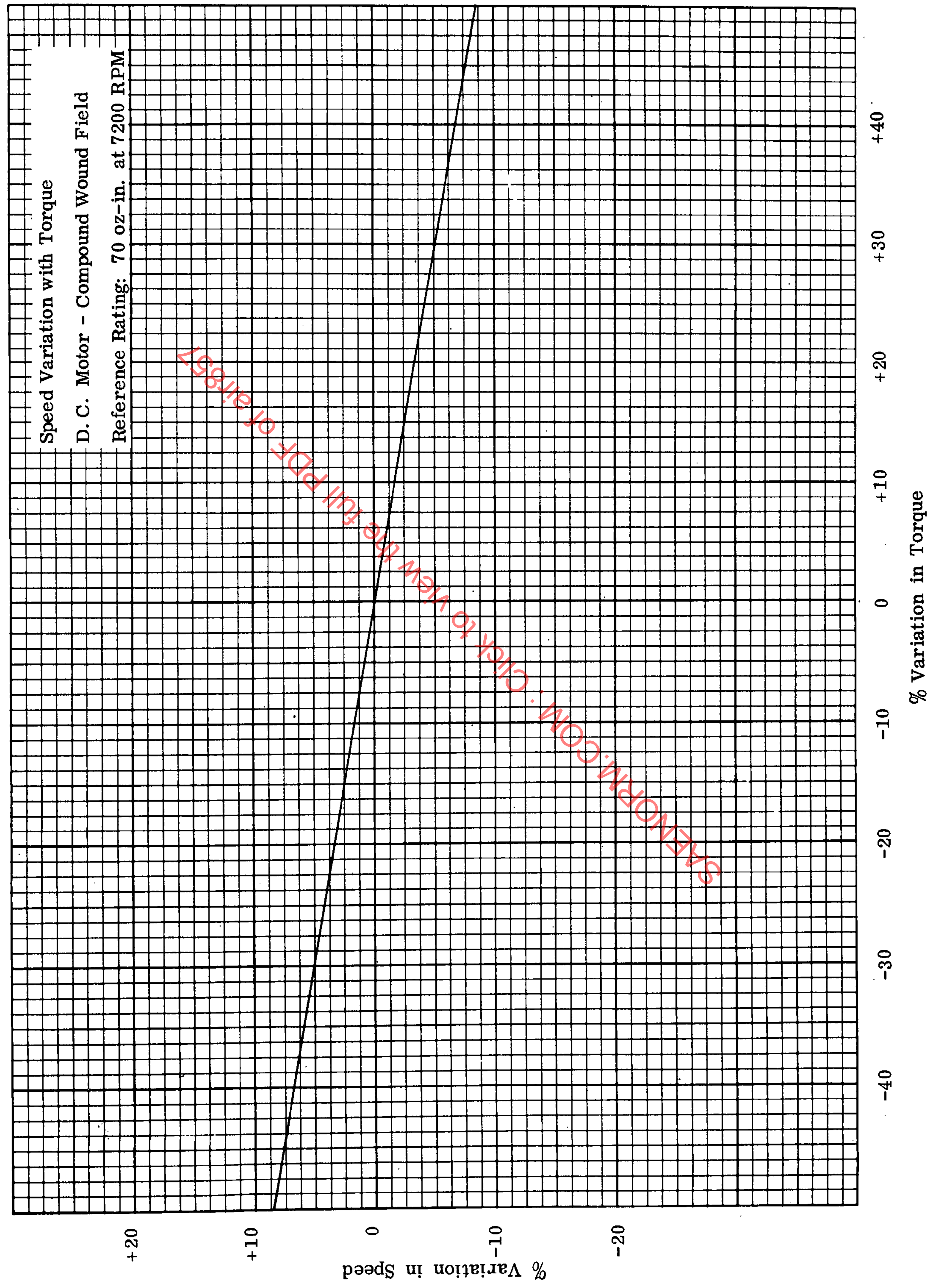
PREPARED BY SUBCOMMITTEE A-2M, MOTORS, OF SAE COMMITTEE A-2, AEROSPACE ELECTRICAL EQUIPMENT

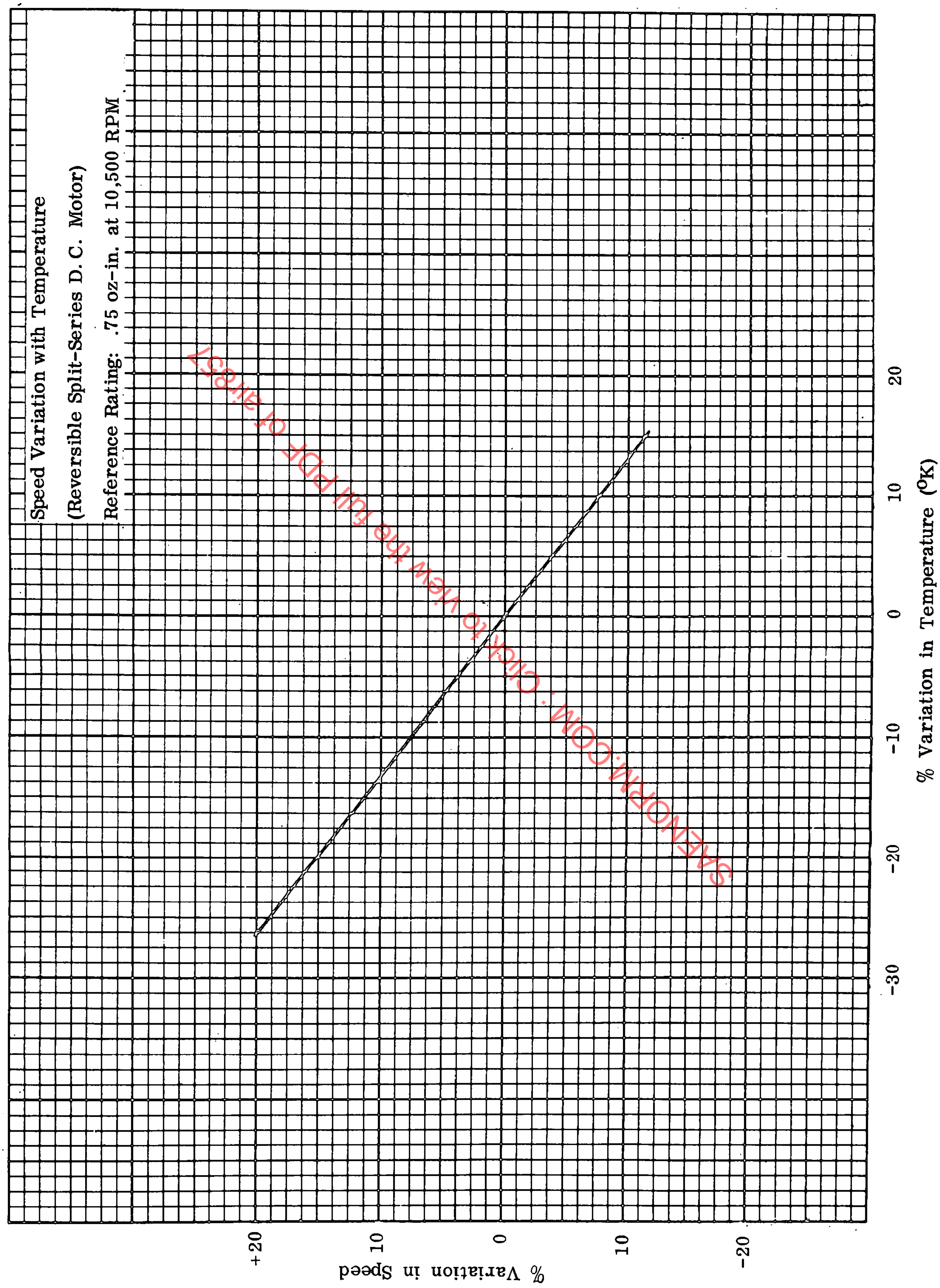

Speed Variation with Torque
(Split-Series D.C. Motor)
Reference Rating: .05 oz-in. at 10,200 RPM
% Variation in Torque
-40
-30
-20
-10
0
10
20
30
40
50
% Variation in Speed
+40
30
20
10
0
10
-20


Speed Variation with Torque
(Split-Series D. C. Motor with Brake)
Reference Rating: 116 oz-in. at 8700 RPM
+60
40
20
0
20
40
-60
% Variation in Speed
-60
-40
-20
0
20
40
60
80
100
% Variation in Torque

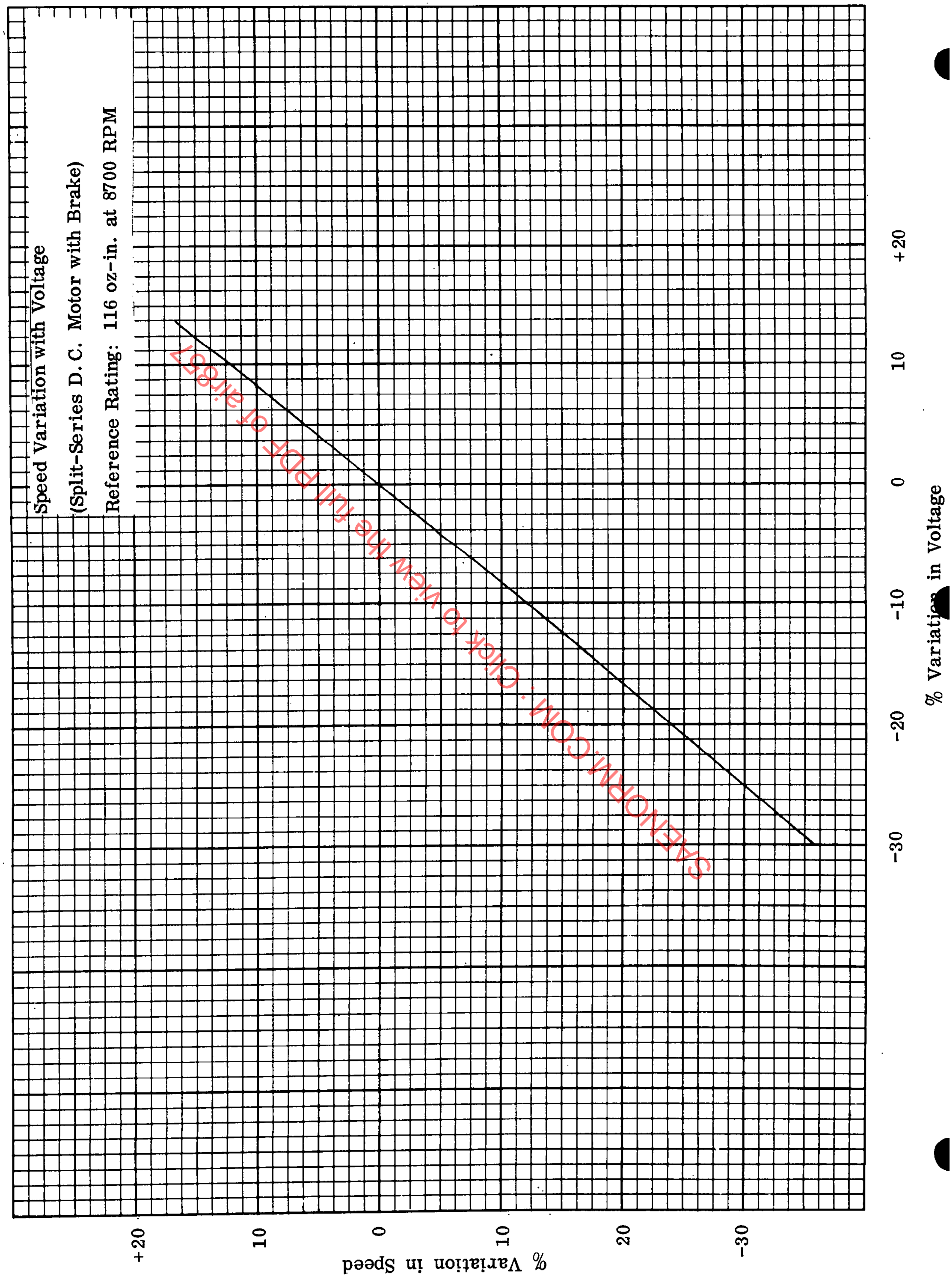

Speed Variation with Torque
(Permanent Magnet D. C. Motor)
Reference Rating: .09 oz-in. at 6200 RPM
% Variation in Torque
-50
-40
-30
-20
-10
0
10
20
30
40
+50
% Variation in Speed
+30
20
10
0
10
20
-30


Speed Variation with Torque
D. C. Motor - Shunt Wound Field
Reference Rating: 12.6 oz-in. at 8000 RPM
+20
+10
0
-10
-20
% Variation in Speed
-40
-30
-20
-10
0
+10
+20
+30
+40
% Variation in Torque


Speed Variation with Torque
D. C. Motor - Compound Wound Field
Reference Rating: 70 oz-in. at 7200 RPM
% Variation in Torque
-40
-30
-20
-10
0
+10
+20
+30
+40
% Variation in Speed
+20
+10
0
-10
-20



Speed Variation with Temperature
(Reversible Split-Series D. C. Motor)
Reference Rating: .75 oz-in. at 10,500 RPM
20
10
0
-10
-20
-30
% Variation in Temperature (°K)
+20
10
0
10
-20
% Variation in Speed


Speed Variation with Voltage
(Split-Series D. C. Motor with Brake)
Reference Rating: 116 oz-in. at 8700 RPM
% Variation in Voltage
+20
10
0
-10
-20
-30
% Variation in Speed
+20
10
0
10
20
-30