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AEROSPACE INFORMATION REPORT

AIR877

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AEROSPACE-PARTICLE COUNT DATA CONVERSION AND EXTRAPOLATION

1. SCOPE:

This Aerospace Information Report (AIR) describes a mathematical model that can be used to analyze particle count data. Particle counts that fit the model can be graphically displayed, converted from one counting size-frequency range to another, and extrapolated to estimate counts beyond the measured range. Derivation, applications and calculations are described.

2. METHOD OUTLINE:

Cumulative particle counts larger than stated size-range are fitted against a modified log-normal distribution function by plotting on special log-log^2 graph paper. Many real particle distributions approximate a straight line, showing good fit for this log-normal model. The resultant plot provides a basis for further analysis, including size-range conversion and extrapolation.

3. MODIFIED LOG-NORMAL MODEL:

Many real particle counts have been shown to approximate a Gaussian distribution with a logarithmic variate. This cumulative distribution function is represented by an equation having the form:

3.1
$$n_c = \frac{\Sigma n}{\sqrt{2\pi}\ \ln \sigma} \int_{\infty}^{\ln x} \exp -\left[\frac{\ln x - \ln M}{\sqrt{2}\ \ln \sigma}\right]^2 d \ln x \qquad \text{(Eq. 1)}$$

Where:
- n_c = Cumulative particle count
- Σn = Total number of particles
- $\ln \sigma$ = The natural logarithm of the geometric standard deviation
- M = The geometric mean particle size
- x = The particle size variate

3.1 (Continued):

Assuming that ln M is small compared to ln x, which means that about half of the particles are smaller than 1 μm in size, the rather formidable expression in Equation 1 may be reduced to a simpler equivalent expression using lumped constants:

3.2
$$\log n_c = \log A - 2.303\ B \log^2 x \qquad \text{(Eq. 2)}$$

Where: n_c = Cumulative particle count expressed as a base -10 logarithm
x = The particle size variable
A = Characteristic intercept-slope constant that describes quantity
B = Characteristic intercept-slope constant that describes distribution

If a particle count fits the mathematical model in Equation 2, data plots on special log-$\log^2$ graph paper are linear and provide a basis for further analysis. The necessary graph paper is made by laying out a logarithmic n-axis and an x-axis proportional to the square of the same logarithmic modulus. (A sample of log-$\log^2$ graph paper is shown in Appendix 1.)

4. DATA PLOTTING:

Contamination particle counts are plotted on log-$\log^2$ graph paper as cumulative counts larger than each lower size-range limit. This method provides a discrete size to be plotted, rather than some "average" value of the size range, and tends to correct classification errors for the smaller size ranges.

EXAMPLE: Contamination analysis according to ARP598 was performed on 100 mL of hydraulic oil. The following microscopic particle counts were obtained.

Size Range (μm)	>100	50-100	25-50	15-25	5-15
No. Particles	4	19	150	450	3780

These frequency counts are converted to corresponding cumulative counts by successive addition from left to right.[1]

Size Limit (μm)	>100	>50	>25	>15	>5
No. Particles	4	23	173	623	4403

The greater number of significant digits in the cumulative counts (4403 particles and 5 μm are not meaningful and may be rounded off (4400) for convenience. These cumulative counts are plotted as shown in Fig. 1.

Many such contamination plots are essentially linear on log-$\log^2$ graph paper showing good fit to the modified log-normal function. The example count has been faired to a straight line because all data points except the largest (4 particles >100 μm) fit quite closely. The largest and smallest particle count size-range usually exhibit the greatest counting error and should be less heavily weighted in drawing the mean curve.

[1]Total particles greater than 100 μm is 4.
Total particles greater than 50 μm is 4 + 19 = 23.
Total particles greater than 25 μm is 23 + 150 = 173, etc.

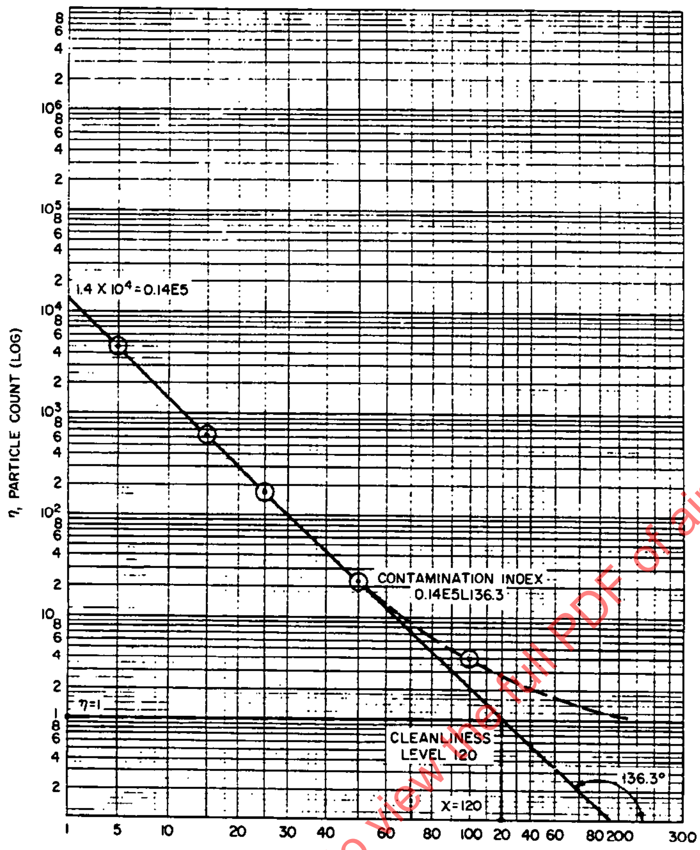

FIGURE 1 - Modified Log-Normal Contamination Plot

4. (Continued):

Deviations from linearity are also significant when based on accurate count data. Downward-concave curves may imply a closed distribution within the counting range or a mechanical breakdown which generates large particles. Upward-concave curves may imply the action of a wear process which preferentially generates small particles. Inflected or multiple curves may imply the presence of more than one type of particle population showing contamination from different sources.

5. CALCULATIONS:

The cumulative particle count plot demonstrated in Fig. 1 may be used for further analysis or modifications of the original data. Data conversion to other particle count size-ranges or extrapolation of data to uncounted size-ranges may be accomplished analytically or graphically. The latest method is easier and is sufficiently accurate for most purposes.

5. (Continued):

EXAMPLE: The particle count given in Section 4 and plotted on Fig. 1 must be converted to equivalent frequency counts in the different micrometer size-ranges of >80, 40-80, 20-40, 10-20, and 5-10. Cumulative particle counts corresponding to these lower micrometer size-range limits of >80, >40, >20, >10, and >5 are read directly from the plot of Fig. 1.

Size Limit (µm)	>80	>40	>20	>10	>5
No. Particles	5	45	320	1450	4400

Counts are read from the straight line plot, not the dashed curve (4.5 particles >80, not 6.5), and are rounded off. Frequency counts are obtained by the reverse process given in Section 4, by successive subtraction from left to right.

Size Range (µm)	>80	40-80	20-40	10-20	5-10
No. Particles	5	40	275	1130	2950

These frequency counts are equivalent to the original ARP598 frequency counts.

Data extrapolation may also be estimated from the plot of Fig. 1. For example, suppose that a pump-wear study or an electrohydraulic servovalve investigation required knowledge of the number of particles in the 1-5 µm size-range. ARP598 data is limited to the size-range above 5 µm. Extrapolation of this data according to Fig. 1 shows a cumulative count of 14 000 particles greater than 1 µm. Subtraction of 4400 particles greater than 5 µm (14 000 - 4400 = 9600), gives 9600 particles in the 1 to 5 µm size-range.

6. ANALYSIS:

Particle count data may be further manipulated after it has been fitted to the modified log-normal model represented by log-log^2 graph paper. The original particle count of Section 4 is repeated below.

Size Range (µm)	>100	50-100	25-50	15-25	5-15
No. Particles	4	19	150	450	3780

Within the errors of the ARP598 method (±20% or more), this tabulation represents the hydraulic oil contamination. The same information may be expressed by an equivalent "contamination index" derived from the plot of Fig. 1. This straight-line plot is uniquely determined by its ordinate intercept as x = 1, n = 14 000 = 1.4 x 10^4 = 0.14 x 10^5 and its angle of inclination 136.3° measured with an ordinary protractor. The contamination index .14E5L136.3, using Fortran format for 0.14 x 10^5, expresses the complete particle count tabulation in concise notation for reference, comparison, communication, or further mathematical analysis. The vector or "phasor" form of the contamination index, .14E5L136.3, encourages mathematical description and analysis of otherwise complex processes such as contamination generation or filtration.