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|  | <b>SURFACE<br/>VEHICLE<br/>RECOMMENDED<br/>PRACTICE</b> | <b>SAE J670 JAN2008</b>           |
|                                                                                  |                                                         | Issued 1952-07<br>Revised 2008-01 |
|                                                                                  |                                                         | Superseding J670e JUL1976         |
| (R) Vehicle Dynamics Terminology                                                 |                                                         |                                   |

## RATIONALE

SAE J670 was last updated over 30 years ago. Since the last revision, the field of vehicle dynamics has changed significantly. New systems such as four-wheel steering and active control have been applied to enhance the performance of vehicles. The terminology for vehicle dynamics needed to be updated to accommodate these new technologies and to make the definitions consistent with current usage in the field. Accordingly, many new terms have been added to the terminology to provide formal definitions for terms that are associated with these new technologies. A number of existing definitions, which were based on front-wheel steer vehicles with passive control, were also revised to accommodate new technologies.

In addition, new SAE and ISO standards have been published since the last revision of SAE J670 that directly relate to topics considered in SAE J670. The content of these new standards also indicated the need to revise SAE J670.

Specifically, in 1987, SAE published J1594, containing aerodynamics terminology previously appearing in SAE J670e. The aerodynamics section of SAE J670e is not included in the revised document, because those terms are now defined in SAE J1594.

In 1991, the International Organization for Standardization (ISO) published a vehicle dynamics vocabulary, ISO 8855. SAE J670e and ISO 8855 are incompatible in several aspects, the most notable being the axis systems defined in the two documents. SAE J670e utilizes an axis system based on aeronautical practice, with positive X forward, positive Y to the right, and positive Z down. ISO 8855 utilizes an axis system with positive X forward, positive Y to the left, and positive Z up. The revised SAE J670 embraces both of these axis orientations. The revised SAE J670 additionally addresses technical shortcomings found in both SAE J670e and ISO 8855 and is a harmonized superset of the two documents.

In 1998, SAE published J2047, containing definitions for tire performance terms that were previously defined in SAE J670e. The revised SAE J670 utilizes many definitions excerpted from SAE J2047, although some of these definitions are revised to enhance their applicability to vehicle dynamics.

Several of the sections of SAE J670e dealing with vibration terminology are not included in the new SAE J670, as the terms that were defined in these sections are commonly defined in engineering textbooks and the definitions are not specific to vehicle dynamics.

Finally, the terminology is extended to include definitions for many suspension and steering components, to enhance communication among vehicle dynamics professionals.

This seventh edition of SAE J670 replaces the preceding edition (SAE J670e) in its entirety.

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## FOREWORD

This terminology is intended to be sufficient to allow meaningful communication between vehicle dynamics professionals who need to describe the static and dynamic characteristics of automobiles, light trucks, and trailers. The terminology is to be used for presenting results and findings concerning the longitudinal, lateral, vertical and rotational dynamic performance of the applicable vehicles as predicted by analyses and simulations or as measured in tests and operational situations.

The purpose of this terminology is not to cover all terms in the manner of a dictionary or terms found in a basic engineering textbook. Rather, this document contains compatible definitions that have been carefully selected to aid in conveying ideas with rigor and accuracy in a readily understood and generally acceptable manner.

The terminology draws upon terms and definitions previously appearing in SAE J670e and ISO 8855 and further develops those that are applicable to the subjects covered by this document. For application to the study of vehicle dynamics, the tire terminology in this document contains pertinent definitions extracted from or based upon SAE J2047. Definitions for many chassis components are also provided.

Many of the definitions in the document contain terms appearing in italic font. Definitions for terms in italic may be found elsewhere in the document.

## INTRODUCTION

ISO 8855:1991 is the international standard corresponding to SAE J670. The scopes of these two standards are different. The scope of SAE J670 is limited to passenger cars and light trucks with two axles, plus those vehicles in combination with single-axle trailers. The scope of ISO 8855 additionally includes heavy commercial vehicles, with multiple axles and multiple units. SAE J670 recognizes axis systems with both Z-Up and Z-Down orientations, while ISO 8855 only recognizes the Z-Up orientation. SAE J670 defines five axis systems: Earth, intermediate, vehicle, tire, and wheel, while ISO 8855 defines four axis systems: Earth, intermediate, vehicle, and wheel (equivalent to the SAE J670 tire system). SAE J670 accommodates four-wheel steering, while ISO 8855 does not. SAE J670 accommodates an inclined road surface that is non-uniform, while ISO 8855 is limited to application on a flat, horizontal road surface. SAE J670 includes definitions for many suspension and steering components, while component definitions are not included in ISO 8855. SAE J670 also defines many more terms (over 600) than are defined in ISO 8855 (approximately 130).

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## 1. SCOPE

The vehicle dynamics terminology presented herein pertains to passenger cars and light trucks with two axles and to those vehicles pulling single-axle trailers. The terminology presents symbols and definitions covering the following subjects: axis systems, vehicle bodies, suspension and steering systems, brakes, tires and wheels, operating states and modes, control and disturbance inputs, vehicle responses, and vehicle characterizing descriptors. The scope does not include terms relating to the human perception of vehicle response.

## 2. REFERENCES

### 2.1 Applicable Publications

The following publications form a part of this specification to the extent specified herein. Unless otherwise specified, the latest issue of SAE publications shall apply.

#### 2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or 724-776-4970 (outside USA), [www.sae.org](http://www.sae.org).

SAE J1594 Vehicle Aerodynamics Terminology

SAE J2047 Tire Performance Technology

SAE J2564 Automotive Stability Enhancement Systems

#### 2.1.2 ISO Publications

Available from ANSI, 25 West 43rd Street, New York, NY 10036, Tel: 212-642-4900, [www.ansi.org](http://www.ansi.org).

ISO 8855:1991 Road vehicles—Vehicle dynamics and road-holding ability—Vocabulary

### 2.2 Related Publications

#### 2.2.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or 724-776-4970 (outside USA), [www.sae.org](http://www.sae.org).

SAE J1451 A Dictionary of Terms for the Dynamics and Handling of Single Track Vehicles (Motorcycles, Mopeds, and Bicycles)

SAE J1982 Nomenclature—Wheels for Passenger Cars, Light Trucks, and Multipurpose Vehicles

SAE M-105 SAE Glossary of Automotive Terms—Second Edition

SAE R-159 Dictionary of Automotive Engineering—Second Edition

### 2.2.2 ISO Publications

Available from ANSI, 25 West 43rd Street, New York, NY 10036, Tel: 212-642-4900, [www.ansi.org](http://www.ansi.org).

|           |                                                                                      |
|-----------|--------------------------------------------------------------------------------------|
| ISO 611   | Road vehicles—Braking of automotive vehicles and their trailers—Vocabulary           |
| ISO 612   | Road vehicles—Dimensions of motor vehicles and towed vehicles—Terms and definitions  |
| ISO 1176  | Road vehicles—Masses—Vocabulary and codes                                            |
| ISO 3833  | Road vehicles—Types—Terms and definitions                                            |
| ISO 3877  | Tyres, valves and tubes—List of equivalent terms—Part 1: Tyres                       |
| ISO 3911  | Wheels and rims for pneumatic tyres—Vocabulary, designation and marking              |
| ISO 4223  | Definitions of some terms used in the tyre industry—Part 1: Pneumatic tyres          |
| ISO 6725  | Road vehicles—Dimensions of two-wheeled mopeds and motorcycles—Terms and definitions |
| ISO 6726  | Mopeds and motorcycles with two wheels—Masses—Vocabulary                             |
| ISO 7237  | Caravans—Masses and dimensions—Vocabulary                                            |
| ISO 11838 | Motorcycle and motorcycle-rider kinematics—Vocabulary                                |

### 2.2.3 Other Publications

Bosch Automotive Handbook

Bosch Automotive Terminology

Tire and Rim Association Year Book

### 3. AXIS AND COORDINATE SYSTEMS

This terminology recognizes axis systems with two existing nominal orientations: the orientation originally defined by SAE J670, which was adopted from aeronautical convention, and the orientation defined in ISO 8855:1991. These axis orientations are equally acceptable, and the selection of the appropriate orientation should be based on the requirements of the analysis or test being performed. The ISO 8855 axis orientation is referred to as the Z-Up orientation, with the traditional SAE J670 axis orientation referred to as the Z-Down orientation. These two axis orientations are shown in Figure 1 below. When definitions are dependent upon the axis orientation, dual definitions are provided, with the definition based on the Z-Up axis orientation appearing first. Many of the terms in this section are also defined in SAE J2047.

The definitions in this section are intended to provide sufficient flexibility to accommodate the broad spectrum of modeling techniques that are currently used to represent the dynamics of vehicles as systems of multiple rigid bodies. Certain commonly used *axis systems* and *coordinate systems* are defined herein. However, there is no intent to limit an analyst or experimentalist to the use of only the *axis systems*, *coordinate systems*, or *reference frames* specifically defined.

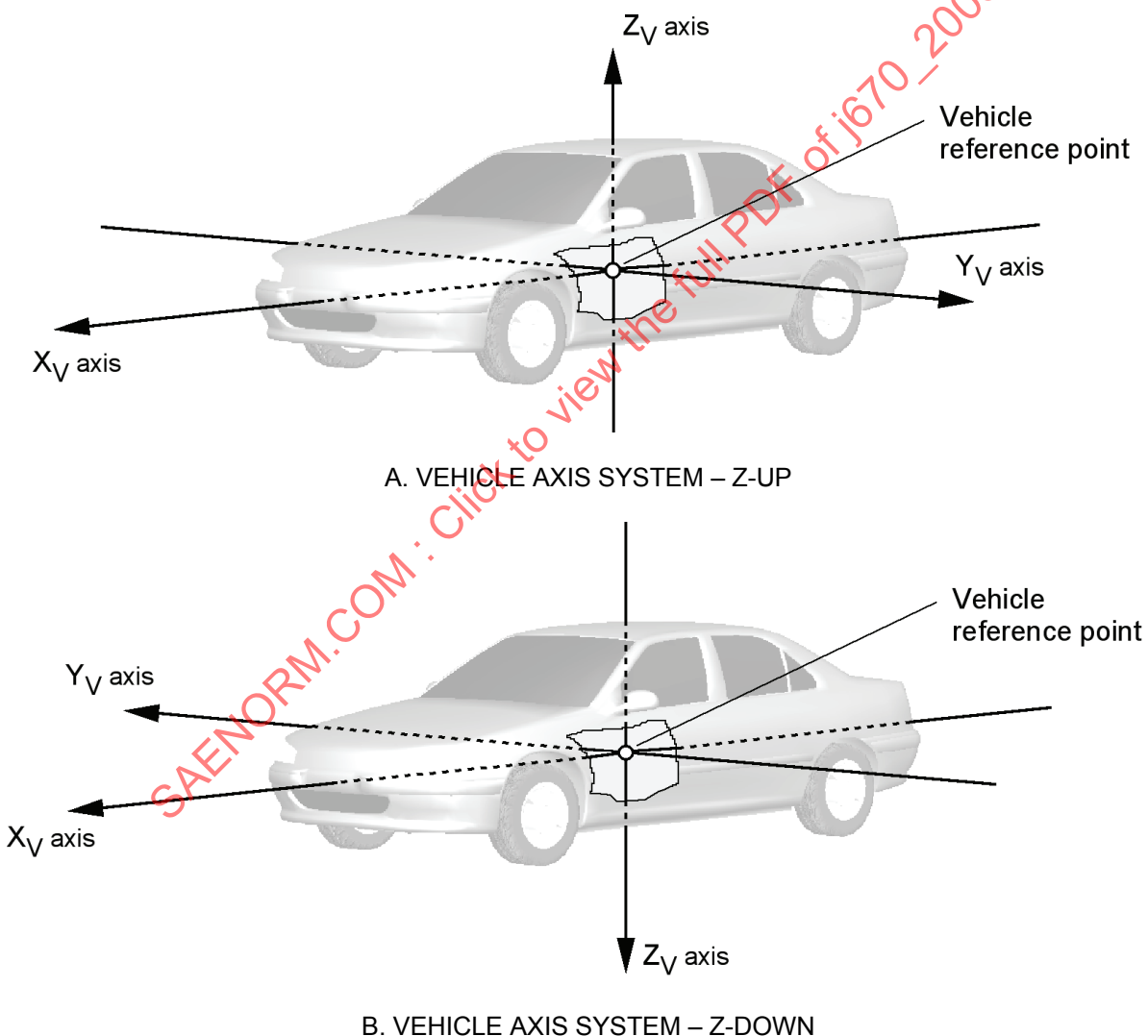


FIGURE 1 – ORIENTATIONS OF AXIS SYSTEMS

- 3.1 Reference Frame—A geometric environment in which all points remain fixed with respect to each other at all times.
- 3.2 Inertial Reference (Newtonian Reference)—A *reference frame* that is assumed to have zero linear and angular acceleration and zero angular velocity. In Newtonian physics, the Earth is assumed to be an inertial reference.
- 3.3 Axis System—A set of three orthogonal directions associated with X, Y, and Z axes. A right-handed axis system is assumed throughout this document, where:

$$\vec{Z} = \vec{X} \times \vec{Y} \quad (\text{Eq. 1})$$

- 3.4 Coordinate System—A numbering convention used to assign a unique ordered trio (x, y, z) of values to each point in a *reference frame*. A coordinate system consists of an *axis system* plus an origin point.
- 3.5 Ground Plane—A horizontal plane in the *inertial reference*, normal to the gravitational vector.
- 3.6 Road Surface—The surface, flat, curved, undulated, or other shape, supporting the *tire* and providing friction necessary to generate *tire shear forces* in the *road plane*.
- 3.7 Road Plane—1) A plane representing the *road surface* within each *tire contact patch*. For an uneven road, a different road plane may exist at each *tire contact patch*. 2) The plane formed when the individual road planes at each *tire contact patch* are essentially coplanar.

NOTE—For a planar *road surface*, the *road plane* will be coincident with the *road surface*. For *road surfaces* with surface contours having a wavelength similar to or less than the size of the *tire contact patch*, as in the case of many ride events, an equivalent *road plane* must be determined. Determination of the equivalent *road plane* is dependent on the requirements of the analysis being performed. The equivalent *road plane* may not be coincident with the actual *road surface* at the *tire contact center*.

- 3.8 Vehicle Plane of Symmetry (Longitudinal Plane of Symmetry)—The median plane of a vehicle that is generally laterally symmetric. This plane typically contains the lateral midpoints of the *axles* and the *hitch point*.
- 3.9 Earth-Fixed Axis System ( $X_E$ ,  $Y_E$ ,  $Z_E$ )—An *axis system* fixed in the *inertial reference*. The  $X_E$  and  $Y_E$  axes are parallel to the *ground plane*. The  $Z_E$  axis is aligned with the gravitational vector. The positive  $Z_E$  axis points upward in the Z-Up orientation. The positive  $Z_E$  axis points downward in the Z-Down orientation. The orientation of the  $X_E$  and  $Y_E$  axes is arbitrary and should be based on the needs of the analysis or test.
- 3.10 Earth-Fixed Coordinate System ( $x_E$ ,  $y_E$ ,  $z_E$ )—A *coordinate system* based on the *Earth-fixed axis system* with an origin that is fixed in the *ground plane*. The location of the origin is generally an arbitrary point defined by the user.
- 3.11 Vehicle Axis System ( $X_V$ ,  $Y_V$ ,  $Z_V$ )—An *axis system* fixed in the *reference frame* of the vehicle *sprung mass*, so that the  $X_V$  axis is substantially horizontal and points forward (with the vehicle at rest), and is parallel to the *vehicle plane of symmetry*. The  $Y_V$  axis is perpendicular to the *vehicle plane of symmetry*. The  $Y_V$  axis points to the left and the  $Z_V$  axis points upward in the Z-Up orientation. The  $Y_V$  axis points to the right and the  $Z_V$  axis points downward in the Z-Down orientation. For articulated vehicles, a separate vehicle axis system may be defined for each unit. See Figure 1.
- 3.12 Vehicle Coordinate System ( $x_V$ ,  $y_V$ ,  $z_V$ )—A *coordinate system* based on the *vehicle axis system* with the origin located at the *vehicle reference point*.
- 3.13 Vehicle Reference Point—A point fixed in the vehicle *sprung mass*. See Figure 1.

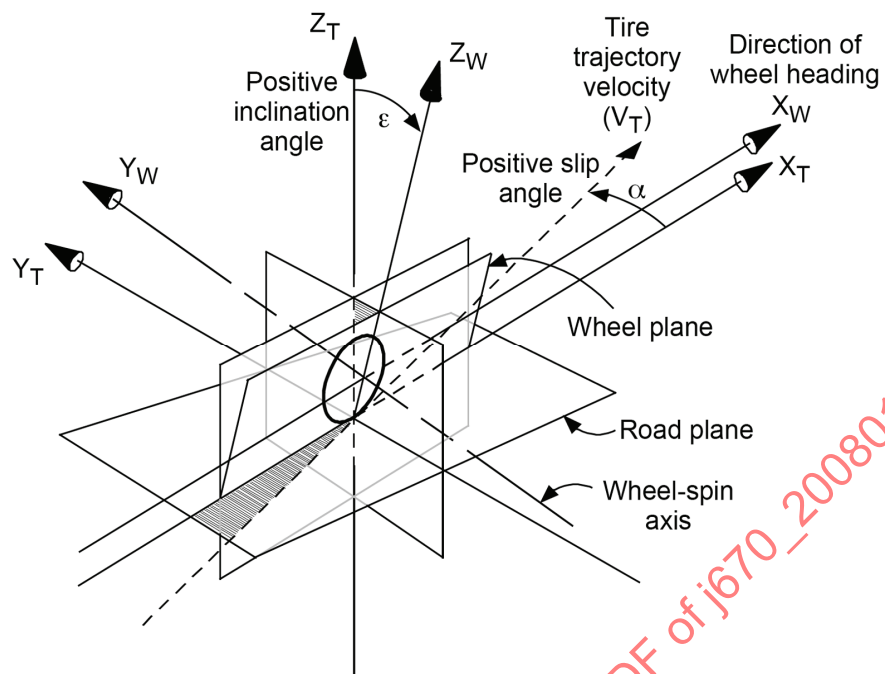
NOTE—The *vehicle reference point* may be defined in a variety of locations, based on the needs of the analysis or test. Commonly used locations include the static *vehicle center of gravity*, the static *sprung-mass center of gravity*, the mid-wheelbase point at the height of the static *vehicle center of gravity*, and the center of the front *axle*.

- 3.14 Intermediate Axis System ( $X, Y, Z$ )—A right-handed orthogonal *axis system* whose  $X$  and  $Y$  axes are parallel to the *ground plane*, with the  $X$  axis aligned with the vertical projection of the  $X_V$  axis onto the *ground plane*. The  $Z$  axis is parallel to the  $Z_E$  axis. The positive  $Z$  axis points upward in the Z-Up orientation. The positive  $Z$  axis points downward in the Z-Down orientation.

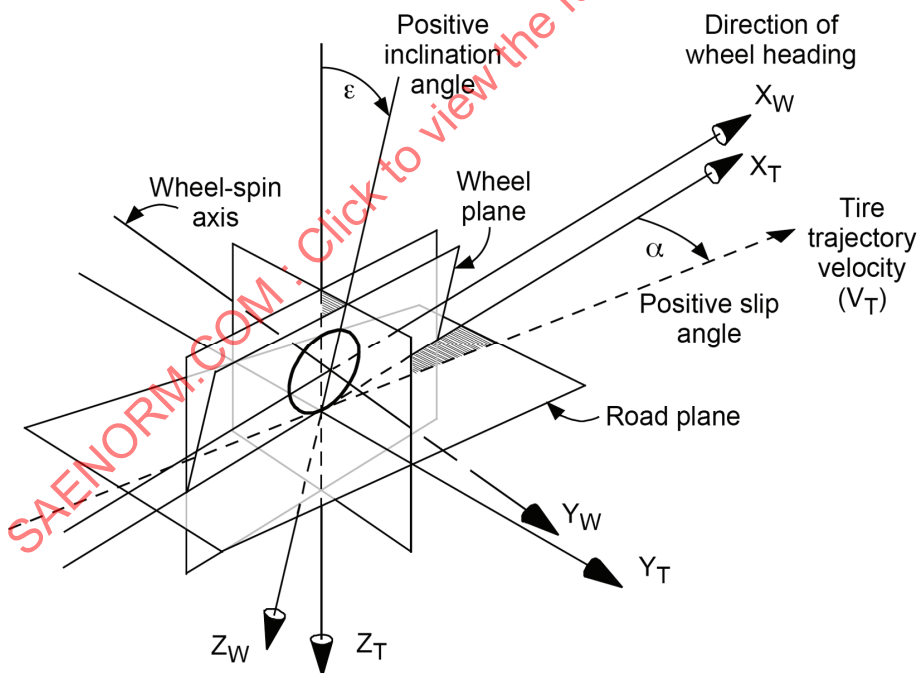
NOTE—The *intermediate axis system* is used to facilitate angular rotations using the *vehicle Euler angles* and the definition of angular orientation terms, the components of force and moment vectors, and the components of translational and angular motion vectors. See 4.2.2.1, 4.3, 4.2.1, 4.2.2.2, and 4.2.2.3, respectively. An *intermediate coordinate system* is not defined herein.

- 3.15 Tire Axis System ( $X_T, Y_T, Z_T$ )—An *axis system* whose  $X_T$  and  $Y_T$  axes are parallel to the local *road plane*, with the  $Z_T$  axis normal to the local *road plane*. The orientation of the  $X_T$  axis is defined by the intersection of the *wheel plane* and the *road plane*. The positive  $Z_T$  axis points upward in the Z-Up orientation. The positive  $Z_T$  axis points downward in the Z-Down orientation. A local tire axis system may be defined at each *wheel*. See Figure 2.

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A. TIRE AND WHEEL AXIS SYSTEMS – Z-UP



B. TIRE AND WHEEL AXIS SYSTEMS – Z-DOWN

FIGURE 2 – TIRE AND WHEEL AXIS SYSTEMS

- 3.16 Tire Coordinate System ( $x_T, y_T, z_T$ )—A *coordinate system* based on the *tire axis system* with the origin fixed at the *contact center*.

NOTE—The Z-Down orientation of the *tire coordinate system* is equivalent to the orientation of the *tire axis system* defined in SAE J2047.

- 3.17 Wheel Axis System ( $X_W, Y_W, Z_W$ )—An *axis system* whose  $X_W$  and  $Z_W$  axes are parallel to the *wheel plane*, whose  $Y_W$  axis is parallel to the *wheel-spin axis*, and whose  $X_W$  axis is parallel to the local *road plane*. The positive  $Z_W$  axis points upward in the Z-Up orientation. The positive  $Z_W$  axis points downward in the Z-Down orientation. A local wheel axis system may be defined for each *wheel*. See Figure 2.

NOTE—This *axis system* definition is not consistent with the definition of wheel axis system found in ISO 8855:1991 or SAE J2047.

- 3.18 Wheel Coordinate System ( $x_W, y_W, z_W$ )—A *coordinate system* based on the *wheel axis system* with the origin fixed at the *wheel center*.

#### 4. VEHICLE

##### 4.1 Geometry and Masses

- 4.1.1 Wheelbase  $L$ —The distance between the *contact centers* of the *tires* on the same side of the vehicle, measured parallel to the X axis, with the vehicle at rest on a horizontal surface, at a prescribed *load condition*, set of *vehicle trim heights*, or set of *suspension trim heights*, with zero *steer angle*.

NOTE—A vehicle may have a different *wheelbase* on the left and right sides by design. It is common practice to average the left and right *wheelbases*; however, the difference may need to be taken into account in performing some analyses. The *wheelbase* typically changes as the *suspension trim heights* change.

- 4.1.2 Trailer Wheelbase—The distance between the vertical projections of the *articulation point* and the center of the trailer *axle* onto the *ground plane*, with the tow-vehicle/trailer combination at rest on a horizontal surface, at a prescribed *load condition*, set of *vehicle trim heights*, or set of *suspension trim heights*.

- 4.1.3 Track (Track Width, Wheel Track)  $T$ —The distance between the *contact centers* of a pair of *tires* on an *axle*, measured parallel to the Y axis, with the vehicle at rest on a horizontal surface, at a prescribed *load condition*, set of *vehicle trim heights*, or set of *suspension trim heights*. For vehicles with dual *tires*, it is the distance between the points centrally located between the *contact centers* of the inner and outer dual *tires*.

NOTE—For vehicles with *independent suspensions*, *track* typically changes as the *suspension trim heights* change.

- 4.1.4 Vehicle Trim Height—A vertical dimension that specifies the location of a fixed point on the vehicle body or chassis relative to ground.

NOTE—A set of congruous *vehicle trim heights* (minimum 3) are necessary to establish the orientation of the vehicle body or chassis relative to ground.

- 4.1.5 Suspension Trim Height—A vertical dimension that specifies the location of a point on a vehicle *suspension* relative to a point on the vehicle *sprung mass*.

- 4.1.6 Articulation Point—The instant center of rotation of the trailer relative to the towing vehicle.

NOTE—For trailers with a simple ball hitch, the *articulation point* is coincident with the center of the hitch ball.

- 4.1.7 Hitch Point(s)—The attachment point(s) of the trailer to the towing vehicle.

4.1.8 Unsprung Weight—All weight that is not carried by the *suspension*, but is supported directly by the *tires*. The unsprung weight includes the weight of the *tires* and *wheels* and all parts that move directly with the *tires* and *wheels*, plus a portion of the weight of the *suspension* linkages, *ride springs*, and driveshafts.

NOTE—It is common practice to include one-half the weight of the *suspension* linkage, *ride springs*, and driveshafts in the *unsprung weight*. For more detailed analyses, the weight distribution of the individual *suspension links* may need to be considered. *Unsprung weight* may also be separately defined for the front and rear *axles* and, further, for *independent suspensions*, for individual *suspension corners*. If *unsprung weight* is defined for subsets of the total unsprung weight, the location being referenced must be explicitly identified (e.g., Left Front Unsprung Weight).

4.1.9 Unsprung Mass—The *unsprung weight* divided by the gravitational constant.

NOTE—*Unsprung mass* may also be separately defined for the front and rear *axles* and, further, for *independent suspensions*, for individual *suspension corners*. If *unsprung mass* is defined for subsets of the total *unsprung mass*, the location being referenced must be explicitly identified (e.g., Left Front Unsprung Mass).

4.1.10 Effective Unsprung Mass—The equivalent mass which reproduces the kinetic energy or inertia forces produced by motion(s) of the unsprung parts while the *sprung mass* is stationary.

NOTE—*Effective unsprung mass* is not the same as the *unsprung mass*, in that it considers effects due to the rotational motion and rotational inertia of the *suspension links*, which usually decreases the apparent *unsprung mass*. This distinction may be important when performing certain dynamic analyses (e.g., ride). The value of *effective unsprung mass* may be different for different directions of excitation. Typically, *effective unsprung mass* is separately defined for the front and rear *axles* and, further, for *independent suspensions*, for individual *suspension corners*. The location being referenced must be explicitly identified (e.g., Left Front Effective Unsprung Mass).

4.1.11 Sprung Weight—All weight that is supported by the *suspension*, including portions of the weight of the *suspension* members. The sprung weight is the *vehicle operating weight* less the *unsprung weight*.

4.1.12 Sprung Mass—1) The *sprung weight* divided by the gravitational constant. 2) The portion of the vehicle supported by the *suspension*, frequently treated as a rigid body.

4.1.13 Base Vehicle Weight—The total weight of the vehicle, including all fluids necessary for normal operation, without fuel and without a *payload*.

4.1.14 Base Vehicle Mass—The *base vehicle weight* divided by the gravitational constant.

4.1.15 Curb Weight—The *base vehicle weight* plus a full tank of fuel.

4.1.16 Curb Mass—The *curb weight* of the vehicle divided by the gravitational constant.

4.1.17 Payload—The weight of the driver, passengers, and cargo at a given *load condition*.

4.1.18 Load Condition—A description of the loaded state of the vehicle being analyzed or tested. It includes descriptions of the fuel load and the *payload*, and the location(s) of the *payload*.

4.1.19 Vehicle Operating Weight *W*—The total weight of the vehicle at a given *load condition*, including the *base vehicle weight*, the weight of the fuel load, and the *payload*.

4.1.20 Vehicle Operating Mass—The *vehicle operating weight* divided by the gravitational constant.

4.1.21 Passenger Mass—The mass representative of a driver or passenger. The conventional mass of a passenger is fixed at 68 kg (150 lb).

4.1.22 Vehicle Center of Gravity (C.G.)—The center of gravity of the total vehicle at a given *load condition*.

NOTE—The symbols  $H$  or  $h$  are commonly used for the height of the *vehicle center of gravity* above ground, with the *suspensions* in equilibrium and the vehicle at rest on a flat, level surface. The symbols  $a$  and  $b$  are commonly used as the longitudinal distances from the front and rear *axle centerlines*, respectively, to the *vehicle center of gravity*.

4.1.23 Sprung-Mass Center of Gravity—The center of gravity of the vehicle *sprung mass* at a given *load condition*.

4.1.24 Mass Moments of Inertia (Total Vehicle)

4.1.24.1 Vehicle Roll Moment of Inertia—The moment of inertia of the total vehicle at a given *load condition*, taken about an axis parallel to the  $X_v$  axis, that passes through the *vehicle center of gravity*.

4.1.24.2 Vehicle Pitch Moment of Inertia—The moment of inertia of the total vehicle at a given *load condition*, taken about an axis parallel to the  $Y_v$  axis, that passes through the *vehicle center of gravity*.

4.1.24.3 Vehicle Yaw Moment of Inertia—The moment of inertia of the total vehicle at a given *load condition*, taken about an axis parallel to the  $Z_v$  axis, that passes through the *vehicle center of gravity*.

4.1.24.4 Vehicle Roll-Yaw Product of Inertia—The product of inertia of the total vehicle at a given *load condition*, taken about the axes used to determine the *vehicle roll moment of inertia* and the *vehicle yaw moment of inertia*.

4.1.25 Mass Moments of Inertia (Sprung Mass)

4.1.25.1 Sprung-Mass Roll Moment of Inertia—The moment of inertia of the vehicle *sprung mass*, taken about an axis parallel to the  $X_v$  axis that passes through the *sprung-mass center of gravity*.

4.1.25.2 Sprung-Mass Pitch Moment of Inertia—The moment of inertia of the vehicle *sprung mass*, taken about an axis parallel to the  $Y_v$  axis that passes through the *sprung-mass center of gravity*.

4.1.25.3 Sprung-Mass Yaw Moment of Inertia—The moment of inertia of the vehicle *sprung mass*, taken about an axis parallel to the  $Z_v$  axis that passes through the *sprung-mass center of gravity*.

4.1.25.4 Sprung-Mass Roll-Yaw Product of Inertia—The product of inertia of the vehicle *sprung mass*, taken about the axes used to determine the *sprung-mass roll moment of inertia* and the *sprung-mass yaw moment of inertia*.

4.1.26 Dynamic Index – Pitch ( $k_{yy}^2/ab$  Ratio – Pitch)—The square of the radius of gyration of the *sprung mass* about an axis parallel to the  $Y_v$  axis that passes through the *sprung-mass center of gravity*, divided by the product of the two longitudinal distances ( $a$  and  $b$ ) from the *sprung-mass center of gravity* to the front and rear *axle centerlines*.

4.1.27 Dynamic Index – Yaw ( $k_{zz}^2/ab$  Ratio – Yaw)—The square of the radius of gyration of the *vehicle operating mass* about an axis parallel to the  $Z_v$  axis that passes through the *vehicle center of gravity*, divided by the product of the two longitudinal distances ( $a$  and  $b$ ) from the *vehicle center of gravity* to the front and rear *axle centerlines*.

## 4.2 Kinematics

### 4.2.1 Translational Motion Variables

In the following paragraphs, velocity and acceleration are relative to the *Earth-fixed axis system* ( $X_E, Y_E, Z_E$ ). They are resolved into components in the *intermediate axis system* ( $X, Y, Z$ ).

NOTE—It is also possible to resolve velocity and acceleration vectors into components in other *axis systems*. For example, velocity and acceleration vectors may be resolved in the *vehicle axis system* ( $X_v, Y_v, Z_v$ ) to produce  $v_{X_v}$ ,  $v_{Y_v}$ ,  $v_{Z_v}$  and  $a_{X_v}$ ,  $a_{Y_v}$ ,  $a_{Z_v}$ . Traditionally,  $v_{X_v}$ ,  $v_{Y_v}$ ,  $v_{Z_v}$  have been represented by the symbols  $u$ ,  $v$ ,  $w$ .

## 4.2.1.1 Velocities

4.2.1.1.1 Vehicle Velocity  $\vec{v}$ —A vector quantity expressing the velocity of the *vehicle reference point*.

4.2.1.1.2 Vehicle Speed—The magnitude of the *vehicle velocity*. In many cases, this is approximated by the *longitudinal velocity*.

4.2.1.1.3 Longitudinal Velocity  $v_X$ —The scalar value of the component of *vehicle velocity* in the direction of the X axis.

$$v_X = \vec{v} \cdot \hat{X} \quad (\text{Eq. 2})$$

4.2.1.1.4 Lateral Velocity  $v_Y$ —The scalar value of the component of *vehicle velocity* in the direction of the Y axis.

$$v_Y = \vec{v} \cdot \hat{Y} \quad (\text{Eq. 3})$$

4.2.1.1.5 Vertical Velocity  $v_Z$ —The scalar value of the component of *vehicle velocity* in the direction of the Z axis.

$$v_Z = \vec{v} \cdot \hat{Z} \quad (\text{Eq. 4})$$

4.2.1.1.6 Horizontal Velocity Vector  $\vec{v}_H$ —The vector projection of *vehicle velocity* onto the X–Y plane.

$$\vec{v}_H = \vec{v} - (\vec{v} \cdot \hat{Z})\hat{Z} = v_X\hat{X} + v_Y\hat{Y} \quad (\text{Eq. 5})$$

4.2.1.1.7 Horizontal Velocity  $v_H$ —The magnitude of the *horizontal velocity vector*.

NOTE—*Horizontal velocity* is also the resultant of *longitudinal velocity* and *lateral velocity*.

## 4.2.1.2 Accelerations

4.2.1.2.1 Vehicle Acceleration  $\vec{a}$ —A vector quantity expressing the acceleration of the *vehicle reference point*.

4.2.1.2.2 Longitudinal Acceleration  $a_X$ —The scalar value of the component of *vehicle acceleration* in the direction of the X axis.

$$a_X = \vec{a} \cdot \hat{X} \quad (\text{Eq. 6})$$

4.2.1.2.3 Lateral Acceleration  $a_Y$ —The scalar value of the component of *vehicle acceleration* in the direction of the Y axis.

$$a_Y = \vec{a} \cdot \hat{Y} \quad (\text{Eq. 7})$$

NOTE—In *steady state*, *lateral acceleration* is equal to the product of *centripetal acceleration* times the cosine of the vehicle's *sideslip angle*. Since in most test conditions the *sideslip angle* is small, for practical purposes *lateral acceleration* may be considered equal to *centripetal acceleration*.

4.2.1.2.4 Vertical Acceleration  $a_Z$ —The scalar value of the component of *vehicle acceleration* in the direction of the Z axis.

$$a_Z = \vec{a} \cdot \hat{Z} \quad (\text{Eq. 8})$$

- 4.2.1.2.5 Tangential Acceleration  $a_T$ —The scalar value of the component of *vehicle acceleration* in the direction of the *horizontal velocity vector*.

$$a_T = \vec{a} \cdot \hat{v}_H \quad (\text{Eq. 9})$$

- 4.2.1.2.6 Centripetal Acceleration  $a_C$ —The scalar value of the component of *vehicle acceleration* in the direction of the horizontal normal to the *horizontal velocity vector*.

$$a_C = \vec{a} \cdot (\hat{Z} \times \hat{v}_H) \quad (\text{Eq. 10})$$

- 4.2.1.2.7 Horizontal Acceleration Vector  $\vec{a}_H$ —The vector projection of *vehicle acceleration* onto the X–Y plane.

$$\vec{a}_H = \vec{a} - (\vec{a} \cdot \hat{Z})\hat{Z} = a_X\hat{X} + a_Y\hat{Y} \quad (\text{Eq. 11})$$

- 4.2.1.2.8 Horizontal Acceleration  $a_H$ —The magnitude of the *horizontal acceleration vector*.

NOTE—*Horizontal acceleration* is also the resultant of *longitudinal acceleration* and *lateral acceleration* or the resultant of *tangential acceleration* and *centripetal acceleration*.

## 4.2.2 Angular Motion Variables

### 4.2.2.1 Angles

The sign of angles resulting from angular rotations is determined in accordance with the right-hand rule.

- 4.2.2.1.1 Vehicle Angular Orientation—The orientation of the *vehicle axis system* ( $X_V, Y_V, Z_V$ ) with respect to the *Earth-fixed axis system* ( $X_E, Y_E, Z_E$ ), for example, as obtained by a sequence of three angular rotations through the *vehicle Euler angles*.

- 4.2.2.1.2 Vehicle Euler Angles ( $\psi, \theta, \phi$ )—A sequence of consecutive angular rotations about the Earth-fixed  $Z_E$  axis, the intermediate Y axis, and the vehicle  $X_V$  axis to obtain the *vehicle angular orientation*. See Table 1.

**TABLE 1 – VEHICLE EULER ANGLES AND STANDARD ORDER OF ROTATION**

| Rotation Order  | Angle Produced by Rotation | Rotation Nature                               |
|-----------------|----------------------------|-----------------------------------------------|
| First Rotation  | Yaw ( $\psi$ )             | $X_E$ axis to the X axis about the $Z_E$ axis |
| Second Rotation | Pitch ( $\theta$ )         | X axis to the $X_V$ axis about the Y axis     |
| Third Rotation  | Roll ( $\phi$ )            | Y axis to the $Y_V$ axis about the $X_V$ axis |

NOTE—There are other possible sets of orientation angles that can be applied to axis rotations. In choosing a sequence of rotations, the second rotation should ideally span a small angle. The sequence of rotations specified above is preferable for use with the vehicle *sprung mass*, where the magnitude of the *pitch angle* is usually small. Other sets of orientation angles could be chosen for use with individual vehicle components (e.g., rotating *wheels*).

- 4.2.2.1.3 Yaw Angle (Heading Angle)  $\psi$ —The angle from the  $X_E$  axis to the X axis, about the  $Z_E$  axis. See Figure 3.

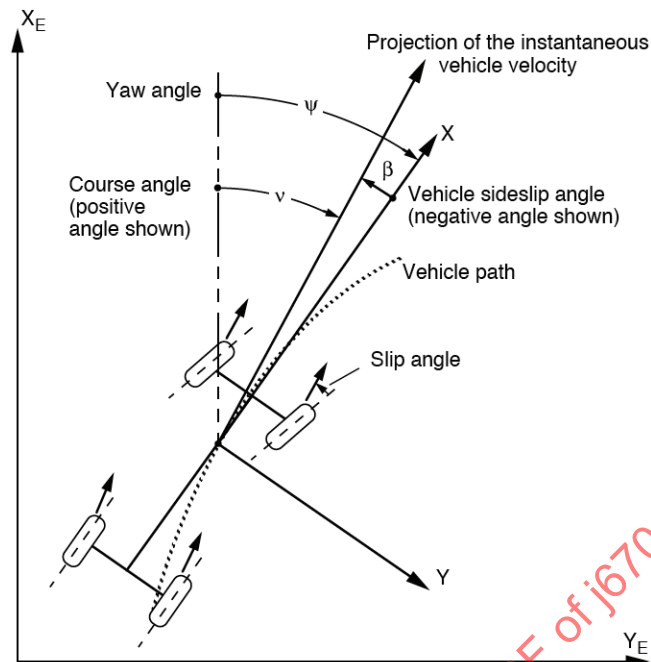


FIGURE 3 – YAW, SIDESLIP, AND COURSE ANGLES  
(Z-Down Axis Orientation)

4.2.2.1.4 Pitch Angle  $\theta$ —The angle from the X axis to the  $X_V$  axis, about the Y axis.

NOTE—The *pitch angle* is not measured relative to the *road plane*, thus a vehicle at rest on an inclined planar *road surface* will have a non-zero *pitch angle*.

4.2.2.1.5 Roll Angle  $\phi$ —The angle from the Y axis to the  $Y_V$  axis, about the  $X_V$  axis.

4.2.2.1.6 Vehicle Roll Angle  $\phi_V$ —The angle from the *ground plane* to the  $Y_V$  axis, about the X axis.

NOTE—The *vehicle roll angle* is different from *roll angle* (4.2.2.1.5) if the *pitch angle* is not zero. The *vehicle roll angle* may be computed using the equation:

$$\sin \phi_V = \sin \phi \cos \theta \quad (\text{Eq. 12})$$

The *vehicle roll angle* is not measured relative to the *road plane*; thus, a vehicle at rest on an inclined planar *road surface* will have a non-zero *vehicle roll angle*.

4.2.2.1.7 Sideslip Angle—At a given point, the angle from the X axis to the vertical projection of the velocity vector of the point onto the *ground plane*, about the Z axis.

4.2.2.1.8 Vehicle Sideslip Angle (Attitude Angle)  $\beta$ —The angle from the X axis to the vertical projection of *vehicle velocity* onto the *ground plane*, about the Z axis. See Figures 3 and 4.

NOTE—*Vehicle sideslip angle* may also be calculated from the *longitudinal velocity*  $v_x$  and the *lateral velocity*  $v_y$ :

$$\beta = \text{Arc tan } \frac{v_y}{v_x} \quad (\text{Eq. 13})$$

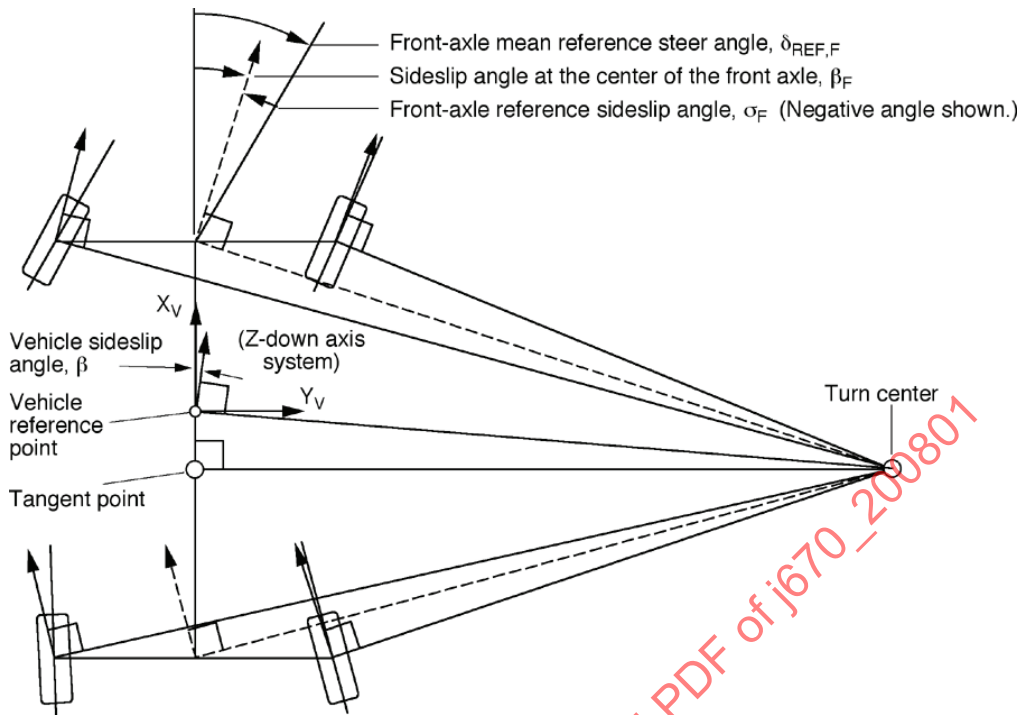


FIGURE 4 – VEHICLE SIDESLIP, REFERENCE STEER, AND AXLE REFERENCE SIDESLIP ANGLES  
(Z-Down axis orientation)

- 4.2.2.1.9 Tangent Point—The point on the X axis whose *sideslip angle* is zero. See Figure 4.
- 4.2.2.1.10 Axle Reference Sideslip Angle  $\sigma_F, \sigma_R$ —The *sideslip angle* at the center of a steered *axle* minus the *mean reference steer angle* for the *axle*. See Figure 4.
- 4.2.2.1.11 Trailer Yaw Articulation Angle  $\Gamma (\Delta\psi)$ —The angle from the trailer  $X_V$  axis to the normal projection of the tow vehicle  $X_V$  axis onto the trailer  $X_V$ – $Y_V$  plane, about the trailer  $Z_V$  axis. The polarity is determined in the trailer *axis system*.
- 4.2.2.1.12 Trailer Pitch Articulation Angle  $\Delta\theta$ —The angle from the trailer  $X_V$  axis to the normal projection of the tow vehicle  $X_V$  axis onto the trailer  $X_V$ – $Z_V$  plane, about the trailer  $Y_V$  axis. The polarity is determined in the trailer *axis system*.
- 4.2.2.1.13 Trailer Roll Articulation Angle  $\Delta\phi$ —The angle from the trailer  $Y_V$  axis to the normal projection of the tow vehicle  $Y_V$  axis onto the trailer  $Y_V$ – $Z_V$  plane, about the trailer  $X_V$  axis. The polarity is determined in the trailer *axis system*.

#### 4.2.2.2 Angular Velocities

In the following paragraphs, angular velocities are relative to the *Earth-fixed axis system* ( $X_E, Y_E, Z_E$ ). They are resolved into components in the *intermediate axis system* ( $X, Y, Z$ ).

NOTE—It is also possible to resolve angular velocity vectors into components in other *axis systems*. For example, angular velocity vectors may be resolved in the *vehicle axis system* ( $X_V, Y_V, Z_V$ ) to produce  $\omega_{X_V}, \omega_{Y_V}, \omega_{Z_V}$ . Traditionally,  $\omega_{X_V}, \omega_{Y_V}, \omega_{Z_V}$  have been represented by the symbols  $p, q, r$ .

- 4.2.2.2.1 Vehicle Angular Velocity  $\vec{\omega}$ —A vector quantity expressing the angular velocity of the *vehicle axis system*.

4.2.2.2.2 Roll Velocity  $\omega_X$ —The scalar value of the X component of *vehicle angular velocity*.

$$\omega_X = \vec{\omega} \bullet \hat{X} \quad (\text{Eq. 14})$$

4.2.2.2.3 Pitch Velocity  $\omega_Y$ —The scalar value of the Y component of *vehicle angular velocity*.

$$\omega_Y = \vec{\omega} \bullet \hat{Y} \quad (\text{Eq. 15})$$

4.2.2.2.4 Yaw Velocity (Yaw Rate)  $\omega_Z$ —The scalar value of the Z component of *vehicle angular velocity*.

$$\omega_Z = \vec{\omega} \bullet \hat{Z} \quad (\text{Eq. 16})$$

#### 4.2.2.3 Angular Accelerations

In the following paragraphs, angular accelerations are relative to the *Earth-fixed axis system* ( $X_E, Y_E, Z_E$ ). They are resolved into components in the *intermediate axis system* ( $X, Y, Z$ ). Suggested symbols are not provided for angular accelerations, since the mathematical symbol normally used for angular acceleration ( $\alpha$ ) is used for tire *slip angle* in vehicle dynamics.

NOTE—It is also possible to resolve angular acceleration vectors into components in other *axis systems*. For example, angular acceleration vectors may be resolved in the *vehicle axis system* ( $X_V, Y_V, Z_V$ ).

4.2.2.3.1 Vehicle Angular Acceleration—A vector quantity expressing the angular acceleration of the *vehicle axis system*.

4.2.2.3.2 Roll Acceleration—The scalar value of the X component of *vehicle angular acceleration*.

4.2.2.3.3 Pitch Acceleration—The scalar value of the Y component of *vehicle angular acceleration*.

4.2.2.3.4 Yaw Acceleration—The scalar value of the Z component of *vehicle angular acceleration*.

#### 4.2.3 Vehicle Trajectory Measures

4.2.3.1 Vehicle Trajectory—The path of a selected point on the vehicle in the *Earth-fixed coordinate system*. This point is usually the *vehicle reference point*.

4.2.3.2 Vehicle Path—The vertical projection of the *vehicle trajectory* onto the *ground plane*.

4.2.3.3 Path Radius  $R$ —The instantaneous radius of curvature of the *vehicle path*.

NOTE—*Path radius* is the perpendicular distance between the *vehicle path* and the *turn center*. The *path radius* of the *vehicle reference point* may be calculated as:

$$R = \frac{v_H^2}{a_C} \quad (\text{Eq. 17})$$

4.2.3.4 Turn Center—The instantaneous center of curvature of the *vehicle path*.

4.2.3.5 Path Curvature  $\kappa$ —The inverse of *path radius*. Path curvature may be calculated as:

$$\kappa = \frac{1}{R} \quad (\text{Eq. 18})$$

4.2.3.6 Normalized Path Curvature—The *wheelbase* divided by the *path radius* of the *tangent point*.

NOTE—*Normalized path curvature* is the small angle approximation, in radians, of the *included Ackermann steer angle*.

4.2.3.7 Course Angle  $\nu$ —The angle from the  $X_E$  axis to the vertical projection of *vehicle velocity* onto the  $X_E$ – $Y_E$  plane. See Figure 3. Course angle can be computed from the *yaw angle*,  $\psi$ , and the *vehicle sideslip angle*,  $\beta$ :

$$\nu = \psi + \beta \quad (\text{Eq. 19})$$

### 4.3 Forces and Moments

External forces and moments acting on the vehicle at any instant may be summed into one resultant force vector and one resultant moment vector. The line of action of the resultant force vector may be moved to any point of interest, providing that the resultant moment vector is altered accordingly. In the following paragraphs, forces and moments are resolved into components in the *intermediate axis system* ( $X$ ,  $Y$ ,  $Z$ ).

NOTE—It is also possible to resolve force and moment vectors into components in other *axis systems*. For example, force and moment vectors may be resolved in the *vehicle axis system* ( $X_v$ ,  $Y_v$ ,  $Z_v$ ) to produce  $F_{X_v}$ ,  $F_{Y_v}$ ,  $F_{Z_v}$  and  $M_{X_v}$ ,  $M_{Y_v}$ ,  $M_{Z_v}$ . Traditionally,  $F_{X_v}$ ,  $F_{Y_v}$ ,  $F_{Z_v}$  have been represented by the symbols  $X$ ,  $Y$ ,  $Z$  and  $M_{X_v}$ ,  $M_{Y_v}$ ,  $M_{Z_v}$  have been represented by the symbols  $L$ ,  $M$ ,  $N$ .

#### 4.3.1 Forces

4.3.1.1 Vehicle Force  $\vec{F}$ —A vector quantity expressing the sum of the external forces acting on the vehicle at any instant, with its line of action passing through the *vehicle reference point*.

4.3.1.2 Longitudinal Force  $F_X$ —The scalar value of the component of *vehicle force* in the direction of the  $X$  axis.

$$F_X = \vec{F} \cdot \hat{X} \quad (\text{Eq. 20})$$

4.3.1.3 Lateral Force  $F_Y$ —The scalar value of the component of *vehicle force* in the direction of the  $Y$  axis.

$$F_Y = \vec{F} \cdot \hat{Y} \quad (\text{Eq. 21})$$

4.3.1.4 Vertical Force  $F_Z$ —The scalar value of the component of *vehicle force* in the direction of the  $Z$  axis.

$$F_Z = \vec{F} \cdot \hat{Z} \quad (\text{Eq. 22})$$

#### 4.3.2 Moments

4.3.2.1 Vehicle Moment  $\vec{M}$ —A vector quantity expressing the sum of the external moments acting on the vehicle at any instant, consistent with the line of action of the *vehicle force*.

4.3.2.2 Roll Moment  $M_X$ —The scalar value of the component of the *vehicle moment* in the direction of the  $X$  axis.

$$M_X = \vec{M} \cdot \hat{X} \quad (\text{Eq. 23})$$

4.3.2.3 Pitch Moment  $M_Y$ —The scalar value of the component of the *vehicle moment* in the direction of the  $Y$  axis.

$$M_Y = \vec{M} \cdot \hat{Y} \quad (\text{Eq. 24})$$

4.3.2.4 Yaw Moment  $M_Z$ —The scalar value of the component of the *vehicle moment* in the direction of the Z axis.

$$M_Z = \vec{M} \cdot \hat{Z} \quad (\text{Eq. 25})$$

## 5. SUSPENSION AND STEERING

### 5.1 General Nomenclature

5.1.1 Suspension—A system that provides ride freedom and kinematic control of the motions of the *wheels* at a given longitudinal position on the vehicle.

5.1.1.1 Independent Suspension—A *suspension* utilizing a separate kinematic control mechanism for the *wheel* on each side of the vehicle.

5.1.1.2 Solid-Axle Suspension—A *suspension* utilizing an essentially rigid *axle*.

5.1.1.3 Suspension Corner—For an *independent suspension*: 1) The portion of the *suspension* that controls the kinematic motion of a single *wheel* (left front, right front, left rear, or right rear). 2) The assemblage of the *suspension*, *wheel*, *tire*, *brake*, and all other components that move with a single *wheel*.

### 5.2 Suspension Components

The terms contained in this section and the following section are defined to facilitate communication between vehicle dynamics professionals and engineers involved in the mechanical design of *suspension* and steering systems. Alternative names in common use have been provided for many of these terms. Additional names may be associated with some of these terms due to differences in national or corporate practice.

5.2.1 Axle—1) An essentially rigid member used to connect *wheels* on opposite sides of the vehicle at a given longitudinal position. The axle/*wheel* assembly has *suspension ride* and *suspension roll* degrees of freedom relative to the *sprung mass*. 2) The imaginary line connecting the *wheel centers* on opposite sides of the vehicle at a given longitudinal position. 3) Less rigorously, the assemblage of the *suspension*, *wheels*, *tires*, *brakes*, and all other components that move with the *wheels* at a given longitudinal position of the vehicle.

5.2.2 Ball Joint (Rod End)—An assemblage of essentially rigid parts, allowing rotational but not translational motion, generally used to connect a *link* or *control arm* to the *knuckle*, or used at each end of a *tie rod*.

5.2.3 Bushing—A compliant part that provides primarily rotational motion between a *link* or *control arm* and the body/frame or the *knuckle*. In its most typical form, it is comprised of two (usually steel) concentric cylinders between which is bonded an elastomeric core. Under load, it allows all three linear and all three angular deflections.

5.2.4 Control Arm (Wishbone)—A *suspension* part generally used to connect a *knuckle* or *axle* to the *sprung mass*. Kinematically, it consists of a revolute joint at its base and a spherical joint or a revolute joint at its apex. It constrains the motion of the apex to a circular path in a plane perpendicular to the hinge axis at its base. In its most typical form, it is roughly triangular with *bushings* having collinear axes at its base and a *ball joint* at its apex. It may take many forms, with a *bushing* or *ball joint* used at any of the three corners.

5.2.5 Damper—A *suspension* part used to provide damping, most commonly in *bounce* and *roll*. A typical damper provides primarily viscous damping, with some attendant Coulomb friction.

5.2.5.1 Shock Absorber—A *damper* that adds negligible kinematic constraint to the *suspension*.

5.2.5.2 Strut—A *damper* that also contributes to rotational constraint of the *knuckle* in camber and caster.

- 5.2.6 Jounce Stop (Jounce Bumper, Bump Stop)—The part that directly limits *jounce* travel of a *suspension*. Jounce stops are usually elastomeric bumpers, but are not limited to this configuration.
- 5.2.7 Rebound Stop—The part that directly limits *rebound* travel of a *suspension*. Rebound stops may be elastomeric bumpers like *jounce stops*, but are often internal stops built into *dampers*.
- 5.2.8 Knuckle (Hub Carrier, Upright)—A *suspension* part that includes the spindle, to which the *control arm(s)*, *link(s)*, *strut*, *ride spring*, and *tie rod* may be attached.
- 5.2.9 Link—An essentially rigid two-force member used to provide kinematic constraint. It typically has a *ball joint* or *bushing* at either end.
- 5.2.9.1 Lateral Link—A *link* used to provide primarily lateral constraint.
- 5.2.9.2 Leading Link—A *link* used to provide primarily longitudinal constraint whose rearward pivot is attached to the *sprung mass*.
- 5.2.9.3 Trailing Link—A *link* used to provide primarily longitudinal constraint whose forward pivot is attached to the *sprung mass*.
- 5.2.10 Ride Spring—A spring designed to provide ride compliance for the *suspension*.

NOTE—Commonly used *ride springs* consist of coil springs, leaf springs, torsion bars, and air springs.

- 5.2.11 Stabilizer Bar (Anti-Roll Bar, Anti-Sway Bar)—A torsional spring which is loaded in *suspension roll* to provide additional *suspension roll stiffness*.

NOTE—In its most typical form, a *stabilizer bar* is “U” shaped with the center section orientated laterally and attached to the *sprung mass* with *bushings*. Each end of the “U” attaches to the *control arm*, *knuckle*, or *strut* of an *independent suspension* or the end of an *axle* of a *solid-axle suspension*. Many permutations exist, using *links*, *ball joints* or *bushings* at different locations or with the center section attached to the *unsprung mass*, in the case of a *solid-axle suspension*.

- 5.2.12 Track Bar (Panhard Rod)—A *link* used to provide the primary lateral constraint between the *axle* and the *sprung mass*.
- 5.2.13 Watt Linkage—A four-bar linkage used to provide the primary lateral constraint between the *axle* and the *sprung mass*. It is typically comprised of a vertical member pivoted on the *axle* and two *lateral links*, each attached to it and to the *sprung mass*.

### 5.3 Steering Components

- 5.3.1 Steering Wheel (Hand Wheel)—The wheel used directly by the driver to steer the vehicle. See Figures 5 and 6.
- 5.3.2 Steering Column—A torsional member carrying rotational motion and torsional loads from the *steering wheel* to the *intermediate shaft*. See Figures 5 and 6.
- 5.3.3 Intermediate Shaft—A member carrying rotational motion and torsional loads from the *steering column* to the *input shaft*. This component generally contains one or more universal joints or flexible couplings and may also contain a slip joint. See Figures 5 and 6.
- 5.3.4 Steering Gear—A gear train and/or mechanism, with housing, carrying steering motion and loads from the *intermediate shaft* to the *tie rods* or *Pitman arm*. A steering gear may have provisions for power assist.
- 5.3.4.1 Rack and Pinion Steering Gear—A *steering gear* employing a rack and pinion gear set. This design transfers rotational motion of the *input shaft* to translational motion of the steering rack. It is used with laterally orientated *tie rods* connected directly to the rack. See Figure 5.

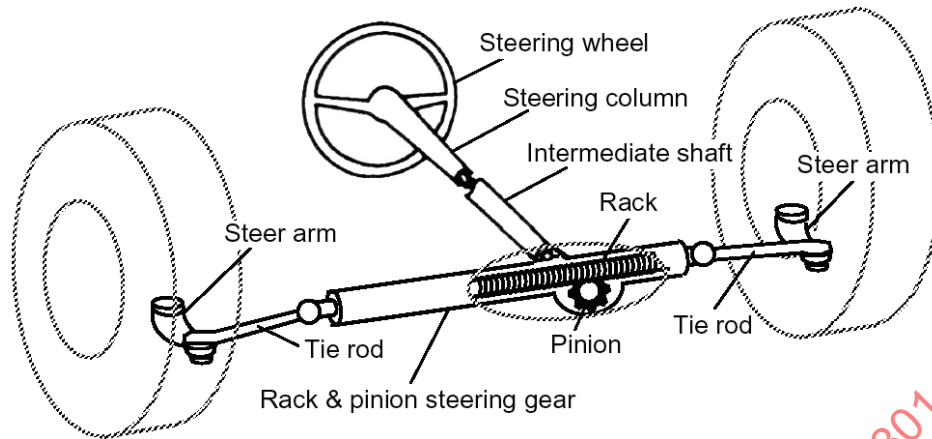


FIGURE 5 – RACK AND PINION STEERING SYSTEM

5.3.4.2 Recirculating-Ball Steering Gear (Rotational-Rotational Steering Gear)—A *steering gear* generally employing a ball screw mechanism to transfer rotational motion from the *input shaft* to rotational motion of the *output shaft*. The ball screw mechanism is not required but is widely used and *steering gears* of this general arrangement (rotational-rotational) are so named. See Figure 6.

NOTE—Many other mechanizations of the *recirculating-ball steering gear* exist (e.g., Acme, worm and sector, worm and roller, cam and peg, etc.), in both manual and power-assisted implementations. These designs are less common in current passenger vehicles, although they are still in use in heavy trucks. The basic mechanical functionality of all these designs is identical, however.

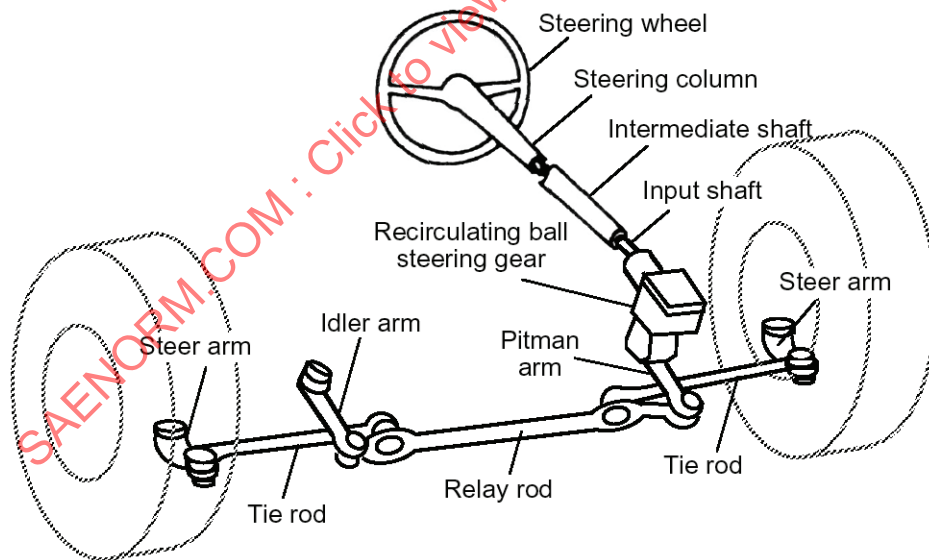


FIGURE 6 – RECIRCULATING-BALL STEERING SYSTEM

5.3.4.3 Input Shaft—A member carrying rotational motion and torsional loads from the *intermediate shaft* into the *steering gear*. The input shaft is considered part of the *steering gear*.

5.3.4.4 Output Shaft—A member carrying rotational motion and torsional loads from the *steering gear* to the *Pitman arm*. The output shaft is considered part of a *recirculating-ball steering gear*.

5.3.5 Steering Linkage—A kinematic linkage used to transfer motion and loads from the *steering gear* to the *knuckles*.

NOTE—Historically, a wide variety of linkages have been used to perform this function. Currently, common practice on passenger cars and light trucks is the use of lateral *tie rods* acting on longitudinal *steer arms*. *Recirculating-ball steering gears* generally utilize a four-bar linkage (*parallelogram linkage*), of which the *Pitman arm* is one *link*, to transmit motion and loads to the *tie rods*. In systems utilizing *rack and pinion steering gears*, the *tie rods* directly connect the steering rack to the *steer arms*.

5.3.5.1 Tie Rod—A lateral *link* connecting the *steer arm* to either the remainder of the *steering linkage* or to the steering rack of a *rack and pinion steering gear*. See Figures 5 and 6.

5.3.5.2 Steer Arm (Knuckle Arm)—A crank, usually integral with the *knuckle*, but possibly attached to it, to which the *steering linkage* is attached. See Figures 5 and 6.

5.3.5.3 Parallelogram Linkage—A *steering linkage* that is characterized by a four-bar linkage consisting of the *Pitman arm*, *relay rod*, *idler arm*, and the vehicle chassis, that transmits motion and loads to the *tie rods*. See Figure 6.

NOTE—Generally, this *steering linkage* design is not a true parallelogram linkage, due to design considerations that provide *Ackermann geometry* corrections. Typical linkages are actually trapezoidal linkages.

5.3.5.3.1 Pitman Arm—A crank that attaches to the *output shaft* of a *recirculating-ball steering gear*. See Figure 6.

5.3.5.3.2 Idler Arm—A *link* that provides a motion constraint in a *parallelogram linkage*. The idler arm is the *link* opposite to the *Pitman arm* and connects the *relay rod* to the chassis. See Figure 6.

5.3.5.3.3 Relay Rod (Center Link)—A *lateral link* that connects the *tie rods* in a *parallelogram linkage*. The motion of the relay rod is constrained by the motion of the *Pitman arm* and the *idler arm*. See Figure 6.

## 5.4 Masses and Inertias

5.4.1 Unsprung-Mass Center of Gravity—The center of gravity of the *unsprung mass* with a given set of *suspension trim heights*.

NOTE—*Unsprung-mass centers of gravity* may also be separately defined for the front and rear *axles* and, further, for *independent suspensions*, for individual *suspension corners*. If centers of gravity are defined for subsets of the total *unsprung mass*, the location being referenced must be explicitly identified (e.g., Left Front Unsprung Mass Center of Gravity).

## 5.4.2 Unsprung-Mass Moments of Inertia

The following definitions apply to the total inertia of the vehicle *unsprung mass* with a given set of *suspension trim heights*.

NOTE—Moments of inertia may also be separately defined for the front and rear *axles* and, further, for *independent suspensions*, for individual *suspension corners*. If inertias are defined for subsets of the total *unsprung mass*, the moments are taken about axes parallel to the vehicle axes ( $X_V$ ,  $Y_V$ ,  $Z_V$ ) that pass through the center of gravity of the *suspension* components under consideration. The location of the inertia must be explicitly identified (e.g., Left Rear Unsprung Mass Pitch Moment of Inertia).

5.4.2.1 Unsprung-Mass Roll Moment of Inertia—The moment of inertia of the *unsprung mass*, taken about an axis parallel to the  $X_V$  axis that passes through the *unsprung-mass center of gravity*.

- 5.4.2.2 Unsprung-Mass Pitch Moment of Inertia—The moment of inertia of the *unsprung mass*, taken about an axis parallel to the  $Y_V$  axis that passes through the *unsprung-mass center of gravity*.
- 5.4.2.3 Unsprung-Mass Yaw Moment of Inertia—The moment of inertia of the *unsprung mass*, taken about an axis parallel to the  $Z_V$  axis that passes through the *unsprung-mass center of gravity*.
- 5.4.3 Spin Moment of Inertia—The moment of inertia of the rotating components at an individual *suspension corner*, taken about the local  $Y_W$  axis. The rotating components typically include the *wheel, tire, brake rotor or brake drum*, and the drive axle or half shaft. The location of the inertia must be explicitly identified (e.g., Right Rear Spin Moment of Inertia).
- 5.4.4 Steer Moment of Inertia—The moment of inertia of the steered components at an individual *suspension corner*, taken about the local *steering axis*. The steered components typically include the *knuckle, wheel, tire, brake rotor or brake drum*, brake apply components (*brake caliper and brake pads or brake shoes, wheel cylinder*, return springs, brake backing plate, etc.), and steering *tie rods*. The location of the inertia must be explicitly identified (e.g., Right Front Steer Moment of Inertia).

## 5.5 Geometry

### 5.5.1 Steer and Camber Angles

- 5.5.1.1 Steer Angle (Road Wheel Steer Angle)  $\delta$ —For each road *wheel*, the angle from the  $X_V$  axis to the *wheel plane*, about the  $Z_V$  axis.

NOTE—The sign of the *steer angle* is determined using the right hand rule. For the same physical case, the sign of *steer angle* is reversed in the Z-Up and Z-Down axis orientations.

- 5.5.1.2 Reference Steer Angle—For each steerable *wheel*, the *steer angle* corresponding to a given *steering-wheel angle*, defined by the kinematics of the steering system in the absence of *suspension roll and tire forces and moments*, but including static *tire vertical load*, with the vehicle at rest at a specified *load condition*, set of *vehicle trim heights*, or set of *suspension trim heights*.

NOTE—If the kinematic relationship between the *steering-wheel angle* and the *steer angle* is a function of vehicle speed, the *reference steer angle* should be determined at the *speed* of interest. Other factors influencing the relationship between *steering-wheel angle* and *steer angle* (e.g., *yaw velocity, lateral acceleration*) are not considered in determining the *reference steer angle*.

- 5.5.1.3 Ackermann Wheel Steer Angle—For each individual *wheel*, the *steer angle* necessary to cause the local  $Y_T$  axis to pass through the vehicle *turn center*. See Figure 7.

NOTE—This definition is more general than the definition used in previous versions of SAE J670 in order to accommodate four-wheel steering. For a front-steer vehicle with the *turn center* lying on a projection of the rear *axle* centerline, this definition accommodates the former definition and defines an *Ackermann wheel steer angle* for both front *wheels*.

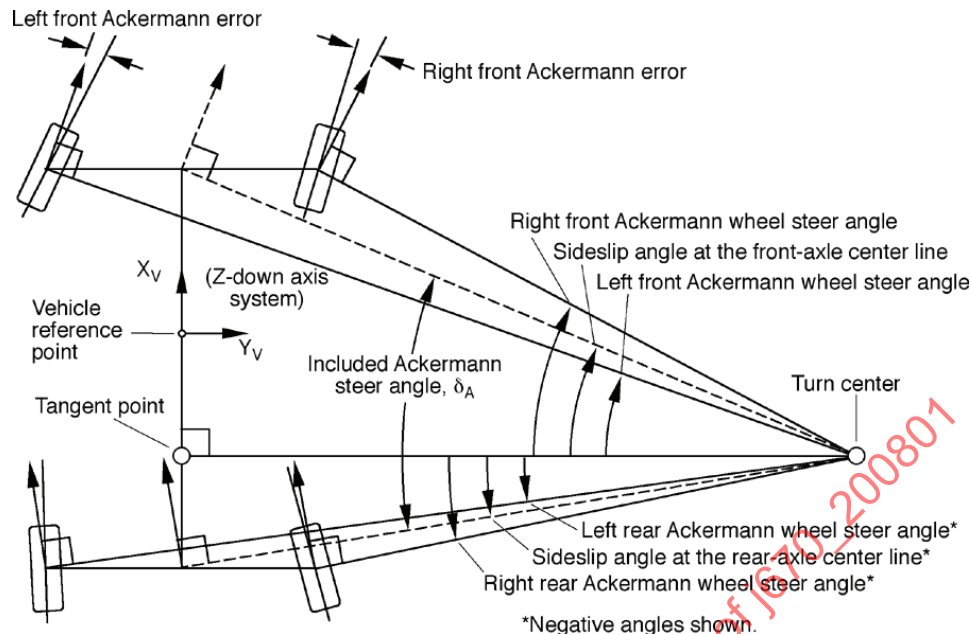


FIGURE 7 – ACKERMANN GEOMETRY  
(Z-Down axis orientation)

- 5.5.1.4 Mean Reference Steer Angle—The average *reference steer angle* for a pair of *wheels* on an *axle*. See Figure 4.
- 5.5.1.5 Included Reference Steer Angle  $\delta_{REF}$ —The *mean reference steer angle* of the front *axle* minus the *mean reference steer angle* of the rear *axle*.
- 5.5.1.6 Included Ackermann Steer Angle  $\delta_A$ —The *sideslip angle* of the centerline of the front *axle* minus the *sideslip angle* of the centerline of the rear *axle*. This is the included angle between the line from the *turn center* to the centerline of the front *axle* and the line from the *turn center* to the centerline of the rear *axle*. See Figure 7.

$$\delta_A = \text{Arc tan } \frac{L_F}{R} - \text{Arc tan } \frac{L_R}{R} \quad (\text{Eq. 26})$$

Where  $L_F$  is the signed distance from the *tangent point* to the front *axle*,  $L_R$  is the signed distance from the *tangent point* to the rear *axle*, and  $R$  is the *path radius* of the *tangent point*.

NOTE—For a *tangent point* located between the *axles*,  $L_F$  will be positive and  $L_R$  will be negative.

- 5.5.1.7 Toe Angle—The angle between the  $X_V$  axis and the *wheel plane*, about the  $Z_V$  axis. The *wheel* has toe-in if the forward portion of the *wheel* is closer to the vehicle centerline than the *wheel center* and has toe-out if it is farther away. By convention, toe-in is considered a positive angle, and toe-out is a negative angle.

NOTE—Static wheel alignment settings for *toe angle*, as well as *static toe*, *camber angle*, *caster angle*, and *steering axis inclination angle*, are normally specified with the vehicle at rest at a given *load condition*, set of *vehicle trim heights*, or set of *suspension trim heights*, with a *steering-wheel angle* of zero, zero *suspension roll angle*, on a horizontal, planar surface.

- 5.5.1.8 Total Toe Angle (Sum Toe Angle)—The total angle between a pair of *wheels* on an *axle*. The value is equal to the sum of the *toe angles* for the two *wheels*.

5.5.1.9 Static Toe—The difference in the transverse distances between the *wheel planes* of a pair of *wheels* on an *axle*, taken at the extreme rear and front points of the tire tread or between corresponding points on the *rim*. The measurement location must be specified. When the distance at the rear is greater, the *wheels* have this amount of toe-in, and where smaller, the *wheels* have toe-out.

NOTE—*Static toe* is equal to the product of *total toe angle* in radians and 1) the *tire overall diameter*, if taken at the tire tread; or 2) the *rim diameter*, if taken at the *rim*.

5.5.1.10 Camber Angle—The angle between the  $Z_V$  axis and the *wheel plane*, about the  $X_V$  axis. It is considered positive when the *wheel* leans outward at the top and negative when it leans inward. See Figure 8.

NOTE—*Camber angle* is a measure of the orientation of the *wheel plane* relative to the vehicle, and is independent of *road plane* geometry. *Camber angle* should not be confused with *inclination angle*, which is a measure of the *wheel plane orientation* relative to the *road plane*. The relation between *camber angle*, *inclination angle*, and *vehicle roll angle* is shown in Figure 8.

View from rear; for  $\theta = \delta_L = \delta_R = 0$ .

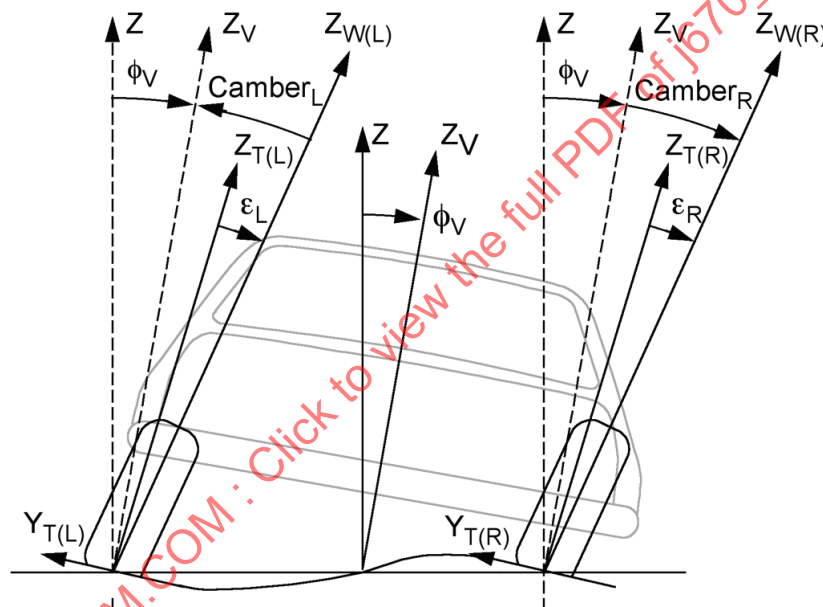


FIGURE 8 – CAMBER, INCLINATION, AND VEHICLE ROLL ANGLES  
(Z-Up axis orientation shown, sign of angles is identical for Z-Down axis orientation)

## 5.5.2 Steering-Axis Geometry

5.5.2.1 Steering Axis (Kingpin Axis)—The axis of rotation of the *knuckle* relative to the vehicle *sprung mass* when steered.

NOTE—The *steering axis* is typically determined with the steering system free of any loads except those associated with the static reference condition. The *steering axis* may shift as the *steer angle* changes, due to *suspension kinematics* and *compliances*.

5.5.2.2 Caster Angle—The angle between the  $Z_V$  axis and the normal projection of the *steering axis* onto the  $X_V$ – $Z_V$  plane. The angle is positive when the top of the *steering axis* is inclined rearward.

- 5.5.2.3 Caster Offset at Wheel Center (Spindle Trail)—The distance, measured parallel to the  $X_V$  axis, from the normal projection of the *wheel center* onto the  $X_V$ – $Z_V$  plane to the normal projection of the *steering axis* onto the  $X_V$ – $Z_V$  plane. This distance is positive if the projection of the *steering axis* is forward of the projection of the *wheel center*.
- 5.5.2.4 Caster Offset at Ground (Caster Trail)—The distance in the  $X_T$  direction from the  $Y_T$ – $Z_T$  plane to the point where the *steering axis* intersects the  $X_T$ – $Y_T$  plane. This distance is positive if the *steering-axis* intersection point is forward of the  $Y_T$ – $Z_T$  plane.
- 5.5.2.5 Steering-Axis Inclination Angle (Kingpin Inclination Angle)—The angle between the  $Z_V$  axis and the normal projection of the *steering axis* onto the  $Y_V$ – $Z_V$  plane. The angle is positive when the top of the *steering axis* is inclined inward.
- 5.5.2.6 Steering-Axis Offset at Wheel Center (Kingpin Offset at Wheel Center)—The distance, measured parallel to the  $Y_V$  axis, from the normal projection of the *wheel center* onto the  $Y_V$ – $Z_V$  plane to the normal projection of the *steering axis* onto the  $Y_V$ – $Z_V$  plane. This distance is positive if the projection of the *steering axis* is inboard of the projection of the *wheel center*.
- 5.5.2.7 Steering-Axis Offset at Ground (Kingpin Offset at Ground, Scrub Radius)—The distance in the  $Y_T$  direction between the *wheel plane* and the point where the *steering axis* intersects the  $X_T$ – $Y_T$  plane. This distance is positive if the *steering-axis* intersection point is inboard of the *wheel plane*.
- 5.5.2.8 Spindle Length—The distance in the  $Y_W$  direction from the *wheel center* to the normal projection of the *steering axis* onto the  $Y_W$ – $Z_W$  plane. This distance is positive if the projection of the *steering axis* is inboard of the *wheel center*.

## 5.6 Suspension Motions

- 5.6.1 Jounce (Compression, Bump)—An upward vertical displacement of the *wheel center* relative to the *sprung mass* from a specified reference *suspension trim height*.
- 5.6.2 Rebound (Extension, Droop)—A downward vertical displacement of the *wheel center* relative to the *sprung mass* from a specified reference *suspension trim height*.
- 5.6.3 Suspension Ride—*Jounce* or *rebound* displacement or velocity of a pair of *wheels* on the same *axle*, which is symmetric with respect to the *vehicle plane of symmetry*.
- 5.6.4 Suspension Roll—*Jounce* or *rebound* displacement or velocity of a pair of *wheels* on the same *axle*, which is antisymmetric with respect to the *vehicle plane of symmetry*.

NOTE—*Suspension ride* and *suspension roll* are qualitative terms describing two special cases of *suspension* vertical motion, which may refer to either displacement or velocity. Any *suspension* displacement or velocity involving *jounce* or *rebound* motion may be viewed as a combination of *suspension ride* and *suspension roll*.

- 5.6.5 Suspension Ride Displacement—The *jounce* or *rebound* displacement produced by *suspension ride*, relative to a pair of reference *suspension trim heights*.
- 5.6.6 Suspension Roll Angle—The angular displacement produced by *suspension roll* relative to a pair of reference *suspension trim heights*. The angular displacement is defined by the normal projection of a line connecting the two *wheel centers* onto the  $Y_V$ – $Z_V$  plane.

## 5.7 Kinematics

Many of the terms defined in this section may be signed. Signs may be assigned based on 1) the direction of the kinematic change and the *axis system* in use, or 2) by the assumed effect of the kinematic change on the *understeer / oversteer gradient*. The sign convention is dependent on the practices of the organization performing the testing or analysis.

Many of the terms that are defined in this section begin with or include the word “*suspension*” (e.g., Suspension Ride Camber). The definitions of such terms typically make reference to *suspension motion* or displacement. Inclusion of the word “*suspension*” indicates that the relative motion or displacement of interest is that of the *wheel center* in the *vehicle axis system* (i.e., motion of the *suspension* alone and not including *tire deflection*). In most cases, a similar term that does not include the word “*suspension*” (e.g., Ride Camber) could be defined. In this case, the relative motion or deflection of interest would be that of the *tire contact center* in the *vehicle axis system* (i.e., including *tire deflections*).

### 5.7.1 Ride Kinematics

5.7.1.1 Front- (Rear-) View Swing Center—For *suspension ride* motion, the instant center of the normal projection of the path of the *wheel center* onto the  $Y_V$ – $Z_V$  plane. A front-view swing center may be defined at each *suspension trim height*.

5.7.1.2 Front- (Rear-) View Swing-Arm Length—The distance from the *front-view swing center* to the normal projection of the point of interest onto the  $Y_V$ – $Z_V$  plane.

NOTE—Commonly used points of interest are the *wheel center* and the *tire contact center*. The point of interest used should be included with the term for clarity (e.g., Front-View Swing-Arm Length to Wheel Center).

5.7.1.3 Front- (Rear-) View Swing-Arm Angle—The angle from the  $Y_V$  axis to the line that passes through the *front-view swing center* and the normal projection of the point of interest onto the  $Y_V$ – $Z_V$  plane.

NOTE—Commonly used points of interest are the *wheel center* and the *tire contact center*. The point of interest used should be included with the term for clarity (e.g., Front-View Swing-Arm Angle to Wheel Center).

5.7.1.4 Side-View Swing Center—For *suspension ride* motion, the instant center of the normal projection of the path of the *wheel center* onto the  $X_V$ – $Z_V$  plane. A side-view swing center may be defined at each *suspension trim height*.

5.7.1.5 Side-View Swing-Arm Length—The distance from the *side-view swing center* to the normal projection of the point of interest onto the  $X_V$ – $Z_V$  plane.

NOTE—Commonly used points of interest are the *wheel center* and the *tire contact center*. The point of interest used should be included with the term for clarity (e.g., Side-View Swing-Arm Length to Wheel Center).

5.7.1.6 Side-View Swing-Arm Angle—The angle from the  $X_V$  axis to the line that passes through the *side-view swing center* and the normal projection of the point of interest onto the  $X_V$ – $Z_V$  plane.

NOTE—Commonly used points of interest are the *wheel center* and the *tire contact center*. The point of interest used should be included with the term for clarity (e.g., Side-View Swing-Arm Angle to Wheel Center).

5.7.1.7 Suspension Ride Camber—The change of *camber angle* resulting from a given *suspension ride displacement*.

5.7.1.8 Suspension Ride Camber Gradient—The rate of change of *camber angle* with respect to *suspension ride displacement* at a given *suspension ride displacement*.

5.7.1.9 Suspension Ride Caster—The change of *caster angle* resulting from a given *suspension ride displacement*.

- 5.7.1.10 Suspension Ride Caster Gradient—The rate of change of *caster angle* with respect to *suspension ride displacement* at a given *suspension ride displacement*.
- 5.7.1.11 Suspension Ride Toe—The change of *toe angle* of a *wheel* resulting from a given *suspension ride displacement*.
- 5.7.1.12 Suspension Ride Toe Gradient—The rate of change of *toe angle* with respect to *suspension ride displacement* at a given *suspension ride displacement*.
- 5.7.1.13 Suspension Ride Steer—The change of *steer angle*,  $\delta$ , of a *wheel* resulting from a given *suspension ride displacement*.
- 5.7.1.14 Suspension Ride Steer Gradient—The rate of change of *steer angle*,  $\delta$ , with respect to *suspension ride displacement* at a given *suspension ride displacement*.
- 5.7.1.15 Net (Average) Suspension Ride Steer—The change in *steer angle* for an *axle* resulting from *suspension ride displacement*. It is the average of the *suspension ride steer* changes for the left and right *wheels*.
- 5.7.1.16 Ride Track Change—The change of *track*,  $T$ , for an *axle* resulting from a given *suspension ride displacement*.
- 5.7.1.17 Ride Track Change Gradient—The rate of change of *track*,  $T$ , for an *axle* with respect to *suspension ride displacement* at a given *suspension ride displacement*.

NOTE—*Ride track change gradient* exists for a symmetrical *independent suspension* when the *front-view swing centers* are not in the *road plane*. For such a *suspension*, half of the *ride track change gradient* is generally of more interest because it is the tangent of each *front-view swing-arm angle* to *contact center*, including a slight error due to *tire deflections*. For asymmetrical *independent suspensions*, the *ride track change gradient* is the sum of contributions from each side of the *suspension* and half its value may not be related to the kinematic properties of either side. In this case, half-track change gradients for each side would equal the tangents of the *front-view swing-arm angle* to *contact center* for each side, again including a slight error due to *tire deflections*. The *ride track change gradient* for solid-*axle suspensions* is zero.

## 5.7.2 Roll Kinematics

- 5.7.2.1 Suspension Roll Camber—The change of *camber angle* resulting from a given *suspension roll angle*.
- 5.7.2.2 Suspension Roll Camber Gradient—The rate of change of *camber angle* with respect to *suspension roll angle* at a given *suspension roll angle*.
- 5.7.2.3 Suspension Roll Inclination—The change of *inclination angle*,  $\epsilon$ , resulting from a given *suspension roll angle*.
- 5.7.2.4 Suspension Roll Inclination Gradient—The rate of change of *inclination angle*,  $\epsilon$ , with respect to *suspension roll angle* at a given *suspension roll angle*.

NOTE—This definition was applied to the term “roll camber coefficient” in previous editions of SAE J670. The magnitude of the *suspension roll inclination gradient* will be different from the magnitude of the *suspension roll camber gradient* defined above, since the *suspension roll inclination gradient* quantifies the angular change of the *wheel* relative to the *road plane*, while the *suspension roll camber gradient* quantifies the angular change of the *wheel* relative to the *sprung mass*.

- 5.7.2.5 Suspension Roll Caster—The change of *caster angle* resulting from a given *suspension roll angle*.
- 5.7.2.6 Suspension Roll Caster Gradient—The rate of change of *caster angle* with respect to *suspension roll angle* at a given *suspension roll angle*.

- 5.7.2.7 Suspension Roll Steer—The change of *steer angle*,  $\delta$ , resulting from a given *suspension roll angle*.
- 5.7.2.8 Suspension Roll Steer Gradient—The rate of change of *steer angle*,  $\delta$ , with respect to *suspension roll angle* at a given *suspension roll angle*.
- 5.7.2.9 Roll Center—The point in the transverse vertical plane through the *wheel centers* on an *axle* at which lateral forces may be applied to the *sprung mass* without producing *suspension roll*.

NOTE—The *roll center* constitutes an idealized kinematic concept and does not necessarily represent a true instantaneous center of rotation of the *sprung mass*.

- 5.7.2.10 Roll Center Height—The height of the *roll center* above a line connecting the tire *contact centers* for an *axle*.
- 5.7.2.11 Roll Axis—The line joining the front and rear *roll centers*.

### 5.7.3 Steering Kinematics

- 5.7.3.1 Steer Camber—The change in *camber angle* resulting from a given *steer angle* displacement.
- 5.7.3.2 Steer Camber Gradient—The rate of change of *camber angle* with respect to *steer angle* at a given *steer angle*.
- 5.7.3.3 Steer Caster—The change in *caster angle* resulting from a given *steer angle* displacement.
- 5.7.3.4 Steer Caster Gradient—The rate of change of *caster angle* with respect to *steer angle* at a given *steer angle*.
- 5.7.3.5 Ackermann Geometry—Steering-system kinematics which provide for the intersection of the *tire axis system*  $Y_T$  axes of the steerable *wheels* at the *turn center* for non-zero *steering-wheel angles* at negligible *lateral acceleration*. See Figure 7.

NOTE—*Ackermann geometry* may be achieved at one or more *steering-wheel angles* during steering articulation. For a front steer only vehicle, the *turn center* is on the line defined by the rear *wheel centers*. For a four-wheel steer vehicle, the *turn center* is generally not on this line. A steering system with *Ackermann geometry* is said to have 100% Ackermann correction, and one with equal *steer angles* (parallel steer) on the steerable *axle* is said to have 0% Ackermann correction. The Ackermann correction may be less than 0% or greater than 100%.

- 5.7.3.6 Ackermann Error—The kinematic inconsistency in *steer angle* between two steerable *wheels* on an *axle* with respect to the *turn center*. See Figure 7.

NOTE—For a front steer only vehicle, *Ackermann error* exists if the *tire axis system*  $Y_T$  axes of the steerable *wheels* do not intersect on the line defined by the rear *wheel centers*. For a four-wheel steer vehicle, *Ackermann error* exists if the *tire axis system*  $Y_T$  axes of the steerable *wheels* do not intersect at a point. If *Ackermann error* exists, an ideal *turn center* may be defined by the intersection of the *tire axis system*  $Y_T$  axes of the inner *wheels*. In this case, *Ackermann error* is the difference between the actual outer *wheel steer angle* and the ideal outer *wheel steer angle*, for which the *tire axis system*  $Y_T$  axis passes through the ideal *turn center*. *Ackermann error* is defined only at the front *axle* for a front steer only vehicle and at both the front and rear *axles* for a four-wheel steer vehicle.

- 5.7.3.7 Steering Ratio—The rate of change of *steering-wheel angle* with respect to the *mean reference steer angle* of a pair of steered *wheels* at a given *steering-wheel position*.
- 5.7.3.8 Overall Steering Ratio—The rate of change of *steering-wheel angle* with respect to *included reference steer angle* at a given *steering-wheel position*.
- 5.7.3.9 Gear Ratio—The rate of change of *input shaft angle* with respect to *output shaft angle* at a given *input shaft angle*, assuming an infinitely stiff *steering gear*.

5.7.3.10 Linkage Ratio—The rate of change of *output shaft* angle with respect to the average *steer angle* of a pair of steered *wheels* at a given *output shaft* angle, assuming an infinitely stiff *steering linkage* with no *suspension roll*.

5.7.3.11 C-Factor (Rack Ratio, Rack Speed)—For a *rack and pinion steering gear*, the rate of change of rack position with respect to *input shaft* angle at a given *input shaft* angle, assuming an infinitely stiff *steering gear*.

NOTE—The *C-Factor* is commonly expressed as the linear rack travel for one complete revolution of the *input shaft*, e.g., 48 mm/rev. For a constant ratio *rack and pinion steering gear*, the *C-Factor* nominally equals the circumference of the pinion pitch circle. This will generally not be true for a variable ratio *steering gear*.

#### 5.7.4 Anti- Characteristics

*Tire shear forces* acting in the *road plane* are reacted by opposing forces in the vehicle *sprung mass* that act through the instant center of rotation of the *suspension* in side view or through the instant center of rotation of the *suspension corner* in front view. These force pairs form couples in side view or in front view. *Suspension anti- characteristics* are a means of quantifying the kinematic properties of *suspensions* that create opposing couples that reduce changes in *suspension* spring force, thereby reducing the *suspension motion* that would otherwise occur because of *longitudinal load transfer* or *lateral load transfer*. *Suspension anti- characteristics* may be quantified as the change in *tire normal force* resulting from a unit change in *tire longitudinal force* or *tire lateral force*. Anti- characteristics may also be quantified by considering the total amount of *longitudinal load transfer* or *lateral load transfer* reacted through the *suspension* of interest. This representation expresses the strength of the anti- property as a percentage, where 0% implies no influence on the change in *suspension* spring load and 100% implies an exact cancellation of the change in *suspension* spring load. The percent anti- can be less than 0% or greater than 100%. For longitudinal anti's, the configuration of the drivetrain or braking system will determine whether *driving torque* or *braking torque* is reacted through the *suspension* or reacted directly by the *sprung mass*, which will affect the magnitude of the anti- characteristic.

5.7.4.1 Anti-Dive—A property of front *suspension* side-view kinematics whereby *tire braking forces* at the front *wheels* are reacted in a manner that reduces changes in *suspension* spring loads relative to those that would have occurred due to the *longitudinal load transfer*.

5.7.4.2 Braking Anti-Lift—A property of rear *suspension* side-view kinematics whereby *tire braking forces* at the rear *wheels* are reacted in a manner that reduces changes in *suspension* spring loads relative to those that would have occurred due to the *longitudinal load transfer*.

5.7.4.3 Acceleration Anti-Lift—A property of front *suspension* side-view kinematics whereby *tire driving forces* at the front *wheels* are reacted in a manner that reduces changes in *suspension* spring loads relative to those that would have occurred due to the *longitudinal load transfer*.

5.7.4.4 Anti-Squat—A property of rear *suspension* side-view kinematics whereby *tire driving forces* at the rear *wheels* are reacted in a manner that reduces changes in *suspension* spring loads relative to those that would have occurred due to the *longitudinal load transfer*.

5.7.4.5 Anti-Roll—A property of *suspension* front-view kinematics whereby *tire lateral forces* are reacted in a manner that reduces changes in *suspension* spring loads relative to those that would have occurred due to *lateral load transfer*.

#### 5.8 Ride and Roll Stiffness

5.8.1 Ride Rate—The rate of change of *tire normal force* with respect to displacement of the *tire contact center* in the  $Z_v$  direction during *suspension ride* motion, at a specified *suspension trim height*.

5.8.2 Suspension Ride Rate (Wheel Rate)—The rate of change of *tire normal force* with respect to displacement of the *wheel center* in the  $Z_v$  direction during *suspension ride* motion, at a specified *suspension trim height*.

NOTE—*Ride rate* and *suspension ride rate* apply to a single *tire* or *wheel* and are, therefore, so-called quarter-car properties. Similar terms could apply to the sum of the rates of both *tires* and *wheels* on an *axle* (i.e., Axle Ride Rate and Axle Suspension Ride Rate).

5.8.3 Roll Stiffness (Roll Rate)—The rate of change of the restoring couple exerted by a *suspension* on the *sprung mass* with respect to the angular change about the  $X_v$  axis of a line connecting the *tire contact centers* of an *axle*, at a specified pair of *suspension trim heights*.

5.8.4 Suspension Roll Stiffness (Suspension Roll Rate)—The rate of change of the restoring couple exerted by a *suspension* on the *sprung mass* with respect to *suspension roll angle*, at a specified pair of *suspension trim heights*.

5.8.5 Vehicle Roll Stiffness—The sum of the separate *roll stiffnesses*.

5.8.6 Roll Stiffness Distribution—The distribution of the *vehicle roll stiffness* between the front and rear *suspensions* expressed as a percentage of the *vehicle roll stiffness*.

NOTE—The terms *vehicle roll stiffness* and *roll stiffness distribution* are both based on the definition of *roll stiffness*. These concepts therefore include the influence of the compliances of the *tires* as well as the *compliances* of the *suspensions*. Similar terms (and concepts) that would exclude the influence of the *tires* and would depend only on the *compliances* of the *suspension* could be defined by using *suspension roll stiffness* as the basis.

## 5.9 Compliances

The terms defined in this section may be signed. Signs may be assigned based on 1) the direction of the compliance change and the polarity of the force or moment producing the change, according to the *axis system* in use, or 2) by the assumed effect of the compliance change on the *understeer / oversteer gradient*. The sign convention is dependent on the practices of the organization performing the testing or analysis.

Tire forces and moments produce compliance deflections. When these deflections are separated into components, the force or moment producing the component deflection is prefixed to the name of the compliance, for example *Lateral Force Compliance Camber*.

Compliant deflections produced by *tire longitudinal force* are affected by the manner in which the wheel-spin degree of freedom is constrained (*driveshaft* or *brakes*).

### 5.9.1 Camber and Steer Compliances

5.9.1.1 Compliance Camber—The *camber angle* displacement of a *wheel* resulting from compliances in *suspension components*, *steering components*, and vehicle structure, produced by *tire forces and moments*.

5.9.1.2 Camber Compliance (Compliance Camber Coefficient)—The rate of change of *inclination angle*,  $\epsilon$ , with respect to a *tire force or moment*.

5.9.1.3 Compliance Steer—The *steer angle* displacement of a *wheel* resulting from compliances in *suspension components*, *steering components*, and vehicle structure, produced by *tire forces and moments*.

5.9.1.4 Steer Compliance (Compliance Steer Coefficient)—The rate of change of *steer angle*,  $\delta$ , with respect to a *tire force or moment*.

## 5.9.2 Other Compliances

- 5.9.2.1 Lateral Compliance at the Wheel Center—The rate of change of the lateral displacement of the *wheel center* resulting from compliances in *suspension components*, *steering components*, and vehicle structure, with respect to a *tire force or moment*.
- 5.9.2.2 Lateral Compliance at the Contact Center—The rate of change of the lateral displacement of the *tire contact center* resulting from compliances in *suspension components*, *steering components*, and vehicle structure, with respect to a *tire force or moment*.
- 5.9.2.3 Longitudinal Compliance—The rate of change of the longitudinal displacement of the *wheel center* resulting from compliances in *suspension components*, *steering components*, and vehicle structure, with respect to a *tire force or moment*.
- 5.9.2.4 Windup Compliance—The rate of change of the angular displacement of the *wheel* about the *wheel-spin axis* resulting from compliances in *suspension components*, *steering components*, and vehicle structure, with respect to a *tire force or moment*.
- 5.9.2.5 Axle Windup Compliance—The rate of change of the angular displacement of the *axle* housing of a *solid-axle suspension* about the *wheel-spin axis* resulting from compliances in *suspension components*, *steering components*, and vehicle structure, with respect to a *tire force or moment*.

## 6. BRAKES

### 6.1 General Nomenclature

- 6.1.1 Brake—A device whose function is to develop *brake torque* that directly opposes the longitudinal motion of a vehicle.
- 6.1.2 Friction Brake—A *brake* that develops *brake torque* by means of friction between a fixed and rotating member. Dynamically, friction brakes convert mechanical energy into heat. Statically, friction brakes use friction to prevent rotation of the *wheels*.
- 6.1.3 Regenerative Brake—A *brake* that develops *brake torque* by converting mechanical energy into stored energy.
- 6.1.4 Disc Brake—A *friction brake* that develops *brake torque* by clamping a disk-shaped rotating member between non-rotating pieces of *friction material*. The direction of the clamping force application is axial.
- 6.1.5 Drum Brake—A *friction brake* that develops *brake torque* by forcing non-rotating *friction material* radially against a drum-shaped rotating member. In nearly all modern automotive applications, the *friction material* is forced outwardly against the inside surface of a rotating *brake drum*.
- 6.1.6 Anti-Lock Brake System—A closed-loop control system designed to regulate *brake torque* to limit the *tire longitudinal slip ratio* during *brake* application. This is typically accomplished by transducers for sensing *wheel* rotation and an actuation system under control of a computer that regulates *brake pressure*. See also SAE J2564.
- 6.1.7 Brake Pressure—The fluid pressure supplying an individual *brake*.
- 6.1.8 Brake Torque—The torque about the *wheel-spin axis* produced by the *brake*. See also 7.8.2.

### 6.2 Brake Components

- 6.2.1 Brake Pedal—The pedal used by the driver to actuate the braking system.
- 6.2.2 Master Cylinder—The hydraulic valve assembly used to convert force from the *brake pedal* to hydraulic pressure for *brake* actuation.

- 6.2.3 Proportioning Valve—A hydraulic valve which reduces *brake pressure* supplied to the rear *brakes*, relative to front *brake pressure*, to maintain *brake balance* in the presence of *longitudinal load transfer*.
- 6.2.4 Brake Caliper—The non-rotating part of a *disc brake* which houses the hydraulic cylinder(s) that clamp the *brake pads* against the *brake rotor* during *brake* actuation.
- 6.2.5 Wheel Cylinder—The hydraulic cylinder(s) in a *drum brake* that force the *brake shoes* to press against the *brake drum*.
- 6.2.6 Friction Material (Brake Lining)—The wearing material used to provide frictional effect between the rotating and non-rotating components of a *friction brake*.
- 6.2.7 Brake Pad—The *friction material*, together with a structural substrate, used in *disc brakes*.
- 6.2.8 Brake Shoe—The *friction material*, together with a structural substrate, used in *drum brakes*.
- 6.2.9 Brake Rotor (Brake Disc)—The rotating disc of a *disc brake* upon which the *brake pads* push during *brake* actuation.
- 6.2.10 Brake Drum—The rotating cylinder of a *drum brake* upon which the *brake shoes* push during *brake* actuation.

### 6.3 Brake Proportioning

- 6.3.1 Brake Balance—A general term used to describe the distribution of *tire braking forces* between the *axles* of a vehicle in relation to the distribution of *tire vertical loads* at a given steady-state *longitudinal acceleration*.

NOTE—A “balanced” brake system provides a distribution of *tire braking forces* that is equal to the distribution of *tire vertical loads* that prevails as the vehicle decelerates. A “front biased” brake system will exhibit a higher ratio of front *tire braking force* to rear *tire braking force* than the ratio of front *tire vertical load* to rear *tire vertical load*. A “rear biased” brake system has the converse *brake balance*.

- 6.3.2 Fixed Proportioning—A braking system that provides a nominally constant ratio of front to rear *brake torque* at all levels of *brake pedal force*.
- 6.3.3 Variable Proportioning—A braking system that alters the ratio of front to rear *brake torque* as a function of a relevant operating condition (e.g., *brake pressure*, *suspension ride displacement*, *longitudinal acceleration*).

## 7. TIRES AND WHEELS

The information contained in this section is derived from SAE J2047 (1998) and is relevant to vehicle dynamics. This section specifically does not contain information on dynamic forces, tire uniformity characteristics, tire noise and vibration, tire tread wear, tire structural degradation, and tire integrity.

Specific tire and rim dimensions and designations are established by tire and rim standards organizations, for example:

The Tire and Rim Association, 175 Montrose West Ave., Copley, OH 44321

Or

European Tyre and Rim Technical Organization, 32<sup>nd</sup> Avenue, Brugmann 1060 Brussels, Belgium.

The specific dimensions and designations are available in the tire and rim standards organizations' yearbooks.

## 7.1 Wheel Nomenclature

7.1.1 Wheel (Road Wheel)—A rotating, load-carrying member, upon which a *tire* is mounted, whose primary function is to support the vehicle and transmit *tire forces and moments*. It typically consists of two major parts: (a) the *rim* and (b) the wheel disc.

7.1.2 Rim—The part of the *wheel* on which the *tire* is mounted and supported. See Figure 9.

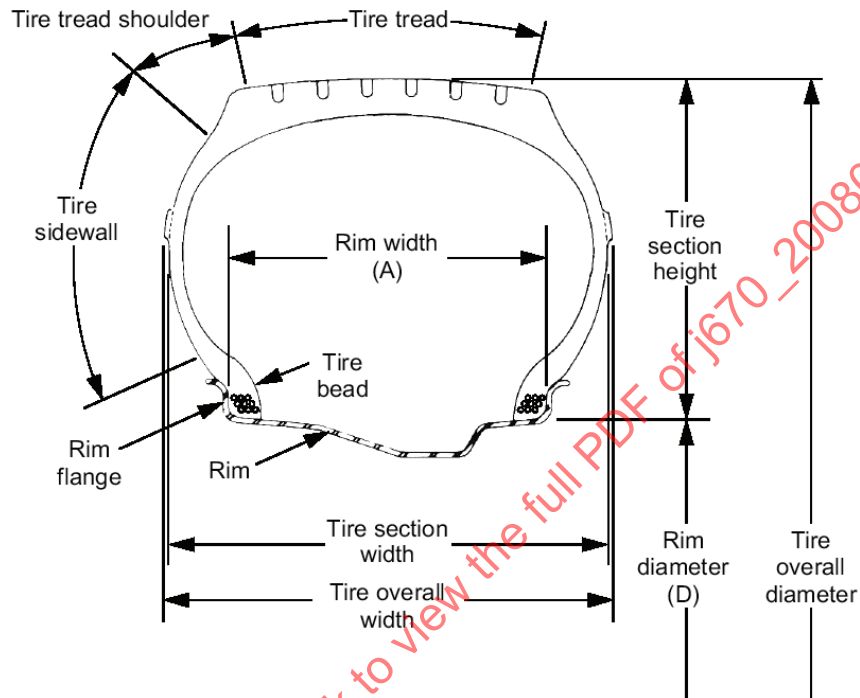


FIGURE 9 – NOMENCLATURE FOR TIRE AND RIM

7.1.2.1 Rim Width A—The distance between the inside surfaces of the *rim flanges*.

7.1.2.2 Rim Diameter (Specified Rim Diameter) D—The diameter at the intersection of the bead seat and the projection of the vertical portion of the *rim flange*.

7.1.2.3 Rim Diameter Designation (Nominal Rim Diameter)—The nominal *rim diameter* assigned for tire/rim matching.

7.1.2.4 Rim Contour Designation—A code comprised of numbers and/or letters designating the width and contour (designated shape) of the *rim*.

EXAMPLE—6J, where 6 is the *rim width* expressed in inches, and J is the contour.

7.1.2.5 Rim Size Designation—*Rim diameter designation* by (x) *rim contour designation*.

EXAMPLE—15 x 6J has a *rim diameter designation* of 15, and a *rim contour designation* of 6J.

7.1.2.6 Rim Flange—The part of the *rim* that provides lateral restraint for the *tire*.

## 7.2 Pneumatic Tire Nomenclature

7.2.1 Tire—A flexible, hollow semi-toroid mounted on the *rim* and filled with compressed gas, usually air. Figure 9 includes a cross-sectional view. The purposes of the tire are to attenuate road impact forces and to produce the vehicle control forces.

7.2.2 Tire Section Width—The width of an unloaded *tire* measured from sidewall to sidewall, excluding width due to protective ribs, bars, and decorations.

NOTE—This measurement is made on a new *tire* inflated to the recommended pressure 24 hours prior to measurement, to account for inflation growth.

7.2.3 Tire Overall Width—The width of an unloaded *tire*, including 24-hour inflation growth, measured from sidewall to sidewall, including width due to protective ribs, bars, and decorations.

7.2.4 Tire Overall (Outside) Diameter—The largest diameter of the unloaded *tire*, including 24-hour inflation growth.

7.2.5 Tire Section Height—The height of the radial cross-section of the *tire*, including 24-hour inflation growth. Tire section height is usually computed by subtracting the *nominal rim diameter* from the *tire overall diameter* and dividing by two.

7.2.6 Tire Aspect Ratio—Ratio of the *tire section height* to the *tire section width* multiplied by 100.

7.2.7 Tire Face—The outwardly directed side of a *tire* if mounted on a vehicle according to the vehicle manufacturer's specification or general practice.

7.2.8 Tire Load Index—A numerical code associated with the maximum *tire load* a *tire* can carry at the speed indicated by its *tire speed symbol* under specified service conditions.

7.2.9 Tire Speed Symbol—An alphabetic code indicating the speed category at which the *tire* can carry a *tire load* corresponding to its *tire load index* under specified service conditions.

7.2.10 Contact Patch (Footprint)—The portion of the *tire* touching the *road surface*.

7.2.11 Tire Designation—The numbers and/or letters indicating *tire* size and service description.

EXAMPLE—P205/60R15 90H is a specific example for a P-metric passenger *tire*. P denotes a passenger *tire*. 205 is the nominal *tire section width* in millimeters. 60 is the nominal *tire aspect ratio*. R denotes radial ply construction. 15 is the *rim diameter designation*. 90 is the *tire load index*. H is the *tire speed symbol*. For additional detail, refer to the appropriate tire and rim technical organization yearbook.

### 7.3 Wheel Plane Geometry

7.3.1 Tire/Wheel Assembly—The assembly composed of the *tire* and *wheel*.

7.3.2 Wheel-Spin Axis—The axis of *wheel* rotation. This axis is coincident with the  $Y_w$  axis.

7.3.3 Wheel Plane—A plane normal to the *wheel-spin axis*, which is located halfway between the *rim flanges*.

7.3.4 Wheel Center—The point at which the *wheel-spin axis* intersects the *wheel plane*.

NOTE—The *wheel center* is the origin of the *wheel coordinate system*.

7.3.5 Contact Line—The intersection of the *wheel plane* and the *road plane*.

7.3.6 Contact Center (Center of Tire Contact)—The intersection of the *contact line* and the normal projection of the *wheel-spin axis* onto the *road plane*.

NOTE—The *contact center* is the origin of the *tire coordinate system*. The *contact center* may not be the geometric center of the *tire contact patch* due to distortion of the *tire* produced by external forces.

7.3.7 Loaded Radius  $R_L$ —The distance from the *wheel center* to the *contact center* at a specified operating condition.

7.3.8 Static Loaded Radius SLR—The *loaded radius* at zero speed.

#### 7.4 Tire Orientation Angles

7.4.1 Wheel Plane Orientation—The angular orientation of the *wheel plane* with respect to the *road plane* and the *tire trajectory velocity*. The *wheel* angular orientation is expressed in terms of *inclination angle* and *slip angle*. See Figures 2, 8 and 10.

7.4.2 Slip Angle  $\alpha$ —The angle from the  $X_T$  axis to the normal projection of the *tire trajectory velocity* onto the  $X_T$ – $Y_T$  plane. See Figure 10.

NOTE—The sign of *slip angle* is determined using the right hand rule. For the same physical case, the sign of *slip angle* is reversed in the Z-Up and Z-Down axis orientations. See Table 2.

7.4.3 Inclination Angle  $\varepsilon$  ( $\gamma$ )—The angle from the  $Z_T$  axis to the  $Z_W$  axis. See Figures 8 and 10.

NOTE—The sign of *inclination angle* is determined using the right hand rule. For the same physical case, the sign of *inclination angle* is the same in the Z-Up and Z-Down axis orientations. See Table 2.

**TABLE 2 – RELATIONSHIPS BETWEEN Z-UP AND Z-DOWN TIRE AXIS SYSTEMS**

|                   | Tire Axis System |                 |
|-------------------|------------------|-----------------|
|                   | Z-Up             | Z-Down          |
| Slip Angle        | $\alpha$         | $= -\alpha$     |
| Inclination Angle | $\varepsilon$    | $= \varepsilon$ |
| Unit Vectors      | $i_{XT}$         | $= i_{XT}$      |
|                   | $i_{YT}$         | $= -i_{YT}$     |
|                   | $i_{ZT}$         | $= -i_{ZT}$     |
| Forces            | $F_{XT}$         | $= F_{XT}$      |
|                   | $F_{YT}$         | $= -F_{YT}$     |
|                   | $F_{ZT}$         | $= -F_{ZT}$     |
| Moments           | $M_{XT}$         | $= M_{XT}$      |
|                   | $M_{YT}$         | $= -M_{YT}$     |
|                   | $M_{ZT}$         | $= -M_{ZT}$     |
|                   | $T_W$            | $= -T_W$        |
| Velocities        | $v_{XT}$         | $= v_{XT}$      |
|                   | $v_{YT}$         | $= -v_{YT}$     |
|                   | $v_{ZT}$         | $= -v_{ZT}$     |
|                   | $\omega_W$       | $= -\omega_W$   |

#### 7.5 Tire Rolling Characteristics

7.5.1 Straight Free-Rolling Tire—A loaded, rolling *tire* moving with zero *braking* or *driving torque* along a linear path with zero *slip angle* and zero *inclination angle*.

7.5.2 Tire Trajectory Velocity  $\vec{v}_T$ —A vector quantity expressing the velocity of the *contact center* relative to the *Earth-fixed axis system*.

7.5.3 Tire Longitudinal Velocity  $v_{XT}$ —The scalar value of the component of the *tire trajectory velocity* in the  $X_T$  direction.

$$v_{XT} = \vec{v}_T \cdot \hat{X}_T \quad (\text{Eq. 27})$$

7.5.4 Tire Lateral Velocity  $v_{YT}$ —The scalar value of the component of the *tire trajectory velocity* in the  $Y_T$  direction.

$$v_{YT} = \vec{v}_T \cdot \hat{X}_T \quad (\text{Eq. 28})$$

7.5.5 Tire Vertical Velocity  $v_{ZT}$ —The scalar value of the component of the *tire trajectory velocity* in the  $Z_T$  direction.

$$v_{ZT} = \vec{v}_T \cdot \hat{Z}_T \quad (\text{Eq. 29})$$

## 7.6 Wheel Spin and Tire Slip

7.6.1 Wheel-Spin Velocity  $\omega_W$ —The angular velocity of the *wheel* about the  $Y_W$  axis.

NOTE—In previous editions of SAE J670, and in the 1998 edition of SAE J2047, positive *wheel-spin velocity* in the Z-Down axis orientation was defined to be consistent with a positive *tire longitudinal velocity*. In this document, the sign of *wheel-spin velocity* is determined using the right-hand rule for both the Z-Up and Z-Down axis orientations. See Appendix C.

7.6.2 Reference Wheel-Spin Velocity  $\omega_{W0}$ —The *wheel-spin velocity* of the *straight free-rolling tire* at a given set of operating conditions.

NOTE—The effects of *tire longitudinal force* and *tire rolling moment* in the straight free-rolling condition are included.

7.6.3 Tire Longitudinal Slip Velocity—The difference between the *wheel-spin velocity* and the *reference wheel-spin velocity*.

7.6.4 Tire Longitudinal Slip Ratio  $S_X$ —The ratio of *tire longitudinal slip velocity* to the *reference wheel-spin velocity*.

$$S_X = \frac{\omega_W - \omega_{W0}}{\omega_{W0}} \quad (\text{Eq. 30})$$

Where both  $\omega_W$  and  $\omega_{W0}$  are determined at the same *tire longitudinal velocity*, *tire load*, and *hot inflation pressure*.

7.6.5 Clockwise Tire Rotation (CW)—Clockwise rotation of the *tire face*.

7.6.6 Counterclockwise Tire Rotation (CCW)—Counterclockwise rotation of the *tire face*.

## 7.7 Standard Loads and Inflation Pressures

7.7.1 Cold Inflation Pressure—The gauge pressure within a *tire* at prevailing ambient temperature with no pressure build-up caused by tire service.

7.7.2 Hot Inflation Pressure (Warm Inflation Pressure)—The equilibrium gauge pressure within a *tire* at a given operating condition.

7.7.3 Tire Load—The load or weight supported by the *tire*.

7.7.3.1 Tire Load Limit (Tire Load Rating)—The maximum *tire load* recommended by a tire and rim standards organization for a particular tire at a given *cold inflation pressure*.

7.8 Wheel Torque  $T_w$ —The external torque exerted upon the *wheel* about the *wheel-spin axis*.

NOTE—In previous editions of SAE J670, and in the 1998 edition of SAE J2047, positive *wheel torque* in the Z-Down axis orientation was defined as a *driving torque* that produced a positive *tire longitudinal force*. In this document, the sign of *wheel torque* is determined using the right-hand rule for both the Z-Up and Z-Down axis orientations. See Appendix C.

7.8.1 Driving Torque—A *wheel torque* that produces a *tire driving force*.

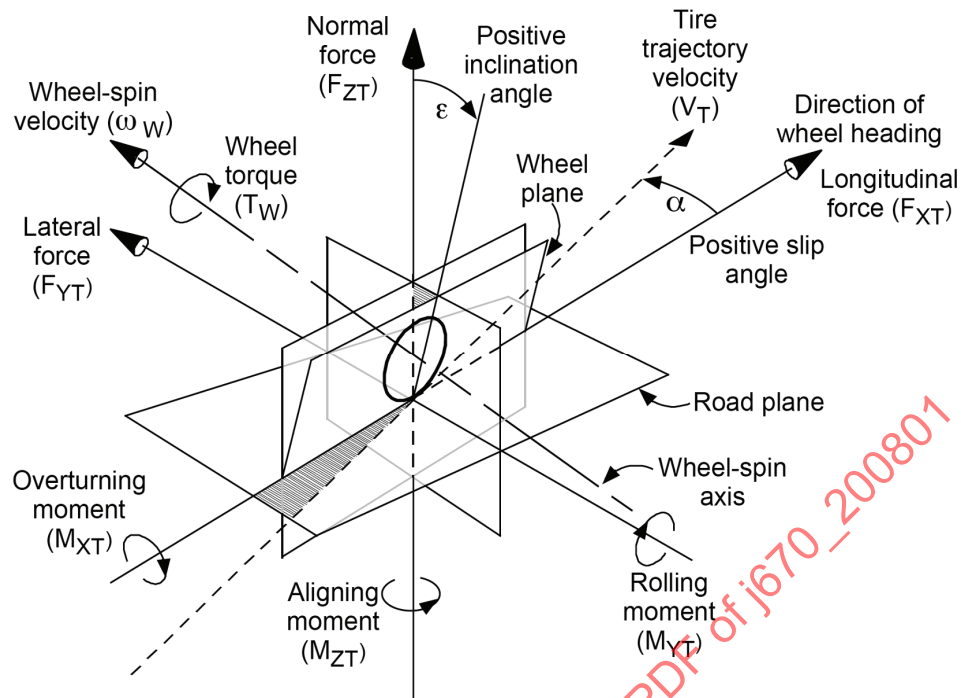
7.8.2 Braking Torque—A *wheel torque* that produces a *tire braking force*. See also 6.1.8 *brake torque*.

## 7.9 Tire Forces and Moments

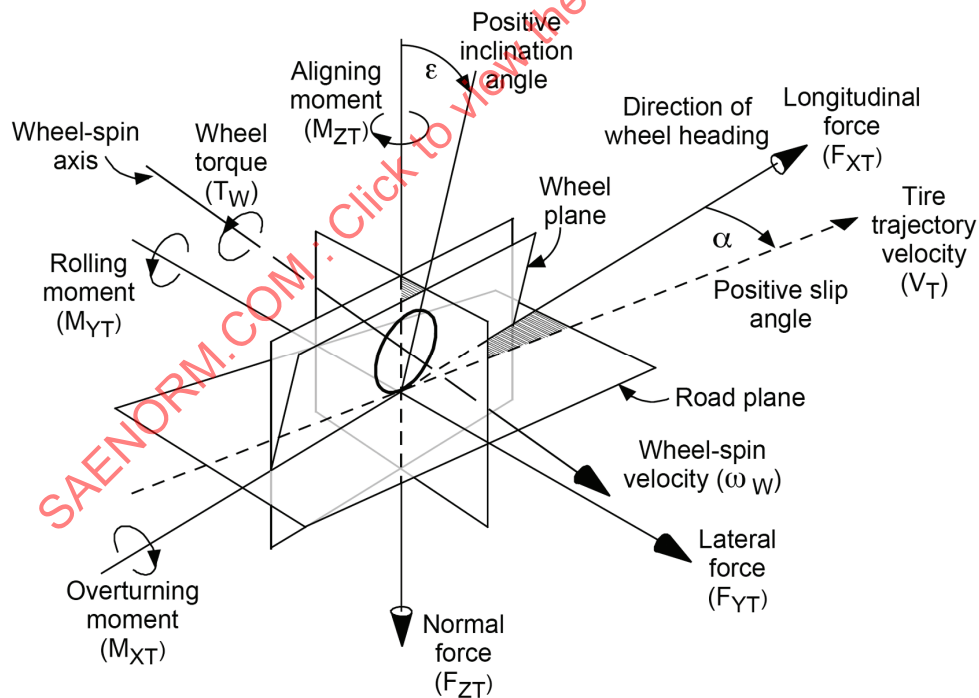
External forces and moments acting on the *tire* at any instant may be summed into one resultant force vector and one resultant moment vector. The line of action of the resultant force vector may be moved to any point of interest, providing that the resultant moment vector is altered accordingly.

The total force and moment exerted on the *tire* by the road can be represented by three force components and three moment components. One force and one moment act in the direction of, and about, each of the three axes in the *tire axis system* described in Section 3. See Figures 2 and 10. The three moments and two of the three forces are generally treated as dependent variables. In tire testing and tire modeling, *tire normal force* is generally treated as an independent variable and the magnitudes of the dependent forces and moments typically vary as a function of the *tire normal force*. Additionally, the magnitudes of the dependent forces and moments are somewhat affected by the direction of tire rotation, although this effect is usually neglected.

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A. TIRE FORCE AND MOMENT NOMENCLATURE – Z-UP



B. TIRE FORCE AND MOMENT NOMENCLATURE – Z-DOWN

FIGURE 10 – TIRE FORCE AND MOMENT NOMENCLATURE

7.9.1 Tire Force  $\vec{F}_T$ —A vector quantity expressing the sum of the forces exerted on the *tire* by the road at any instant, with its line of action passing through the *contact center*.

7.9.2 Tire Normal Force (Tire Radial Force)  $F_{ZT}$ —The scalar value of the component of the *tire force* in the direction of the  $Z_T$  axis.

$$F_{ZT} = \vec{F}_T \cdot \hat{Z}_T \quad (\text{Eq. 31})$$

7.9.3 Tire Vertical Load—The absolute value of *tire normal force*.

7.9.4 Tire Shear Force Vector  $\vec{F}_{XYT}$ —The vector projection of the *tire force* onto the  $X_T$ – $Y_T$  plane.

$$\vec{F}_{XYT} = \vec{F}_T - (\vec{F}_T \cdot \hat{Z}_T) \hat{Z}_T = F_{XT} \hat{X} + F_{YT} \hat{Y} \quad (\text{Eq. 32})$$

7.9.5 Tire Shear Force  $F_{XYT}$ —The magnitude of the *tire shear force vector*.

NOTE—*Tire shear force* is also the resultant of *tire longitudinal force* and *tire lateral force*.

7.9.6 Tire Longitudinal Force (Tire Fore-Aft Force)  $F_{XT}$ —The scalar value of the component of the *tire force* in the direction of the  $X_T$  axis.

$$F_{XT} = \vec{F}_T \cdot \hat{X}_T \quad (\text{Eq. 33})$$

7.9.6.1 Tire Driving Force—The magnitude of a positive *tire longitudinal force*.

7.9.6.2 Tire Braking Force—The magnitude of a negative *tire longitudinal force*.

7.9.7 Tire Lateral Force (Tire Side Force)  $F_{YT}$ —The scalar value of the component of the *tire force* in the direction of the  $Y_T$  axis.

$$F_{YT} = \vec{F}_T \cdot \hat{Y}_T \quad (\text{Eq. 34})$$

NOTE—For small *slip* and *inclination angles*, *tire lateral force* can be approximated by:

$$F_{YT} = -C_{Y\alpha} \cdot \alpha + C_{Y\varepsilon} \cdot \varepsilon \quad (\text{Eq. 35})$$

7.9.8 Tire Moment  $\vec{M}_T$ —A vector quantity expressing the sum of the external moments acting on the *tire* at any instant, consistent with the line of action of the *tire force*.

7.9.9 Tire Overturning Moment  $M_{XT}$ —The scalar value of the component of the *tire moment* in the direction of the  $X_T$  axis.

$$M_{XT} = \vec{M}_T \cdot \hat{X}_T \quad (\text{Eq. 36})$$

7.9.10 Tire Rolling Moment (Tire Rolling Resistance Moment)  $M_{YT}$ —The scalar value of the component of the *tire moment* in the direction of the  $Y_T$  axis.

$$M_{YT} = \vec{M}_T \cdot \hat{Y}_T \quad (\text{Eq. 37})$$

7.9.11 Tire Aligning Moment (Tire Aligning Torque)  $M_{ZT}$ —The scalar value of the component of the *tire moment* in the direction of the  $Z_T$  axis.

$$M_{ZT} = \vec{M}_T \cdot \hat{Z}_T \quad (\text{Eq. 38})$$

## 7.10 Pull Forces and Moments

Tire pull forces and moments are intrinsic tire properties arising from asymmetries in tire material, construction, or geometry due to either deliberate design choices, manufacturing irregularities, or both.

7.10.1 Lateral Force Offset  $F_{YT0}$ —The *tire lateral force* exerted on the *straight free-rolling tire*. It is typically different for *clockwise tire rotation* and *counterclockwise tire rotation*. See Figures 11 and 12.

NOTE—The *lateral force offset* is the sum of two components. One, *conicity lateral force*, behaves like a force arising from *inclination angle*. It is directed toward a particular side of the *tire* regardless of the direction of tire rotation. For example, if it were directed from the *wheel plane* toward the *tire face*, *conicity lateral force* has this direction regardless of whether the *tire* is rotating clockwise or counterclockwise. However, in the *tire axis system*, the *conicity lateral force* would change sign depending on the direction of tire rotation. The other component, *plysteer lateral force*, behaves like a force arising from *slip angle*. Its direction relative to the *tire face* depends on the direction of rotation. For example, if the *plysteer lateral force* were directed from the *wheel plane* toward the *tire face* in *clockwise tire rotation*, it would be directed away from the *tire face* in *counterclockwise tire rotation*. In the *tire axis system*, the sign of the *plysteer lateral force* would be independent of the direction of tire rotation.

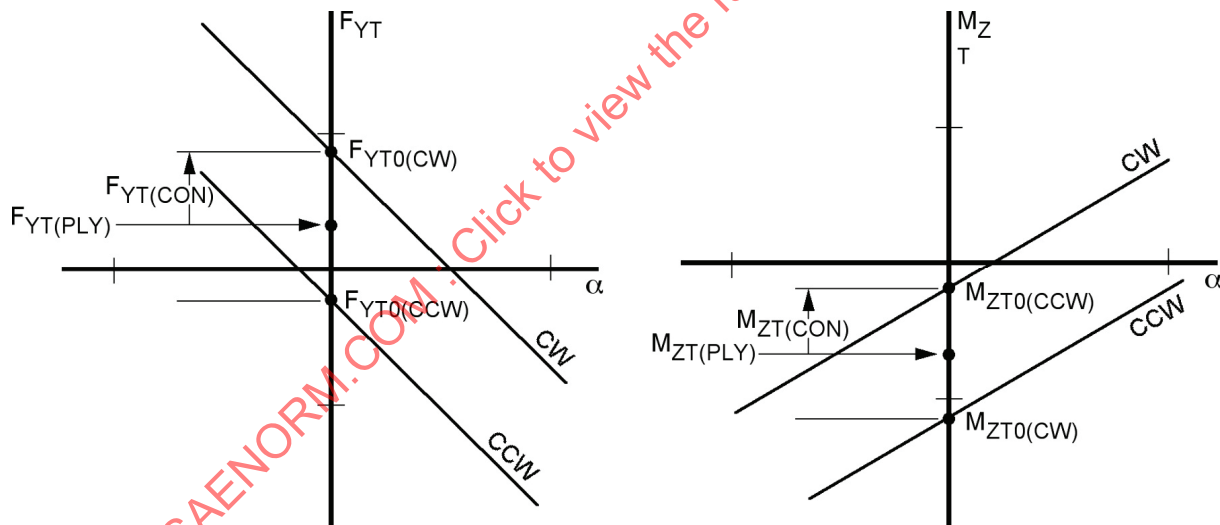


FIGURE 11 – COMPONENTS OF TIRE CONICITY AND PLYSTEER

7.10.1.1 Conicity Lateral Force  $F_{YT(CON)}$ —The portion of the *lateral force offset* that is dependent upon the direction of tire rotation. It is illustrated in Figure 11 and can be computed from the *lateral force offsets* in *clockwise* and *counterclockwise tire rotation* according to the following equation, which applies in the *tire axis system*:

$$F_{YT(CON)} = 0.5[F_{YT0}(CW) - F_{YT0}(CCW)] \quad (\text{Eq. 39})$$

NOTE—The equation shown above provides the correct magnitude and sign for *conicity lateral force* for a *tire* installed on the right side of the vehicle, with the *tire face* outboard, regardless of the *axis system* being used (Z-Up or Z-Down). For the same *tire* installed on the left side of the vehicle, with the *tire face* outboard, the sign of *conicity lateral force* is reversed.

- 7.10.1.2 Plysteer Lateral Force  $F_{YT(PLY)}$ —The portion of the *lateral force offset* that is independent of the direction of tire rotation. It is illustrated in Figure 11 and can be computed from the *lateral force offsets* in *clockwise* and *counterclockwise tire rotation* according to the following equation, which applies in the *tire axis system*:

$$F_{YT(PLY)} = 0.5[F_{YT0(CW)} + F_{YT0(CCW)}] \quad (\text{Eq. 40})$$

- 7.10.2 Residual Lateral Force  $F_{YTR}$ —The *tire lateral force* at the *slip angle* for which the *tire aligning moment* of the *straight free-rolling tire* at zero *inclination angle* is zero. It is typically different for *clockwise tire rotation* and *counterclockwise tire rotation*. See Figure 12.

- 7.10.2.1 Conicity Residual Lateral Force  $F_{YTR(CON)}$ —The portion of the *residual lateral force* that is dependent upon the direction of tire rotation. It is computed from the *residual lateral forces* in *clockwise* and *counterclockwise tire rotation* according to the following equation, which applies in the *tire axis system*:

$$F_{YTR(CON)} = 0.5[F_{YTR(CW)} - F_{YTR(CCW)}] \quad (\text{Eq. 41})$$

NOTE—The equation shown above provides the correct magnitude and sign for *conicity residual lateral force* for a *tire* installed on the right side of the vehicle, with the *tire face* outboard, regardless of the *axis system* being used (Z-Up or Z-Down). For the same *tire* installed on the left side of the vehicle, with the *tire face* outboard, the sign of *conicity residual lateral force* is reversed.

- 7.10.2.2 Plysteer Residual Lateral Force  $F_{YTR(PLY)}$ —The portion of the *residual lateral force* that is independent of the direction of tire rotation. It is computed from the *residual lateral forces* in *clockwise* and *counterclockwise tire rotation* according to the following equation, which applies in the *tire axis system*:

$$F_{YTR(PLY)} = 0.5[F_{YTR(CW)} + F_{YTR(CCW)}] \quad (\text{Eq. 42})$$

- 7.10.3 Aligning Moment Offset  $M_{ZT0}$ —The *tire aligning moment* exerted on the *straight free-rolling tire*. It is typically different for *clockwise tire rotation* and *counterclockwise tire rotation*. See Figures 11 and 12.

NOTE—The *aligning moment offset* is the sum of two components. One, *conicity aligning moment*, behaves like a moment arising from *inclination angle*. It tries to turn the *tire* in the direction of the *conicity lateral force*. Thus, in the *tire axis system*, it reverses in sense when the direction of tire rotation reverses. The other component, *plysteer aligning moment*, behaves like a moment arising from *slip angle*. It tries to turn the *tire* to eliminate the *plysteer lateral force*. Its sense is independent of the direction of tire rotation.

- 7.10.3.1 Conicity Aligning Moment  $M_{ZT(CON)}$ —The portion of the *aligning moment offset* that is dependent upon the direction of tire rotation. It is illustrated in Figure 11 and can be computed from the *aligning moment offsets* in *clockwise* and *counterclockwise tire rotation* according to the following equation, which applies in the *tire axis system*:

$$M_{ZT(CON)} = 0.5[M_{ZT0(CW)} - M_{ZT0(CCW)}] \quad (\text{Eq. 43})$$

NOTE—The equation shown above provides the correct magnitude and sign for *conicity aligning moment* for a *tire* installed on the right side of the vehicle, with the *tire face* outboard, regardless of the *axis system* being used (Z-Up or Z-Down). For the same *tire* installed on the left side of the vehicle, with the *tire face* outboard, the sign of *conicity aligning moment* is reversed.

- 7.10.3.2 Plysteer Aligning Moment  $M_{ZT(PLY)}$ —The portion of the *aligning moment offset* that is independent of the direction of tire rotation. It is illustrated in Figure 11 and can be computed from the *aligning moment offsets* in *clockwise* and *counterclockwise tire rotation* according to the following equation, which applies in the *tire axis system*:

$$M_{ZT(PLY)} = 0.5[M_{ZT0(CW)} + M_{ZT0(CCW)}] \quad (\text{Eq. 44})$$

7.10.4 Residual Aligning Moment  $M_{ZTR}$ —The *tire aligning moment* at the *slip angle* for which the *tire lateral force* of the *straight free-rolling tire* at zero *inclination angle* is zero. It is typically different for *clockwise tire rotation* and *counterclockwise tire rotation*. See Figure 12.

NOTE—The *residual aligning moment* is the sum of two components. One, *conicity residual aligning moment*, reverses in sense in the *tire axis system* when the direction of tire rotation reverses. The other component, *plysteer residual aligning moment*, is independent of the direction of tire rotation.

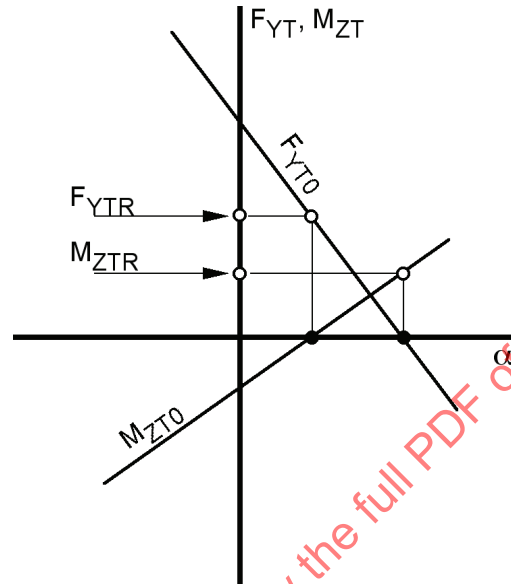


FIGURE 12 – TIRE RESIDUAL LATERAL FORCE AND ALIGNING MOMENT

7.10.4.1 Conicity Residual Aligning Moment  $M_{ZTR(CON)}$ —The portion of the *residual aligning moment* that is dependent upon the direction of tire rotation. It is illustrated in Figure 13 and can be computed using the following equation:

$$M_{ZTR(CON)} = 0.5[M_{ZTR(CW)} - M_{ZTR(CCW)}] \quad (\text{Eq. 45})$$

NOTE—The equation shown above provides the correct magnitude and sign for *conicity residual aligning moment* for a *tire* installed on the right side of the vehicle, with the *tire face* outboard, regardless of the *axis system* being used (Z-Up or Z-Down). For the same *tire* installed on the left side of the vehicle, with the *tire face* outboard, the sign of *conicity residual aligning moment* is reversed.

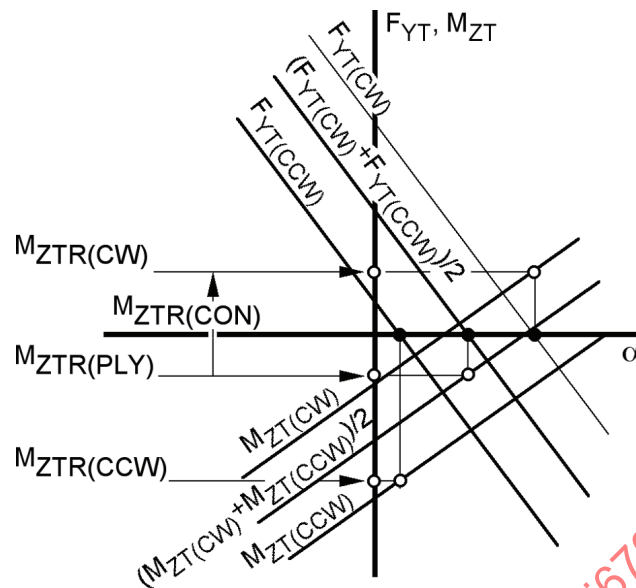


FIGURE 13 – TIRE CONICITY RESIDUAL ALIGNING MOMENT

7.10.4.2 Plysteer Residual Aligning Moment  $M_{ZTR(PLY)}$ —The portion of the *residual aligning moment* that is independent of the direction of tire rotation. It is illustrated in Figure 13 and can be computed using the following equation:

$$M_{ZTR(PLY)} = 0.5[M_{ZTR(CW)} + M_{ZTR(CCW)}] \quad (\text{Eq. 46})$$

#### 7.11 Properties of Forces in the Road Plane

Force coefficients are determined by dividing a *tire force* by *tire normal force*.

7.11.1 Tire Shear Force Coefficient (Tire Normalized Shear Force)—The absolute value of the ratio of *tire shear force* to *tire normal force*.

7.11.2 Tire Longitudinal Force Coefficient (Tire Normalized Longitudinal Force)—The ratio of the *tire longitudinal force* to the absolute value of *tire normal force*.

7.11.2.1 Tire Driving Force Coefficient (Tire Driving Coefficient, Tire Normalized Driving Force)—The absolute value of the ratio of *tire driving force* to *tire normal force*.

7.11.2.2 Tire Braking Force Coefficient (Tire Braking Coefficient, Tire Normalized Braking Force, Adhesion Utilization)—The ratio of the *tire braking force* to the absolute value of *tire normal force*.

NOTE—The *tire driving force coefficient* is a positive number and the *tire braking force coefficient* is a negative number.

7.11.3 Tire Lateral Force Coefficient (Tire Normalized Lateral Force)—The ratio of the *tire lateral force* to the absolute value of the *tire normal force*.

7.11.4 Tire Lateral Force Load Sensitivity—The first derivative of *tire lateral force* with respect to the absolute value of *tire normal force* at any given value of *slip angle*.

7.11.5 Tire Longitudinal Stiffness (Tire Braking / Driving Stiffness, Tire Longitudinal Force / Longitudinal Slip Gradient)  $C_{XS}$  ( $C_s$ )—The first derivative of *tire longitudinal force* with respect to *tire longitudinal slip ratio*.

NOTE—*Tire longitudinal stiffness* is usually determined at zero *slip angle*, zero *inclination angle*, and a *tire longitudinal slip ratio* of zero.

7.11.6 Tire Cornering Stiffness  $C_{Y\alpha}$  ( $C_{\alpha}$ )—The negative of the first derivative of *tire lateral force* with respect to *slip angle*.

NOTE—*Tire cornering stiffness* is usually determined at zero *slip angle*, zero *inclination angle*, with zero *wheel torque*.

7.11.7 Tire Inclination Stiffness (Tire Camber Stiffness)  $C_{Y\epsilon}$  ( $C_{\epsilon}$ ,  $C_{Y\gamma}$ ,  $C_{\gamma}$ )—The first derivative of *tire lateral force* with respect to *inclination angle*.

NOTE—*Tire inclination stiffness* is usually determined at zero *slip angle*, zero *inclination angle*, with zero *wheel torque*.

7.11.8 Tire Longitudinal Stiffness Coefficient (Tire Braking / Driving Stiffness Coefficient)  $C_{XSC}$  ( $C_{SC}$ )—The ratio of the *tire longitudinal stiffness* to the absolute value of *tire normal force*.

7.11.9 Tire Cornering Stiffness Coefficient (Tire Cornering Coefficient)  $C_{Y\alpha C}$  ( $C_{\alpha C}$ )—The ratio of the *tire cornering stiffness* to the absolute value of *tire normal force*. It is the initial slope of the *tire lateral force coefficient* diagram.

7.11.10 Tire Inclination Stiffness Coefficient (Tire Camber Stiffness Coefficient, Tire Camber Coefficient)  $C_{Y\epsilon C}$  ( $C_{\epsilon C}$ ,  $C_{Y\gamma C}$ ,  $C_{\gamma C}$ )—The ratio of the *tire inclination stiffness* to the absolute value of *tire normal force*.

7.11.11 Tire Relaxation Length—The distance rolled by a *tire* while *tire lateral force* builds to a particular percentage of the steady-state *tire lateral force*, following a step change in *slip angle* or other independent variable.

NOTE—*Tire relaxation length* is analogous to the time constant of a first-order system. Common usage refers to the behavior after a step change in *slip angle* from zero. The behavior is typically modeled by an exponential rise. In this model, *tire relaxation length* is the distance rolled associated with the change from the *tire lateral force* at the initiation of the unit-step *slip angle* to 63.2% of the change in steady-state *tire lateral force*.

7.11.12 Tire Lateral Force Response Phase Angle—The phase angle between a sinusoidal input of *slip angle* or other independent variable and the sinusoidal *tire lateral force* response.

## 7.12 Normal Force Properties

7.12.1 Tire Deflection—The amount by which the *tire section height* is reduced due to *tire normal force*.

7.12.2 Tire Normal Stiffness (Tire Radial Stiffness, Tire Spring Rate)  $k_{ZT}$ —The first derivative of the absolute value of *tire normal force* with respect to *tire deflection*.

NOTE—This definition is for a static stiffness, which implies stepwise or slowly varying changes in *tire normal force*. The definition is applicable for either a non-rolling or a rolling *tire*. Because the normal stiffness of a non-rolling and a rolling *tire* may be different, the operating condition should be explicitly stated. Other operating conditions influencing *tire normal stiffness* may include tire inflation pressure, *tire longitudinal velocity*, *slip angle*, *inclination angle*, and *wheel torque*.

## 7.13 Moment Properties

Moment coefficients are determined by dividing a *tire moment* by *tire normal force*.

7.13.1 Tire Overturning Moment Coefficient (Tire Normalized Overturning Moment)—The ratio of the *tire overturning moment* to the absolute value of the *tire normal force*.

7.13.2 Tire Rolling Moment Coefficient (Tire Normalized Rolling Moment)—The ratio of the *tire rolling moment* to the absolute value of the *tire normal force*.

- 7.13.3 Tire Aligning Moment Coefficient (Tire Normalized Aligning Moment)—The ratio of the *tire aligning moment* to the absolute value of the *tire normal force*.
- 7.13.4 Tire Aligning Moment Load Sensitivity—The first derivative of *tire aligning moment* with respect to the absolute value of *tire normal force* at any given value of *slip angle*.
- 7.13.5 Tire Overturning Stiffness  $C_{XX\alpha}$ —The first derivative of the *tire overturning moment* with respect to *slip angle*.
- 7.13.6 Tire Overturning Inclination Stiffness  $C_{XX\epsilon}$  ( $C_{XX\gamma}$ )—The first derivative of the *tire overturning moment* with respect to *inclination angle*.
- 7.13.7 Tire Aligning Stiffness  $C_{ZZ\alpha}$  ( $C_M$ )—The first derivative of the *tire aligning moment* with respect to *slip angle*.

NOTE—*Tire aligning stiffness* is usually determined at zero *slip angle*, zero *inclination angle*, with zero *wheel torque*.

- 7.13.8 Tire Aligning Inclination Stiffness  $C_{ZZ\epsilon}$  ( $C_{ZZ\gamma}$ )—The first derivative of the *tire aligning moment* with respect to *inclination angle*.

NOTE—*Tire aligning inclination stiffness* is usually determined at zero *slip angle*, zero *inclination angle*, with zero *wheel torque*.

- 7.13.9 Tire Aligning Stiffness Coefficient  $C_{ZZ\alpha C}$ —The ratio of the *tire aligning stiffness* to the absolute value of the *tire normal force*.

#### 7.14 Tire/Road Friction

In the following definitions, the term "steady-state operating conditions" refers to *road surface*, *tire load*, *tire longitudinal velocity*, temperature, inflation pressure, *slip angle*, *inclination angle*, *tire longitudinal slip ratio*, etc.

- 7.14.1 Tire Friction Ellipse—The locus of points of a plot of *tire lateral force* versus *tire longitudinal force* (or *tire lateral force coefficient* versus *tire longitudinal force coefficient*) for a *tire* at a particular set of steady-state operating conditions, with all conditions held fixed except one, which is independently varied.

NOTE—Typically, tire measurements to develop a *tire friction ellipse* are performed by independently varying *tire longitudinal slip ratio* while holding all other operating conditions fixed, including *slip angle*. By incrementally changing *slip angle* and repeating the measurement, a series of *tire friction ellipses* may be developed, with each describing the steady-state lateral and longitudinal force generating capability of the *tire* with a given set of operating conditions.

- 7.14.2 Tire Friction Ellipse Boundary—The locus of points describing the outermost envelope of a series of *tire friction ellipse* plots for a given *tire*, with a given set of steady-state operating conditions. This boundary describes the combined lateral and longitudinal force generation capability limits of the *tire*.

- 7.14.3 Peak Coefficient of Friction  $\mu_p$ —The maximum values of the *tire shear force coefficient* attainable by a *tire* with a given set of steady-state operating conditions: 1) during the application of a *braking torque* at zero *slip angle*; 2) during the application of a *driving torque* at zero *slip angle*; or 3) when cornering with zero applied *wheel torque*.

NOTE—Typically, the *peak coefficients of friction* determined in the three pure modes of braking, driving, and cornering will have different values; moreover, the values for cornering may differ for positive and negative *slip angles*. The *peak coefficients of friction* determined in these pure modes define the intersections of the *tire friction ellipse boundary* with the  $F_{XT}$ – $F_{YT}$  axes.

- 7.14.4 Slide Coefficient of Friction  $\mu_s$ —The values of the *tire shear force coefficient* for a *tire* at a given set of steady-state operating conditions that is: 1) sliding across the surface without angular velocity (locked by braking); 2) spinning because of a heavy *driving torque* application; or 3) skidding sideways at a 90° *slip angle*.

NOTE—While the sliding tire condition is well defined for both braking and cornering (locked *wheel* and 90° *slip angle*, respectively), the spinning tire condition is open-ended. For practical purposes, the *slide coefficient of friction* for the spinning tire condition may be determined at a *tire longitudinal slip ratio* of 1.0 (*tire* spinning so that the *wheel-spin velocity* is twice the *reference wheel-spin velocity*).

- 7.14.5 Peak-to-Slide Ratio  $\mu_p/\mu_s$ —The ratio of the *peak coefficient of friction* to the *slide coefficient of friction* for a *tire* at a set of steady-state operating conditions in a given operating mode (braking, driving, or cornering).

NOTE—*Peak-to-slide ratio* is often used to characterize a given *road surface*. In reality, it is a property of the *road surface*, *tire*, and the steady-state operating conditions.

## 8. STATES AND MODES

### 8.1 Equilibrium

- 8.1.1 Steady State (Trim)—State of a vehicle wherein the sum of the applied external forces and moments, and the inertial forces and moments which balance them, form an unchanging force and moment system in the ( $X_v$ ,  $Y_v$ ,  $Z_v$ ) and ( $X$ ,  $Y$ ,  $Z$ ) *reference frames* over an arbitrarily long time period. Used as the reference state for analysis of dynamic vehicle stability and control characteristics.

NOTE—An arbitrarily long time period as used in the above definition is a time period that is long relative to the longest time constant of the system being examined.

- 8.1.2 Transient State—State of a vehicle wherein the applied external forces and moments, the control positions, or the vehicle motion responses are varying with time.

### 8.2 Stability

- 8.2.1 Non-Oscillatory Stability (Asymptotic Stability)—Stability characteristic at a prescribed *steady state* if, following any small temporary *disturbance* or *control input*, the vehicle will return to the initial *steady-state* condition without oscillation.
- 8.2.2 Neutral Stability—Stability characteristic at a prescribed *steady state* if, following any small temporary *disturbance* or *control input*, the resulting motion of the vehicle remains close to, but does not return to, the initial *steady-state* condition.
- 8.2.3 Oscillatory Stability—Stability characteristic at a prescribed *steady state* if a small temporary *disturbance* or *control input* causes an oscillatory *vehicle response* of decreasing amplitude and a return to the initial *steady-state* condition.
- 8.2.4 Non-Oscillatory Instability (Divergent Instability)—Stability characteristic at a prescribed *steady state* if a small temporary *disturbance* or *control input* causes an ever-increasing *vehicle response* without oscillation. See Figure 14.
- 8.2.5 Oscillatory Instability—Stability characteristic at a prescribed *steady state* if a small temporary *disturbance* or *control input* causes an oscillatory *vehicle response* of ever-increasing amplitude about the initial *steady-state* condition.

### 8.3 Control Modes

The following definitions are generic. Typically, they refer to the driver-vehicle interfaces used for the application of *steering*, *braking*, or *acceleration control inputs*.

- 8.3.1 Position Control—The mode of vehicle control wherein *control inputs* or restraints in the form of displacements are placed at some control point in the system, independent of the force required.
- 8.3.2 Fixed Control—The mode of vehicle control wherein the position of some point in the control system is held fixed. This is a special case of *position control*.
- 8.3.3 Force Control—The mode of vehicle control wherein *control inputs* or restraints in the form of forces are placed upon some control point in the system, independent of the displacement required.
- 8.3.4 Free Control—The mode of vehicle control wherein no restraint is placed on the control system. This is a special limit case of *force control*.
- 8.3.5 Closed-Loop Control (Feedback Control)—The mode of vehicle control wherein information about the *vehicle response* is fed back to the input controller (driver or mechanical actuator) for comparison with the desired *vehicle response*, and the *control inputs* are modified to reduce the error between the actual and desired *vehicle response*.
- 8.3.6 Open-Loop Control—The mode of vehicle control wherein *control inputs* are independent of the resulting *vehicle response*.

## 9. INPUTS AND RESPONSES

### 9.1 Inputs

- 9.1.1 Control Input—A positioning of, or application of force to, an element of the vehicle (typically, within the steering, braking or propulsion systems) for the purpose of maintaining, or inducing a change in, motion of the vehicle.
- 9.1.1.1 Steering Control Input—Positioning of, or application of force to, an element of the steering system (e.g., *steering wheel*, front *wheels*, *Pitman arm*, steering rack) for the purpose of maintaining, or inducing a change in, the direction of motion of the vehicle.
- 9.1.1.1.1 Steering-Wheel Angle (Hand-Wheel Angle)  $\delta_{SW}$  ( $\delta_H$ )—The angular displacement of the *steering wheel* measured from a predetermined straight-ahead position. When using the Z-Up axis orientation, counterclockwise rotation of the *steering wheel* is positive. When using the Z-Down axis orientation, clockwise rotation of the *steering wheel* is positive.
- NOTE—Straight ahead may be determined as the *steering-wheel angle* that produces zero *yaw velocity* under a given set of initial conditions or as the *steering-wheel angle* that provides a zero net *steer angle*.
- 9.1.1.1.2 Steering-Wheel Torque (Hand-Wheel Torque)  $M_{SW}$  ( $M_H$ )—The moment applied to the *steering wheel*, usually by the driver, about its axis of rotation.
- 9.1.1.1.3 Steering-Wheel Rim Force (Hand-Wheel Rim Force)  $F_{SW}$  ( $F_H$ )—*Steering-wheel torque* divided by the average value of *steering-wheel* radius, measured to the center of the *steering-wheel* rim.
- 9.1.1.2 Braking Control Input—Positioning of, or application of force to, an element of the braking system (e.g., *brake pedal*, *master cylinder*) for the purpose of maintaining, or inducing a change in, *vehicle speed*.
- 9.1.1.2.1 Brake Pedal Force—The force applied to the *brake pedal*, usually by the driver, tangential to the *brake pedal* pivot axis.
- 9.1.1.2.2 Brake Pedal Displacement (Brake Pedal Travel)—The linear displacement of the center of the *brake pedal* relative to its position with zero *brake pedal* force applied, measured along a chord.

9.1.1.3 Acceleration Control Input—Positioning of, or application of force to, an element of the propulsion system (e.g., accelerator pedal, throttle lever) for the purpose of maintaining, or inducing a change in, *vehicle speed*.

9.1.1.3.1 Accelerator Pedal Position—The angular displacement of the accelerator pedal relative to its position at idle.

9.1.1.3.2 Throttle Position—The angular displacement of the engine throttle lever relative to its position at idle.

NOTE—*Accelerator pedal position* and *throttle position* are often expressed in percent of wide-open throttle.

9.1.1.4 Driver Control Input—A *control input* initiated by actions of the human driver.

9.1.1.5 Automatic Control Input—A *control input* initiated by an automatic control system within the vehicle.

9.1.2 Control System Input—An event (e.g., approach of another vehicle, change in *yaw velocity*) that induces a change in the signal of a sensor (e.g., a radar range sensor or a *yaw velocity* transducer) that is part of a system that generates *automatic control inputs*.

9.1.3 Driver Settings—Parameters of a system that generate *automatic control inputs* and are established by the driver (e.g., cruise control set speed).

9.1.4 Disturbance Input—An influence on the vehicle that induces a change in motion of the vehicle.

9.1.4.1 Aerodynamic Disturbance Input—A change in wind direction or speed relative to the vehicle, resulting in a change in aerodynamic forces acting on the vehicle.

9.1.4.2 Load-Shift Disturbance Input—A shift of mass within a vehicle.

9.1.4.3 Road Disturbance Input—A change in road properties (e.g., friction, surface roughness, profile, elevation, gradient), resulting in a change in forces or moments at the tire/road interface.

9.1.4.4 Tire/Wheel Disturbance Input—Rotation of a non-uniform or out-of-balance *tire/wheel assembly*, resulting in oscillating forces or moments at the tire/road interface or at the wheel spindle.

9.1.5 Equivalent Time of Initiation—The time at which a *control*, *control system*, or *disturbance input*, in the form of a ramp-step, reaches a user-defined percentage of the difference between the initial and final constant values.

NOTE—This time is considered equivalent to the time of initiation of a true step input and is used as the reference for computing *rise time*, *peak response time*, and *settling time*. Commonly used percentages are 5, 10, and 50.

## 9.2 Responses

9.2.1 Vehicle Response—The vehicle motion resulting from *control* or *disturbance inputs* to the vehicle.

9.2.2 Control Response—The vehicle motion resulting from a *control input*.

9.2.3 Disturbance Response—The vehicle motion resulting from a *disturbance input*.

9.2.4 Steady-State Response—The *steady state* associated with a set of constant inputs.

9.2.4.1 Gain (Sensitivity)—The rate of change in the *steady-state response* of a motion variable with respect to a *control input* at a given *steady-state* condition.

NOTE—Many *gain* measures may be created using this generic definition. Commonly used *gains* are defined elsewhere in this document. *Gain* measures not in common use should be created by referencing both the output and input in the *gain* name. For example, "Vehicle Roll Angle / Steering-Wheel Angle Gain" would be the rate of change of *vehicle roll angle* with respect to *steering-wheel angle*.

9.2.4.2 Gradient—The rate of change in the *steady-state response* of a variable with respect to a motion variable (typically, lateral acceleration) at a given *steady-state* condition.

NOTE—Many *gradient* measures may be created using this generic definition. Commonly used *gradients* are defined elsewhere in this document. *Gradient* measures not in common use should be created by referencing both the output and input in the *gradient* name. For example, "Steering-Wheel Torque / Lateral Acceleration Gradient" would be the rate of change of *steering-wheel torque* with respect to *lateral acceleration*.

9.2.5 Transient Response—The *transient state* resulting from a set of time-varying inputs.

NOTE—The inputs may include step, ramp-step, sinusoidal, pulse, or other time-varying *control* or *disturbance inputs*. Responses may include *lateral acceleration*, *yaw velocity*, *vehicle speed*, *vehicle roll angle*, *vehicle sideslip angle*, *path curvature*, *yaw angle*, or many others.

9.2.5.1 Dynamic Gain (Dynamic Sensitivity)—The ratio of a measure of the magnitude of the *transient response* of a motion variable to a measure of the magnitude of an input, in a specified transient maneuver.

NOTE—This definition is applicable for periodic or aperiodic inputs, linear or nonlinear responses, and analysis in either the time domain or frequency domain. Many *dynamic gain* measures may be created using this generic definition.

9.2.5.2 Dynamic Gradient—The ratio of a measure of the magnitude of the *transient response* of a motion variable to a measure of the magnitude of another motion variable (typically, *lateral acceleration*), in a specified transient maneuver.

NOTE—This definition is applicable for periodic or aperiodic inputs, linear or nonlinear responses, and analysis in either the time domain or frequency domain. Many *dynamic gradient* measures may be created using this generic definition.

9.2.6 Rise Time (Response Time)—The interval between the *equivalent time of initiation* of a ramp-step input and the time at which the *transient response* of a motion variable first reaches a user-defined percentage of the difference between the final and initial *steady-state* values.

NOTE—A commonly used percentage for determining *rise time* is 90. If a percentage other than 90 is used to determine *rise time*, the percentage should be specified (e.g., 63-Percent Rise Time).

9.2.7 Peak Response Time—The interval between the *equivalent time of initiation* of a ramp-step input and the time at which the *transient response* of a motion variable reaches its peak value, provided there is an *overshoot*.

9.2.8 Settling Time—The interval between the *equivalent time of initiation* of a ramp-step input and the time at which the *transient response* of a motion variable last enters and remains within a user-defined tolerance band around the final *steady-state* value. The band is expressed as a percentage of the difference between the final and initial *steady-state* values.

NOTE—Commonly used percentages for the tolerance band are  $\pm 2$ ,  $\pm 5$ , and  $\pm 10$ .

9.2.9 Overshoot—For an underdamped system, the difference between the peak value and the final *steady-state* value of a motion variable in response to a ramp-step input.

9.2.10 Percent Overshoot—The ratio of the *overshoot* to the difference between the final and initial *steady-state* values of a motion variable, expressed as a percentage.

9.2.11 Bandwidth—The frequency at which the response has declined 3 dB from its low-frequency value.

9.2.12 Peak to Steady-State Ratio—The ratio of the peak frequency response to the low-frequency value.

## 10. VEHICLE LONGITUDINAL RESPONSE

10.1 Longitudinal Load Transfer—The net *tire vertical load* transferred from one *axle* to the other *axle*. It can be caused by *longitudinal acceleration*, road pitch, drivetrain torque, or other effects.

### 10.2 Descriptors of Steady-State Longitudinal Response

#### 10.2.1 Gain Measures

Many gain measures may be defined using the generic definition for *gain (sensitivity)* in 9.2.4.1. The following definitions are in common use.

10.2.1.1 Brake Pedal Force Gain—The absolute value of the rate of change of *longitudinal acceleration*,  $a_x$ , with respect to *brake pedal force*, on a level road at a given *steady-state longitudinal acceleration* condition.

10.2.1.2 Accelerator Pedal Position Gain—The rate of change of *longitudinal acceleration*,  $a_x$ , with respect to *accelerator pedal position*, on a level road at a given *steady-state longitudinal acceleration* condition.

#### 10.2.2 Gradient Measures

Many gradient measures may be defined using the generic definition for *gradient* in 9.2.4.2. The following definitions are in common use.

10.2.2.1 Pitch Angle Gradient—The rate of change of *pitch angle*,  $\theta$ , with respect to *longitudinal acceleration*,  $a_x$ , on a level road at a given *steady-state longitudinal acceleration* condition.

NOTE—*Pitch angle gradients* determined under acceleration and braking will typically have different magnitudes.

10.2.2.2 Suspension Trim Height Gradient—The mean rate of change of the *suspension trim heights* on an *axle* with respect to *longitudinal acceleration*,  $a_x$ , on a level road at a given *steady-state longitudinal acceleration* condition.

NOTE—*Suspension trim height gradients* determined under acceleration and braking will typically have different magnitudes.

### 10.3 Descriptors of Transient Longitudinal Response

10.3.1 Pitch Angle Overshoot—The difference between the peak value and the final *steady-state* value of *pitch angle*,  $\theta$ , in response to a ramp-step *acceleration* or *braking control input*.

### 10.4 Descriptors of Transient Brake System Response

10.4.1 Brake Apply Rise Time—The interval between the time at which a ramp-step *braking control input* reaches 50% of its final value and the time at which the *braking control response* reaches 90% of its *steady-state response*.

NOTE—*Brake apply rise time* is a measure of the response speed of braking to a *braking control input* (e.g., *brake pedal force*). Typical *control responses* include *brake pressure*, *brake torque*, and vehicle *longitudinal acceleration*.

10.4.2 Brake Release Time—The interval between the time at which a *braking control input* is removed and the time at which the change in *braking control response* reaches 90% of its *steady-state* magnitude.

NOTE—*Brake release time* is a measure of the response speed of braking to removal of a *braking control input* (e.g., *brake pedal force*). Typical *control responses* include *brake pressure*, *brake torque*, and vehicle *longitudinal acceleration*.

## 10.5 Characterizing Descriptors of Braking Performance

10.5.1 Adhesion Utilization—See 7.11.2.2 *Tire Braking Force Coefficient*.

10.5.2 Braking Efficiency—The absolute value of the maximum steady state *longitudinal acceleration* (in g units) without *wheel lockup* divided by the nominal *peak coefficient of friction* in braking, expressed as a percentage.

NOTE—*Braking efficiency* is a general concept to approximately characterize the adhesion utilization of a vehicle relative to the prevailing *tire/road friction*. Accordingly, the *peak coefficient of friction* used is a representative value for the *tires* on the vehicle. Because *peak coefficient of friction* is often not well known, *braking efficiency* may exceed 100%.

## 11. VEHICLE LATERAL RESPONSE

### 11.1 Lateral Load Transfer

The terms in this subsection are most useful in *steady-state* conditions.

11.1.1 Tire Lateral Load Transfer—The *tire vertical load* transferred from one *tire* to the other on an *axle*. It can be caused by *lateral acceleration*, road camber, drivetrain torque, or other effects.

11.1.2 Total Tire Lateral Load Transfer—The sum of the *tire lateral load transfers* for the front and rear *axles*.

11.1.3 Tire Lateral Load Transfer Distribution—The ratio of the *tire lateral load transfer* on the front *axle* to the *total tire lateral load transfer*, usually expressed as a percentage.

11.1.4 Overturning Couple—The *roll moment* on the vehicle with respect to a central, longitudinal axis in the *ground plane* due to the effects of *lateral acceleration*, *vertical acceleration*, and *roll acceleration*.

11.1.5 Overturning Couple Distribution—The portion of the *overturning couple* reacted by the front *tires* expressed as a percentage of the *overturning couple*.

### 11.2 Ranges of Directional Response

11.2.1 On-Center Range—The *lateral acceleration* range associated with operation on a nearly straight *vehicle path* (typically  $\pm 1 \text{ m/s}^2$ ), over which *vehicle response* to *steering control inputs* is not proportional.

11.2.2 Linear Range—The *lateral acceleration* range over which *vehicle response* to *steering control inputs* is nearly proportional (beyond the *on-center range*, typically to about  $\pm 4 \text{ m/s}^2$ ).

11.2.3 Nonlinear Range—The *lateral acceleration* range over which *vehicle response* to *steering control inputs* is not proportional (beyond the *linear range*).

### 11.3 Descriptors of Steady-State Directional Response

Many of the descriptors characterizing vehicle directional stability and control come from classical proving-ground tests or linear analyses using simple vehicle models such as bicycle models. Thus, the definitions that follow may not rigorously include the influences of certain *compliances*, aerodynamics, traction/braking, active directional control systems, etc.

#### 11.3.1 Gain or Sensitivity Measures

Many gain and sensitivity measures may be defined using the generic definition for *gain (sensitivity)* in 9.2.4.1. The following definitions are in common use.

- 11.3.1.1 Lateral Acceleration Gain (Steering Sensitivity)—The rate of change of *lateral acceleration*,  $a_Y$ , with respect to *steering-wheel angle*,  $\delta_{SW}$ , on a level road at a given *steady-state* condition. The value may be determined at either constant radius or constant speed. Values determined at these two conditions will generally differ. This is the reciprocal of *steering-wheel angle gradient*.

NOTE—Steady-state *lateral acceleration* is a function of *longitudinal velocity*  $v_X$  and *path radius*  $R$  such that a differential change in *lateral acceleration* may, in general, be expressed as:

$$da_Y = \frac{\partial a_Y}{\partial v_X} dv_X + \frac{\partial a_Y}{\partial R} dR \quad (\text{Eq. 47})$$

An incremental change in steady-state *lateral acceleration* may be brought about: 1) by a change of *path radius* at a specified *vehicle speed* ( $dv_X = 0$  and  $dR \neq 0$ ) (i.e., the constant-speed method); 2) by a change of *vehicle speed* at a specified *path radius* ( $dR = 0$  and  $dv_X \neq 0$ ) (i.e., the constant-radius method); or 3) at least in concept, by changes in both *vehicle speed* and *path radius* according to a specified relationship ( $dR = f(dv_X)$ ). The change in *steer angle* required to produce a given change in *lateral acceleration* (and, therefore, the lateral acceleration gain) may depend on the method. Therefore, the method as well as the nominal *steady-state* condition should always be indicated along with the lateral acceleration gain.

In the *steady-state* condition, *lateral acceleration* is equal to the product of *centripetal acceleration* times the cosine of the *vehicle sideslip angle*. Since in most test conditions the *vehicle sideslip angle* is small, for practical purposes, the *lateral acceleration* can be considered equal to the *centripetal acceleration*.

- 11.3.1.2 Lateral Acceleration Moment Gain (Control Moment Sensitivity)—The rate of change of *lateral acceleration*,  $a_Y$ , with respect to *steering-wheel torque*,  $M_{SW}$ , on a level road at a given *steady-state* condition. This is the reciprocal of *steering-wheel torque gradient*.
- 11.3.1.3 Yaw Velocity Gain—The rate of change of *yaw velocity*,  $\omega_Z$ , with respect to *steering-wheel angle*,  $\delta_{SW}$ , on a level road at a given *steady-state* condition.
- 11.3.1.4 Sideslip Angle Gain—The rate of change of *vehicle sideslip angle*,  $\beta$ , with respect to *steering-wheel angle*,  $\delta_{SW}$ , on a level road at a given *steady-state* condition.
- 11.3.1.5 Path Curvature Gain—The rate of change of *path curvature*,  $\kappa$ , with respect to *steering-wheel angle*,  $\delta_{SW}$ , on a level road at a given *steady-state* condition.
- 11.3.1.6 Steering-Wheel Torque Gain (Steering Stiffness)—The rate of change of *steering-wheel torque*,  $M_{SW}$ , with respect to *steering-wheel angle*,  $\delta_{SW}$ , on a level road at a given *steady-state* condition.

## 11.3.2 Gradient Measures

Many gradient measures may be defined using the generic definition for *gradient* in 9.2.4.2. The following definitions are in common use.

- 11.3.2.1 Steering-Wheel Angle Gradient (Hand-Wheel Angle Gradient)—The rate of change of *steering-wheel angle*,  $\delta_{SW}$ , with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition. This is the reciprocal of *lateral acceleration gain*.
- 11.3.2.2 Steering-Wheel Torque Gradient (Hand-Wheel Torque Gradient)—The rate of change of *steering-wheel torque*,  $M_{SW}$ , with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition. This is the reciprocal of *lateral acceleration moment gain*.
- 11.3.2.3 Reference Steer Angle Gradient—The rate of change of *included reference steer angle*,  $\delta_{REF}$ , with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition.

11.3.2.4 Ackermann Steer Angle Gradient—The rate of change of *included Ackermann steer angle*,  $\delta_A$ , with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition.

NOTE—For the conventional four-wheel vehicle with no rear steer, the *Ackermann steer angle gradient* is traditionally defined as  $(180/\pi)$  times the *wheelbase* divided by the square of the *vehicle speed*, commonly expressed in degrees per g.

11.3.2.5 Roll Angle Gradient—The rate of change of *vehicle roll angle*,  $\phi_V$ , with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition.

11.3.2.6 Suspension Roll Angle Gradient—The rate of change of *suspension roll angle* with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition.

11.3.2.7 Sideslip Angle Gradient—The rate of change of *vehicle sideslip angle*,  $\beta$ , with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition.

11.3.2.8 Path Curvature Gradient—The rate of change of *path curvature*,  $\kappa$ , with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition.

11.3.2.9 Trailer Yaw Articulation Angle Gradient (Articulation Angle Gradient)—The rate of change of *trailer yaw articulation angle*,  $\Delta\psi$ , with respect to *lateral acceleration*,  $a_Y$ , on a level road at a given *steady-state* condition.

### 11.3.3 Understeer and Oversteer

11.3.3.1 Understeer / Oversteer Gradient  $U$ —The quantity obtained by subtracting the *Ackermann steer angle gradient* from the *reference steer angle gradient*, on a flat, level road at a given *steady-state* condition.

NOTE—In general, in *steady-state* cornering, the *included reference steer angle* may be a function of the *included Ackermann steer angle* and *lateral acceleration*:

$$\delta_{REF} = f(\delta_A, a_Y) \quad (\text{Eq. 48})$$

For a vehicle whose response is linearized about a *steady-state* condition, the *included reference steer angle* would depend on the *included Ackermann steer angle* and *lateral acceleration* according to the following relationship:

$$\delta_{REF} = \delta_A + Ua_Y \quad (\text{Eq. 49})$$

It is under these conditions that the *understeer / oversteer gradient* definition is strictly defined.

For a simplified vehicle without aerodynamic forces and moments, without rear steer (except via *suspension compliances*) or with a fixed front/rear steer ratio relationship, and within the *linear range*, the *understeer / oversteer gradient* is a constant, which is independent of speed, *lateral acceleration* and *path radius*. However, aerodynamic forces and moments and certain active directional control systems cause the *understeer / oversteer gradient* to be a function of speed and/or *path radius*, even within the *linear range*.

Different test methods utilize different forms of the above equation. For a constant radius test, the *included Ackermann steer angle* is constant, the *Ackermann steer angle gradient* is zero, and:

$$U = \frac{d\delta_{REF}}{da_Y} \quad (\text{Eq. 50})$$

For a constant steer angle test, the *included reference steer angle* is constant, the *reference steer angle gradient* is zero, and:

$$U = -\frac{d\delta_A}{da_Y} \quad (\text{Eq. 51})$$

For a constant speed test, neither *reference steer angle gradient* nor *Ackermann steer angle gradient* is zero, and the general format applies:

$$U = \frac{d\delta_{REF}}{da_Y} - \frac{d\delta_A}{da_Y} \quad (\text{Eq. 52})$$

It is common practice in both theory and testing to approximate *included Ackermann steer angle* with *normalized path curvature*.

11.3.3.2 Neutral Steer—A vehicle is neutral steer at a given *steady-state* condition, if the *reference steer angle gradient* equals the *Ackermann steer angle gradient*. The *understeer / oversteer gradient* is zero. See Figure 14.

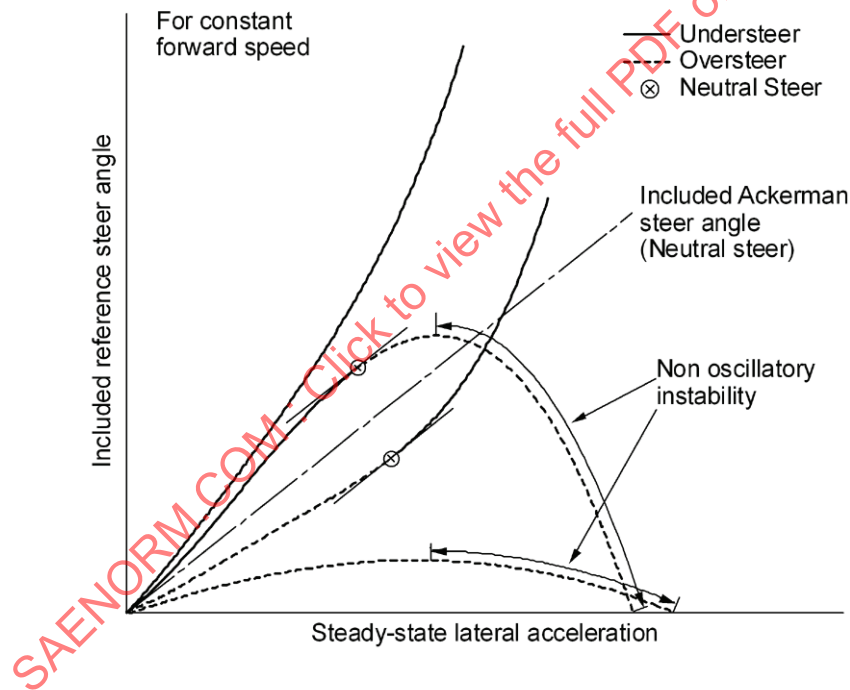


FIGURE 14 – STEER PROPERTIES

NOTE—As implied by Figure 14, the vehicle is a nonlinear system and may not have the same characteristics at all *steady-state* conditions. Consequently, it is necessary to specify the specific conditions (*vehicle speed, included reference steer angle, lateral acceleration*) when making a determination of the vehicle's *understeer / oversteer* characteristics.

11.3.3.3 Understeer—A vehicle is understeer at a given *steady-state* condition, if the *reference steer angle gradient* is greater than the *Ackermann steer angle gradient*. The *understeer / oversteer gradient* is positive. See Figure 14.

11.3.3.4 Oversteer—A vehicle is oversteer at a given *steady-state* condition, if the *reference steer angle gradient* is less than the *Ackermann steer angle gradient*. The *understeer / oversteer gradient* is negative. See Figure 14.

- 11.3.3.5 Cornering Compliance  $D_F$ ,  $D_R$ —For an axle, the negative of the derivative of the axle reference sideslip angle with respect to lateral acceleration at a given steady-state condition.

$$D_F = -\frac{d\sigma_F}{da_Y} \quad (\text{Eq. 53})$$

$$D_R = -\frac{d\sigma_R}{da_Y} \quad (\text{Eq. 54})$$

NOTE—The understeer / oversteer gradient is related to the cornering compliances by the relation:

$$U = D_F - D_R \quad (\text{Eq. 55})$$

Where  $D_F$  is the front cornering compliance and  $D_R$  is the rear cornering compliance.

#### 11.3.4 Stability Measures

- 11.3.4.1 Neutral Steer Line—A set of points in the vehicle plane of symmetry at which an arbitrary external lateral force applied to the sprung mass produces no change in steady-state yaw velocity.
- 11.3.4.2 Static Margin—The horizontal distance from the vehicle center of gravity to the neutral steer line, divided by the wheelbase. It is positive if the vehicle center of gravity is forward of the neutral steer line.
- 11.3.4.3 Stability Factor  $K$ —The relationship between path curvature,  $\kappa$ , yaw velocity,  $\omega_Z$ , or lateral acceleration,  $a_Y$ , and included reference steer angle,  $\delta_{REF}$ , for a linear vehicle. The following three relations are equivalent:

$$\frac{\kappa}{\delta_{REF}} = \frac{1/L}{1 + Kv_X^2} \quad (\text{Eq. 56})$$

$$\frac{\omega_Z}{\delta_{REF}} = \frac{v_X / L}{1 + Kv_X^2} \quad (\text{Eq. 57})$$

$$\frac{a_Y}{\delta_{REF}} = \frac{v_X^2 / L}{1 + Kv_X^2} \quad (\text{Eq. 58})$$

Where  $v_X$  is the longitudinal velocity and  $L$  is the wheelbase. If  $K=0$  the vehicle is neutral steer, if  $K>0$  the vehicle is understeer, and if  $K<0$  the vehicle is oversteer. The stability factor,  $K$ , is related to the understeer / oversteer gradient by the relation:

$$K = \frac{U}{Lg} * \frac{\pi}{180} \quad (\text{Eq. 59})$$

#### 11.3.5 Characterizing Speeds

- 11.3.5.1 Tangent Speed—The longitudinal velocity at which the vehicle sideslip angle is zero, under steady-state operating conditions.

NOTE—Because the vehicle sideslip angle is defined at the vehicle reference point, tangent speed will be affected by the choice of the vehicle reference point.