



SURFACE VEHICLE INFORMATION REPORT

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A Recommended Method of Analytically Determining
the Competence of Hydraulic Telescopic Cantilevered Crane Booms

RATIONALE

Due to advances in computer design and analysis, few people will likely use the guidance in SAE J1078 for the design of new cranes. While still relevant, the process defined in SAE J1078 is mature and not likely to change in the foreseeable future.

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1. SCOPE

This analysis applies to crane types as covered by ASME B30.5.

1.1 Purpose

This calculation method has been established to illustrate an analysis to determine the competence of hydraulic telescopic cantilevered crane booms.

2. REFERENCES

2.1 Applicable Documents

The following publications form a part of this specification to the extent specified herein. Unless otherwise indicated, the latest issue of SAE publications shall apply.

2.1.1 ASME Publications

Available from ASME, P.O. Box 2900, 22 Law Drive, Fairfield, NJ 07007-2900, Tel: 800-843-2763 (U.S./Canada), 001-800-843-2763 (Mexico), 973-882-1170 (outside North America), www.asme.org.

ASME B30.5 Mobile and Locomotive Cranes

2.1.2 AISI Publications

Available from American Iron and Steel Institute, 25 Massachusetts Avenue, NW, Suite 800, Washington, DC 20001, Tel: 202-452-7100, www.steel.org.

AISI, "Specification for the Design of Cold-Formed Steel Structural Members," 1968 edition. In addition, "Commentary on the 1968 Edition," by George Winter and Supplementary Information Part II.

SAENORM.COM : Click to view the full PDF of J1078_202303

2.1.3 Other Publications

AISC, "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," adopted February 12, 1969. In addition, Supplement Nos. 1, 2, 3, and Commentary with additions and revisions where applicable.

Column Research Council, "Guide to Design Criteria for Metal Compression Members," Second Printing, 1960.

"USS Steel Design Manual," by R. L. Brockenbrough and B. G. Johnston, November 1968 printing.

3. NOMENCLATURE

- a = Clear distance between transverse stiffeners on side plate; also the ratio of the material yield of the web to the material yield of the compression flange
- A = Actual area of section
- A_e = Total effective area of section used in calculating F_a (see Appendix E for illustration)
- A_f = Area of compression flange
- A_i = Area based on inside dimensions of section (see Appendix D for illustration)
- A_m = Area based on mean dimensions of section (see Appendix D for illustration)
- A_o = Area based on outside dimensions of section (see Appendix D for illustration)
- A_{st} = Cross-sectional area of stiffener or pair of stiffeners
- A_w = Area of both webs
- b = Actual width of stiffened and unstiffened compression elements whether flange or web (see Appendix F for illustration)
- b_e = Effective width of stiffened compression element (see Appendix D for illustration)
- b_f = Actual flange width (see Appendix E for illustration)
- b_m = Mean width of section or $b_w - t_w$ (see Appendix D for illustration)
- b_w = Overall width of section (see Appendix E for illustration)
- C_t = Distance from neutral axis to extreme tension fiber of box section (see Appendix D for illustration)
- C_c = Distance from neutral axis to compressive fiber of box section (see Appendix D for illustration)
- C_b = Bending coefficient dependent upon moment gradient; equal to (see Equation 1)

$$1.75 + 1.05 \left(\frac{M_{x \min}}{M_{x \max}} \right) + 0.3 \left(\frac{M_{x \min}}{M_{x \max}} \right)^2 \quad (\text{Eq. 1})$$

but not more than 1.3 (see Appendix C for illustration)

- C_c = Column slenderness ratio dividing elastic and inelastic buckling equal to (see Equation 2)

$$\sqrt{\pi^2 E / (F_y - \sigma_{rc})} \quad (\text{Eq. 2})$$

C'_c = Effective column slenderness ratio dividing elastic and inelastic buckling equal to (see Equation 3)

$$\sqrt{\frac{\pi^2 E}{Q_s Q_a (F_y - \sigma_{re})}} \quad (\text{Eq. 3})$$

C_m = Coefficient applied to bending term in the interaction formula and dependent upon column curvature caused by applied moments; use 0.85

C_{mx} = 0.85

C_{my} = 0.85

C_v = Ratio of “critical” web stress, according to linear buckling theory, to the shear yield stress of web material

d = Overall depth of section (see Appendix E for illustration)

D = Factor depending upon type of transverse stiffeners

E = Modulus of elasticity, 29500 ksi (203400 MPa)

f = Computed axial and bending compression stress on appropriate flange or web

f_a = Computed axial stress based on total section area

f_b = Computed bending stress about the appropriate axis

f_c = Sum of the computed axial and side bending compressive stresses

f_{bx} = Computed bending stress about the x-x axis

f_{by} = Computed bending stress about the y-y axis

f_s = Sum of the computed torsional and vertical shear stress

f_v = Computed average web or flange shear stress

f_{vs} = Total shear transfer of stiffener(s), kips per inch of length (Pa/m)

F_a = Allowable axial stress permitted in the absence of a bending moment

F_b = Allowable bending stress for the appropriate axis

F_{bx} = Allowable bending stress about the x-x axis if this bending moment alone existed

F'_{bx} = Allowable bending stress in compression flange of box sections as reduced for hybrid sections or because of large web depth-to-thickness ratio

F_{by} = Allowable bending stress about the y-y axis if this bending moment alone existed

F'_e = Euler stress divided by factor of safety; equal to (see Equation 4)

$$\frac{12 \pi^2 E}{23 (Kl/r)^2} \quad (\text{Eq. 4})$$

F'_{ex} = Same as F'_e about the x-x axis

F'_{ey} = Same as F'_e about the y-y axis

F_v = Allowable web shear stress

F_y = Specified minimum yield stress of material being used, based on “yield stress” or yield strength, whichever is applicable

g = Wind load, lb/in², $g = 0.004 \text{ (mph)}^2/144$

G = Shear modulus of elasticity 11300 ksi

h = Clear distance between flanges (see Appendix D for illustration)

h_m = Mean height of section $d - (t_c + t_t)/2$ (see Appendix E for illustration)

h_v = Vertical height of horizontal stiffener

H_o = Height to boom foot pin from ground

H_p = Height to center of pressure on boom

H_r = Reference height at which wind velocity is measured (20 feet in U.S.)

I_x = Area moment of inertia about the x-x axis

I_y = Area moment of inertia about the y-y axis

I_{st} = Moment of inertia of a pair of intermediate stiffeners, or a single intermediate stiffener, with reference to an axis in the plane of the web

I_{xe} = Effective moment of inertia about the x-x axis

I_{ye} = Effective moment of inertia about the y-y axis

J = Torsional constant; equal to (see Appendix D for other equations) (see Equation 5)

$$\frac{4(b_m)^2(h_m)^2}{\frac{2h_m}{t_w} + \frac{b_m}{t_c} + \frac{b_m}{t_t}}$$

(Eq. 5)

k = Coefficient relating linear buckling strength of a plate to its dimensions and conditions of edge support

K = Effective length factor for cantilevered sections; use the value 2 unless a smaller one can be justified

K_t = Torsional length factor for cantilevered sections; use the value 4/3

ℓ = Dimensional lengths of boom

L = Distance from tip to section in question

L_b = Actual unbraced length of section in the plane of bending

M	Bending moment about the appropriate axis
M_1	Constant moment load about the x-x axis resulting from eccentric loading on the head
M_2	Constant moment load about the y-y axis resulting from the side loading on the head
$M_{x\min}$	Smaller moment at end of unbraced length of beam-column at tip
$M_{x\max}$	Larger moment at end of unbraced length of beam-column at section in question
M_x	Bending moment about the x-x axis
M_y	Bending moment about the y-y axis
N	Number of parts of line
p	Wind velocity exponent
P	Externally applied load at the tip
P_a	Axial load applied to section
P_x	Lateral loading component (side load)
P_y	Vertical loading component
P_z	Axial loading component
Q_a	Ratio of effective profile area of an axially loaded member to its total profile area of A_e/A
Q_s	Axial stress reduction factor for unstiffened elements of a section (see Appendix F)
r	Radius of gyration for appropriate axis
r_b	Radius of gyration about the axis of concurrent bending, computed on the basis of actual cross-sectional area
R	Load radius from centerline of rotation to centerline of load
R_h	Hoist cylinder reaction
R_x	Reaction loads in the lateral direction
R_y	Reaction loads in the vertical direction
R_z	Reaction loads in the axial direction
S_x	Strong axis section modulus with c taken to the compressive side
S_y	Weak axis section modulus with c taken to the compressive side
S_{xe}	Effective strong axis section modulus with c taken to the compressive side
S_{ye}	Effective weak axis section modulus with c taken to the compressive side
t	Thickness of flange or web in compression (see Appendix D for illustration)

t_c	Thickness of compression flange (see Appendix D for illustration)
t_t	Thickness of tension flange (see Appendix D for illustration)
t_w	Thickness of web (see Appendix D for illustration)
T	Torsional moment
V_p	Wind velocity (mph) at center of pressure height H_p
V_r	Wind velocity (mph) at reference height H_r
V_x	Static shear load on section in the lateral direction
V_y	Static shear load on section in the vertical direction
w	Component weight, lb/in
W	Total component weight
x	Subscript relating symbol to strong axis bending
Y	Ratio of yield stress of web steel to that of yield stress of stiffener steel
y	Subscript relating symbol to weak axis bending
z	Subscript relating symbol to axial loading
α	Boom centerline elevation angle relative to a horizontal plane, or the ratio of web yield stress to flange yield stress
θ	Angle between a line perpendicular to the boom axis and the hoist cylinder axis
σ_{rc}	Residual compressive stress; equal to $0.5 E_y$ in lieu of specific information on steel used
u	Poisson's ratio; equal to 0.3

4. CRITERIA

Calculations shall include the dead weight loads, rated load, and a minimum side load of 2% of the rated load at the rated load radius. The side load provides for "normal" conditions of machine operation. In addition, the effect of the wind on the boom should be considered, as is provided for in the calculations.

- 4.1 The factors of safety used herein are the recommended factors of the AISC "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," adopted February 12, 1969.
- 4.2 The boom shall be deemed competent when the solution of the interaction equations provided herein yield a value equal to or less than one.

5. LOADS AND FORCES

- 5.1 The 2% side load provides for "normal" conditions of boom motion. No allowances have been made for dynamic loads, duty cycle operation, effects of the wind on the load lifted, or operations other than lifting crane service.
- 5.2 All forces and loads are expressed in pounds (kg). Dimensions are in inches (m). Stresses both allowable and calculated are in units of ksi (MPa). Also, the modulus of elasticity is expressed in units of ksi (MPa).

6. ANALYTICAL DETERMINATION OF STRESSES AND CRITICAL LOADS

6.1 Applicability

This analysis is applicable to multi-sectioned “box” type booms, which are totally enclosed and cantilevered beyond the base section.

6.2 Basis for Analysis

The equations presented in this analysis are based on laterally unsupported beam column formulas, the solution of which are combined in interaction equations. In determining the section properties, the effective width of the plates in compression are used. The areas covered in this analysis consist of axial and torsional loading, bidirectional bending, and panel buckling. Of primary importance in the analysis are the compressive stress calculations.

The work of this committee is not intended to cover all design concepts, but rather a basic system. However, other design configurations may use alternative calculation methods when substantiated with suitable test data.

6.3 Summary

Where strain gage results are available, they should be used to supplement the analytical data.

7. LOAD MOMENT DIAGRAMS AND EQUATIONS

7.1 Assumptions Used on Load Moment Equations

- 7.1.1 Wind force is negligible on head (should include effects if jib used).
- 7.1.2 Torque is created by the side load P on the head (would also be applicable for a jib).
- 7.1.3 Equations are still applicable if jib used but dimensions, weights, and center of gravity to be adjusted accordingly.
- 7.1.4 $P_y = P \cos \alpha$; $P_z = P \sin \alpha$.
- 7.1.5 Winch rope fleet angle and angle relative to boom is negligible.
- 7.1.6 Wind force is uniformly distributed along the exposed length of the side of the section with its reaction at the center (is a valid assumption since each section considered individually).
- 7.1.7 That the dimensions are to the reaction points and that the tips of each section beyond these points are small in length and will not affect the validity of the equations.
- 7.1.8 That the axial stresses produced by the friction forces due to the section reaction points from one to the next are small in comparison to the other stresses, that the section support cylinders carry the axial loads.
- 7.1.9 That equations and formulations appearing in the foregoing analysis are for the boom in the extended position—Figures 1, 2, 3, 4, 5, 6, and 7. Partially retracted positions will require reformulation of some equations; as an example, in Figure 4 when l_{11} is zero or negative, the cylinder no longer takes the axial load at the section being considered. The moment equations would then appear as those written for reference Figure 3. Similar changes would appear in the axial load, reactions, and shear force equations.

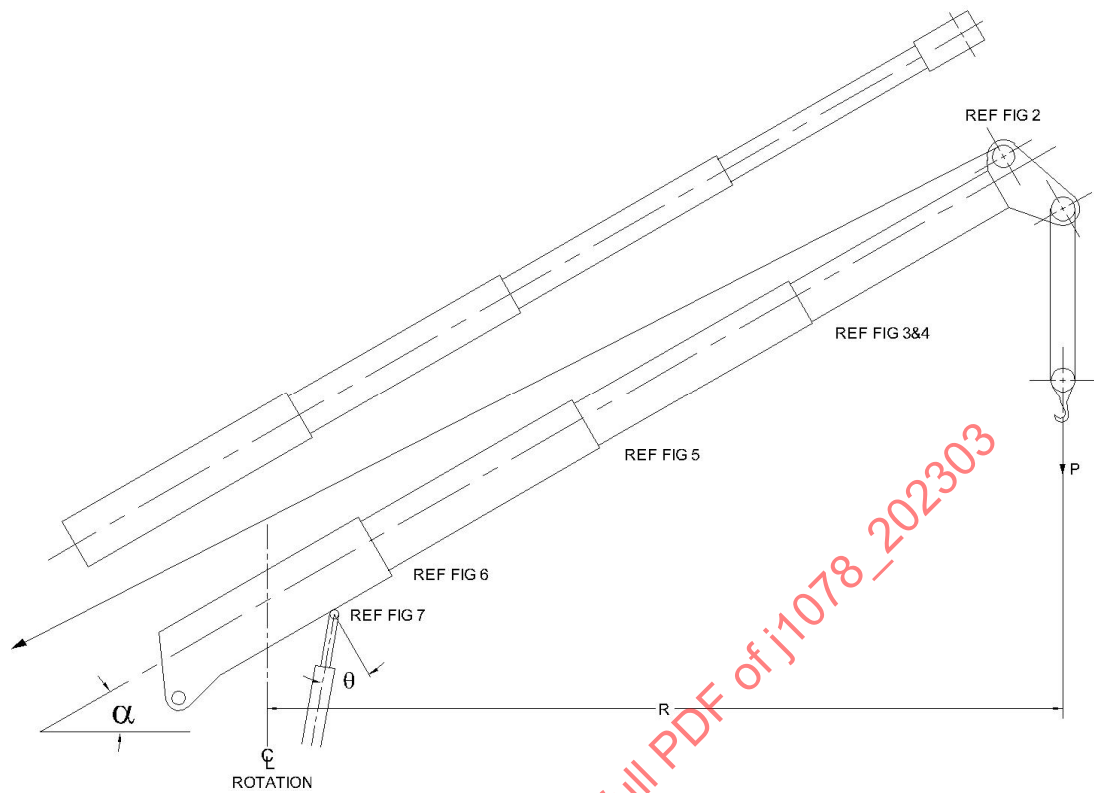


Figure 1 - Loading diagram - boom assembly

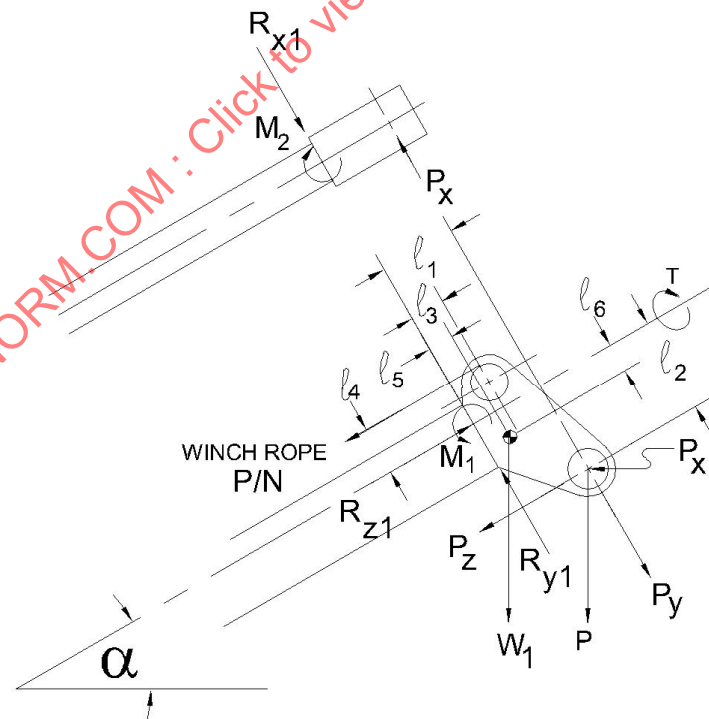


Figure 2 - Load moment diagram - head section

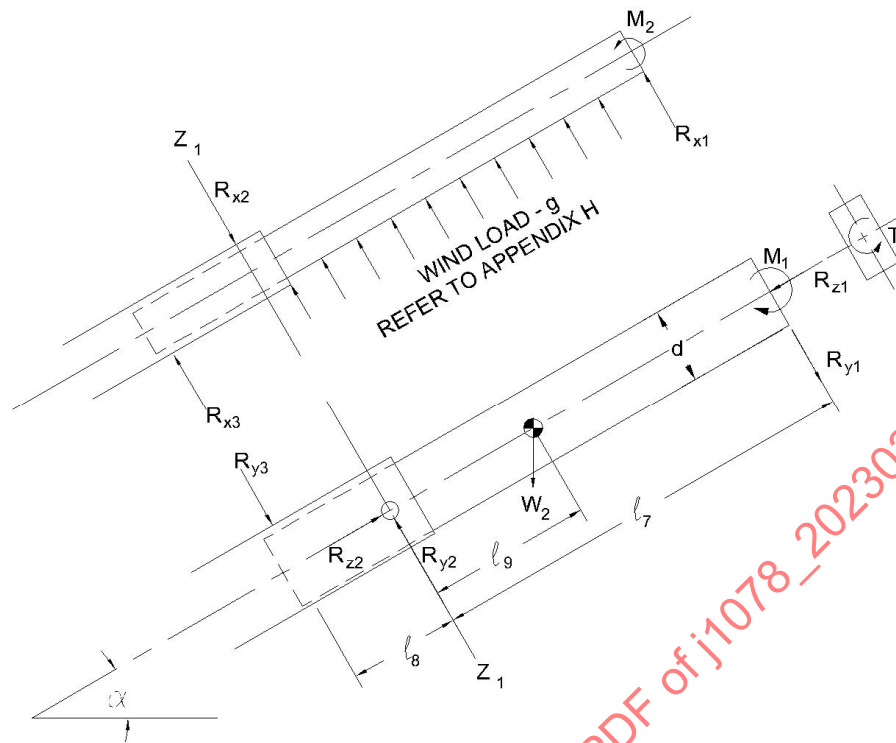


Figure 3 - Load moment diagram - tip section

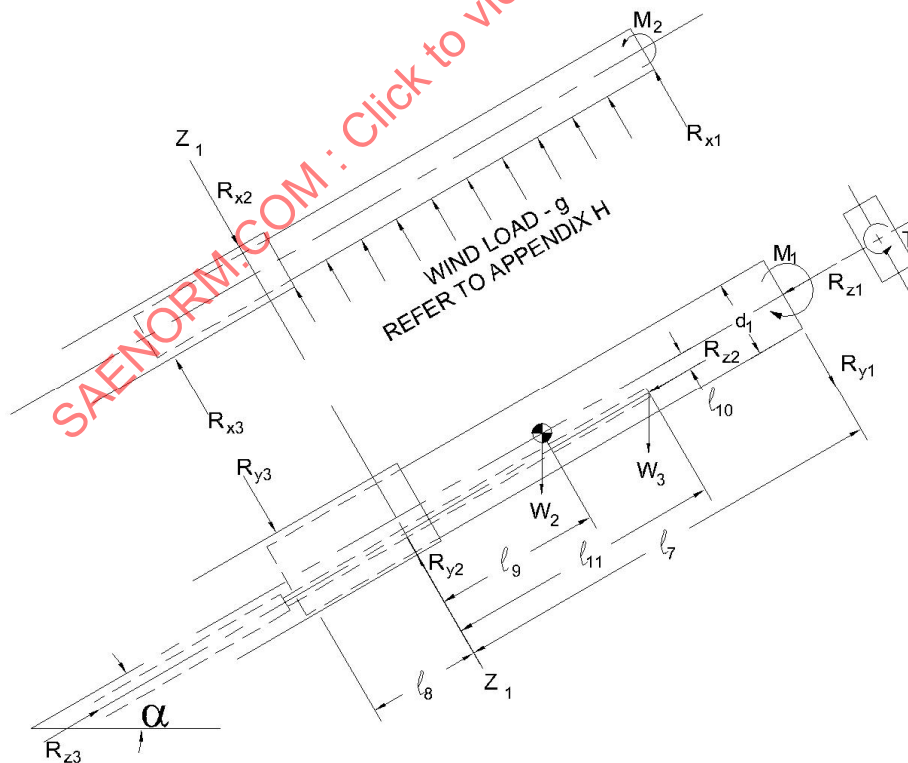


Figure 4 - Load moment diagram - alternate tip section

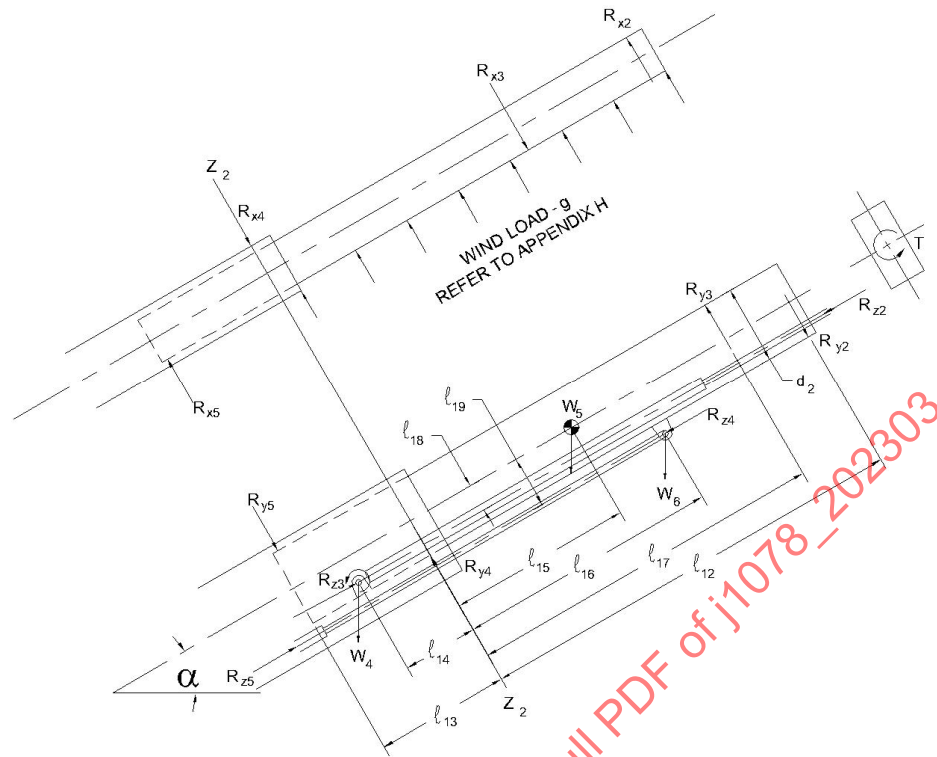


Figure 5 - Load moment diagram - intermediate section

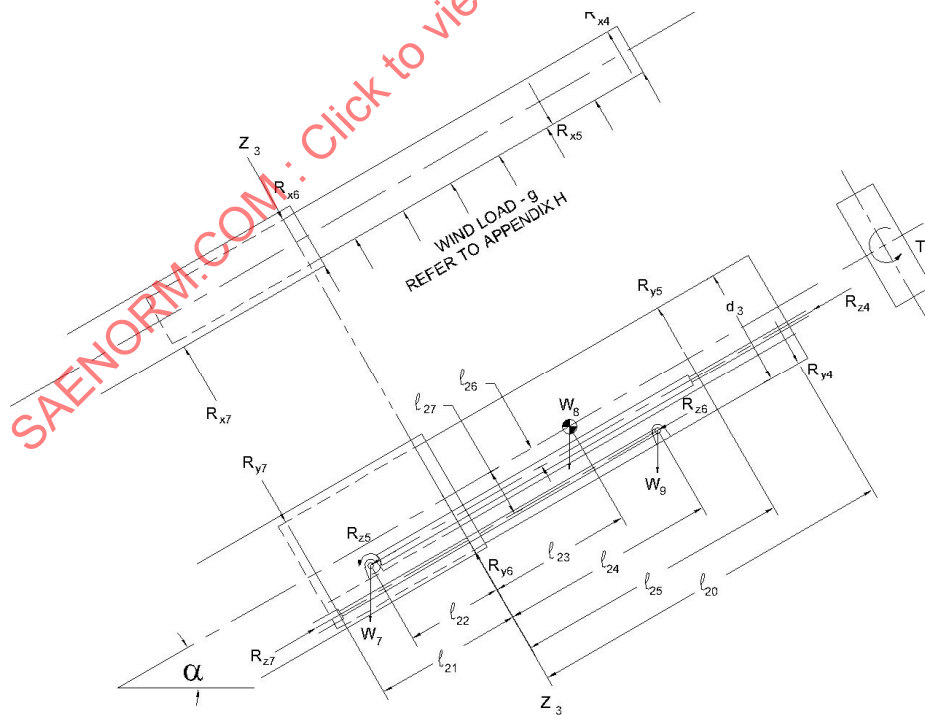


Figure 6 - Load moment diagram - intermediate section

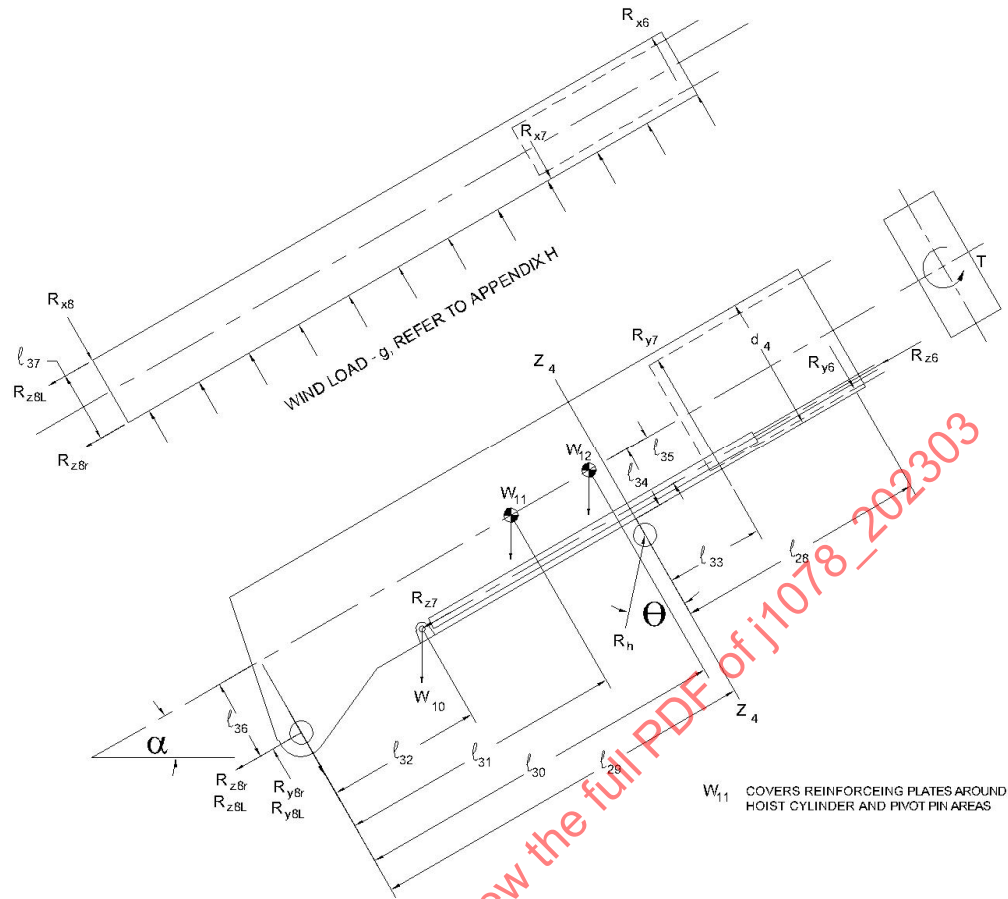


Figure 7 - Load moment diagram - base section

7.2 Figure 2

Load moment equations for reaction of head forces on tip section (see Equations 6, 7, 8, and 9).

Moment (Equation 6):

$$\begin{aligned} M_1 &= P_y l_1 + P_z l_2 - P \sqrt{N} l_4 + W_1 [l_5 \cos \alpha + l_6 \sin \alpha] \\ M_2 &= P_x l_1 \\ T &= P_x l_2 \end{aligned} \quad (\text{Eq. 6})$$

Axial load (Equation 7):

$$R_{Z1} = P \sqrt{N} + P_z + W_1 \sin \alpha \quad (\text{Eq. 7})$$

Shear loads (Equation 8):

$$\begin{aligned} V_x &= -R_{x1} = P_x \\ V_y &= -R_{y1} = W_1 \cos \alpha + P_y \end{aligned} \quad (\text{Eq. 8})$$

Side load (Equation 9):

$$P_x = 0.02 P \quad (\text{Eq. 9})$$

7.3 Figure 3

Load moment equations for tip section at Section Z₁ - Z₁ (see Equations 10, 11, 12, 13, 14, 15, and 16).

Moments (Equation 10):

$$\begin{aligned} M_x &= M_1 + R_{y1} l_7 + \frac{0.5 W_2 \cos \alpha l_7^2}{l_7 + l_8} \\ M_y &= M_2 + R_{x1} l_7 + 0.5 g d_1 l_7^2 \\ T &= P_x l_2 \end{aligned} \quad (\text{Eq. 10})$$

Axial load on pin (Equation 11):

$$R_{z2} = R_{z1} + W_2 \sin \alpha \quad (\text{Eq. 11})$$

Axial load on section (Equation 12):

$$P_{ar} = R_{z1} + W_2 \sin \alpha \frac{l_7}{l_7 + l_8}; P_{aL} = W_2 \sin \alpha \frac{l_8}{l_7 + l_8} \quad (\text{Eq. 12})$$

Vertical reactions (Equation 13):

$$R_{y3} = \frac{M_x}{l_8} - 0.5 W_2 \cos \alpha \frac{l_8}{l_7 + l_8}; R_{y2} = R_{y1} + R_{y3} + W_2 \cos \alpha \quad (\text{Eq. 13})$$

Lateral reactions (Equation 14):

$$R_{x3} = \frac{M_y}{l_8}; R_{x2} = R_{x1} + R_{x3} + g d_1 l_7 \quad (\text{Eq. 14})$$

Vertical shear forces (Equation 15):

$$V_{yr} = R_{y1} + W_2 \cos \alpha \frac{l_7}{l_7 + l_8}; V_{yL} = R_{y3} + W_2 \cos \alpha \frac{l_8}{l_7 + l_8} \quad (\text{Eq. 15})$$

Lateral shear forces (Equation 16):

$$V_{xr} = R_{x1} + g d_1 l_7; V_{xL} = R_{x3} \quad (\text{Eq. 16})$$

NOTE: Subscripts r and L refer to right and left of Section Z₁ - Z₁.

7.4 Figure 4

Load moment equations for alternate tip section at Section Z₁ - Z₁ (see Equations 17, 18, 19, 20, 21, 22, and 23).

Moments (Equation 17):

$$\begin{aligned} M_x &= M_1 + R_{y1} l_7 + 0.5 W_2 \cos \alpha \frac{l_7^2}{l_7 + l_8} + W_3 \cos \alpha l_{11} \mp R_{z2} l_{10} \\ M_y &= M_2 + R_{x1} l_7 + 0.5 g d_1 l_7^2 \\ T &= P_x l_2 \end{aligned} \quad (\text{Eq. 17})$$

Axial load on cylinder support (Equation 18):

$$R_{z2} = R_{z1} + W_2 \sin \alpha \quad (\text{Eq. 18})$$

Axial load on section (Equation 19):

$$P_{ar} = P_{aL} = \frac{W_2 l_8}{l_7 + l_8} \sin \alpha \quad (\text{Eq. 19})$$

Vertical reactions (Equation 20):

$$R_{y3} = \frac{M_x}{l_8} - 0.5 W_2 \cos \alpha \frac{l_8}{l_7 + l_8}; R_{y2} = R_{y1} + R_{y3} + W_2 \cos \alpha + W_3 \cos \alpha \quad (\text{Eq. 20})$$

Lateral reactions (Equation 21):

$$R_{x3} = \frac{M_y}{l_8}; R_{x2} = R_{x1} + R_{x3} + g d_1 l_7 \quad (\text{Eq. 21})$$

Vertical shear forces (Equation 22):

$$V_{yr} = R_{y1} + W_2 \cos \alpha \frac{l_7}{l_7 + l_8} + W_3 \cos \alpha; V_{yL} = R_{y3} + W_2 \cos \alpha \frac{l_8}{l_7 + l_8} \quad (\text{Eq. 22})$$

Lateral shear forces (Equation 23):

$$V_{xr} = R_{x1} + g d_1 l_7; V_{xL} = R_{x3} \quad (\text{Eq. 23})$$

NOTE: Subscripts r and L refer to right and left of Section Z₁ - Z₁.

7.5 Figure 5

Load moment equations for intermediate section at Section Z₂ - Z₂ (see Equations 24, 25, 26, 27, 28, 29, and 30).

Moments (Equation 24):

$$\begin{aligned} M_x &= R_{y2}l_{12} - R_{y3}l_{17} + W_6 \cos \alpha l_{16} + 0.5 W_5 \cos \alpha \frac{l_{12}^2}{l_{12} + l_{13}} \mp W_5 \sin \alpha l_{19} \mp R_{z3}(l_{19} - l_{18}) \\ M_y &= R_{x2}l_{12} - R_{x3}l_{17} + 0.5 g d_2 l_{12}^2 \\ T &= P_x l_2 \end{aligned} \quad (\text{Eq. 24})$$

Axial load on cylinder supports (Equation 25):

$$R_{z3} = R_{z2} + (W_3 + W_4) \sin \alpha; R_{z4} = R_{z3} + W_5 \sin \alpha \quad (\text{Eq. 25})$$

Axial load on section (Equation 26):

$$P_{ar} = P_{aL} = R_{z3} + W_5 \sin \alpha \frac{l_{13}}{l_{12} + l_{13}} \quad (\text{Eq. 26})$$

Vertical reactions (Equation 27):

$$R_{y5} = \frac{M_x}{l_{13}} - W_4 \cos \alpha \frac{l_{14}}{l_{13}} - 0.5 W_5 \cos \alpha \frac{l_{12}}{l_{12} + l_{13}}; R_{y4} = R_{y2} - R_{y3} + R_{y5} + \cos \alpha (W_4 + W_5 + W_6) \quad (\text{Eq. 27})$$

Lateral reactions (Equation 28):

$$R_{x5} = \frac{M_y}{l_{13}}; R_{x4} = R_{x2} - R_{x3} + R_{x5} + g d_2 l_{12} \quad (\text{Eq. 28})$$

Vertical shear forces (Equation 29):

$$\begin{aligned} V_{yr} &= R_{y2} - R_{y3} + W_6 \cos \alpha + W_5 \cos \alpha \frac{l_{12}}{l_{12} + l_{13}} \\ V_{yL} &= R_{y5} + W_4 \cos \alpha + W_5 \cos \alpha \frac{l_{13}}{l_{12} + l_{13}} \end{aligned} \quad (\text{Eq. 29})$$

Lateral shear forces (Equation 30):

$$V_{xr} = R_{x2} - R_{x3} + g d_2 l_{12}; V_{xL} = R_{x5} \quad (\text{Eq. 30})$$

NOTE: Subscripts r and L refer to the right and left of Section Z₂ - Z₂.

7.6 Figure 6

Load moment equations for intermediate section at Section Z₃ - Z₃ (see Equations 31, 32, 33, 34, 35, 36, and 37).

Moments (Equation 31):

$$\begin{aligned} M_x &= R_{y4} l_{20} - R_{y5} l_{25} + W_9 \cos \alpha l_{24} + 0.5 W_8 \cos \alpha \frac{l_{20}^2}{l_{20} + l_{21}} \mp W_8 \sin \alpha l_{27} \mp R_{z5} (l_{27} - l_{26}) \\ M_y &= R_{x4} l_{20} - R_{x5} l_{25} + 0.5 g d_3 l_{20}^2 \\ T &= P_x l_2 \end{aligned} \quad (\text{Eq. 31})$$

Axial load on cylinder supports (Equation 32):

$$R_{z5} = R_{z4} + (W_6 + W_7) \sin \alpha; R_{z6} = R_{z5} + W_8 \sin \alpha \quad (\text{Eq. 32})$$

Axial load on section (Equation 33):

$$P_{ar} = P_{aL} = P_{z5} + W_8 \sin \alpha \frac{l_{21}}{l_{20} + l_{21}} \quad (\text{Eq. 33})$$

Vertical reactions (Equation 34):

$$\begin{aligned} R_{y7} &= \frac{M_x}{l_{21}} - W_7 \cos \alpha \frac{l_{22}}{l_{21}} - 0.5 W_8 \cos \alpha \frac{l_{21}}{l_{20} + l_{21}} \\ R_{y6} &= R_{y4} - R_{y5} + R_{y7} + \cos \alpha (W_7 + W_8 + W_9) \end{aligned} \quad (\text{Eq. 34})$$

Lateral reactions (Equation 35):

$$R_{x7} = \frac{M_y}{l_{21}}; R_{x6} = R_{x4} - R_{x5} + R_{x7} + g d_3 l_{20} \quad (\text{Eq. 35})$$

Vertical shear forces (Equation 36):

$$\begin{aligned} V_{yr} &= R_{y4} - R_{y5} + W_9 \cos \alpha + W_8 \cos \alpha \frac{l_{20}}{l_{20} + l_{21}} \\ V_{yL} &= R_{y7} + W_7 \cos \alpha + W_8 \cos \alpha \frac{l_{21}}{l_{20} + l_{21}} \end{aligned} \quad (\text{Eq. 36})$$

Lateral shear forces (Equation 37):

$$V_{xr} = R_{x4} - R_{x5} + g d_3 l_{20}; V_{xL} = R_{x7} \quad (\text{Eq. 37})$$

NOTE: Subscripts r and L refer to right and left of Section Z₃ - Z₃.

7.7 Figure 7

Load moment equations for base section at Section Z₄ - Z₄ (see Equations 38, 39, 40, 41, 42, 43, 44, 45, and 46).

Moments (Equation 38):

$$\begin{aligned} M_x &= R_{y6} l_{28} - R_{y7} l_{33} + 0.5 w_{12} \cos \alpha \frac{l_{28}^2}{l_{28} + l_{29}} \\ M_y &= R_{x6} l_{28} - R_{x7} l_{33} + 0.5 g d_4 l_{28}^2 \\ T &= P_x l_2 \end{aligned} \quad (\text{Eq. 38})$$

Axial load on cylinder support (Equation 39):

$$R_{z7} = R_{z6} + (W_9 + W_{10}) \sin \alpha \quad (\text{Eq. 39})$$

Axial load on section (Equation 40):

$$P_{ar} = P_{aL} = W_{12} \sin \alpha \frac{l_{28}}{l_{28} + l_{29}} \quad (\text{Eq. 40})$$

Vertical shear force (Equation 41):

$$\begin{aligned} V_{yr} &= R_{y6} - R_{y7} + W_{12} \cos \alpha \frac{l_{28}}{l_{28} + l_{29}} \\ V_{yL} &= R_{y8r} + R_{y8L} + (W_{10} + W_{11}) \cos \alpha + W_{12} \cos \alpha \frac{l_{28}}{l_{28} + l_{29}} \end{aligned} \quad (\text{Eq. 41})$$

Lateral shear force (Equation 42):

$$V_{xr} = R_{x6} - R_{x7} + g d_4 l_{28}; V_{xL} = R_{x8} - g d_4 l_{29} \quad (\text{Eq. 42})$$

Hoist cylinder reactions (Equation 43):

$$\begin{aligned} R_h &= [R_{y6}(l_{28} + l_{29}) - R_{y7}(l_{29} + l_{33}) + W_{10} l_{32} \cos \alpha \\ &\quad + W_{11}(l_{31} \cos \alpha - l_{36} \sin \alpha) + W_{12}(l_{30} \cos \alpha - l_{36} \sin \alpha) \\ &\quad - R_{z7}(l_{36} - l_{35})] / \left[l_{29} - \left(\frac{l_{36} - l_{34}}{\cot \alpha \tan \theta} \right) \right] \cos \theta \end{aligned} \quad (\text{Eq. 43})$$

Pivot pin loading:

Lateral reaction (Equation 44):

$$R_{x8} = R_{x6} - R_{x7} + g d_4 (l_{28} + l_{29}) \quad (\text{Eq. 44})$$

Axial reactions (Equation 45):

$$\begin{aligned}
 R_{z8r} &= \frac{R_h \sin \theta}{2} + R_{x6} \frac{l_{28} + l_{29}}{l_{37}} - R_{x7} \left(\frac{l_{29} + l_{33}}{l_{37}} \right) \\
 &\quad - \frac{R_{z7} - (W_{11} + W_{12}) \sin \alpha}{2} + \frac{0.5 g d_4 (l_{28} + l_{29})^2}{l_{37}} \\
 R_{z8L} &= \frac{R_h \sin \theta}{2} - R_{x6} \frac{l_{28} + l_{29}}{l_{37}} + R_{x7} \left(\frac{l_{29} + l_{33}}{l_{37}} \right) \\
 &\quad - \frac{R_{z7} - (W_{11} + W_{12}) \sin \alpha}{2} - \frac{0.5 g d_4 (l_{28} + l_{29})^2}{l_{37}}
 \end{aligned}
 \tag{Eq. 45}$$

Vertical reactions (Equation 46):

$$\begin{aligned}
 R_{y8r} &= 0.5 [R_h \cos \theta + R_{y7} - R_{y6} - (W_{10} + W_{11} + W_{12}) \cos \alpha] \\
 &\quad - P_x \frac{l_2 - l_{36}}{l_{37}} + g d_4 (l_{28} + l_{29}) \frac{l_{36}}{l_{37}} \\
 R_{y8L} &= 0.5 [R_h \cos \theta + R_{y7} - R_{y6} - (W_{10} + W_{11} + W_{12}) \cos \alpha] \\
 &\quad + P_x \frac{l_2 - l_{36}}{l_{37}} - g d_4 (l_{28} + l_{29}) \frac{l_{36}}{l_{37}}
 \end{aligned}
 \tag{Eq. 46}$$

NOTE: Subscripts r and L refer to right and left of Section Z₄ - Z₄.

8. CALCULATION PROCEDURE

8.1 Step 1 - Preliminary Data

- a. Provide description of geometry and loading, such as boom length, working radius, boom angle, rated load, etc.
- b. Identify boom arrangement.
 1. Generate shear and moment diagrams.
 2. Solve for forces and moments from Section 4.
- c. Identify boom section for analysis.
 1. Determine material properties.
 2. Determine section properties.

8.2 Step 2 - Calculation of Section Properties, Based on Compressive Stresses, the Actual Stress, and the Allowable Stress

8.2.1 To Determine Section Properties

a. Determine if plates in compression are fully effective at yield.

1. For vertical bending loads, compute the b/t ratio for the compressive flange.
2. For side bending loads, compute the b/t ratio for the compressive web.
3. For axial loads, compute the b/t ratio for both webs and both flanges. If $b/t \leq 184/\sqrt{f}$, where $f = 0.6F_y$, then the entire section will be fully effective at yield. The properties can then be computed based on the actual section. Proceed to 8.2.2 for the allowable stress computations. If $b/t > 184/\sqrt{f}$, for any or all plates, the section may still be fully effective for the actual stress. (Refer to AISC 1.9.2.2.)

b. Determine if plates in compression are fully effective at the actual stress.

1. Compute actual stresses based on full section properties.

- i. For compression flange, $f = f_a + f_{bx}$.
- ii. For compression web, $f = f_a + f_{bx}$.
- iii. For axial (all plates), $f = f_a$.

Use this calculated stress for f and recompute the b/t ratios.

NOTE: For the axial case, the effective widths will be different. See Appendix E.

If $b/t \leq 184/\sqrt{f}$, for all plates, the entire section is fully effective at a stress level 1.67 times the actual stress. Proceed to 8.2.2 for the allowable stress computations. If $b/t > 184/\sqrt{f}$, for any one or all plates, the section is not fully effective for stress level f and an effective width calculation must be made for each plate that exceeds this ratio.

c. Determine effective width of plates that have b/t ratio greater than $184/\sqrt{f}$.

1. Calculate the effective width of plates which are not fully effective accordingly:

$$b_c = \frac{253 t}{\sqrt{f}} \left(1 - \frac{50.3}{(b/t)\sqrt{f}} \right) \quad \text{AISC (C3-1)}$$

where f is the actual stress computed from 8.2.1.b.1.i, 8.2.1.b.1.ii, and 8.2.1.b.1.iii.

2. Calculate new section properties A_e , S_{xe} , and S_{ye} based on the effective widths b_e .

NOTE: The effective widths be used in computing A_e do not require an iterative solution because the stress f_a is based on the actual area A .

3. Recompute new stress levels based on new properties.
4. Recompute new effective widths based on new stress levels.
5. Continue until stress level stabilizes; approximately three iterations. Proceed to 8.2.2 for the allowable stress computations.

8.2.2 To Determine Allowable Stresses

a. Allowable axial stress F_a :

NOTE: If the stress is a tensile value, then $F_a = 0.6F_y$; proceed to 8.2.2.b.

1. Factor Q_a :

$$Q_a = \frac{\text{effective area } (A_e)}{\text{actual area } (A)} \quad \text{AISC (C4)}$$

where:

A_e = the effective area of all stiffened elements, both flanges and webs, corresponding to the actual stress

If all plates are fully effective, $Q_a = 1$.

2. Factor Q_s ; see Appendix F to determine if this computation must be made. Applies to outstanding plates free on one edge.

$$C_c = \sqrt{\frac{\pi^2 E}{Q_s Q_a (F_y - \sigma_{rc})}} \quad \text{AISC (C5)}$$

where:

$$Q_s Q_a \leq 1.0.$$

4. Compute (KL/r) of both axis and use the largest KL/r value for the F_a calculation:

$$r = \sqrt{\frac{I}{A}}$$

5. If $(KL/r) < C'_c$ —inelastic range:

$$F_a = \frac{Q_s Q_a \left[1 - \frac{\sigma_{rc} (KL/r)^2}{F_y (C_c)^2} \right] F_y}{5/3 + 3/8 \left(\frac{KL/r}{C_c} \right) - 1/8 \left(\frac{KL/r}{C_c} \right)^3} \quad \text{AISC (C5-1)}$$

6. If $(KL/r) \geq C'_c$ —elastic range:

$$F_a = \frac{12 \pi^2 E}{23 (KL/r)^2} \quad \text{AISC (1.5-2)}$$

NOTE: L as used previously is the distance from the outer end of the section in question to the point where the stresses are to be calculated in that section.

b. Allowable compressive bending stresses F_b for x and y directions considering lateral torsional buckling.

1. Inelastic lateral buckling check:

$$(KL/r)_{\text{equiv.}} = \sqrt{\frac{5.1 K_t L S_x}{\sqrt{J I_y}}} / \sqrt{C_b} \quad \text{CRC(4.7)}$$

where:

$$K_t = 4/3$$

L = distance from tip to section in question.

$$C_b = 1.75 + 1.05 \left(\frac{M_{x\min}}{M_{x\max}} \right) + 0.3 \left(\frac{M_{x\min}}{M_{x\max}} \right)^2; \quad 1.0 \leq C_b \leq 1.3$$

CRC p. 101

$M_{x\min}$ = the moment at the tip

$M_{x\max}$ = the moment at the section in question at L

NOTE: Clockwise moments are positive. Counterclockwise moments are negative. See Appendix C for further discussion.

2. Compute first check on allowable compressive stresses:

i. If $(KL/r)_{\text{equiv.}} \leq \sqrt{\frac{102\,000}{F_y}}$

$$F_{bx} = 0.6F_y$$

$$F_{by} = 0.6F_y$$

$$\text{If } \sqrt{\frac{102\,000}{F_y}} < (KL/r)_{\text{equiv.}} \leq \sqrt{\frac{510\,000}{F_y}}$$

$$F_{bx} = F_y \left(\frac{2}{3} - \frac{5.1 K_t L S_x F_y}{1\,530\,000 C_b \sqrt{J I_y}} \right) \leq 0.6 F_y \quad \text{AISC(1.5-6a)}$$

ii.

$$F_{by} = 0.6F_y$$

iii. If $\left(\frac{KL}{r} \right)_{\text{equiv.}} > \sqrt{\frac{510\,000}{F_y}}$

$$F_{bx} = 170\,000 / (KL/r)^2_{\text{equiv.}} \quad \text{AISC (1.5-6b)}$$

$$F_{by} = 0.6F_y$$

If there are unstiffened elements on the section that result in a value for Q_s less than one, then F_b shall be the smaller value $0.6F_y Q_s$ or that provided by 8.2.2.b.2.i, 8.2.2.b.2.ii, and 8.2.2.b.2.iii multiplied by Q_s , whichever is applicable.

c. Determine if a further reduction in F_{bx} is required.

1. If web $(h/t) > 760/\sqrt{F_{bx}}$, then:

$$F_{bx} = F_{bx} \left[1.0 - 0.0005 \frac{A_w}{A_f} \left(\frac{h}{t} - \frac{760}{\sqrt{F_{bx}}} \right) \right] \quad \text{AISC (1.10-5)}$$

If web $(h/t) > 760 \sqrt{5.4}/\sqrt{F_{bx}}$ and horizontal stiffeners are used and placed at 0.4 the distance between the compression flange and the neutral axis as measured from the compression flange (see Appendix G), then:

$$F_{bx} = F_{bx} \left[1.0 - 0.0005 \frac{A_w}{A_f} \left(\frac{h}{t} - \frac{760 \sqrt{5.4}}{\sqrt{F_{bx}}} \right) \right]$$

2. And if the section is a hybrid, F_{bx} in either flange shall not exceed the previous or:

$$F_{bx} = F_{bx} \left(\frac{12 + (A_w/A_f)(3a - a^3)}{12 + 2A_w/A_f} \right) \quad \text{AISC (1.10-6)}$$

where:

$a = F_y$ of web/ F_y of flange.

8.3 Step 3 - Solution to the Interaction Equation(s) for the Compressive Stresses

NOTE: The actual stresses f_{bx} and f_{by} are based on the effective section properties, if applicable, S_{xe} and S_{ye} . The axial stress f_a is based on the total area (A) of the section. Also, the f_a term may be positive for some sections. See 8.2.2.a.

a. If $f_a/F_a \leq 0.15$, then compute:

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \quad \text{AISC (1.6-2)}$$

b. If $f_a/F_a > 0.15$, then compute both 1 and 2:

$$1. \quad \frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \quad \text{AISC (1.6-1b)}$$

$$2. \quad \frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F_{ey}}\right) F_{by}} \leq 1.0 \quad \text{AISC (1.6-1a)}$$

where:

$$C_{mx} = C_{my} = 0.85$$

$$F_e = \frac{12 \pi^2 E}{23 \left(\frac{KL_b}{r_b} \right)^2}$$

Do for both x and y axes using their corresponding r_b .

8.4 Step 4 - Calculation of the Actual and Allowable Shear Stress in the Webs

8.4.1 To Determine the Actual Stresses

$$f_a = V_y/2bt + T/2A_m t$$

where:

bt = the area of one web

A_m = the area based on the mean dimensions of the section (see Appendix E)

8.4.2 To Determine the Allowable Shear Stress F_v

a. No stiffeners on the web plates if:

1. $h/t < 260$ and/or:

$$h/t \leq \frac{14\,000}{\sqrt{F_y(F_y + 16.5)}} \quad \text{AISC (1.10.2)}$$

where:

F_y = material yield of the compression flange

2. $k = 5.34$

$$C_v = \frac{45\,000k}{F_y(h/t)^2}$$

- 3.

$$\text{If } C_v > 0.8, \text{ then } C_v = \frac{190}{(h/t)\sqrt{F_y}} \quad \text{AISC (1.10-1)}$$

$$F_v = \frac{F_y C_v}{2.89} \leq 0.4 F_y \quad \text{AISC (1.10-1)}$$

- 4.

5. $f_s \leq F_v$

b. If 8.4.2.a is not met, then proceed accordingly; stiffeners are required.

1. If $a/h \leq 1.5$, then h/t may be as much as $\frac{2000}{\sqrt{F_y}}$. If $a/h < 1.0$, then h/t may be as much as $\frac{2500}{\sqrt{F_y}}$. Where F_y is the same as before, otherwise h/t is limited to $\frac{14\,000}{\sqrt{F_y(F_y + 16.5)}}$; in any case $a/h \leq \left(\frac{260}{h/t}\right)^2$ to a maximum of three.

If $a/h < 1.0$; $k = 4 + 5.34 / (a/h)^2$ AISC (1.10.5.2)

2. If $a/h > 1.0$ (up to 3); $k = 5.34 + 4 / (a/h)^2$

$C_v = \frac{45\,000k}{F_y(h/t)^2}$; however, if $C_v > 0.8$, then

$$C_v = \frac{190}{h/t} \sqrt{\frac{k}{F_y}}$$

3.

4. If $C_v \leq 1.0$, then:

$$F_v = \frac{F_y}{2.89} \left[C_v + \frac{1 - C_v}{1.15 \sqrt{1 + (a/h)^2}} \right] \leq 0.4 F_y \quad \text{AISC (1.10-2)}$$

5. If $C_v > 1.0$, then:

$$F_v = \frac{F_y C_v}{2.89} \leq 0.4 F_y$$

6. $f_s \leq F_v$

c. Stiffener properties:

1. Required cross-sectional area:

$$A_{st} = \frac{1 - C_v}{2} \left(\frac{a}{h} - \frac{(a/h)^2}{\sqrt{1 + (a/h)^2}} \right) Y \cdot D \cdot h \cdot t \quad \text{AISC (1.10-3)}$$

where:

$Y = F_y$ of web/ F_y of stiffener (see Appendix G)

$D = 1.0$ for stiffeners in pairs

$D = 1.8$ for single angle stiffeners

$D = 2.4$ for single plate stiffeners

NOTE: If F_v was computed from IIB-4 previously, then:

$$A_{st} = A_{st} \frac{f_s}{F_v}$$

2. Required moment of inertia with reference to an axis in the plane of the web:

$$I_{st} \geq \left(\frac{h}{50}\right)^4 \quad \text{AISC (1.10.5.4)}$$

3. Required weld to connect stiffener to the web:

$$f_{vs} \geq h \sqrt{\left(\frac{F_y}{340}\right)^3} \quad \text{AISC (1.10-4)}$$

where:

F_y = material yield of web

f_{vs} = kips per lineal inch

NOTE: If F_v was computed from IIB-4 previously, then:

$$f_{vs} = f_{vs} \frac{f_s}{F_v}$$

8.5 Step 5 - Calculation for Tensile Stresses

8.5.1 Actual Stress Without Stiffeners

$$-f_a + f_{bx} + f_{by} \leq 0.6 F_y$$

NOTE: If the section is a hybrid, then the limit is F'_{bx} from 8.2.2.c.1 and/or 2. On some sections f_a may be positive.

- 8.5.2 If stiffeners are used and if F_v came from 8.4.2.b.4, then the bending tensile stress is limited accordingly.

$$f_{bx} + f_{by} \leq \left(0.825 - 0.375 \frac{f_s}{F_v}\right) \leq 0.6 F_y \quad \text{AISC (1.10-7)}$$

- 8.5.3 If the flanges and webs are of ASTM A514 steel and stiffeners are used, and if $f_{bx} + f_{by} > 0.75 F_{bx}$ or F_{bx} , if applicable, then f_s shall not exceed F_v as computed from 8.4.2.b.5.

9. NOTES

9.1 Revision Indicator

A change bar (I) located in the left margin is for the convenience of the user in locating areas where technical revisions, not editorial changes, have been made to the previous issue of this document. An (R) symbol to the left of the document title indicates a complete revision of the document, including technical revisions. Change bars and (R) are not used in original publications, nor in documents that contain editorial changes only.

APPENDIX A

Critical flexural stress due to lateral-torsional buckling for the compression flange.

$$\left(\frac{KL}{r}\right)_{\text{equiv}}^2 = \frac{5.1 K_t L S_x}{\sqrt{J I_y}} / C_b \quad (\text{Eq. A1})$$

(Section 1.5.1.4.4 Commentary on AISC.)

where:

L = distance from tip to section in question

S_x = major axis section modulus to the compressive flange

I_y = minor axis moment of inertia

J = torsional constant of the beam cross section

$K_t = 4/3$ cantilevered section with no end restraint

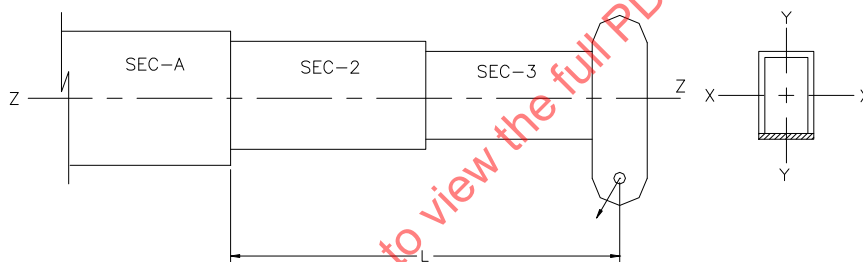


Figure A1

$$F_{bx} = F_y \left(\frac{2}{3} - \frac{5.1 K_t L S_x F_y}{1\,530\,000 C_b \sqrt{J I_y}} \right) \leq 0.6 F_y$$

substitute $(KL/r)_{\text{equiv}}$ for (L/r) to obtain expression for F_{bx} .

(See Appendix C for value of C_b .)

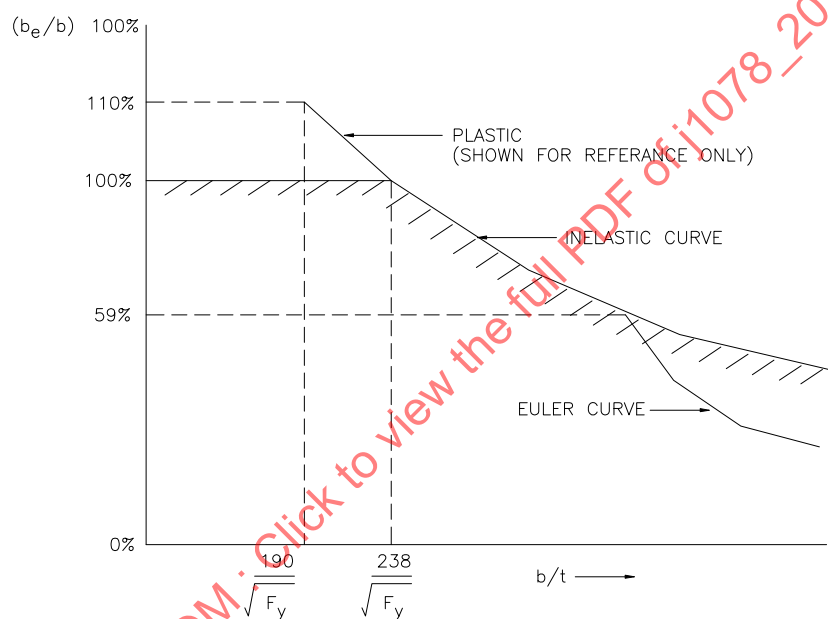
APPENDIX B

Relationship between relative effectiveness and width-thickness ratio for a stiffened compression element based on $0.60 F_y$ compressive stress level, uniformly distributed. (Safety factor = 1.67 for $k_c = 4.0$.)

$$b/t = 1.9 \sqrt{\frac{0.6 E}{f_{\max}}} \left(1 - (0.91) \frac{(0.415)}{w/t} \sqrt{\frac{0.6 E}{f_{\max}}} \right) \quad (\text{Eq. B1})$$

where:

$$f_{\max} \leq 0.6 F_y$$



b_e – EFFECTIVE WIDTH OF COMPRESSIN ELEMENT
 b – ACTURAL WIDTH OF COMPRSSION ELEMENT
 t – THICKNESS OF COMPRESSION ELEMENT

Figure B1

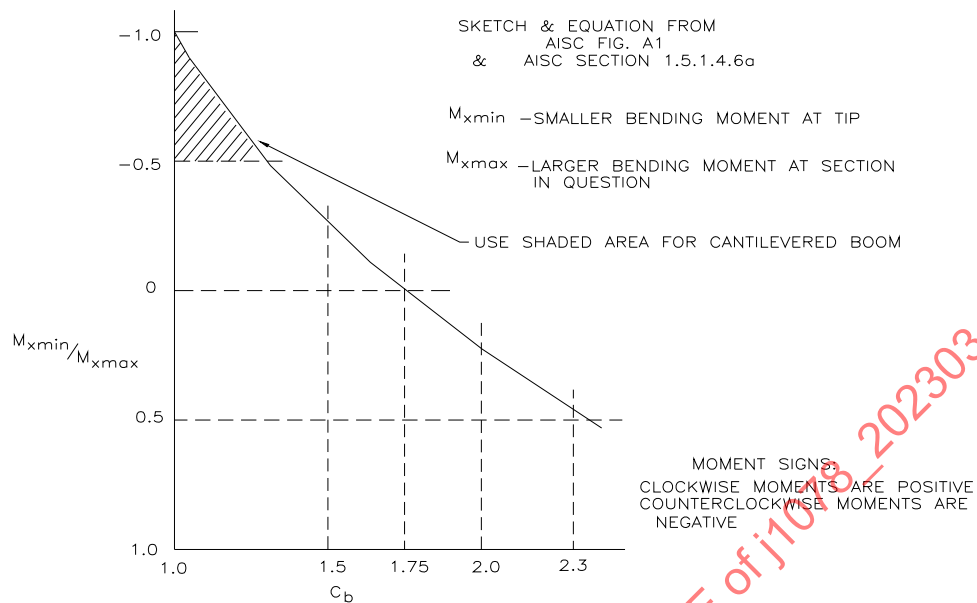
APPENDIX C - BENDING COEFFICIENT C_B 

Figure C1

Equation:

$$C_b = 1.75 + 1.05 \left(\frac{M_{x\min}}{M_{x\max}} \right) + 0.3 \left(\frac{M_{x\min}}{M_{x\max}} \right)^2$$

(Eq. C1)

where:

$$1 \leq C_b \leq 1.3$$

(CRC p. 101.)

(Other reference: AISI Commentary - Flexural members chapter, subject - lateral buckling.)

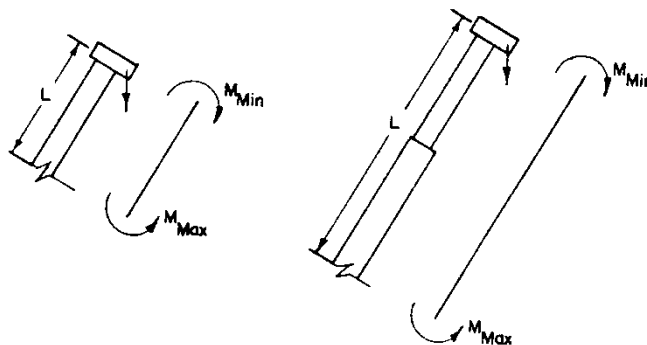


Figure C2

APPENDIX D - TORSIONAL CONSTANT J FOR CLOSED RECTANGULAR SECTIONS

General equation:

$$J = \frac{2(A_o + A_i)A_m}{\frac{2h_m}{t_w} + \frac{b_m}{t_1} + \frac{b_m}{t_2}} \quad (\text{Eq. D1})$$

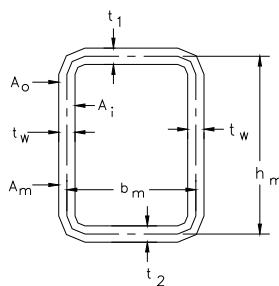


Figure D1

Case I - For thin wall sections:

Top and bottom flanges are the same thickness.

$$J = \frac{2t_1 t_w b_m^2 h_m^2}{b_m t_w + h_m t_1} \quad (\text{Eq. D2})$$

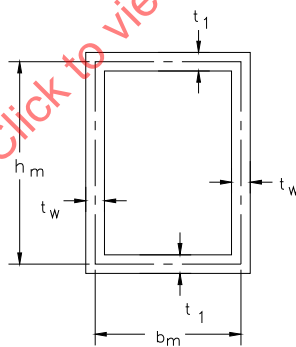


Figure D2

Case II - For thin wall sections:

Top and bottom flanges are not the same thickness.

$$J = \frac{4b_m^2 h_m^2}{\left(\frac{2h_m}{t_w} + \frac{b_m}{t_1} + \frac{b_m}{t_2}\right)} \quad (\text{Eq. D3})$$

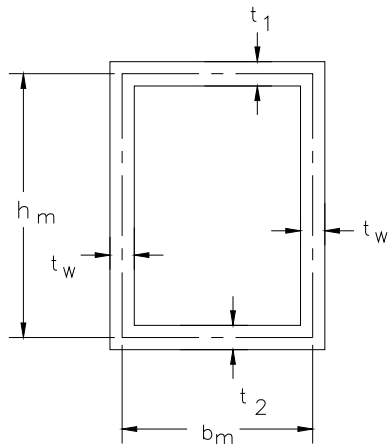


Figure D3

$$A_m = b_m \times n_m$$

$$A_i = b_f \times h$$

$$A_o = b_w \times d$$

(Eq. D4)

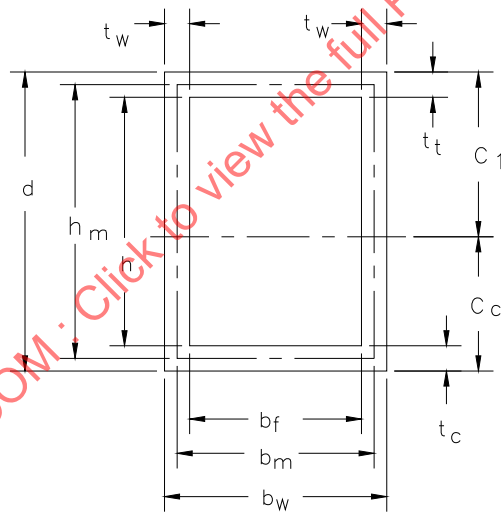
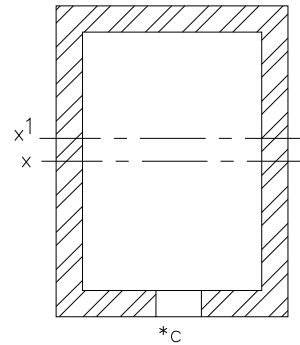


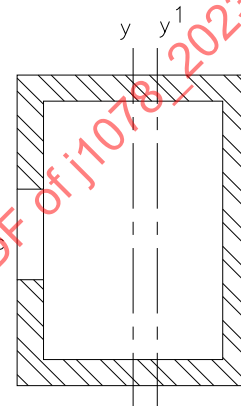
Figure D4

APPENDIX E - EFFECTIVE WIDTH OF CROSS SECTION

S_{xe} : BASED ON EFFECTIVE AREA



S_{ye} : BASED ON EFFECTIVE AREA. (An EQUAL AREA MAY BE CONSERVATIVELY REMOVED FROM THE TENSILE SIDE TO MAKE THE SECTION SYMMETRICAL AND THUS FACILITATE THE SECTION MODULUS CALCULATION)



NOTE: *c INDICATES COMPRESSION SIDE

Figure E1