

AEROSPACE RECOMMENDED PRACTICE

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Superseding ARP1533

Procedure for the Analysis and Evaluation of Gaseous Emissions from Aircraft Engines

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1. SCOPE:

Aerospace Recommended Practice (ARP) 1533 is a procedure for the analysis and evaluation of the measured composition of the exhaust gas from aircraft engines. Measurements of carbon monoxide, carbon dioxide, total hydrocarbon, and the oxides of nitrogen are used to deduce emission indices, fuel-air ratio, combustion efficiency, and exhaust gas thermodynamic properties. The emission indices (EI) are the parameters of critical interest to the engine developers and the atmospheric emissions regulatory agencies because they relate engine performance to environmental impact.

While this procedure is intended to guide the analysis and evaluation of the emissions from aircraft gas turbine engines, the methodology may be applied to the analysis of the exhaust products of any hydrocarbon / air combustor. Some successful applications include:

- Aircraft engine combustor development rig tests (aviation kerosene fueled)
- Stationary source combustor development rig tests (natural gas and diesel fueled)
- Afterburning military engine tests (JP fueled)
- Internal combustion aircraft engine diagnostics (AVGAS fueled)

Each application may be characterized by very different measured emissions levels (parts per million versus percent by volume) but this common approach solves the same basic combustion chemical equation.

This revision of ARP1533 assumes that major advances will occur in gas analysis technology in the near future. New instruments will be accepted by the regulatory agencies such that it will no longer be appropriate to specify the measurement method for each chemical species.

The matrix method of solving the combustion chemical equation is recommended because of all the potential variations in exhaust gas measurement requirements. Changes in the fuel type, addition of diluents, addition of measured species, and options for wet or dry basis measurements are most easily handled by revising individual matrix row equations. Matrix solution software is widely available on personal computers. However, derivation of the algebraic solution of the chemical equation is retained for traceability to previous versions of this document. New sections have been added to this document that pertain to data quality checks, measurement uncertainty, and water content calculations.

2. REFERENCES:

2.1 SAE Publications:

Available from SAE, 400 Commonwealth Drive, Warrendale, PA 15096-0001.

2.1.1 ARP1256, Procedure for the Continuous Sampling and Measurement of Gaseous Emissions from Aircraft Turbine Engines

2.2 Other References:

2.2.1 Kilpatrick, D. A., Calculation Methodology for Basic Engine Exhaust Gaseous Emission Parameters, National Gas Turbine Establishment Memorandum M81002, February, 1981

2.2.2 NOAA/NASA/USAF, U.S. Standard Atmosphere, 1976, U.S. Government Printing Office, Washington, DC, October, 1976

2.2.3 Shawn P. Heneghan and Charles W. Frayne, Propagation of Errors for Combustion Analysis using Emission Analyzer Data, AIAA-2000-0955, January, 2000

2.2.4 Bob Hardy, ITS-90 Formulations for Vapor Pressure, Frost Point Temperature, Dewpoint Temperature, and Enhancement Factors in the Range -100 to 100 °C, published in The Proceedings of the Third International Symposium on Humidity & Moisture, Teddington, London, England, April 1998

2.2.5 Humidity – Part 1: Terms, definitions and formulae, British Standard BS 1339-1:2002

2.3 Definitions:

AIRCRAFT GAS TURBINE ENGINE: Any gas turbine engine used for aircraft propulsion or power generation, including those commonly called turbojet, turbofan, turboprop, or turboshaft type engines.

COMBUSTION EFFICIENCY: The percentage ratio of the energy actually released by the combustion process to the energy which would be realized if all the carbon in the fuel were oxidized to carbon dioxide and the hydrogen to water vapor.

CONCENTRATION: The volume fraction of the component of interest in the gas mixture expressed as volume percentage or as parts per million by volume (ppmV).

EMISSION INDEX: The mass of emissions of a given constituent per unit mass of fuel, multiplied by 1000.

2.3 (Continued):

FUEL-AIR RATIO: The mass rate of fuel flow to the engine divided by the mass rate of dry airflow through the engine.

GASEOUS EMISSIONS: The carbon monoxide, carbon dioxide, nitric oxide, nitrogen dioxide, and hydrocarbon in the exhaust from a gas turbine engine.

MOLE FRACTION: The volume concentration of a gas per unit volume of the gas mixture of which it is a part. In the context of the measurements discussed in this procedure, "volume concentration (or volume fraction)" and "molar concentration (or mole fraction)" are synonymous.

NET HEAT OF COMBUSTION: The energy released per unit mass of fuel due to its complete oxidation at constant pressure as measured by cooling the products to the initial temperature without condensation of the water vapor formed in the reaction.

NO_x: Oxides of nitrogen, specifically, the sum of nitric oxide (NO) and nitrogen dioxide (NO₂).

PARTS PER MILLION (ppm): The unit volume concentration of a gas per million unit volumes of the gas mixture of which it is a part. (Also applicable to weight measurements but only volume relationships are referred to in these procedures.)

PARTS PER MILLION CARBON (ppmC): The mole fraction of hydrocarbon in the exhaust multiplied by 10⁶. Thus, 1 ppm of methane is indicated as 1 ppmC. To convert ppm concentration of the hydrocarbon to an equivalent ppmC value, multiply ppm concentration by the number of carbon atoms per molecule of the gas. For example, 1 ppm propane translates as 3 ppmC hydrocarbon; 1 ppm hexane as 6 ppmC hydrocarbon.

TOTAL HYDROCARBON: The total of hydrocarbon compounds of all classes and molecular weights.

2.4 Nomenclature and Suggested Values:

[CO] = Mole fraction concentration of carbon monoxide in the exhaust

[CO₂] = Mole fraction concentration of carbon dioxide in the exhaust

C_mH_nO_pN_qS_r = the chemical formula of one mole of hydrocarbon fuel

[C_xH_y] = Mole fraction concentration of total hydrocarbon in the exhaust

[H₂O] = Mole fraction concentration of water in the exhaust

[NO] = Mole fraction concentration nitric oxide in the exhaust

[NO₂] = Mole fraction concentration nitrogen dioxide in the exhaust

[NO_x] = Mole fraction concentration of the oxides of nitrogen in the exhaust

[O₂] = Mole fraction concentration of oxygen in the exhaust

[SO₂] = Mole fraction concentration of sulfur dioxide in the exhaust

[Z] = Mole fraction concentration of constituent Z in exhaust

DiluentZ = A diluent such as nitrogen or water that is added to the combustion process

EI_Z = Emission index of constituent Z, g/kg fuel (lb/1000 lb fuel)

f = Water vapor saturation pressure enhancement factor.

F/A = Fuel-air ratio by weight

H_c = Net heat of combustion of fuel at constant pressure J/kg (Btu/lb)

h = Water content of the inlet air, moles of water vapor per mole of dry inlet air

h_{sd} = Water content of the semi-dry exhaust sample leaving the dryer, moles of water vapor per mole of dry sample gas

ICAO = International Civil Aviation Organization

J = Oxygen interference coefficient for effect of O₂ on the measurement of CO₂

K = Ratio of wet concentration to completely dry concentration

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2.4 (Continued):

L	= Interference coefficient for effect of CO_2 on the measurement of CO
L'	= Interference coefficient for effect of CO_2 on the measurement of NO and NO_x
$\dot{m}_n$	= mass flow rate of fuel "n"
M	= Interference coefficient for effect of H_2O on the measurement of CO
M'	= Interference coefficient for effect of H_2O on the measurement of NO and NO_x
M_{AIR}	= Molecular weight of dry air = 28.965
M_{C}	= Atomic weight of carbon = 12.011
M_{H}	= Atomic weight of hydrogen = 1.008
M_{N}	= Atomic weight of nitrogen = 14.0067
M_{O}	= Atomic weight of oxygen = 15.9994
M_{S}	= Atomic weight of sulfur = 32.0600
M_{Z}	= Atomic weight of constituent Z
P_{N}	= Moles of exhaust constituent "N" per mole of fuel
P_{T}	= Total moles of exhaust products
PHYG	= Gas sample pressure where the frost or dew point temperature measurement is made.
PWV	= Ideal water saturation vapor pressure at the measured frost or dew point temperature.
PWVE	= Effective water saturation vapor pressure at the gas sample pressure, PHYG
R	= Mole fraction of O_2 in dry inlet air = 0.20948
S	= Mole fraction of nitrogen (0.78084) plus mole fraction of argon (0.00934) in the dry inlet air = 0.79018

2.4 (Continued):

T = Mole fraction of carbon dioxide in dry inlet air = 0.00034

THYG = Frost or dew point temperature measured with a hygrometer.

U = Mole fraction of methane in dry inlet air

X = Moles of dry air / mole of fuel

α = Atomic hydrogen-carbon ratio of the fuel = n/m

α, β = Intermediate terms in the calculation of the enhancement factor, f

η = NO_x converter efficiency

η_b = Combustion efficiency, %

2.4.1 Subscripts:

d = Completely dry basis

m,n,p,q,r = Molar constants for fuel, C_mH_nO_pN_qS_r

ms = Measured value

sd = Semidry basis

w = Wet basis

x,y = Molar constants selected for the unburned hydrocarbon in exhaust, C_xH_y.

z = Constituent Z

3. INTRODUCTION:

The exhaust gas composition measurements required by this procedure include carbon monoxide (CO), carbon dioxide (CO₂), nitric oxide (NO), oxides of nitrogen (NO_x), total hydrocarbon (C_xH_y), as well as ambient air and sample dew point temperatures. The measured value of oxygen (O₂) content is not required by the analysis but is generally accepted as essential for assessing data quality. Measurement of sulfur dioxide (SO₂) is also not required but has been included herein to illustrate the addition of other species that may become significant and measurable. The instruments used for these measurements may be single-gas analyzers or single analyzers capable of measuring multiple gas compositions. Whichever the approach, each instrument must be fully characterized in terms of limitations in linearity, drift, interferences, repeatability, and biases. Some analyzers require a dried sample while others do not. Correct bookkeeping is required to ensure mixing wet basis, semi-dry and dry basis mole fractions does not occur. These issues are further explained in Section 5.2.

Ideally, the analysis and evaluation of emissions data should not require facility – measured parameters. However, when mixed fuels and diluents are used, facility metered flow rates are required to specify the number of moles of each reactant.

This procedure is valid for analysis of hydrocarbon / air exhaust products. Regardless of the hydrocarbon fuel; the chemical formula, molecular weight, and the lower heating value must be known. Typical values of these parameters for several fuels are provided in Table 1. However, precise values should be obtained from analysis of pre-and post-test fuel samples or from reference literature. The fuel chemical formula may be expanded to include the oxygen, nitrogen and sulfur in addition to carbon and hydrogen for a more rigorous balance of the reactants and products of combustion.

TABLE 1 - Typical Values of Hydrocarbon Fuel Properties

Fuel	Chemical Formula	Molecular Weight	H/C Ratio	Lower Heating Value, BTU/lbm
Jet A	C _{11.6} H ₂₂	161.5	1.897	18,521
JP-4	C _{8.5} H _{16.9}	119.1	1.988	18,747
JP-5	C _{7.16} H _{13.87}	100	1.937	18,300
JP-8	C _{10.9} H _{20.9}	152.0	1.917	18,646
JP-10	C ₁₀ H ₁₆	136	1.6	18,137
AVGAS	C _{7.68} H _{16.8}	112	2.187	18,700
Natural Gas	C _{1.04} H _{4.01}	16.53	3.855	20,680
#2 Diesel	C _{7.24} H _{12.90}	100	1.782	18,318

The properties of standard dry air are presented in Table 2. A baseline sample of the inlet air should be analyzed and the frost or dew point temperature measured. When large deviations from standard air are encountered (i.e. high inlet air water content, higher ambient carbon dioxide), actual concentrations should replace the dry air concentrations.

3. (Continued):

TABLE 2 - Properties of Standard Air (Ref. 2.2.2)

Component	Chemical Formula	Molecular Weight	Percent by Volume
Nitrogen	N ₂	28.0134	78.084 (dry)
Oxygen	O ₂	31.9998	20.948 (dry)
Argon	Ar	40	0.934 (dry)
Carbon dioxide	CO ₂	44.0098	0.034 (dry)
Water vapor	H ₂ O	18.015	Must be measured
Total		28.9659 (dry)	100 (dry)

Data quality checks have been incorporated into this procedure to encourage the use of near real time automated evaluation of the emissions measurement process. While there may be others, the checklist included in Section 7 has proven to aid in the early identification of emissions measurement problems.

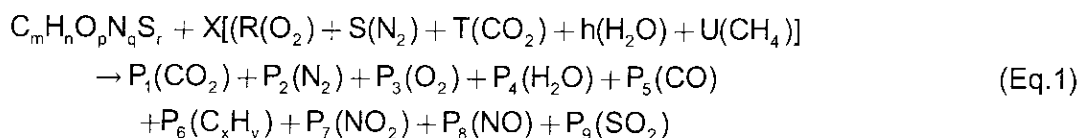
Calculations of emissions measurement uncertainties are increasingly necessary to show compliance with very low atmospheric emissions regulations. A methodology for the propagation of measurement errors in the calculations of the emissions indices (Ref. 2.2.3) is provided in Appendix A.

The method of calculating sample water content from measured frost and dew point temperatures (Ref. 2.2.4) is provided in Appendix B.

Two complete sample calculations are provided in Appendix C. The first calculation is the one presented in the version of ARP1533 issued in 1996. It is retained in this version for traceability of existing emissions analysis codes. The second calculation illustrates the expanded method of calculations with the matrix solution.

4. COMBUSTION CHEMICAL EQUATION:

The chemical equation for the combustion of one mole of hydrocarbon fuel and atmospheric air is:



where: P_1 through P_9 are the number of moles of CO₂, N₂, O₂, H₂O, CO, C_xH_y, NO₂, NO, and SO₂ respectively.

4. (Continued):

The composition of the hydrocarbon in the exhaust gas may be defined by at least three options between the SAE and ICAO methodologies. Equivalent fuel (CH_x , $x = 1$, $y = n/m$) implies that the hydrocarbon in the exhaust gas have the same formulation as the fuel (SAE method). Equivalent methane (CH_4 , $x = 1$, $y = 4$) implies that all of the hydrocarbon in the exhaust gas is methane (ICAO method). Characterized fuel (C_xH_y , $x \neq m$, $y \neq n$) requires some knowledge of the composition of the hydrocarbon in the exhaust gas. The methodology described here assumes that $x = m$ and $y = n$. The analyst must clearly state which assumption was applied.

The combustion equation makes no allowance for the small amounts of hydrogen, free carbon, various species of hydrocarbon, and the oxides of nitrogen (other than nitric oxide and nitrogen oxide) that may be present in the exhaust. Argon is unchanged during the combustion reaction. This procedure combines nitrogen and argon together for a total percent by volume of 79.018. The water content of the inlet air, h , is expressed as moles of water vapor per mole of dry air.

The $\text{P}_9(\text{SO}_2)$ term may be added to Eq. 1 when the fuel has significant sulfur content. However, SO_2 is not a mandatory measurement for the calculation of emission indices.

4.1 Modified Combustion Chemical Equation:

Occasionally fuels are mixed or diluted during the development of new engines. The basic chemical equation must be modified to account for the additional terms. In the case of multiple fuels, the metered mass flow rates of each fuel ($\dot{m}_1$ and $\dot{m}_2$) are used to calculate the total per-mole of fuel properties such as H/C ratio, net heating value and molecular weight for the combined fuel.

$$\frac{\dot{m}_1 [\text{C}_m\text{H}_n\text{O}_p\text{N}_q\text{S}_r]_1 + \dot{m}_2 [\text{C}_m\text{H}_n\text{O}_p\text{N}_q\text{S}_r]_2}{\dot{m}_1 + \dot{m}_2} = [\text{C}_m\text{H}_n\text{O}_p\text{N}_q\text{S}_r]_T$$

In the case of diluents (nitrogen, steam, etc.), the metered mass flow rates of the diluent and fuel are used to calculate the number of moles of diluent per-mole of fuel for use in the chemical equation.

$$\text{C}_m\text{H}_n\text{O}_p\text{N}_q\text{S}_r + X[(R(\text{O}_2) + S(\text{N}_2) + T(\text{CO}_2) + h(\text{H}_2\text{O}) + U(\text{CH}_4))] + \left(\frac{\dot{m}_Z}{\dot{m}_Z} * \frac{M_{\text{Fuel}}}{\dot{m}_{\text{Fuel}}} \right) (\text{DiluentZ})$$

5. MATRIX SOLUTION OF THE COMBUSTION CHEMICAL EQUATION:

5.1 Basic Matrix:

Eleven simultaneous equations are required to solve for the eleven unknowns, $P_1 - P_9$, P_T and X , in Eq. 1. The procedure for writing the eleven simultaneous equations is the same whether multiple fuels and /or diluents are used or not. The equations for a single fuel and no diluents are defined as follows:

Carbon balance:

$$m + (T + U)X = P_1 + P_5 + xP_6 \quad (\text{Eq.2})$$

Hydrogen balance:

$$n + (2h + 4U)X = 2P_4 + yP_6 \quad (\text{Eq.3})$$

Oxygen balance:

$$p + (2R + 2T + h)X = 2P_1 + 2P_3 + P_4 + P_5 + 2P_7 + P_8 + 2P_9 \quad (\text{Eq.4})$$

(NOTE: The measured oxygen content is not used in the solution of the combustion equation.)

Nitrogen balance:

$$q + 2SX = 2P_2 + P_7 + P_8 \quad (\text{Eq.5})$$

Sulfur balance:

$$r = P_9 \quad (\text{Eq.6})$$

Moles of carbon dioxide:

$$P_1 = P_T[\text{CO}_2]_w \quad (\text{Eq.7})$$

Moles of carbon monoxide:

$$P_5 = P_T[\text{CO}]_w \quad (\text{Eq.8})$$

Moles of total hydrocarbon:

$$xP_6 = P_T[\text{C}_x\text{H}_y]_w \quad (\text{Eq.9})$$

5.1 (Continued):

Moles of the oxides of nitrogen:

$$P_7 + P_8 = P_T [\text{NO}_x]_w \quad (\text{Eq.10})$$

Moles of nitric oxide:

$$P_8 = P_T [\text{NO}]_w \quad (\text{Eq.11})$$

Moles of water:

$$P_4 = [\text{H}_2\text{O}]_w P_T \quad (\text{Eq.12})$$

Where total number of moles of product:

$$P_T = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 \quad (\text{Eq.13})$$

Equations 2 through 11 and 13 are solved for the constant terms then entered into the 11 x 11 Matrix A (Variables) and the 1x 11 Matrix B (Constants) in Table 3 . Row 1 from Equation 2 reads as $P_1 + P_5 + xP_6 - (T+U)X = m$. Row 10 from Equation 13 reads as $-P_T + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 = 0$. The inverse of Matrix A is multiplied by Matrix B to obtain the 1 x 11 Matrix C which contains the eleven unknowns, $P_1 - P_9$, P_T and X. Equations 7 through 11 use concentrations on a wet basis. Measurements made on a semidry basis need to be converted to measurements on a wet basis.

The elements of Matrix C are wet basis.

TABLE 3 - Matrix A (Equations 2 – 11 and 13) and Matrix B

Equation	Matrix A											Matrix B
	P _T	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈	P ₉	X	Constants
2	0	1	0	0	0	1	x	0	0	0	-T-U	m
3	0	0	0	0	2	0	y	0	0	0	-2h-4U	n
4	0	2	0	2	1	1	0	2	1	2	-2R-2T - h	p
5	0	0	2	0	0	0	0	1	1	0	-2S	q
6	0	0	0	0	0	0	0	0	0	1	0	r
7	[CO ₂] _w	-1	0	0	0	0	0	0	0	0	0	0
8	[CO] _w	0	0	0	0	-1	0	0	0	0	0	0
9	[C _x H _y] _w	0	0	0	0	0	-x	0	0	0	0	0
10	[NO _x] _w	0	0	0	0	0	0	-1	-1	0	0	0
11	[NO] _w	0	0	0	0	0	0	0	1	0	0	0
13	-1	1	1	1	1	1	1	1	1	1	0	0

5.2 Measurements, Interferences and Corrections:

In a perfect world all gas analyzers would provide precise, unambiguous measurements of gas composition. In reality, depending on the type of analyzer being used, the emissions measurer must be concerned with:

- The interference of carbon dioxide and water on the measured concentrations of carbon monoxide and the oxides of nitrogen.
- The interference of oxygen on the measured concentration of carbon dioxide.
- The efficiency of the converter that converts the oxides of nitrogen to nitric oxide.
- The basis of measurement (semidry or wet)

This section of the procedure addresses these issues and modifies the equations in Matrix A. Effects such as that of oxygen upon carbon monoxide have been determined to be negligible for analyzers currently in use.

- 5.2.1 Interferences of Carbon Dioxide, Water and Oxygen: The measurement of a single gaseous specie may be affected by the presence of other gaseous species. For example, carbon dioxide and water vapor in the exhaust interfere with the non-dispersive infrared measurement of CO and the chemiluminescent measurement of NO. An interference coefficient is required that quantifies the parts per volume of species [A] that is not measured due to the parts per volume of interfering species [B] that is present. The convention used in defining these effects is that the correction is added to the measured value (or added to unity and multiplied by the measured value where appropriate) to give the best estimate of the true value. Thus, an interference that reduces the reading is expressed as a positive interference effect. L, L', M and M' are the interference coefficients for analyzers that should be determined under test conditions. Representative values are provided below. Reference 2.2.1 provides some guidance in the selection of the interference coefficients (below). However, interference coefficients should be measured for each instrument.

$$L = -1.3 \times 10^{-4}, \text{ mole fraction CO per mole fraction CO}_2$$

$$M = -4.5 \times 10^{-4}, \text{ mole fraction CO per mole fraction H}_2\text{O}$$

$$L' = 0.14, \text{ mole fraction of NO per mole fraction CO}_2$$

$$M' = 0.28, \text{ mole fraction of NO per mole fraction H}_2\text{O}$$

$$J = 0.07, \text{ mole fraction of CO}_2 \text{ per mole fraction of O}_2$$

Correct the CO measurement for the CO₂ and H₂O interferences as follows:

$$P_5 = [\text{CO}]_{\text{ms}} P_T + LP_1 + MP_4 \quad (\text{Eq.14})$$

Correct the NO_x measurement for the CO₂ and H₂O interferences as follows:

$$P_7 + P_8 = [\text{NO}_x]_{\text{ms}} P_T + L'P_1 + M'P_4 \quad (\text{Eq.15})$$

Correct the NO measurement for the CO₂ and H₂O interferences as follows:

$$P_8 = [\text{NO}]_{\text{ms}} P_T + L'P_1 + M'P_4 \quad (\text{Eq.16})$$

Correct the CO₂ measurement for the O₂ interference as follows:

$$P_1 = [\text{CO}_2]_{\text{ms}} P_T + JP_3 \quad (\text{Eq.17})$$

5.2.1 (Continued):

To include these interference effects in the matrix solution:

For CO, Equation 14 becomes:

$$[\text{CO}]_{\text{ms}} P_T + L P_1 + M P_4 - P_5 = 0 \quad (\text{Eq.18})$$

For NO_x, Equation 15 becomes:

$$[\text{NO}_x]_{\text{ms}} P_T + L' P_1 + M' P_4 - P_7 - P_8 = 0 \quad (\text{Eq.19})$$

For NO, Equation 16 becomes:

$$[\text{NO}]_{\text{ms}} P_T + L' P_1 + M' P_4 - P_8 = 0 \quad (\text{Eq.20})$$

For CO₂, Equation 17 becomes:

$$[\text{CO}_2]_{\text{ms}} P_T - P_1 + J P_3 = 0 \quad (\text{Eq.21})$$

5.2.2 NO_x Converter Efficiency: A chemiluminescent analyzer can be used to measure both nitric oxide, NO, and the oxides of nitrogen, NO_x. The nitrogen dioxide concentration, NO₂, is assumed to be the difference between the measured values of NO_x and NO. In the NO_x setting, the entire sample flows through a converter, which converts the NO₂ to NO such that all of the NO_x is measured as NO. In the NO setting, the converter is bypassed and only the sampled NO is measured. The NO_x converter has an efficiency, η , typically between 0.9 and 1.0. Reference 2.1.1 gives the procedure for determining this efficiency. The measured value of NO_x must be corrected for the quantity of NO₂ that was not converted to NO. From the definition of NO_x converter efficiency:

$$[\text{NO}_x]_{\text{ms}} = \eta [\text{NO}_2] + [\text{NO}] \quad (\text{Eq.22})$$

Moles of nitrogen dioxide:

$$P_7 = P_T [\text{NO}_2]_w \quad (\text{Eq.23})$$

Moles of nitric oxide:

$$P_8 = P_T [\text{NO}]_w \quad (\text{Eq.11})$$

5.2.2 (Continued):

Correct the NO_x measurement for converter efficiency as follows:

$$[\text{NO}_x]_{\text{ms}} P_T = \eta P_7 + P_8 \quad (\text{Eq.24})$$

The NO_x converter efficiency effects only P₇. Combine Equations 19 and 24 to include the interference effects and the NO_x converter efficiency in the matrix solution:

$$[\text{NO}_x]_{\text{ms}} P_T + L'P_1 + M'P_4 - \eta P_7 - P_8 = 0 \quad (\text{Eq.25})$$

- 5.2.3 Sample Drying: Some gas analysis systems require a cold trap or membrane dryer in the sample line ahead of the analyzers. Since these dryers do not remove the water vapor completely, the concentrations after the dryer are referred to as semidry. The frost or dew point temperature at the dryer exit is measured and the water vapor saturation pressure is obtained from the appropriate tables. Appendix B provides a method of calculating sample water content based on Reference 2.2.4. This method applies to inlet air frost or dew point temperatures as well as frost or dew point temperatures measured before the gas analysis system dryers as long as the local sample pressure is used to correct the ideal vapor pressure.

The water content of the sample after the sample dryer is closely approximated for very dry samples as:

$$h_{\text{sd}} = \frac{P_{4\text{sd}}}{P_T - P_4} \quad (\text{Eq.26})$$

The number of moles of semidry sample is greater than the number of moles of dry sample by P_{4sd}:

$$P_T - P_4 + P_{4\text{sd}} = (P_T - P_4)(1 + h_{\text{sd}}) \quad (\text{Eq.27})$$

5.2.3 (Continued):

The semidry concentrations of CO, CO₂, NO, NO_x, H₂O and O₂ are:

$$[\text{CO}]_{\text{sd}} = P_5 / (P_T - P_4 + h_{\text{sd}}) = P_5 / ((P_T - P_4)(1 + h_{\text{sc}})) \quad (\text{Eq.28})$$

$$[\text{CO}_2]_{\text{sd}} = P_1 / ((P_T - P_4)(1 + h_{\text{sd}})) \quad (\text{Eq.29})$$

$$[\text{NO}]_{\text{sd}} = P_8 / ((P_T - P_4)(1 + h_{\text{sd}})) \quad (\text{Eq.30})$$

$$[\text{NO}_x]_{\text{sd}} = P_7 + P_8 / ((P_T - P_4)(1 + h_{\text{sd}})) \quad (\text{Eq.31})$$

$$[\text{H}_2\text{O}]_{\text{sd}} = (P_4)_{\text{sd}} / ((P_T - P_4)(1 + h_{\text{sd}})) = h_{\text{sd}} / (1 + h_{\text{sd}}) \quad (\text{Eq.32})$$

$$[\text{O}_2]_{\text{sd}} = P_3 / ((P_T - P_4)(1 + h_{\text{sd}})) \quad (\text{Eq.33})$$

Combining the interference and sample drying effects on the measured concentration of CO, Eq. 8 in Matrix A is replaced by:

$$([\text{CO}]_{\text{ms}}(1 + h_{\text{sd}}) + Mh_{\text{sd}})(P_T - P_4) + LP_1 - P_5 = 0 \quad (\text{Eq.34})$$

Combining the interference and sample drying effects on the measured concentration of CO₂, Eq. 7 in Matrix A is replaced by:

$$[\text{CO}_2]_{\text{ms}}(1 + h_{\text{sd}})(P_T - P_4) - P_1 + (J[\text{CO}_2]_{\text{ms}})P_3 = 0 \quad (\text{Eq.35})$$

Combining the interference and sample drying effects on the measured concentration of NO, Eq. 11 in Matrix A is replaced by:

$$[\text{NO}]_{\text{ms}}(1 + h_{\text{sd}})(1 + M')(P_T - P_4) + L'[\text{NO}]_{\text{ms}}P_1 - P_8 = 0 \quad (\text{Eq.36})$$

Combining the interference, sample drying effects, and NO_x converter efficiency on the measured concentration of NO_x, Eq. 10 in Matrix A is replaced by:

$$[\text{NO}_x]_{\text{ms}}(1 + h_{\text{sd}})(1 + M')(P_T - P_4) + L'[\text{NO}_x]_{\text{ms}}P_1 - \eta P_7 - P_8 = 0 \quad (\text{Eq.37})$$

The equations that account for the interferences and the sample drying process have been incorporated into Matrix A in Table 4 to provide a comprehensive solution of the eleven simultaneous equations.

TABLE 4 - Expanded Matrix A and Matrix B

Equation	Matrix A											Matrix B
	P _T	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈	P ₉	X	Constants
2	0	1	0	0	0	1	x	0	0	0	-T-U	m
3	0	0	0	0	2	0	y	0	0	0	-2h-4U	n
4	0	2	0	2	1	1	0	2	1	2	-2R	p
5	0	0	2	0	0	0	0	1	1	0	-2T - h	q
6	0	0	0	0	0	0	0	0	0	1	-2S	r
35	(1+h _{sd})[CO ₂] _{ms}	-1	0	J[CO ₂] _{ms}	-(1+h _{sd})[CO ₂] _{ms}	0	0	0	0	0	0	0
34	(1+h _{sd})[CO] _{ms} + h _{sd} M	L	0	0	-(1+h _{sd})[CO] _{ms} - h _{sd} M	-1	0	0	0	0	0	0
9	[C _x H _y] _{ms}	0	0	0	0	0	-x	0	0	0	0	0
37	(1+(h _{sd})(1+M')) L'[NO _x] _{ms} · [NO _x] _{ms}	0	0	0	-(1+(h _{sd})(1+M')) · [NO _x] _{ms}	0	0	-η	1	0	0	0
36	(1+(h _{sd})(1+M')) L'[NO] _{ms} · [NO] _{ms}	0	0	0	-(1+(h _{sd})(1+M')) · [NO] _{ms}	0	0	0	-1	0	0	0
13	-1	1	1	1	1	1	1	1	1	1	0	0

6. CALCULATION OF GASEOUS EMISSIONS PARAMETERS:

6.1 Conversion From Wet to Dry Basis:

To convert product coefficients from the wet basis (w) to the completely dry basis (d), multiply the wet basis coefficient by K:

where:

$$K = \frac{P_T}{P_T - P_4} \quad (\text{Eq.38})$$

$$[\text{CO}_2]_d = KP_1 \quad (\text{Eq.39})$$

$$[\text{N}_2]_d = KP_2 \quad (\text{Eq.40})$$

$$[\text{O}_2]_d = KP_3 \quad (\text{Eq.41})$$

$$[\text{CO}]_d = KP_5 \quad (\text{Eq.42})$$

6.1 (Continued):

$$[C_x H_y]_d = KP_6 \quad (\text{Eq.43})$$

$$[NO_2]_d = KP_7 \quad (\text{Eq.44})$$

$$[NO]_d = KP_8 \quad (\text{Eq.45})$$

6.2 The fuel-air ratio, F/A , is the ratio of the mass flow rate of the fuel ($C_m H_n$) to dry air. The equation presented below ignores the presence of trace species in the fuel and the air:

$$\begin{aligned} F/A &= \frac{(1 \cdot \text{mole} \cdot \text{fuel})(\text{molecular} \cdot \text{wt.} \cdot \text{of} \cdot \text{fuel})}{(\text{moles} \cdot \text{dry} \cdot \text{air})(\text{molecular} \cdot \text{wt.} \cdot \text{of} \cdot \text{dry} \cdot \text{air})} \\ &= \frac{mM_c + nM_H}{X * M_{AIR}} = \frac{m(M_c + \alpha M_H)}{X * M_{AIR}} \end{aligned} \quad (\text{Eq.46})$$

6.3 The emissions index of constituent Z , El_z , is the ratio of the mass of constituent Z to 1000 mass units of fuel consumed. It is commonly referred to as the mass of Z per 1000 mass units of fuel.

$$\begin{aligned} El_z &= \left[\frac{\text{mass} \cdot \text{rate} \cdot \text{of} \cdot Z}{\text{mass} \cdot \text{rate} \cdot \text{of} \cdot \text{fuel}} \right] (1000) \\ &= \left[\frac{\text{moles of } Z}{\text{moles of fuel}} \right] \left[\frac{\text{molecular wt. of } Z}{\text{molecular wt. of fuel}} \right] (1000) \end{aligned} \quad (\text{Eq.47})$$

Thus

$$El_{CO} = \frac{P_5 * M_{CO} * 10^3}{m(M_c + \alpha M_H)} \quad (\text{Eq.48})$$

$$El_{NO} = \frac{P_8 * M_{NO_2} * 10^3}{m(M_c + \alpha M_H)} \quad (\text{Eq.49})$$

$$El_{NOx} = \frac{(P_7 + P_8) * M_{NO_2} * 10^3}{m(M_c + \alpha M_H)} \quad (\text{Eq.50})$$

6.3 (Continued):

NOTE: The molecular weight used in the calculation of the emission index of nitric oxide and the oxides of nitrogen is the molecular weight of nitrogen dioxide.

$$EI_{C_xH_y} = \frac{x * P_6 * M_{C_xH_y} * 10^3}{m(M_C + \alpha M_H)} \quad (\text{Eq.51})$$

6.4 The combustion efficiency, η_b , is calculated on an enthalpy basis by subtracting the inefficiencies due to unburned hydrocarbon and CO from 100%. It neglects the effects of NO_x and H_2 and the dissociation of combustion products.

$$\eta_b = \left[1.00 - 4.346 \frac{EI_{CO}}{H_c} - \frac{EI_{C_xH_y}}{1000} \right] * 100 \quad (\text{Eq.52})$$

where:

H_c = Net heat of combustion of fuel (sometimes referred to as the lower heating value) in Btu/lb

In SI units:

$$\eta_b = \left[1.00 - 10109 \frac{EI_{CO}}{H_c} - \frac{EI_{C_xH_y}}{1000} \right] * 100 \quad (\text{Eq.53})$$

where:

H_c is in J/kg

Reference 2.2.1 is the source of much of the subject matter of this document. Refer to 2.2.1 for more detailed discussions of the various details relating to the calculations.

7. CALCULATION OF DATA QUALITY INDICATORS:

The expense associated with the measurement of emissions from aircraft engines can be minimized if instrumentation problems are quickly identified and corrected. Data quality checks have been incorporated into this procedure to encourage the use of near real time automated assessment of the emissions measurement process. While there may be others, those identified below have proven valuable in the early identification of emissions measurement problems.

7.1 Sample Stability:

The standard deviation of a set of gaseous species measurements is an indicator that both the source and the analyzer were stable during the period of time when the emissions measurements were collected. For example, when n scans of gas analyzer data are collected over a ten second interval, the standard deviation of the n samples about the average value should not exceed the instrument manufacturer's quoted repeatability. The target value for this data quality indicator should account for known measurement oscillations as well as instrumentation measurement precision.

$$[Z]_{\text{SDEV}} = \sqrt{\frac{\sum [Z]_{\text{ms}}^2 - \frac{(\sum [Z]_{\text{ms}})^2}{n}}{n-1}} \quad (\text{Eq.54})$$

7.2 Oxygen Balance:

Even though a measurement of the oxygen concentration is not required to solve the combustion chemical equation, the measured semi-dry oxygen concentration may be compared to the calculated semidry basis oxygen concentration. The target value for the oxygen balance is 0.1 moles O_2 or less. The oxygen analyzer is usually the last one to stabilize so an acceptable oxygen balance is another indication that the emission source and the CO_2 and O_2 analyzers were stable.

$$\text{O}_2\text{Balance} = [\text{O}_2]_{\text{dry}} - [\text{O}_2]_{\text{measured semidry}} * \left(1 + \frac{[\text{H}_2\text{O}]_{\text{w}}}{100} - h_{\text{sd}} \right) \quad (\text{Eq.55})$$

7.3 Carbon Balance:

Solution of the chemical equation results in the P-terms which are the number of moles of the product species. The number of carbon atoms in the product species is forced to be equal to the number of carbon atoms in the reactant species. However, a comparison of the facility-metered carbon inlet flow to the carbon atoms specified by the P-terms is a method of validating the fuel chemical formula, the ability of the sampling system to capture a representative sample, the facility fuel and air flow meters and the CO₂ gas analyzer health. The target value for the carbon balance is 1.0 ± 0.05 .

Calculate the ratio of the number of carbon atoms flowing into the combustor to the number of carbon atoms measured in the exhaust. Note that the ppmC levels of carbon monoxide (P5) and unburned hydrocarbon (P6) have been ignored. Also note that W_{water} appears in the total flow rate term, (W_{fuel} + W_{air} + W_{water}). This accounts for any water injection. When the total flow into the combustor is known, the wet basis carbon dioxide mole fraction and the sample molecular weight should be used in the calculation of CARBAL.:

$$\text{CARBAL} = \left(\frac{\text{MW}_{\text{fuel}}}{\text{W}_{\text{fuel}} * m} + \frac{\text{MW}_{\text{AIR}}}{\text{W}_{\text{air}} * T} \right) * \frac{(\text{W}_{\text{fuel}} + \text{W}_{\text{air}} + \text{W}_{\text{water}}) * [\text{CO}_2]_{\text{wet}}}{\text{MW}_{\text{sample}} * 100} \quad (\text{Eq.56})$$

W_{fuel} = the facility-metered fuel flow rate

W_{air} = the facility-metered air flow rate

W_{water} = the facility-metered water injection flow rate

m = the number of moles of carbon per mole of fuel from the most recent fuel analysis

T = the number of moles of CO₂ per mole of dry air

MW_{sample} = the molecular weight of the exhaust gas sample

MW_{fuel} = the molecular weight of the fuel

[CO₂]_{wet} = the wet basis carbon dioxide mole fraction, percent by volume

7.4 Fuel / Air Ratio Balance:

A comparison of the fuel / air ratio calculation (Eq. 48) with the ratio of the facility – metered fuel and air mass flow rates is a higher level indicator of emissions data quality, especially the CO₂ measurement. Problems with the sample extraction process will be obvious here as well. The target value for the fuel / air ratio balance is 5% or less.

$$\text{FARBAL} = \frac{(\text{FAR}_{\text{emissions}} - \text{FAR}_{\text{facility}})}{\text{FAR}_{\text{facility}}} * 100 \quad (\text{Eq.57})$$

7.5 NO/NO_x Ratio:

For a given combustor, the ratio of NO/NO_x will not vary over wide ranges of FAR. Once an average value, typically 0.7 to 0.8, is established, changes in this ratio may be an indication of problems with either the NO or NO_x analyzers.

$$\text{NORatio} = \frac{[\text{NO}]_w}{[\text{NO}_x]_w} \quad (\text{Eq.58})$$

7.6 Trend Plots:

Trend plots that correlate calculated parameters are useful in detecting analyzer anomalies such as incorrect range settings or volume flow problems. When mixed fuels or diluents are used, the absence of step-shifts in the trend plots will be useful in confirming that the analysis routine has correctly accounted for the additional reactants. For example a plot of carbon balance vs. fuel / air ratio balance will accentuate sampling problems or fuel flow measurement problems.

8. CALCULATION OF MEASUREMENT UNCERTAINTY:

Calculations of emissions measurement uncertainties are necessary to show compliance with very low atmospheric emissions regulations. The propagation of errors analysis is a straightforward method of determining the measurement uncertainty of a calculated value, $f(x_i)$ (e.g. the emission index), that is based on a set of input measured variables x_i . Of course, each of the input measurements is subject to its own error, $\frac{\Delta x_i}{x_i}$, depending upon the quality of the instrumentation used. Two methods of determining the uncertainty of the output calculations are presented here. The first is based on an analytic derivation, the details of which are summarized in Appendix A.1. This method yields a precise calculation that requires knowing the functional form, tedious differentiation, and assumes linearity over the range of errors for accuracy. The second method uses a stochastic process that estimates the uncertainty of the measurements. The stochastic method is simple to set up and yields correct results even for non-linear functional ranges. This method is discussed in Appendix A.2. Comparisons of propagated error using both methods, the analytic and stochastic, have shown their equivalency.

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SAE ARP1533 Revision A

APPENDIX A CALCULATION OF MEASUREMENT UNCERTAINTY

A.1 ANALYTIC ANALYSIS OF THE MEASUREMENT UNCERTAINTY:

To calculate the error associated with the emission indices, fuel / air ratio, and efficiency, results of ARP1533 as summarized in Equations A1 - A5 are used with the general partial differential summation for error calculation as presented by Bevington¹ (Equation A6). The solutions of which are given in Equations A7 - A11. For the purposes of this error analysis we have assumed that the error is associated only with the measurements of the mole fractions ([CO], [CO₂], [NO_x], and [C_xH_y]).

$$EI_{CO} = \left(\frac{[CO]}{\text{sum}} \right) \left(\frac{10^3 M_{CO}}{M_C + \alpha M_H} \right) \left(1 + \frac{TX}{m} \right) \quad (\text{Eq.A1})$$

$$EI_{NOx} = \left(\frac{[NOx]}{\text{sum}} \right) \left(\frac{10^3 M_{NOx}}{M_C + \alpha M_H} \right) \left(1 + \frac{TX}{m} \right) \quad (\text{Eq.A2})$$

$$EI_{CxHy} = \left(\frac{[C_xH_y]}{\text{sum}} \right) \left(\frac{10^3 M_{C_xH_y}}{M_C + \alpha M_H} \right) \left(1 + \frac{TX}{m} \right) \quad (\text{Eq.A3})$$

$$\frac{F}{A} = \left(\frac{(1+h)(\text{sum}) - T}{1 - \frac{\alpha}{4}(\text{sum})} \right) \left(\frac{M_C + \alpha M_H}{M_{Air}} \right) \quad (\text{Eq.A4})$$

$$\eta_b = \left(1.00 - 4.346 \frac{EI(CO)}{H_c} - \frac{EI(CxHy)}{1000} \right) 100 \quad (\text{Eq.A5})$$

where,

$$X = m \left(\frac{1 - \frac{\alpha}{4}(\text{sum})}{(1+h)(\text{sum}) - T} \right)$$

¹ Bevington, P. R. "Data reduction and error analysis for the physical sciences" McGraw Hill, New York, NY, 1969.

A.1 (Continued):

And "sum" is $[CO] + [CO_2] + [C_xH_y]$.

In general, for a dependent variable $y = f(x_1, x_2, x_3, \dots, x_n)$, the square of the expected relative error in y due to the relative errors in the x_i ($i = 1$ to n) is given by Equation A6.

$$\left(\frac{\Delta y}{y}\right)^2 = \sum_{\text{all } i} \left(\frac{\partial \ln y}{\partial \ln x_i}\right)^2 \left(\frac{\Delta x_i}{x_i}\right)^2 \quad (\text{Eq. A6})$$

where,

$$\frac{\Delta f}{f} = \text{fractional error in } f$$

Applying Equation A6 to Equations A1 - A5 yields Equations A7 - A11 given in Section A.1.1.

A.1.1 Solutions to Analytic Derivation of Uncertainty Measurements:

Equation (Eq. A7)

$$\left(\frac{\Delta EI_{CO}}{EI_{CO}}\right)^2 = \left(\frac{\partial \ln(EI_{CO})}{\partial \ln(CO)}\right)^2 \left(\frac{\Delta [CO]}{[CO]}\right)^2 + \left(\frac{\partial \ln(EI_{CO})}{\partial \ln(CO_2)}\right)^2 \left(\frac{\Delta [CO_2]}{[CO_2]}\right)^2 + \left(\frac{\partial \ln(EI_{CO})}{\partial \ln(C_xH_y)}\right)^2 \left(\frac{\Delta [C_xH_y]}{[C_xH_y]}\right)^2$$

where,

$$\frac{\partial \ln(EI_{CO})}{\partial \ln(CO)} = \frac{[CO_2] + [C_xH_y]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4}\right)}{[(1 + h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T]^2} \cdot [CO]$$

$$\frac{\partial \ln(EI_{CO})}{\partial \ln(CO_2)} = \frac{-[CO_2]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4}\right)}{[(1 + h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T]^2} \cdot [CO_2]$$

$$\frac{\partial \ln(EI_{CO})}{\partial \ln(C_xH_y)} = \frac{-[C_xH_y]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4}\right)}{[(1 + h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T]^2} \cdot [C_xH_y]$$

A.1.1 (Continued):

Equation (Eq.A8)

$$\left(\frac{\Delta EI_{NOx}}{EI_{NOx}} \right)^2 = \left(\frac{\partial \ln(EI_{NOx})}{\partial \ln(CO)} \right)^2 \left(\frac{\Delta [CO]}{[CO]} \right)^2 + \left(\frac{\partial \ln(EI_{NOx})}{\partial \ln(CO_2)} \right)^2 \left(\frac{\Delta [CO_2]}{[CO_2]} \right)^2 + \left(\frac{\partial \ln(EI_{NOx})}{\partial \ln(C_xH_y)} \right)^2 \left(\frac{\Delta [C_xH_y]}{[C_xH_y]} \right)^2 + \left(\frac{\partial \ln(EI_{NOx})}{\partial \ln(NOx)} \right)^2 \left(\frac{\Delta [NOx]}{[NOx]} \right)^2$$

where,

$$\frac{\partial \ln(EI_{NOx})}{\partial \ln(CO)} = \frac{-[CO]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4} \right)}{\left[(1 + h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T \right]^2} \cdot [CO]$$

$$\frac{\partial \ln(EI_{NOx})}{\partial \ln(CO_2)} = \frac{-[CO_2]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4} \right)}{\left[(1 + h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T \right]^2} \cdot [CO_2]$$

$$\frac{\partial \ln(EI_{NOx})}{\partial \ln(C_xH_y)} = \frac{-[C_xH_y]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4} \right)}{\left[(1 + h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T \right]^2} \cdot [C_xH_y]$$

$$\frac{\partial \ln(EI_{NOx})}{\partial \ln(NOx)} = 1$$

Equation (Eq.A9)

$$\left(\frac{\Delta EI_{C_xH_y}}{EI_{C_xH_y}} \right)^2 = \left(\frac{\partial \ln(EI_{C_xH_y})}{\partial \ln(CO)} \right)^2 \left(\frac{\Delta [CO]}{[CO]} \right)^2 + \left(\frac{\partial \ln(EI_{C_xH_y})}{\partial \ln(CO_2)} \right)^2 \left(\frac{\Delta [CO_2]}{[CO_2]} \right)^2 + \left(\frac{\partial \ln(EI_{C_xH_y})}{\partial \ln(C_xH_y)} \right)^2 \left(\frac{\Delta [C_xH_y]}{[C_xH_y]} \right)^2$$

A.1.1 (Continued):

where,

$$\frac{\partial \ln(EI_{C_xH_y})}{\partial \ln(CO)} = \frac{-[CO]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4}\right)}{[(1+h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T]^2} \cdot [CO]$$

$$\frac{\partial \ln(EI_{C_xH_y})}{\partial \ln(CO_2)} = \frac{-[CO_2]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4}\right)}{[(1+h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T]^2} \cdot [CO_2]$$

$$\frac{\partial \ln(EI_{C_xH_y})}{\partial \ln(C_xH_y)} = \frac{[CO] + [CO_2]}{([CO] + [CO_2] + [C_xH_y])} + \frac{T}{m + TX} \cdot \frac{(-m) \cdot \left(1 + h - T \frac{\alpha}{4}\right)}{[(1+h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T]^2} \cdot [C_xH_y]$$

Equation (Eq.A10)

$$\left(\frac{\Delta(F/A)}{(F/A)} \right)_{\text{Effluent}}^2 = \left(\frac{\partial \ln(F/A)}{\partial \ln(CO)} \right)^2 \left(\frac{\Delta CO}{CO} \right)^2 + \left(\frac{\partial \ln(F/A)}{\partial \ln(CO_2)} \right)^2 \left(\frac{\Delta CO_2}{CO_2} \right)^2 + \left(\frac{\partial \ln(F/A)}{\partial \ln(C_xH_y)} \right)^2 \left(\frac{\Delta C_xH_y}{C_xH_y} \right)^2$$

where,

$$\frac{\partial \ln(F/A)}{\partial \ln(x_i)} = [x_i] \cdot \left[\frac{4 \cdot (1+h) - \alpha T}{[(1+h) \cdot ([CO] + [CO_2] + [C_xH_y]) - T] \cdot [4 - \alpha([CO] + [CO_2] + [C_xH_y])]} \right]$$

Equation (Eq.A11)

$$\left(\frac{\Delta \eta_b}{\eta_b} \right)^2 = \left(\frac{\partial \ln(\eta_b)}{\partial \ln(EI_{CO})} \right)^2 \left(\frac{\Delta EI_{CO}}{EI_{CO}} \right)^2 + \left(\frac{\partial \ln(\eta_b)}{\partial \ln(EI_{C_xH_y})} \right)^2 \left(\frac{\Delta EI_{C_xH_y}}{EI_{C_xH_y}} \right)^2$$

A.1.1 (Continued):

where,

$$\frac{\partial \ln(\eta_b)}{\partial \ln(EI_{CO})} = \frac{-EI_{CO}}{\left(\frac{100H_c}{4.346} - EI_{CO} - \frac{H_c}{4346} EI_{C_xH_y} \right)}$$

$$\frac{\partial \ln(\eta_b)}{\partial \ln(EI_{C_xH_y})} = \frac{-EI_{C_xH_y}}{\left(1000 - \frac{4346}{H_c} EI_{CO} - EI_{C_xH_y} \right)}$$

Finally, the simple calculation of the error analysis for the input fuel/air ratio (that is the measurement of F/A based on the mass flow recorders is given by Equation A12. This is a second method of calculating the fuel air ratio. Comparing the input and output F/A ratio is important because it shows an internal consistency in the experiment and establishes the crucial parameter describing the combustion condition.

The difference between the two calculations should be less than the square root of the sum of the errors in Equations A4 and A12. If the calculations yield significantly different results, then the experimenter must decide which results are in error. It is our experience that the error is frequently associated with poor sampling of the combustion effluent. This is an excellent quality check on the entire system.

Equation (Eq.A12)

$$\left(\frac{\Delta(F/A)}{(F/A)} \right)_{\text{input}}^2 = \left(\frac{\partial \ln(F/A)}{\partial \ln(F)} \right)^2 \left(\frac{\Delta F}{F} \right)^2 + \left(\frac{\partial \ln(F/A)}{\partial \ln(A)} \right)^2 \left(\frac{\Delta A}{A} \right)^2$$

where,

$$\frac{\partial \ln(F/A)}{\partial \ln(F)} = 1$$

$$\frac{\partial \ln(F/A)}{\partial \ln(A)} = -1$$

This method yields a precise calculation of the uncertainty. However, to extend the analysis to the uncertainty of any other output variable, or to include the error associated with other input parameters (such as h) would require a detailed analysis of the fairly complicated partial differential equation. This formulation also assumes linear behavior of the function in the region of x_i . As a result, the formula does not give a correct answer when the function is sharply curved (generally unimportant in this analysis), or the errors are large. The stochastic process discussed in Section A.2 overcomes all these limitations.

A.2 STOCHASTIC METHODOLOGY FOR DETERMINATION OF UNCERTAINTY:

To overcome the difficulties associated with determining the propagation of errors (applying Equation A6 to determine Equations A7 - A11), a computer program can be designed that performs this calculation in a statistical sense. It takes as inputs the following:

- The measured variables [CO], [CO₂], [C_xH_y], [NO], and [NO_x],
- The scale used to make these mole fraction measurements,
- The expected percent of full scale error associated with the measurement of variables [CO], [CO₂], [C_xH_y], [NO], and [NO_x],
- Measured value "h" determined from the hygrometer reading,
- "T", the amount of CO₂ in dry air,
- The formula for fuel C_mH_n and unburned hydrocarbon C_xH_y.

The propagated error calculation is accomplished by adding "noise" to the input values. To do this, each input variable is allowed to vary with a Gaussian distribution about the input mean value. In practice, it is usually sufficient to add noise to only the mole fraction measurements, as in most real systems these are the major sources of error. The mean of the Gaussian distribution is set equal to the input measured value, and the standard deviation of the distribution is set equal to the expected error (percentage of full scale error multiplied by the scale). There are many standard routines for generating Gaussian variables of a given mean and standard deviation. In Microsoft Excel®, such a Gaussian distributed random variable can be calculated by using the NORMINV function specifically, as shown in Equation A13.

$$\text{Variable} = \text{NORMINV}(\text{RAND}(), \mu, \sigma) \quad (\text{Eq.A13})$$

Where,

RAND() assigns a random number (uniform distribution from 0 to 1).

The mean value, μ , is set to the actual emissions analyzer reading.

The standard deviation, σ , is set equal to the % full-scale error times the scale used.

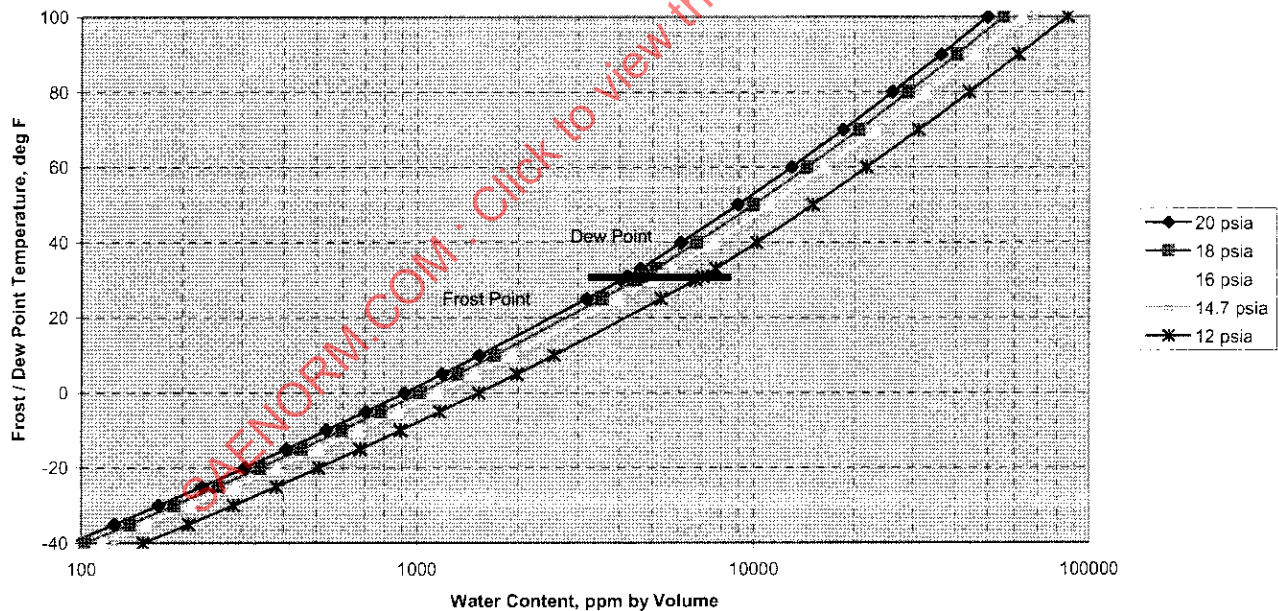
A.2 (Continued):

A set of 250 separate "noisy" random variables of the input parameters are generated and 250 separate calculations of the output values are calculated. That is the matrix equation discussed in Section 5.1 using the comprehensive matrix given in Table 4 must be solved 250 times, and the results used to calculate the appropriate emissions indices 250 times. The propagated error is then calculated as the standard deviation of the set of output parameters. These 250 random calculations provide sufficient precision to the error measurement, while not significantly slowing the calculation process.

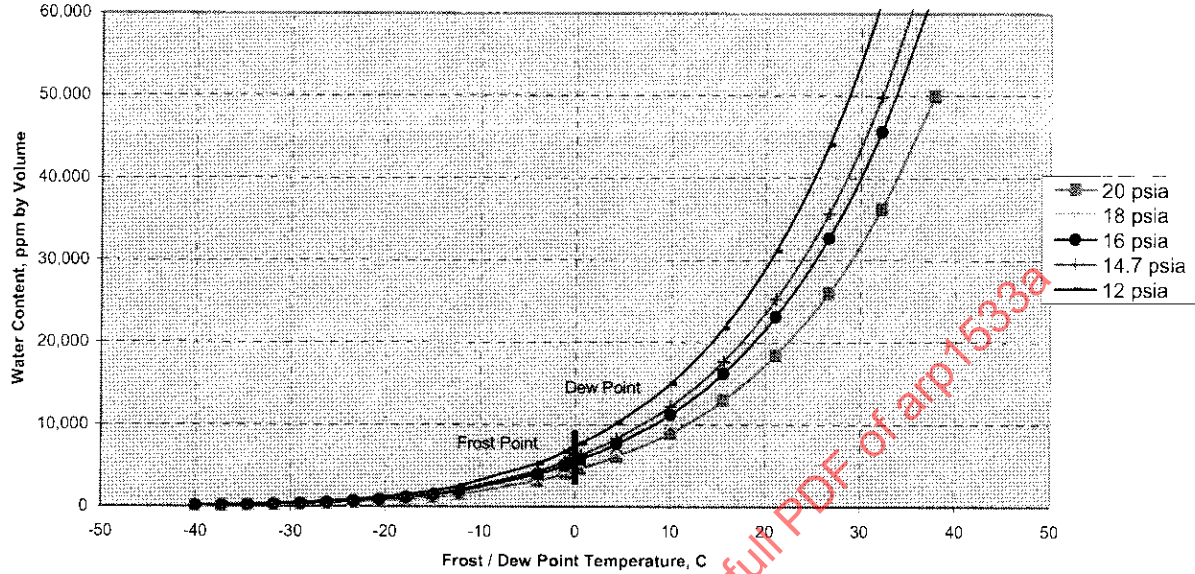
Alternatively, Equations A1 - A5 can be used to calculate the errors by this methodology. Again, a set of 250 separate "noisy" mole fractions is generated, and the output parameter is calculated 250 times. The error is then calculated as the standard deviation of the 250 output calculations. The calculation time for a set of variables using the spreadsheet is under a second and would be significantly shorter in a "compiled" language. (note that we have not done any careful calibration of the calculation speed, or optimization of the randomization calculations).

APPENDIX B CALCULATION OF SAMPLE WATER CONTENT FROM FROST OR DEW POINT TEMPERATURE MEASUREMENTS

In this document, water content must be known in terms of the number of moles of water per mole of dry air (h), the number of moles of water per mole of semidry sample (h_{sd}), and the number of moles of water per mole of fuel (P_4). These mole ratios are also referred to as molal ratios, volume ratios, or mixing ratio by volume (Ref. 2.2.5). Molal ratio is equal to the ratio of the vapor partial pressure to the pressure of dry air or sample. Therefore the actual water vapor partial pressure must be known. Typically, the sample frost or dew point temperature (THYG) is measured with a hygrometer at the gas sampling system pressure (PHYG). Empirical correlations are used to calculate the ideal water vapor saturation pressure (PWV) from the measured frost or dew point temperature at standard atmosphere pressure (14.695 psia). An enhancement factor (f) that accounts for varying gas sampling system pressure and the presence of other gas constituents is multiplied by the ideal saturation vapor pressure to obtain the effective water vapor saturation vapor pressure (PWVE). Increasing sample pressure increases the saturation temperature and decreasing sample pressure decreases the saturation temperature for constant water content.



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The empirical correlation presented by Hardy (Ref. 2.2.4) is recommended here because it is based on the ITS-90 temperature scale with traceability back to similar correlations of the IPTS-68 temperature scale that gained the largest international acceptance.

Measured dew point temperature, THYG, deg F converted to deg C and deg K

Ideal saturation vapor pressure at standard conditions, PWV, Pa

over water for THYG in deg K

$$PWV = \exp \left(\sum_{i=0}^6 g_i * THYG^{i-2} + g_7 * \ln THYG \right) \quad (\text{Eq.B1})$$

or over ice with THYG in deg K

$$PWV = \exp \left(\sum_{i=0}^4 k_i * THYG^{i-1} + k_5 * \ln THYG \right) \quad (\text{Eq.B2})$$

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where

g0	-2.8366E+03	
g1	-6.0281E+03	
g2	1.9543E+01	
g3	-2.7378E-02	
g4	1.6262E-05	
g5	7.0229E-10	
g6	1.8680E-13	
g7	2.7150E+00	
k0		-5.8666E+03
k1		2.2329E+01
k2		1.3939E-02
k3		-3.4262E-05
k4		2.7041E-08
k5		6.7064E-01

Measured sample pressure, PHYG, psia converted to Pa

Enhancement factor, f :

$$\alpha = \sum_{i=0}^3 A_i * THYG^i \quad \text{with THYG in deg C} \quad (\text{Eq.B3})$$

$$\beta = \exp \left(\sum_{i=0}^3 B_i * THYG^i \right) \quad \text{with THYG in deg C} \quad (\text{Eq.B4})$$

	For Water, -50 to 0 °C	For Water, 0 to 100 °C	For Ice, -100 to -50 °C	For Ice, -50 to 0 °C
A ₀	3.62183x10 ⁻⁴	3.53624x10 ⁻⁴	9.8830022x10 ⁻⁴	3.61345x10 ⁻⁴
A ₁	2.6061244x10 ⁻⁵	2.9328363x10 ⁻⁵	5.7429701x10 ⁻⁵	2.9471685x10 ⁻⁵
A ₂	3.866777x10 ⁻⁷	2.6168979x10 ⁻⁷	8.9023096x10 ⁻⁷	5.2191167x10 ⁻⁷
A ₃	3.8268958x10 ⁻⁹	8.5813609x10 ⁻⁹	6.2038841x10 ⁻⁹	5.0194210x10 ⁻⁹
B ₀	-1.07604x10 ¹	-1.07588x10 ¹	-1.0415113x10 ¹	-1.07401x10 ¹
B ₁	6.3987441x10 ⁻²	6.3268134x10 ⁻²	9.1177156x10 ⁻²	7.3698447x10 ⁻²
B ₂	-2.6351566x10 ⁻⁴	-2.5368934x10 ⁻⁴	5.1128274x10 ⁻⁵	-2.6890021x10 ⁻⁴
B ₃	1.6725084x10 ⁻⁶	6.3405286x10 ⁻⁷	3.5499292x10 ⁻⁶	1.5395086x10 ⁻⁶

$$f = \exp \left[\alpha * \left(1 - \frac{PWV}{PHYG} \right) + 3 * \left(\frac{PHYG}{PWV} - 1 \right) \right] \quad (\text{Eq.B5})$$

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Effective saturation vapor pressure, PWVE, Pa

$$PWVE = PWV * f \quad (\text{Eq.B6})$$

Water content, ppm

$$h \text{ or } h_{sd} = \frac{PWVE}{(PHYG - PWVE)} * 1 \times 10^6 \quad (\text{Eq.B7})$$

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APPENDIX C SAMPLE CALCULATIONS

Two sample calculations are provided. The first sample calculation uses data from earlier versions (1996-01 and before) of ARP1533 presented in the new format. In addition to providing values to validate existing data reduction programs, this sample illustrates how to manipulate the matrix for:

- analysis of the fuel $C_mH_nO_pN_qS_r$, where p , q , and $r = 0$.
- wet measurements of NO and NOx
- deleted measurements like SO₂ where one row and one column of the matrix are not required.

Sample Calculation #1

In this sample the fuel is JP-4 where $m = 9.5$, $n = 19.0$. The concentrations of C_xH_y , NO and NOx are wet measurements and the concentrations of CO and CO₂ are semi-dry measurements. Matrix A has been simplified to account for these variations. This sample does not include data quality checks.

Average fuel formulation from chemical analysis of the pre- and posttest samples.

Fuel Name:	JP-4
Fuel Formula:	$C_{9.5}H_{19.0}$
Fuel lower heating value (FLHV), BTU/mass	18,730
Moles of carbon in fuel, m	9.5
Moles of hydrogen in fuel, n	19.0
Moles of oxygen in fuel, p	0
Moles of nitrogen in fuel, q	0
Moles of sulfur in fuel, r	0
Ratio of H to C in fuel	2.00

Average air formulation from ambient air composition measurements.

Mole fraction of O ₂ in air, R	0.20948	moles O ₂ / mole dry air
Mole fraction of N ₂ and Ar in air, S	0.79020	moles N ₂ & Ar / mole dry air
Mole fraction of CO ₂ in air, T	0.00032	moles CO ₂ / mole dry air
Mole fraction of CH ₄ in air, U	0	moles CH ₄ / mole dry air
Inlet air water content, h	0.00883	moles H ₂ O / mole dry air

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Instrument interferences from laboratory checks on instruments.

CO ₂ interference on CO (L):	-1.3e-4
H ₂ O interference on CO (M):	-4.5e-4
CO ₂ interference on NO (L'):	0.14
H ₂ O interference on NO (M'):	0.28
O ₂ interference on CO ₂ (J):	0.09
NO _x converter efficiency (ETA):	0.975

NOTE: The oxygen interference coefficient, J = 0.09, assumes that the CO₂ calibration gas uses N₂ as the diluent.

Water content of inlet air and semi-dry sample.

Inlet air water content, h	0.00884	moles H ₂ O/mole dry air
Semidry sample water content, hsd	0.000017	moles H ₂ O/mole dry air

Measurement summary.

Species		Measurement
CO, ppm	semi-dry	500
CO ₂ , percent	semi-dry	2
C _x H _y , ppmC	wet	225
NO, ppm	wet	9
NO _x , ppm	wet	20

Simultaneous Equations: Matrix A

	PT	P1	P2	P3	P4	P5	P6	P7	P8	X
Eq 2	0	1	0	0	0	1	1.0	0	0	-0.000320
Eq 3	0	0	0	0	2	0	2.0	0	0	-0.017686
Eq 4	0	2	0	2	1	1	0	2	1	-0.428443
Eq 5	0	0	2	0	0	0	0	1	1	-1.5804
Eq 35	0.020121	-1	0	0.0018	-0.0201214	0	0	0	0	0
Eq 34	0.000500	-1.30E-04	0	0	-5.00E-04	-1	0	0	0	0
Eq 9	0.000225	0	0	0	0	0	-1.0	0	0	0
Eq 37	0.000020	0.0000028	0	0	0.0000056	0	0	-0.95	-1	0
Eq 36	0.000009	0.00000126	0	0	0.00000252	0	0	0	-1	0
Eq 13	-1	1	1	1	1	1	1	1	1	0

Constants: Matrix B

	Constants
Eq 2	9.5
Eq 3	19.0
Eq 4	0
Eq 5	0
Eq 35	0
Eq 34	0
Eq 9	0
Eq 37	0
Eq 36	0
Eq 13	0

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Results: Matrix C

		mole / mole fuel
Total (wet)	PT	469.03
carbon dioxide	P1	9.315
nitrogen	P2	363.53
oxygen	P3	82.385
water vapor	P4	13.463
carbon monoxide	P5	0.2267
total hydrocarbon	P6	0.1055
nitrogen dioxide	P7	0.00549
nitric oxide	P8	0.004267
air	X	460.05
Total (dry)	PT- P4	455.57

Analysis Summary

Species	Formula	Measured Concentration	Wet Basis Concentration	Dry Basis Concentration	EI [Z] (kg Z/1000 Kg fuel)
oxygen, % calculated	O ₂	-	17.565	18.084	
carbon dioxide, % semi-dry	CO ₂	2.00	1.986	2.045	
carbon monoxide, ppm semi-dry	CO	500	483.4	497.6	47.65
nitrogen, % calculated	N ₂	-	77.51	79.796	
water vapor, % calculated	H ₂ O	-	2.87	0	
total hydrocarbon, ppmC wet	CH ₂	225	225.0	231.6	11.11
nitrogen dioxide, ppm calculated	NO ₂	-	11.70	12.05	
oxides of nitrogen, ppm wet	NO _x	9	9.10	9.37	1.47
nitric oxide, ppm wet	NO	20	20.80	21.42	3.37

Gaseous Emissions Parameters

Emissions- derived fuel/air ratio	0.009998
Combustion efficiency, %	97.78

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Sample Calculation #2

In this sample the fuel is Jet A with a comprehensive fuel analysis. The concentration of C_xH_y is measured wet and the concentrations of CO , CO_2 , O_2 , NO , and NO_x are semi-dry measurements. The matrices include a column and row to account for sulfur in the fuel and air even though SO_2 is not a measured product.

Average fuel formulation from chemical analysis of the pre- and posttest samples.

Fuel Name:	Jet A
Fuel Formula:	C 7.158 H 13.919 O 0.00004 N 0 S 0.0012
Fuel lower heating value, BTU/lbm	18,550.5
Fuel Molecular Weight, lbm/mole	100.04
Moles of carbon in fuel, m	7.1576
Moles of hydrogen in fuel, n	13.9187
Moles of oxygen in fuel, p	0.00004
Moles of nitrogen in fuel, q	0
Moles of sulfur in fuel, r	0.0012
Ratio of H to C in fuel	1.945

Average air formulation from periodic ambient air composition measurements.

Mole fraction of O_2 in air, R	0.20687	moles O_2 / mole air
Mole fraction of N_2 and Ar in air, S	0.78036	moles N_2 & Ar / mole air
Mole fraction of CO_2 in air, T	0.00032	moles CO_2 / mole air
Mole fraction of CH_4 in air, U	0.0000037	moles CH_4 / mole air
Mole fraction of SO_2 in air, V	0.000001	moles SO_2 / mole air
Ambient air dew point hygrometer operating pressure, $PHYG_1$	14.20	psia
Dew point temp of inlet air, $THYG_1$	50	deg F
Effective vapor pressure, $PWVE_1$	0.177	psia
Water content of inlet air, h_1	0.01261	moles H_2O / mole air

Instrument interferences from laboratory checks on instruments.

CO_2 interference on CO (L):	-1.3e-4
H_2O interference on CO (M):	-4.5e-4
CO_2 interference on NO (L'):	0.14
H_2O interference on NO (M'):	0.28
O_2 interference on CO_2 (J):	0.09
NO_x converter efficiency (ETA):	0.975

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Sample water content before and after the dryer.

Pre-dryer dew point hygrometer operating pressure, PHYG ₂	14.2	psia
Dew point temp before dryer, THYG ₂	73	deg F
Effective vapor pressure, PWVE ₂	0.400	psia
Pre-dryer water content, h ₂	0.02897	moles H ₂ O/mole dry sample
Post-dryer dew point hygrometer operating pressure, PHYG ₃	14.2	psia
Dew point temp after dryer, THYG ₃	-23	deg F
Effective vapor pressure, PWVE ₃	0.0058	psia
Post-dryer water content, h ₃ or hsd	0.00041	moles H ₂ O/mole dry sample

Facility-metered flow rates.

Engine power setting	79 %
Fuel flow rate	0.243 lb/sec
Air flow rate	29.65 lb/sec

Measurement summary.

Species		Analyzer Range	Measurement	Standard Deviation
CO, ppm	semi-dry	1000	193.67	0.61
CO ₂ , percent	semi-dry	10	1.77	0.12
O ₂ , percent	semi-dry	25	18.61	0.21
C _x H _y , ppmC	wet	1000	85.5	1.30
NO, ppm	semi-dry	300	23.77	0.98
NO _x , ppm	semi-dry	300	32.57	1.67

NOTE: A number of data scans have been recorded such that the reported measurement is the average value and the standard deviation represents the stability of the emissions source and the analyzer during that time period.

Simultaneous Equations, Matrix A

	PT	P1	P2	P3	P4	P5	P6	P7	P8	P9	X
Eq 2	0	1	0	0	0	1	1.000	0	0	0	-0.00032
Eq 3	0	0	0	0	2	0	1.945	0	0	0	-0.02526
Eq 4	0	2	0	2	1	1	0	2	1	2	-0.42699
Eq 5		0	2	0	0	0	0	1	1	0	-1.56072
Eq 6	0	0	0	0	0	0	0	0	0	1	0
Eq 35	0.017707	-1	0	0.001593	-0.017707	0	0	0	0	0	0
Eq 34	0.000194	-0.00013	0	0	-1.94E-04	-1	0	0	0	0	0
Eq 9	0.000086	0	0	0	0	0	-1.0	0	0	0	0
Eq 37	0.000033	4.5598E-06	0	0	-3.2587E-05	0	0	-0.975	-1	0	0
Eq 36	0.000024	3.3278E-06	0	0	-2.3782E-05	0	0	0	-1	0	0
Eq 13	-1	1	1	1	1	1	1	1	1	1	0